DIGITAL IMAGE PROCESSING

ASSIGNMENT-2

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1. Convolution of $w * I$

$$w = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

output size $= M+N-1$
$\Rightarrow 3+2-1$
$\Rightarrow 4$.

$w * I = I * w$ — Convolution is commutative.

$$\begin{array}{c|cccc} & (1,1) & (1,2) & (1,3) & (1,4) \\ \hline (1,1) & 0 & 0 & 0 & 0 \\ (2,1) & 0 & 1 & 0 & 0 \\ (3,1) & 0 & 0 & 1 & 0 \\ (4,1) & 0 & 0 & 0 & 0 \end{array} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

I $\qquad$ w

To compute pixel values after convolution at points in I
- invert the filter w.
- place the centre of the filter where convolution is computed.
- Compute element wise multiplication and sum them to get value at that point.

Compute at (1,1)

$$\begin{bmatrix} - & - & - \\ - & \boxed{0} & 0 \\ - & 0 & 1 \end{bmatrix} .* \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 1$$

(1,2)

$$\begin{bmatrix} - & - & - \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} .* \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 0$$

(1,3)

$$\begin{bmatrix} - & - & - \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} .* \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

(1,4)

$$\begin{bmatrix} - & - & - \\ 0 & 0 & - \\ 0 & 0 & - \end{bmatrix} .* \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 0$$

Similarly for other points (2,1), (2,3) (2,4), (3,1), (3,2), (3,4), (4,1), (4,2), (4,3) convolution at these points evaluate to 0.

(2,2)

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} .* \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 1$$

(3,3)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} .* \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = -1$$

(4,4)

$$\begin{pmatrix} 1 & 0 & - \\ 0 & 0 & - \\ - & - & - \end{pmatrix} .* \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = -1$$

Output $\Rightarrow$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

3b Express Unsharp Masking as $f(x,y) * w(x,y)$

① Blur the original → $f(x,y) * w_1(x,y)$ where $w_1(x,y)$ is Box filter.

② Subtract the blurred from original = $\underbrace{f(x,y) - f(x,y) * w_1(x,y)}_{g_{mask}}$

③ Add the mask back to original

Unsharp Masked Output $= f(x,y) + g_{mask} = f(x,y) + [f(x,y) - f(x,y) * w_1(x,y)]$

$= 2f(x,y) - f(x,y) * w_1(x,y)$

convolution

Output $= f(x,y) * \underbrace{[2\,\delta(x,y) - w_1(x,y)]}_{w(x,y)}$

filter $w(x,y)$ for unsharp mask $= \boxed{2\,\delta(x,y) - w_1(x,y)}$.

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4. Inverse Fourier Transform of $\overset{\delta(t)}{=} \underbrace{\delta(\omega - k\omega_0)}_{s_1(t)} + \underbrace{\delta(\omega + k\omega_0)}_{s_2(t)}$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega.$$

Let $s_1(t) = \delta(\omega - k\omega_0)$ $\quad s_2(t) = \delta(\omega + k\omega_0)$

$$x_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega - k\omega_0)\, e^{j\omega t}\, d\omega$$

$$\boxed{x_1(t) = \frac{1}{2\pi}\, e^{j(k\omega_0)t}}$$

$$x_2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega + k\omega_0)\, e^{j\omega t}\, d\omega.$$

$$\boxed{x_2(t) \Rightarrow \frac{1}{2\pi}\, e^{-j(k\omega_0)t}}$$

$$x(t) = x_1(t) + x_2(t) = \frac{1}{2\pi}\left[e^{j(k\omega_0)t} + e^{-j(k\omega_0)t}\right]$$

$$\boxed{x(t) = \frac{1}{\pi}\cos(k\omega_0 t)}$$

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