Formal Methods（形式化方法）

Lecture 10. An Application of Sequence（序列的应用）

智能与计算学部 章衡

2021年上学期

Outline

1. A Simple Text Editor

2. Extended Text Editor

A simple text editor: Types and descriptions of states

$$[Char, Report]$$

Report ::= "Okay" | "At top of document" | "At bottom of document"

A simple text editor: Types and descriptions of states

[Char, Report]

Report ::= “Okay” | “At top of document” | “At bottom of document”

```
┌─ Doc1 ────────────────────────────────────
│ left, right : seq Char
└───────────────────────────────────────────
```

A simple text editor: Types and descriptions of states

$$[\mathit{Char}, \mathit{Report}]$$

$$\mathit{Report} ::= \text{“Okay”} \mid \text{“At top of document”} \mid \text{“At bottom of document”}$$

Doc1

$\mathit{left}, \mathit{right} : \mathrm{seq}\ \mathit{Char}$

ΔDoc1

Doc1

Doc1$'$

A simple text editor: Types and descriptions of states

$$[\mathrm{Char}, \mathrm{Report}]$$

$$\mathrm{Report} ::= \text{“Okay”} \mid \text{“At top of document”} \mid \text{“At bottom of document”}$$

Doc1

$\mathrm{left}, \mathrm{right} : \mathrm{seq}\ \mathrm{Char}$

$\Delta\mathrm{Doc1}$

Doc1

$\mathrm{Doc1}'$

$\Xi\mathrm{Doc1}$

$\Delta\mathrm{Doc1}$

$\mathrm{left} = \mathrm{left}'$

$\mathrm{right} = \mathrm{right}'$

A simple text editor: Descriptions of operations (1)

InitDoc1′

$\mathrm{Doc1}'$

$\mathit{left}' = \langle\rangle$

$\mathit{right}' = \langle\rangle$

A simple text editor: Descriptions of operations (1)

$$
\begin{array}{|l}
\text{InitDoc1}' \\
\hline
\text{Doc1}' \\
\hline
\text{left}' = \langle\rangle \\
\text{right}' = \langle\rangle \\
\hline
\end{array}
$$

$$
\begin{array}{|l}
\text{Success} \\
\hline
\text{rep!} : \text{Report} \\
\hline
\text{rep!} = \text{“Okay”} \\
\hline
\end{array}
$$

A simple text editor: Descriptions of operations (2)

DeleteLeftDoc1

$\Delta \mathrm{Doc1}$

$\mathrm{left} \neq \langle\rangle$

$\mathrm{left}' = \mathrm{front}\,\mathrm{left}$

$\mathrm{right}' = \mathrm{right}$

A simple text editor: Descriptions of operations (2)

$$\begin{array}{|l}
\hline
\textrm{DeleteLeftDoc1} \\
\Delta \textrm{Doc1} \\
\hline
\textrm{left} \neq \langle\rangle \\
\textrm{left}' = \textrm{front}\ \textrm{left} \\
\textrm{right}' = \textrm{right} \\
\hline
\end{array}$$

$$\begin{array}{|l}
\hline
\textrm{ErrorAtTop} \\
\Xi \textrm{Doc1} \\
\textrm{rep!} : \textrm{Report} \\
\hline
\textrm{left} = \langle\rangle \\
\textrm{rep!} = \textrm{“At top of document”} \\
\hline
\end{array}$$

A simple text editor: Descriptions of operations (2)

$$
\begin{array}{|l}
\multicolumn{1}{l}{\text{DeleteLeftDoc1}} \\ \hline
\Delta \mathrm{Doc1} \\ \hline
\mathrm{left} \neq \langle\rangle \\
\mathrm{left}' = \mathrm{front}\ \mathrm{left} \\
\mathrm{right}' = \mathrm{right} \\ \hline
\end{array}
$$

$$
\begin{array}{|l}
\multicolumn{1}{l}{\text{ErrorAtTop}} \\ \hline
\Xi \mathrm{Doc1} \\
\mathrm{rep!} : \mathrm{Report} \\ \hline
\mathrm{left} = \langle\rangle \\
\mathrm{rep!} = \text{“At top of document”} \\ \hline
\end{array}
$$

$$\mathrm{DoDeleteLeftDoc1} \mathrel{\widehat{=}} (\mathrm{DeleteLeftDoc1} \wedge \mathrm{Success}) \vee \mathrm{ErrorAtTop}$$

A simple text editor: Descriptions of operations (3)

MoveLeftDoc1

$\Delta \mathrm{Doc1}$

$$\begin{array}{l}
\mathrm{left} \neq \langle\rangle \\
\mathrm{left}' = \mathrm{front}\,\mathrm{left} \\
\mathrm{right}' = \langle \mathrm{last}\,\mathrm{left} \rangle \frown \mathrm{right}
\end{array}$$

A simple text editor: Descriptions of operations (3)

MoveLeftDoc1

$$\Delta \mathrm{Doc1}$$

$$\begin{aligned}
&\mathrm{left} \neq \langle\rangle \\
&\mathrm{left}' = \mathrm{front}\,\mathrm{left} \\
&\mathrm{right}' = \langle \mathrm{last}\,\mathrm{left} \rangle \frown \mathrm{right}
\end{aligned}$$

$$\mathrm{DoMoveLeftDoc1} \mathrel{\widehat{=}} (\mathrm{MoveLeftDoc1} \land \mathrm{Success}) \lor \mathrm{ErrorAtTop}$$

A simple text editor: Descriptions of operations (4)

DeleteRightDoc1

$\Delta \mathrm{Doc1}$

$$
\begin{array}{l}
\mathrm{right} \neq \langle\rangle \\
\mathrm{right}' = \mathrm{tail}\ \mathrm{right} \\
\mathrm{left}' = \mathrm{left}
\end{array}
$$

A simple text editor: Descriptions of operations (4)

DeleteRightDoc1

$\Delta Doc1$

$right \neq \langle\rangle$
$right' = tail\ right$
$left' = left$

ErrorAtBottom

$\Xi Doc1$
$rep! : Report$

$right = \langle\rangle$
$rep! = \text{“At bottom of document”}$

A simple text editor: Descriptions of operations (4)

DeleteRightDoc1

$\Delta \mathrm{Doc1}$

$\mathrm{right} \neq \langle\rangle$
$\mathrm{right}' = \mathrm{tail}\ \mathrm{right}$
$\mathrm{left}' = \mathrm{left}$

ErrorAtBottom

$\Xi \mathrm{Doc1}$
$\mathrm{rep}! : \mathrm{Report}$

$\mathrm{right} = \langle\rangle$
$\mathrm{rep}! =$ “At bottom of document”

$\mathrm{DoDeleteRightDoc1} \mathrel{\widehat{=}} (\mathrm{DeleteRightDoc1} \land \mathrm{Success}) \lor \mathrm{ErrorAtTBottom}$

A simple text editor: Descriptions of operations (5)

MoveRightDoc1

$\Delta \mathrm{Doc1}$

$\mathrm{right} \neq \langle\rangle$

$\mathrm{left}' = \mathrm{left} \frown \langle \mathrm{head}\ \mathrm{right} \rangle$

$\mathrm{right}' = \mathrm{tail}\ \mathrm{right}$

A simple text editor: Descriptions of operations (5)

MoveRightDoc1

$\Delta \mathrm{Doc1}$

$$\mathrm{right} \neq \langle\rangle$$

$$\mathrm{left}' = \mathrm{left} \frown \langle \mathrm{head\ right} \rangle$$

$$\mathrm{right}' = \mathrm{tail\ right}$$

$$\mathrm{DoMoveRightDoc1} \mathrel{\widehat{=}} (\mathrm{MoveRightDoc1} \land \mathrm{Success}) \lor \mathrm{ErrorAtBottom}$$

A simple text editor: Descriptions of operations (6)

InsertLeftDoc1

$\Delta \mathrm{Doc1}$

$\mathrm{ch?} : \mathrm{Char}$

$$\mathrm{left}' = \mathrm{left} \frown \langle \mathrm{ch?} \rangle$$

$$\mathrm{right}' = \mathrm{right}$$

A simple text editor: Descriptions of operations (6)

InsertLeftDoc1

$\Delta \mathrm{Doc1}$

$\mathrm{ch?} : \mathrm{Char}$

$\mathrm{left}' = \mathrm{left} \frown \langle \mathrm{ch?} \rangle$

$\mathrm{right}' = \mathrm{right}$

InsertRightDoc1

$\Delta \mathrm{Doc1}$

$\mathrm{ch?} : \mathrm{Char}$

$\mathrm{right}' = \langle \mathrm{ch?} \rangle \frown \mathrm{right}$

$\mathrm{left}' = \mathrm{left}$

A simple text editor: Descriptions of operations (6)

$$\begin{array}{|l}
\multicolumn{1}{l}{\text{InsertLeftDoc1}} \\
\Delta \text{Doc1} \\
ch? : \text{Char} \\
\hline
left' = left \frown \langle ch? \rangle \\
right' = right \\
\hline
\end{array}$$

$$\begin{array}{|l}
\multicolumn{1}{l}{\text{InsertRightDoc1}} \\
\Delta \text{Doc1} \\
ch? : \text{Char} \\
\hline
right' = \langle ch? \rangle \frown right \\
left' = left \\
\hline
\end{array}$$

$$\text{DoInsertLeftDoc1} \mathrel{\widehat=} \text{InsertLeftDoc1} \land \text{Success}$$

$$\text{DoInsertRightDoc1} \mathrel{\widehat=} \text{InsertRightDoc1} \land \text{Success}$$

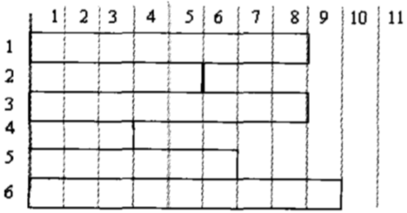
Outline

1. A Simple Text Editor

2. Extended Text Editor

Extended text editor: Unbounded display

$$\mathrm{Line} == \mathrm{seq}(\mathrm{Char} \setminus \{\backslash n\})$$



Text editor with unbounded display

$$\begin{array}{|l}
\text{UnboundedDisplay} \\
\hline
doclines : \mathrm{seq}_1\ Line \\
ln, col : \mathbb{N}_1 \\
\hline
ln \leqslant \sharp doclines \\
col \leqslant \sharp(doclines\ ln) + 1 \\
\hline
\end{array}$$

Extended text editor: Operator flatten

$$\begin{array}{|l}
\mathrm{flatten} : \mathrm{seq}_1\,\mathrm{Line} \rightarrow \mathrm{seq}\,\mathrm{Char} \\
\hline
\forall\, \mathrm{ln} : \mathrm{Line};\, \mathrm{lns} : \mathrm{seq}_1\,\mathrm{Line} \bullet \\
\qquad \mathrm{flatten}\,\langle \mathrm{ln} \rangle = \mathrm{ln} \land \\
\qquad \mathrm{flatten}(\langle \mathrm{ln} \rangle \frown \mathrm{lns}) = (\mathrm{ln} \frown \langle \backslash \mathrm{n} \rangle) \frown (\mathrm{flatten}\,\mathrm{lns})
\end{array}$$

Extended text editor: Operator flatten

$$
\begin{array}{|l}
\text{flatten} : \text{seq}_1\,\text{Line} \rightarrow \text{seq}\,\text{Char} \\
\hline
\forall\, \text{ln} : \text{Line}; \text{lns} : \text{seq}_1\,\text{Line} \bullet \\
\qquad \text{flatten}\,\langle \text{ln} \rangle = \text{ln} \land \\
\qquad \text{flatten}(\langle \text{ln} \rangle \frown \text{lns}) = (\text{ln} \frown \langle \backslash\text{n} \rangle) \frown (\text{flatten}\,\text{lns})
\end{array}
$$

Example (flatten)

Let $\text{lns} == \langle \langle a, b \rangle, \langle c, d, e \rangle, \langle f \rangle \rangle$. Then we have

$$\text{flatten}\,\text{Ins} = \langle a, b, \backslash\, n, c, d, e, \backslash\, n, f \rangle.$$

Extended text editor: Operator flatten

$$
\begin{array}{|l}
\text{flatten} : \text{seq}_1\,\text{Line} \rightarrow \text{seq}\,\text{Char} \\
\hline
\forall\, \text{ln} : \text{Line}; \text{lns} : \text{seq}_1\,\text{Line} \bullet \\
\qquad \text{flatten}\,\langle \text{ln} \rangle = \text{ln} \wedge \\
\qquad \text{flatten}(\langle \text{ln} \rangle \frown \text{lns}) = (\text{ln} \frown \langle \backslash \text{n} \rangle) \frown (\text{flatten}\,\text{lns})
\end{array}
$$

Example (flatten)

Let lns $== \langle \langle a, b \rangle, \langle c, d, e \rangle, \langle f \rangle \rangle$. Then we have

$$\text{flatten}\,\text{Ins} = \langle a, b, \backslash\, n, c, d, e, \backslash\, n, f \rangle.$$

- flatten is a bijection from $\text{seq}_1\,\text{Line}$ to $\text{seq}\,\text{Char}$,
- Thus $\text{flatten}^{\sim}$ is also a total function from $\text{seq}\,\text{Char}$ to $\text{seq}_1\,\text{Line}$.

Extended text editor: Descriptions of states

Doc2

Doc1
UnboundedDisplay

$$\mathit{left} \frown \mathit{right} = \mathit{flatten}\ \mathit{doclines}$$

$$\mathit{ln} = \#(\mathit{left} \rhd \{\backslash n\}) + 1$$

$$\mathit{col} = \#\mathit{left} - \#(\mathit{flatten}\ (1..(\mathit{ln} - 1) \upharpoonleft \mathit{doclines}))$$

Extended text editor: Examples

Example

Let $left == \langle a, b, \backslash n, c, d, e, \backslash n, f\rangle$, $right == \langle g, h, \backslash n, i, j\rangle$. Then

$$flatten^{\sim}\ left = \langle\langle a, b\rangle, \langle c, d, e\rangle, \langle f\rangle\rangle$$

Extended text editor: Examples

Example

Let $\mathrm{left} == \langle a, b, \backslash n, c, d, e, \backslash n, f \rangle$, $\mathrm{right} == \langle g, h, \backslash n, i, j \rangle$. Then

$$\mathrm{flatten}^{\sim}\ \mathrm{left} = \langle \langle a, b \rangle, \langle c, d, e \rangle, \langle f \rangle \rangle$$

Since $\mathrm{left} \frown \mathrm{right} = \mathrm{flatten}\ \mathrm{doclines}$, we thus have

$$\mathrm{doclines} = \mathrm{flatten}^{\sim}(\mathrm{left} \frown \mathrm{right}) = \langle \langle a, b \rangle, \langle c, d, e \rangle, \langle f, g, h \rangle, \langle i, j \rangle \rangle$$

Extended text editor: Descriptions of operations

$$
\begin{aligned}
\mathrm{DoDeleteLeftDoc2} &\mathrel{\widehat{=}} \mathrm{DoDeleteLeftDoc1} \land \Delta\mathrm{Doc2}\\
\mathrm{DoMoveLeftDoc2} &\mathrel{\widehat{=}} \mathrm{DoMoveLeftDoc1} \land \Delta\mathrm{Doc2}\\
\mathrm{DoInsertLeftDoc2} &\mathrel{\widehat{=}} \mathrm{DoInsertLeftDoc1} \land \Delta\mathrm{Doc2}\\
\mathrm{DoDeleteRightDoc2} &\mathrel{\widehat{=}} \mathrm{DoDeleteRightDoc1} \land \Delta\mathrm{Doc2}\\
\mathrm{DoMoveRightDoc2} &\mathrel{\widehat{=}} \mathrm{DoMoveRightDoc1} \land \Delta\mathrm{Doc2}\\
\mathrm{DoInsertRightDoc2} &\mathrel{\widehat{=}} \mathrm{DoInsertRightDoc1} \land \Delta\mathrm{Doc2}
\end{aligned}
$$

Exercise

We change the definition of Report as follows:

$$\text{Report} ::= \text{“Okay”} \mid \text{“Nonexist”} \mid \text{“At top of document”} \mid \text{“At bottom of document”}$$

Please design a schema to describe the operation of search, defined as follows:

Given a character x? as input, let the output be “Nonexist” if x? does not appear in the text, and “Okay” otherwise. In addition, let the cursor stays at the first occurrence of x? (i.e., the first character in “right” is x?, and x? does not appear in “left”) if the output is “Okay”.