Logic: What is it and Why study it?

--An Elementary Introduction to Logic

Logic: What Is It?

- Logic is a normative and/or prescriptive science
  - Logic deals with what entails what or what follows from what, and aims at determining which are the correct conclusions of a given set of premises, i.e., to determine which arguments are valid.
  - It is a normative science to evaluate various arguments.
- Logic is NOT a science dealing with thinking
  - "It is a familiar misconception to believe that to do mathematical logic is to be engaged primarily in formal thinking. The important point is rather to make precise the concept of formal and thereby be able to reason mathematically about formal systems. And this adds a new dimension to mathematics."
    - H. Wang, "Popular Lectures on Mathematical Logic," Van Nostrand Reinhold,1981,Dover Publications,1993.

Reasoning: What Is It?

- **Reasoning as a process of drawing new conclusions**
  - **Reasoning is the process of drawing new conclusions from given premises, which are already known facts or previously assumed hypotheses to provide some evidence for the conclusions.**
- **Notes**
  - **“process”, “new conclusions”, “premises”, “evidence”**
- **Reasoning as an ordered process**
  - **In general, a reasoning consists of a number of arguments (or inferences) in some order.**
- **Notes**
  - **“arguments”, “inferences”, “order”**

Reasoning: What Is It

- **Reasoning as a way to acquire new knowledge**
  - **Reasoning is the process of going from what we do know (the premises) to what we previously did not know (the new conclusions).**
- **Reasoning as a way to expand our knowledge**
  - **Reasoning is intrinsically *ampliative*, i.e., it has the function of enlarging or extending some things, or adding to what is already known or assumed.**

Reasoning : An Example

- Example
  - (1) All rational number are expressible as ratio of integers
  - (2) π is not expressible as a ratio of integers
  - Therefor,
    - (3)π is a not a rational number.
    - (4)π is a number.
    - Therefore,
      - (5) There exists at least one non-rational number.

The Characteristics of Reasoning

- **Evidential relation between premises and conclusions**
  - The premises of a reasoning are supposed to present evidence for the conclusions of that reasoning.
  - Though the premises of a reasoning are intended to provide some evidence for the conclusions of that reasoning, they need not actually do so.
- **New conclusions**
  - The conclusions of a reasoning are supposed to be new to the premises of that reasoning.
  - How to define the notion of 'new' formally and satisfactorily is still a difficult philosophical problem until now.

The Characteristics of Reasoning

- **The correctness and/or validity of reasoning**
  - Good and bad reasonings
  - Correct and incorrect reasonings
  - Valid and invalid reasonings
- **Fundamental problems about reasoning**
  - What is a good, correct, valid reasoning and how we do it?
  - What is the criterion by which one can decide whether or not the conclusion of a reasoning is really new to the premises of that reasoning?
  - What is the criterion by which one can decide whether or not the premises of a reasoning really provide some evidence for the conclusions of that reasoning?
  - What is an argument (or inference)?

Argument: What Is It?

- Argument as a set of statements
  - An argument is a set of statements (or declarative sentences) of which one statement is intended as the conclusion, and one or more statements, called “premises,” are intended to provide some evidence for the conclusion.
- Argument as an evidential relation
  - An argument is a conclusion standing in relation to its supporting evidence.
  - In an argument, a claim is being made that there is some sort of evidential relation between its premises and its conclusion: the conclusion is supposed to follow from the premises, or equivalently, the premises are supposed to entail the conclusion.

Argument: An Example

- An example(an argument by Sherlock Holmes):
  - Premises
    - This is a large hat.
    - Someone is the owner of this hat.
    - The owners of large hats are people with large heads.
    - People with large brains are highly intellectual.
  - Conclusion
    - The owner of this hat is highly intellectual.

Proving : What Is It?

- **Definitions of ‘proving’ in dictionary[The Oxford English Dictionary, 2nd Edition]**
  - **“The action of showing to be true, genuine, or valid; demonstration.”**
  - **“The action of establishing a claim.”**
- **Proving**
  - ***Proving* is the process of finding a *justification* for an explicitly *specified statement* from given premises, which are already known facts or previously assumed hypotheses.**
  - **A *proof* is a *description* of a found justification**

Reasoning and Proving: Intrinsic Difference?

- **Typical pattern of reasoning**
  - From A, B, C, …, what we can say?
  - Before reasoning, we do not know what conclusion we can draw from the premises.
- **Typical pattern of proving**
  - From A, B, C, …, can we say D?
  - Before proving, we do know what statement we have to justify from the premises.

Deduction

- **Definitions of 'deduction' in dictionary[The Oxford English Dictionary, 2nd Edition]**
  - **"The process of deducing or drawing a conclusion from a principle already known or assumed; spec. in Logic, inference by reasoning from generals to particulars; opposed to induction."**
- **Deductive argument (reasoning)**
  - **A *deductive argument* is an argument in which the premises are intended to provide absolute support (evidence) for the conclusion.**
  - **Ex. : If *A* then *B*, *A*, therefore *B*.**

Induction

- **Definitions of 'induction' in dictionary[The Oxford English Dictionary, 2nd Edition]**
  - **"The process of inferring a general law or principle from the observation of particular instances (opposed to deduction, q.v.)."**
- **Inductive argument (reasoning)**
  - **An *inductive argument* is an argument in which the premises are intended to provide some degree of support (evidence) for the conclusion.**
  - **Ex. : If A_1 is a B, A_2 is a B, …, A_n is a B, then all A are B.**

Abduction

- **Definitions of 'abduction' in dictionary[The Oxford English Dictionary, 2nd Edition]**
  - **"A syllogism, of which the major premise is certain, and the minor only probable, so that the conclusion has only the probability of the minor."**
- **Abductive argument (reasoning)**
  - **An *abductive argument* is an argument that produces a hypothesis as its end result.**
  - **"The surprising fact, C, is observed. But if A were true, C would be a matter of course. Hence, there is reason to suspect that A is true." [C. S. Peirce]**
  - **Ex. : If *A* then *C*, *C*, therefore *A*.**

Deduction, Induction, and Abduction

- **"if … then …" in deduction, induction, and abduction**
  - If A then B, A, therefore B.
  - If A_1 is a B, A_2 is a B, …, A_n is a B, then all A are B.
  - If A then C, C, therefore A.
- **The logic of scientific discovery : three major approaches**
  - The hypothetico-deductive account [Popper, Hempel]
  - The inductive-probability account [Reichenbach, Salmon]
  - The abductive inference account [Peirce, Hanson]

Formal Logic Systems

Formal logic system

- ***Formal logic system*** **L** $=_{df}$ **(*F*(L),** $\vdash_L$**)**
- ***F*(L) : A *formal language* (call the '*object language*'), which is the set of all *well-formed formulas* of L**
- $\vdash_L$ **: A *logical consequence relation* among the formulas of *F*(L), defined as** $2^{F(L)} \rightarrow F(L)$**, such that for premises** $P \subseteq F(L)$ **and conclusion** $C \in F(L)$**,** $P \vdash_L C$ **means that within the framework of L, *C validly follows from P*, or equivalently, *P validly entails C*.**

Notes

- **'**$\vdash_L$**' is read as 'the turnstile with subscript L'.**
- **Both *F*(L) and** $\vdash_L$ **have to be defined in detail.**

Formal Logic Systems

- **Logic theorems**
  - For a formal logic system (F(L), $\vdash_L$), a *logical theorem* T of L is a formula such that $\phi \vdash_L$ T where ϕ is the empty set.
  - *Th*(L): the set of all logical theorems of L.
- **Notes**
  - *Th*(L) is completely determined by $\vdash_L$.
  - It is *Th*(L) that characterizes the logic L, i.e. if *Th*(L) = *Th*(L'), then we consider the L and L' to be the same logic system.

Formal Theory

- **L-theory with premises P**
  - **Let (F(L), $\vdash_L$) be a formal logic system and $P \subseteq F$(L) be a non-empty set of *sentences* (i.e., *closed well-formed formulas*). A *formal theory* with premises P based on L, called *a L-theory with premises P* and denoted by $T_L(P)$, is defined as**

$$T_L(P) =_{df} Th(L) \cup Th_L^{e}(P)$$

$$Th_L^{e}(P) =_{df} \{\, et \mid P \vdash_L et \text{ and } et \notin Th(L) \}$$

  - **Th(L): The *logical part*.**
  - **$Th_L^{e}(P)$: *The empirical part*, any element of $Th_L^{e}(P)$ is called an *empirical theorem*.**
  - **P: The *empirical axioms*.**

Formal Theory

- ***Formal theory: What is it ?***
  - **A formal theory is a representation of an area of the real world characterized by premises P in a symbolic world characterized by logic L.**
  - **An area of the real world characterized by premises P may be represented by different formal theories characterized by different logics.**