Artificial Intelligence

Two-player, zero-sum, perfect-information

Games

(Chapter 5)

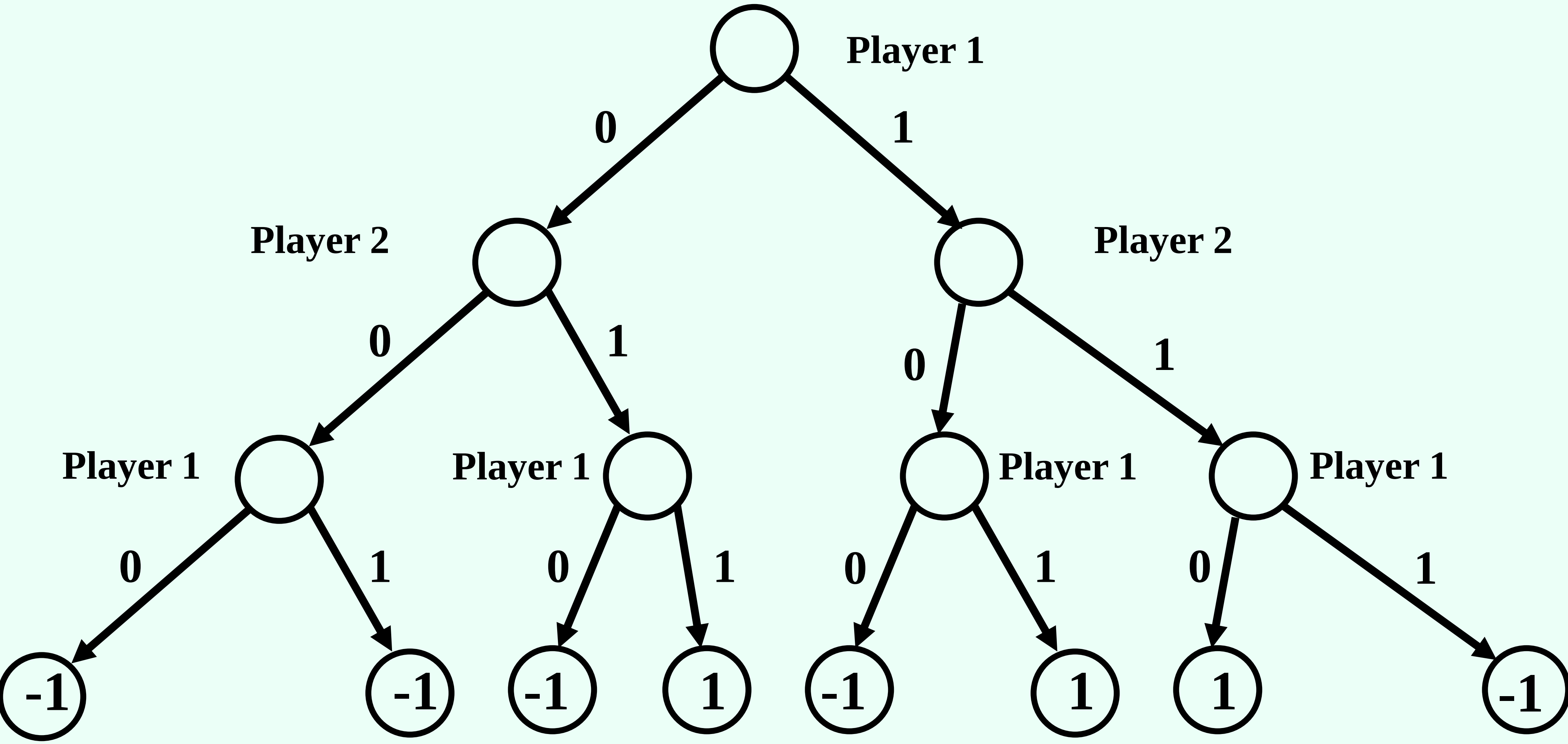
Instructor: Qiang Yu

Game playing

- Rich tradition of creating game-playing programs in AI
- Many similarities to search
- Most of the games studied
  - have two players,
  - are zero-sum: what one player wins, the other loses
  - have perfect information: the entire state of the game is known to both players at all times
- E.g., tic-tac-toe, checkers, chess, Go, backgammon, …
- Will focus on these for now
- Recently more interest in other games
  - Esp. games without perfect information; e.g., poker
    - Need probability theory, game theory for such games

"Sum to 2" game

- Player 1 moves, then player 2, finally player 1 again
- Move = 0 or 1
- Player 1 wins if and only if all moves together sum to 2

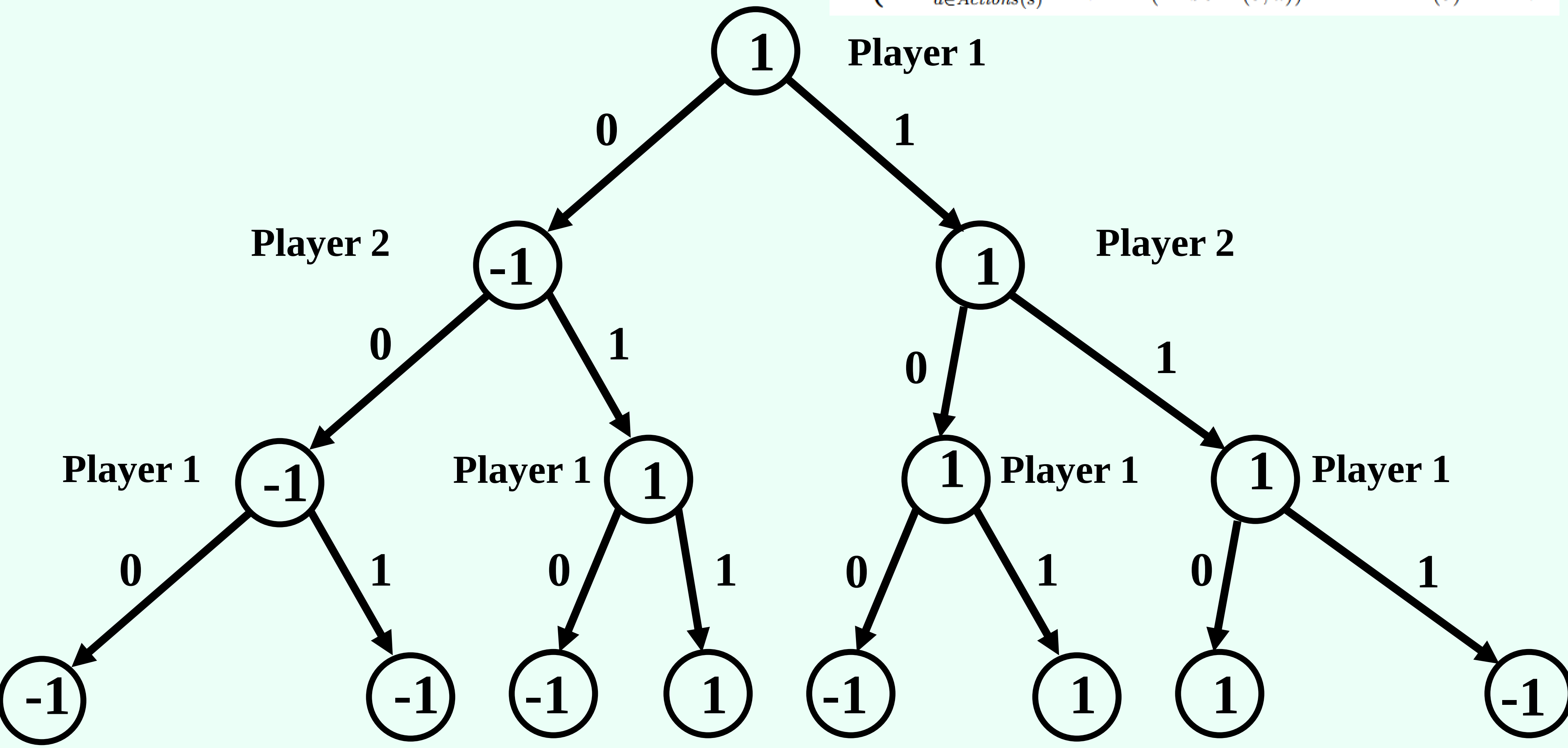


Player 1's utility is in the leaves; player 2's utility is the negative of this

Backward induction (aka. minimax)

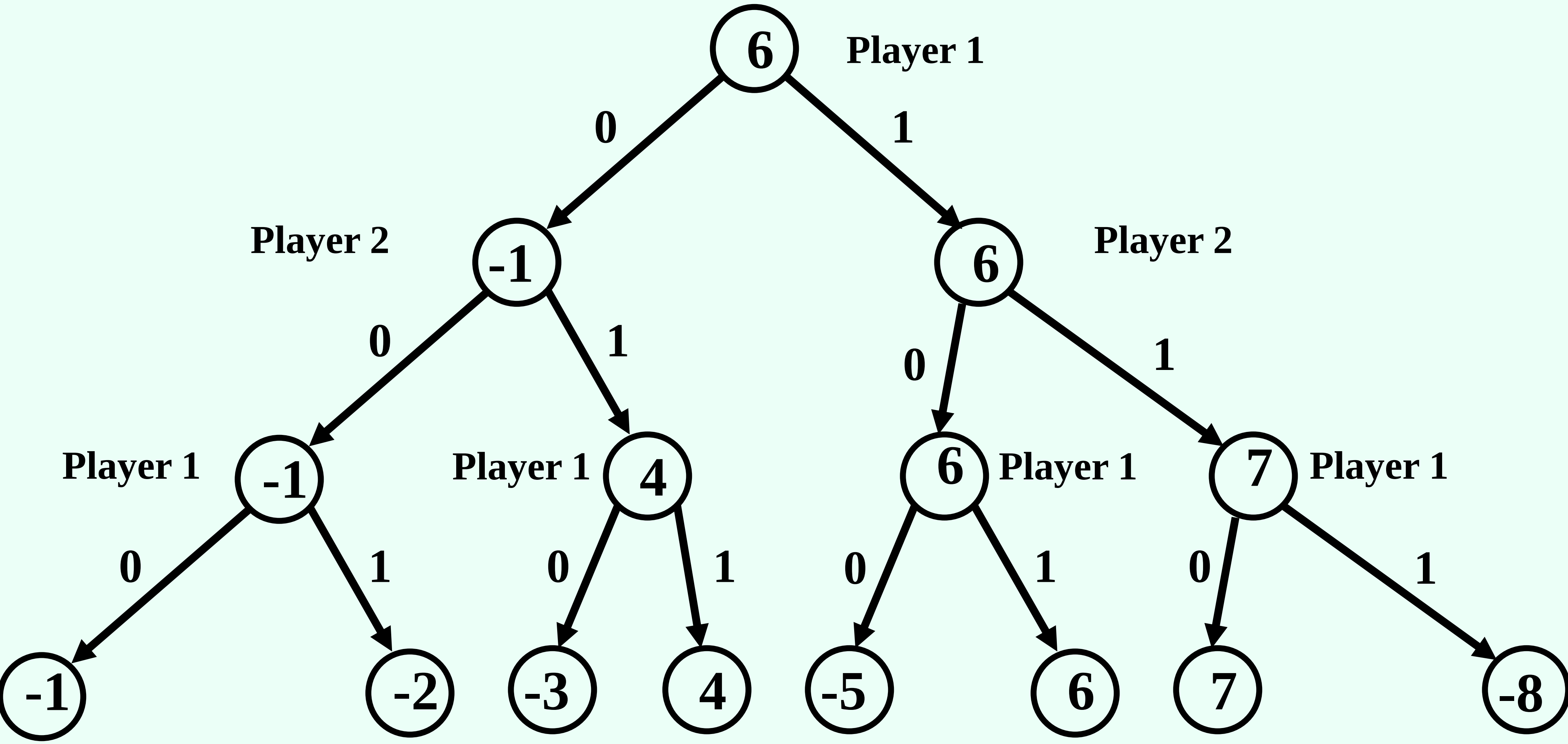
- From leaves upward, analyze best decision for player at node, give node a value
  - Once we know values, easy to find optimal action (choose best value)

$$\textsc{Minimax}(s) = \begin{cases} \textsc{Utility}(s) & \text{if } \textsc{Terminal-Test}(s) \\ \max_{a \in Actions(s)} \textsc{Minimax}(\textsc{Result}(s,a)) & \text{if } \textsc{Player}(s) = \textsc{max} \\ \min_{a \in Actions(s)} \textsc{Minimax}(\textsc{Result}(s,a)) & \text{if } \textsc{Player}(s) = \textsc{min} \end{cases}$$



Modified game

- From leaves upward, analyze best decision for player at node, give node a value



A recursive implementation

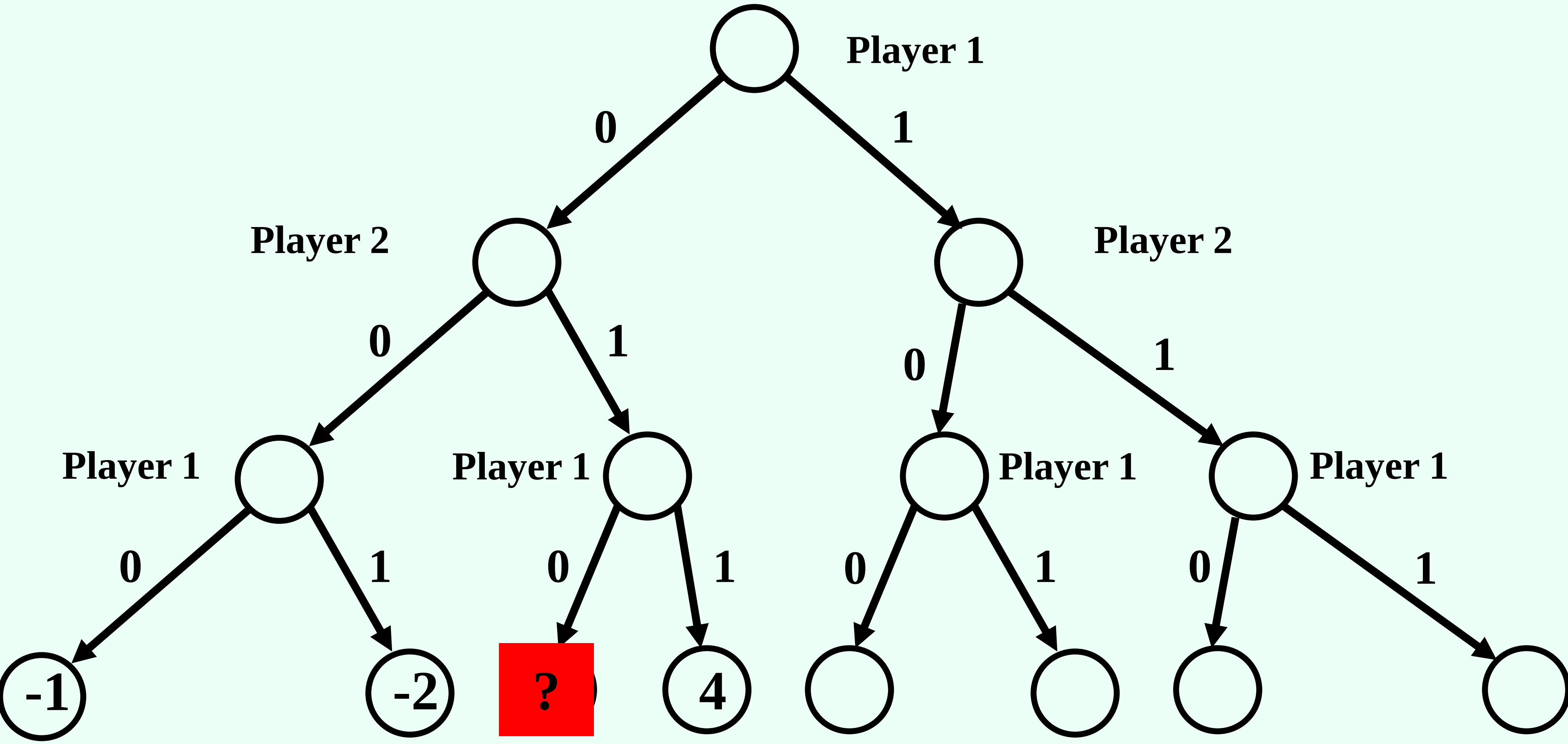
- **Value(state)**
- If state is terminal, return its value
- If (player(state) = player 1)
  - v := -infinity
  - For each action
    - v := max(v, Value(successor(state, action)))
  - Return v
- Else
  - v := infinity
  - For each action
    - v := min(v, Value(successor(state, action)))
  - Return v

$$\textsc{Minimax}(s) = \begin{cases} \textsc{Utility}(s) & \text{if } \textsc{Terminal-Test}(s) \\ \max_{a \in Actions(s)} \textsc{Minimax}(\textsc{Result}(s, a)) & \text{if } \textsc{Player}(s) = \textsc{max} \\ \min_{a \in Actions(s)} \textsc{Minimax}(\textsc{Result}(s, a)) & \text{if } \textsc{Player}(s) = \textsc{min} \end{cases}$$

Space? Time?

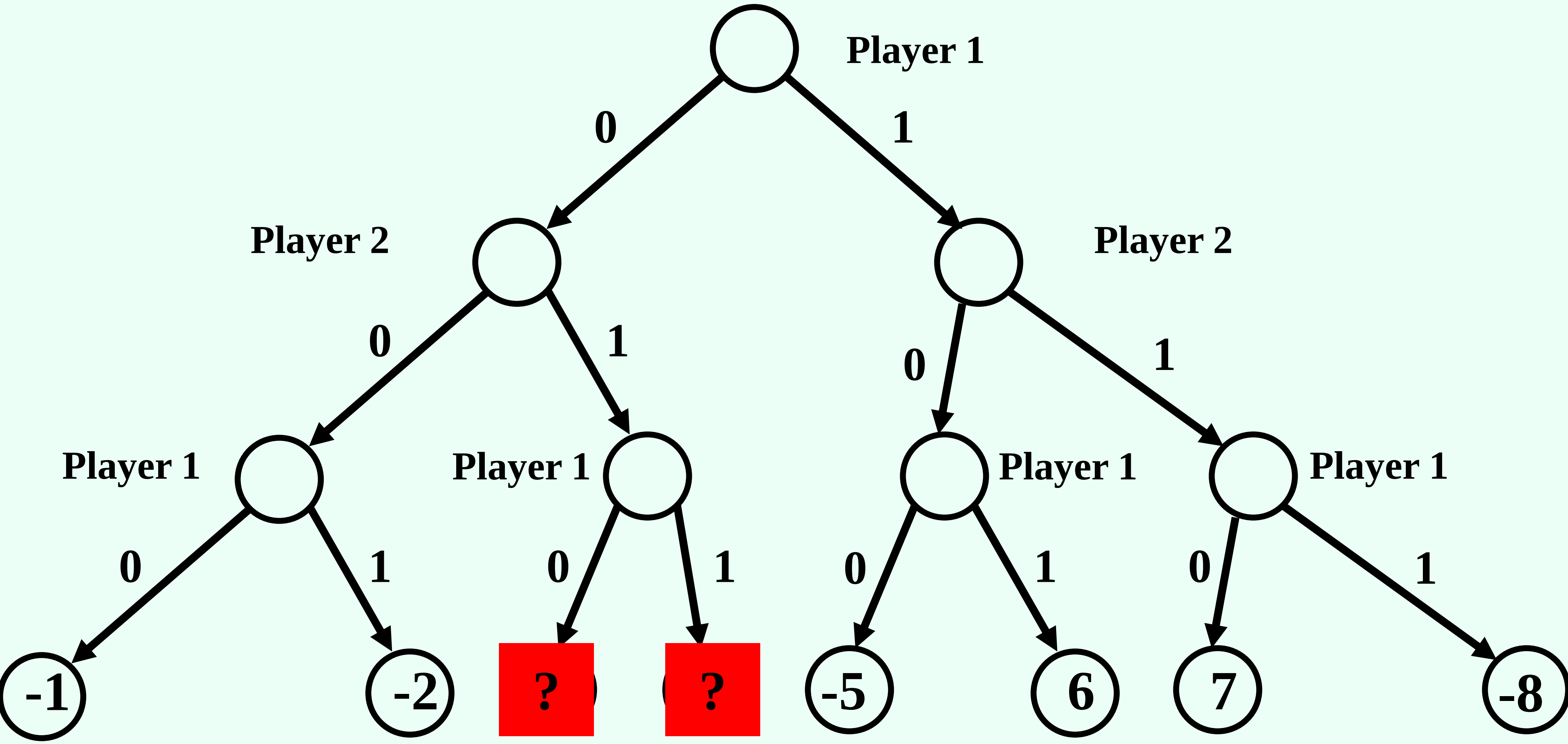
Do we need to see all the leaves?

- Do we need to see the value of the question mark here?



Do we need to see all the leaves?

- Do we need to see the values of the question marks here?

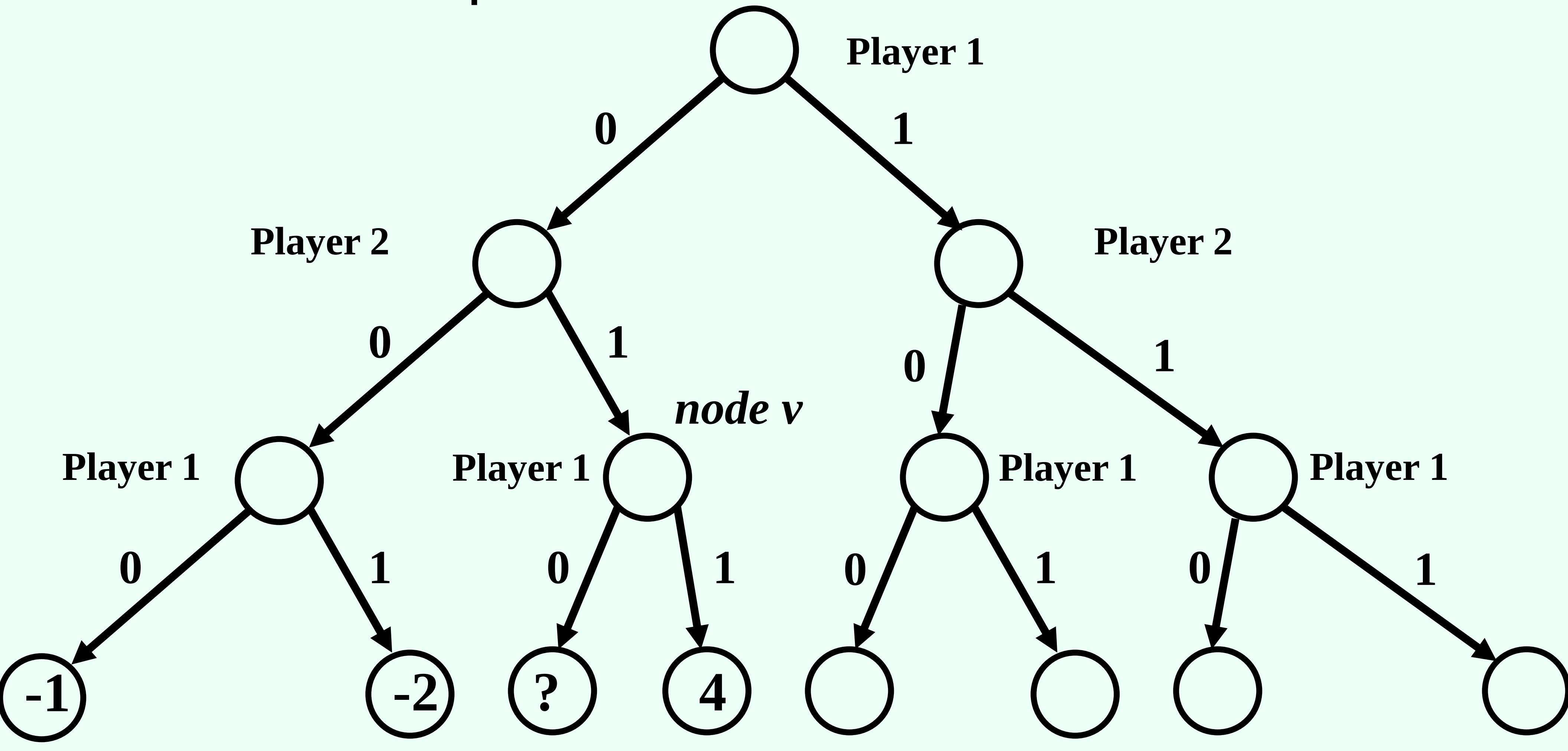


Alpha-beta pruning

- Pruning = cutting off parts of the search tree (because you realize you don't need to look at them)
  - When we considered A* we also pruned large parts of the search tree
- Maintain alpha = value of the best option for player 1 along the path so far
- Beta = value of the best option for player 2 along the path so far

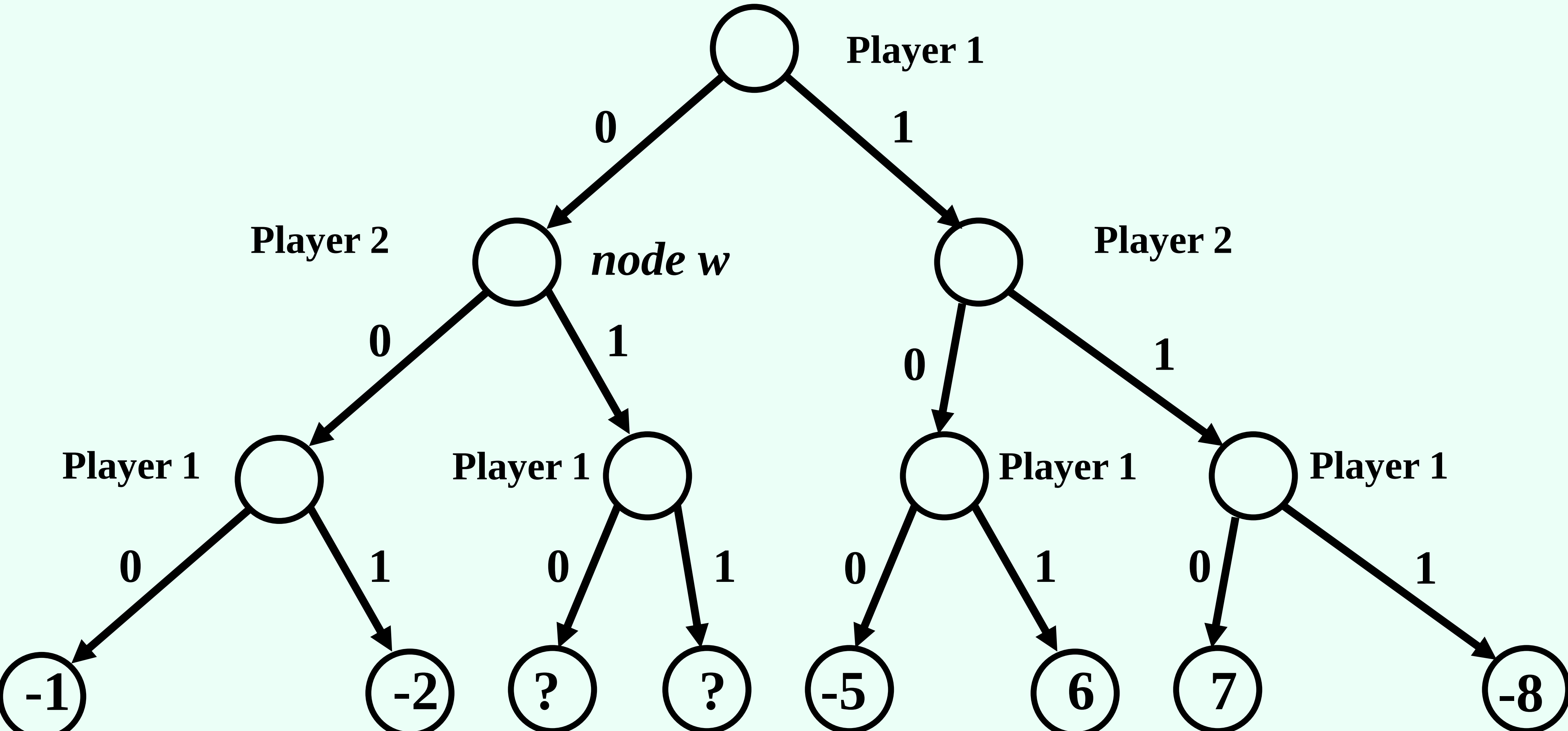
Pruning on beta

- Beta at node v is -1
- We know the value of node v is going to be at least 4, so the -1 route will be preferred
- No need to explore this node further



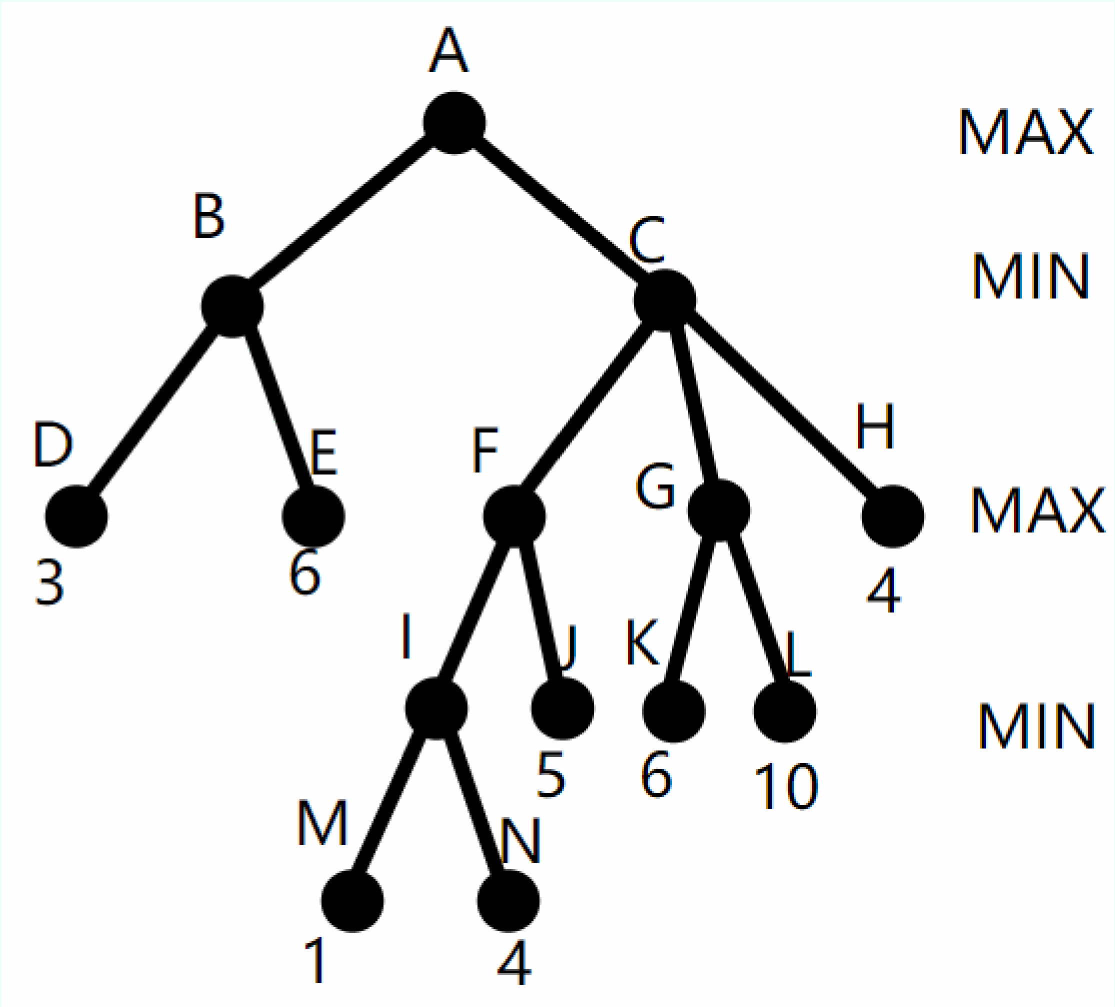
Pruning on alpha

- Alpha at node w is 6
- We know the value of node w is going to be at most -1, so the 6 route will be preferred
- No need to explore this node further

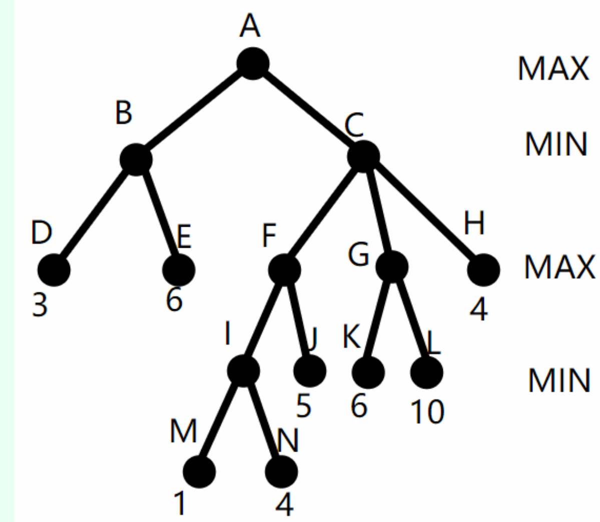


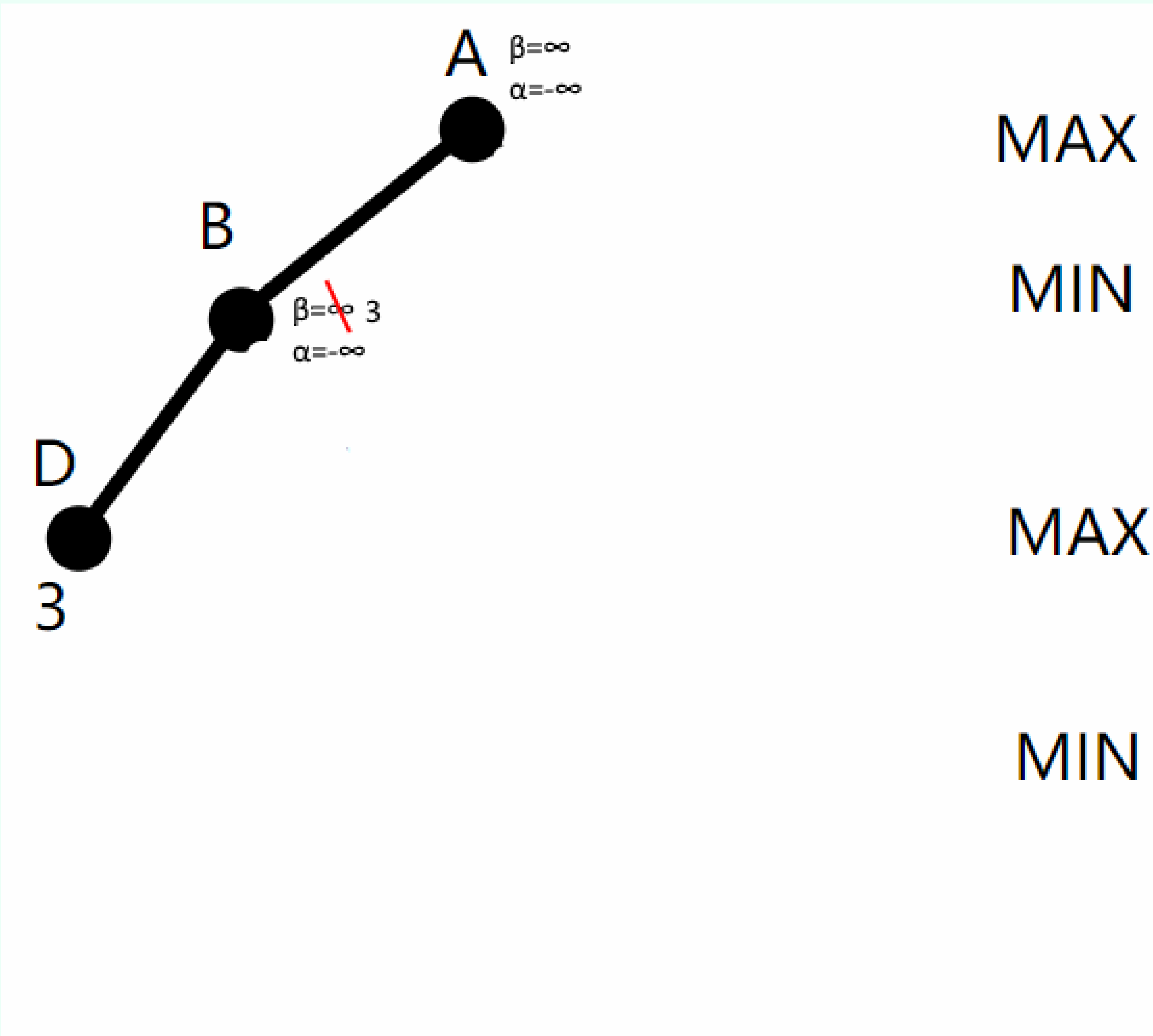
Example

- Define Search order: from left to right



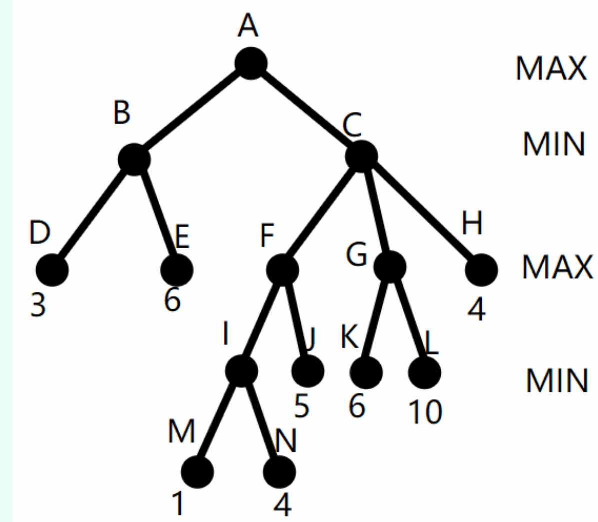
Example

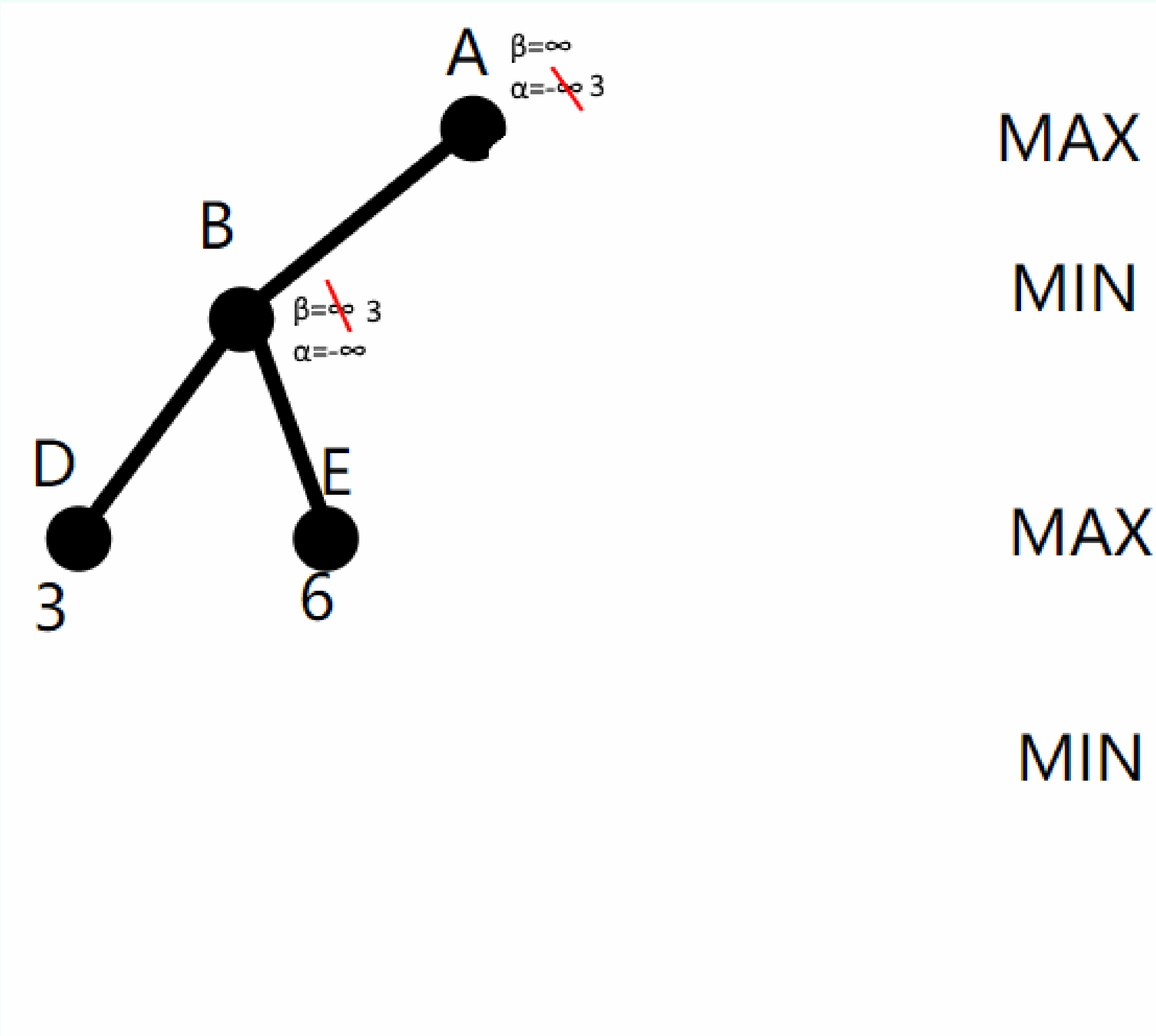




从根节点开始，初始化根节点的 $\alpha=-\infty$，$\beta=\infty$，向左边的子节点展开。到D节点时，得到D的值为3，返回B节点，由于B节点是Min节点，所以更新B节点的β值为$\min(\infty, 3)=3$

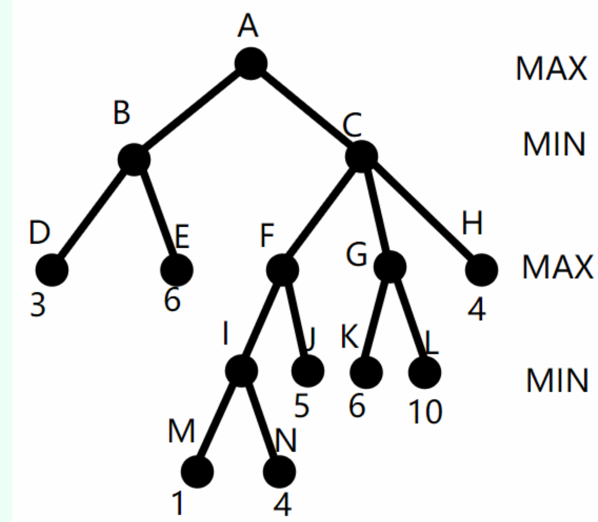
Example

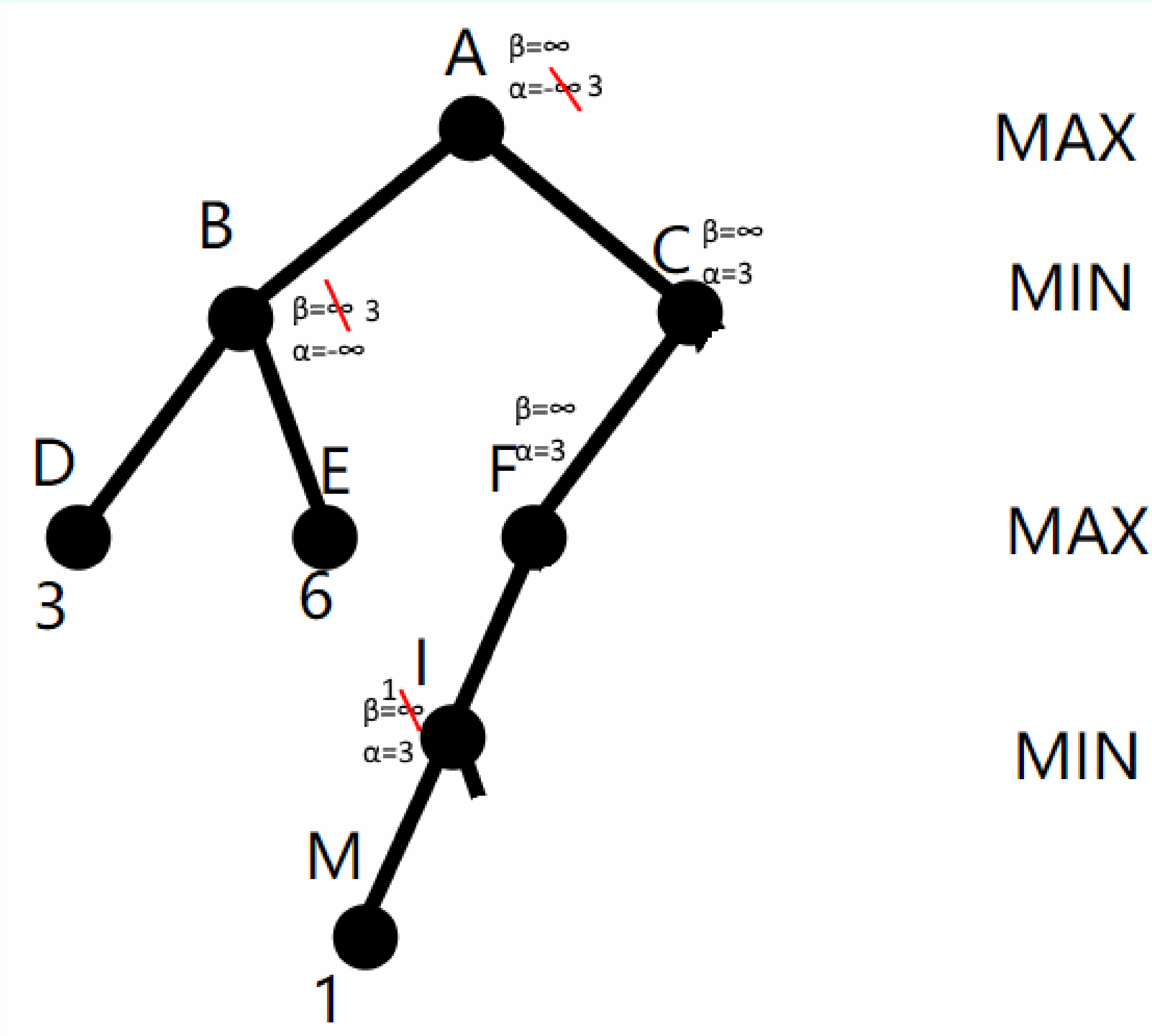




接下来从B到E，再从E返回B，min(3,6)仍为3。从B返回根节点A，A是 Max 节点，更新A的α值为 max(-∞, 3[B的β值])=3

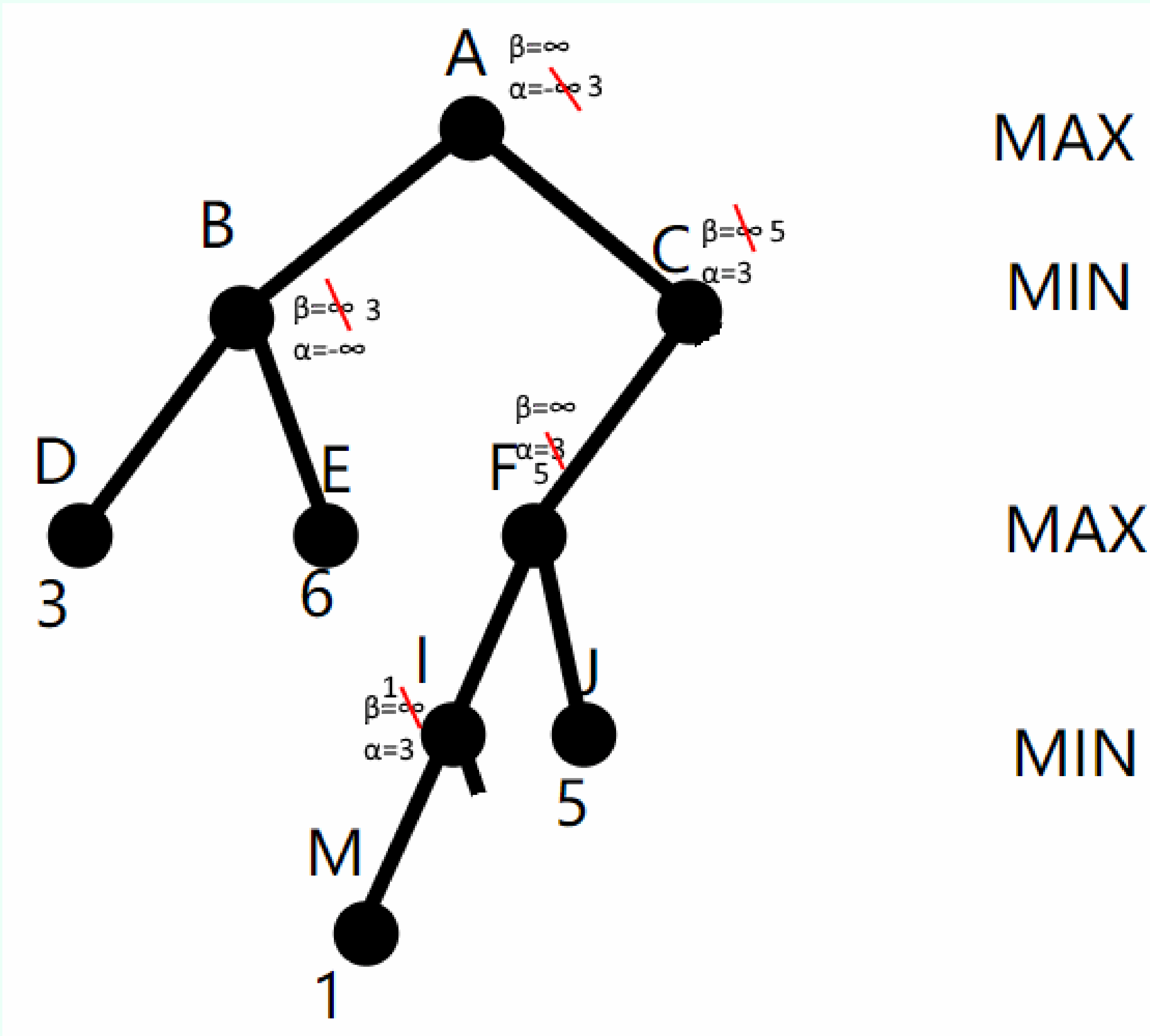
Example

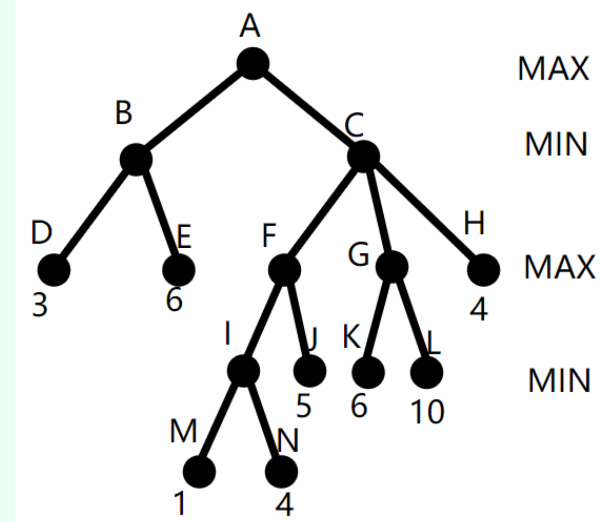




接下来从根节点往右深入，把根节点的β=∞, α=3 依次传给C、F、I，从 I 深入 M 再返回时，I 是 Min 节点，更新 I 的 β = min(∞, 1) = 1。留意到此时 I 的 α=3 ＞ β，所以无需再探索 I 的剩余子节点，把未探索的子节点剪掉.

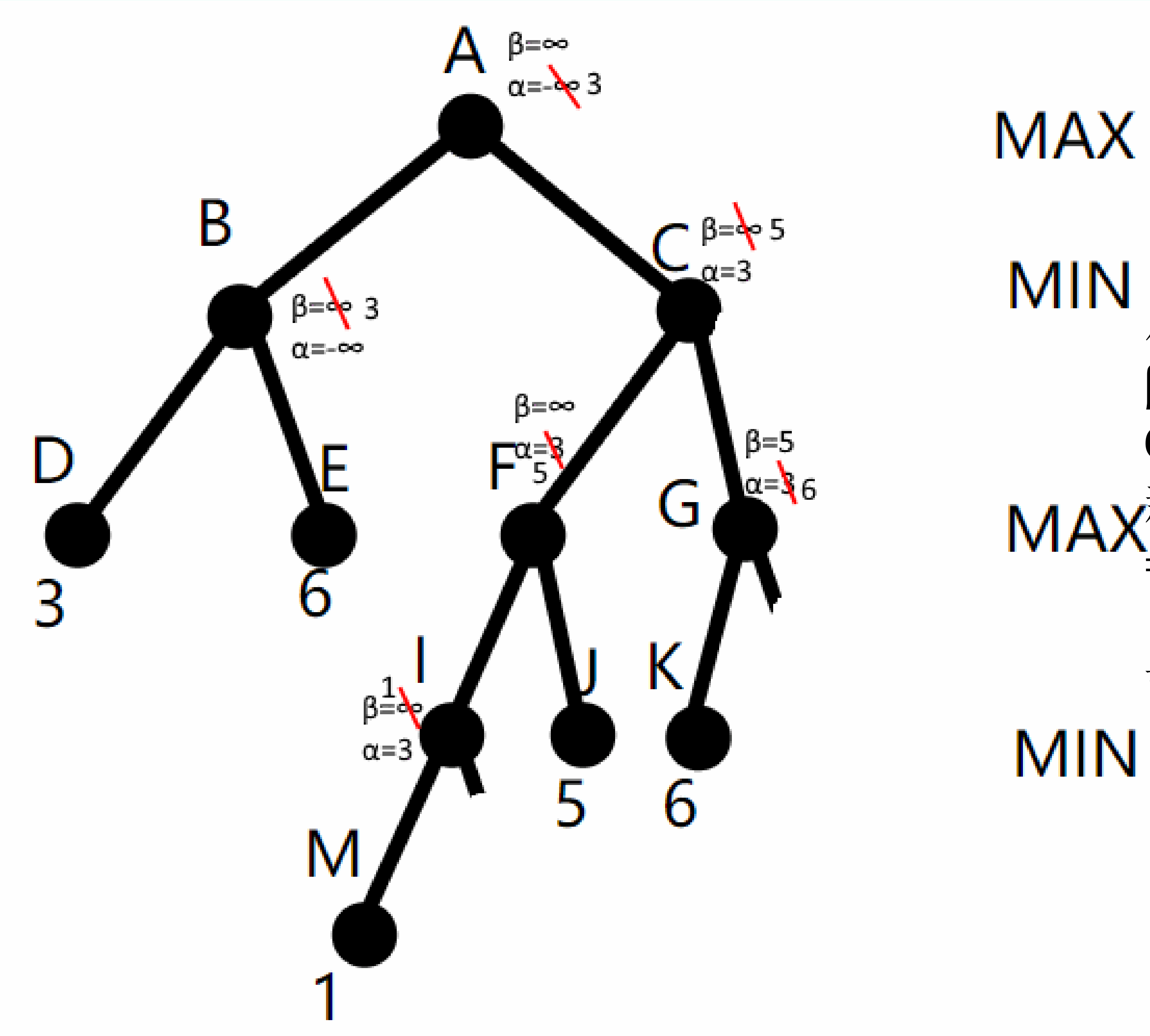
Example

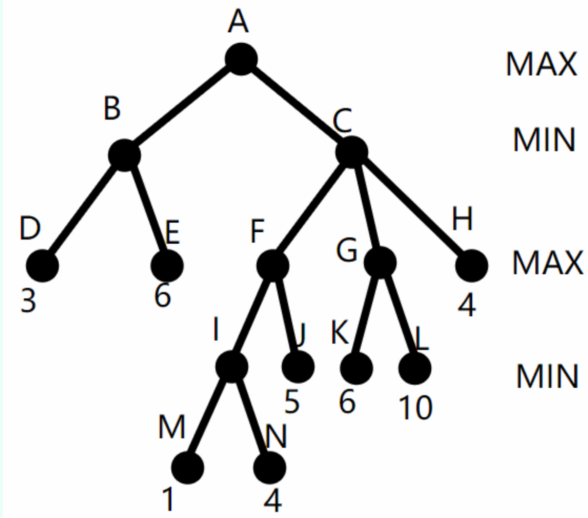




从节点 I 返回到F，F 的 α 值仍为 max(3, 1)=3不变。从F到J再返回，更新 F 的 α = max(3, 5) = 5。从 F 返回 C，更新 C 的 β = min(∞, 5) = 5

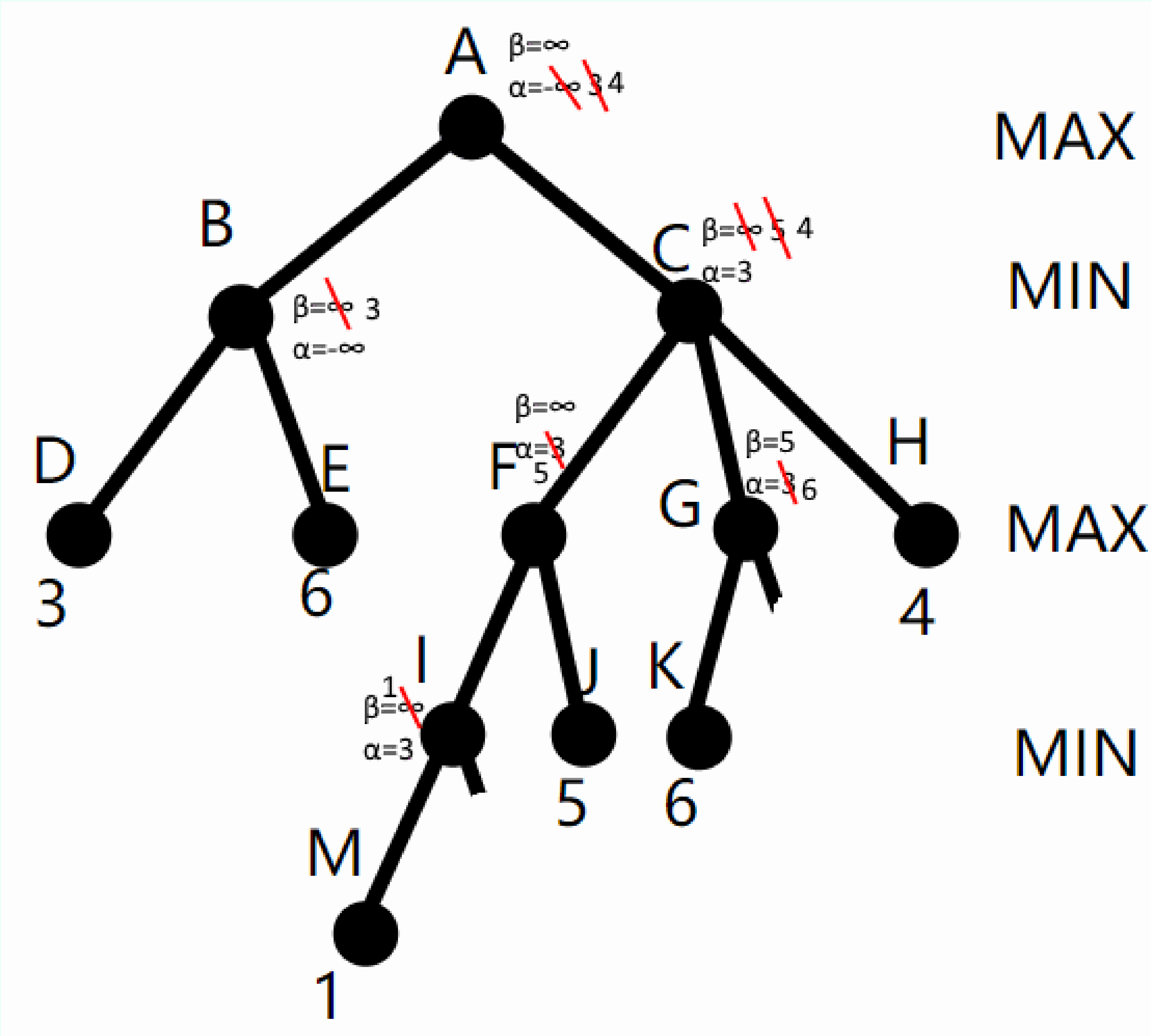
Example

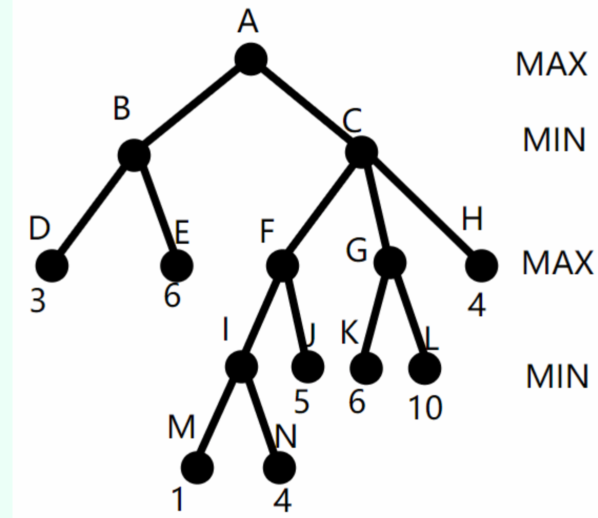




从 C 到 G，把 C 的 β=5, α=3 传给 G，从 G 到 K 再返回 G，更新 G 的 α = max(3, 6) = 6。注意到 G 的 α=6 >β，把 G 的其余子节点剪掉.

Example





从 G 返回 C，C 的 β = min(5, 6) = 5 不变。从 C 到 H 然后返回，C 的 β = min(5, 4) = 4。
最后，从 C 返回根节点 A，A 是 Max 节点，A 的 α = max(3, 4[C的β值]) = 4，

Modifying recursive implementation to do alpha-beta pruning

- **Value(state, alpha, beta)**
- If state is terminal, return its value
- If (player(state) = player 1)
  - v := -infinity
  - For each action
    - v := max(v, Value(successor(state, action), alpha, beta))
    - If v >= beta, return v
    - alpha := max(alpha, v)
  - Return v
- Else
  - v := infinity
  - For each action
    - v := min(v, Value(successor(state, action), alpha, beta))
    - If v <= alpha, return v
    - beta := min(beta, v)
  - Return v

Benefits of alpha-beta pruning

- Without pruning, need to examine $O(b^m)$ nodes
- With pruning, depends on which nodes we consider first
- If we choose a random successor, need to examine $O(b^{3m/4})$ nodes
- If we manage to choose the best successor first, need to examine $O(b^{m/2})$ nodes
  - Practical heuristics for choosing next successor to consider get quite close to this
- Can effectively look twice as deep!
  - Difference between reasonable and expert play

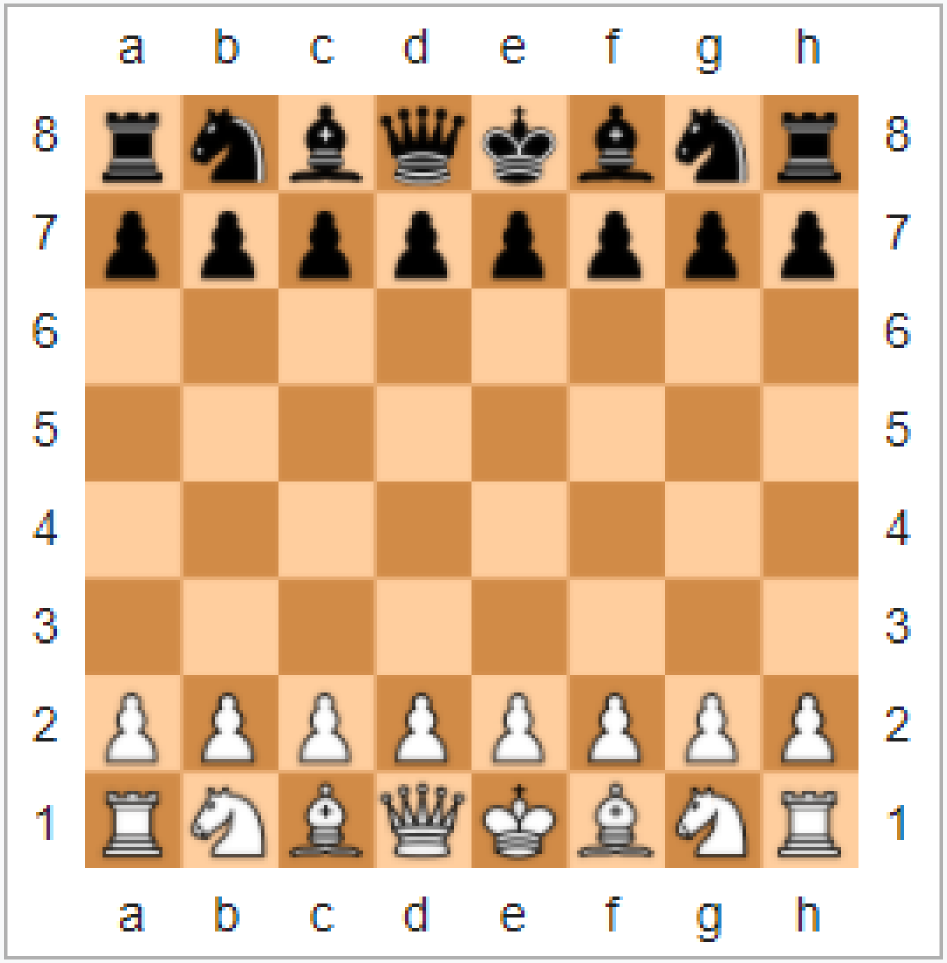
Repeated states

- As in search, multiple sequences of moves may lead to the same state
- Again, can keep track of previously seen states (usually called a transposition table in this context)
  - May not want to keep track of **all** previously seen states…

Using evaluation functions

- Most games are too big to solve even with alpha-beta pruning
- Solution: Only look ahead to limited depth (nonterminal nodes)
- Evaluate nodes at depth cutoff by a heuristic (aka. evaluation function)
- E.g., chess:
  - Material value: queen worth 9 points, rook 5, bishop 3, knight 3, pawn 1
  - Heuristic: difference between players’ material values

Chess example

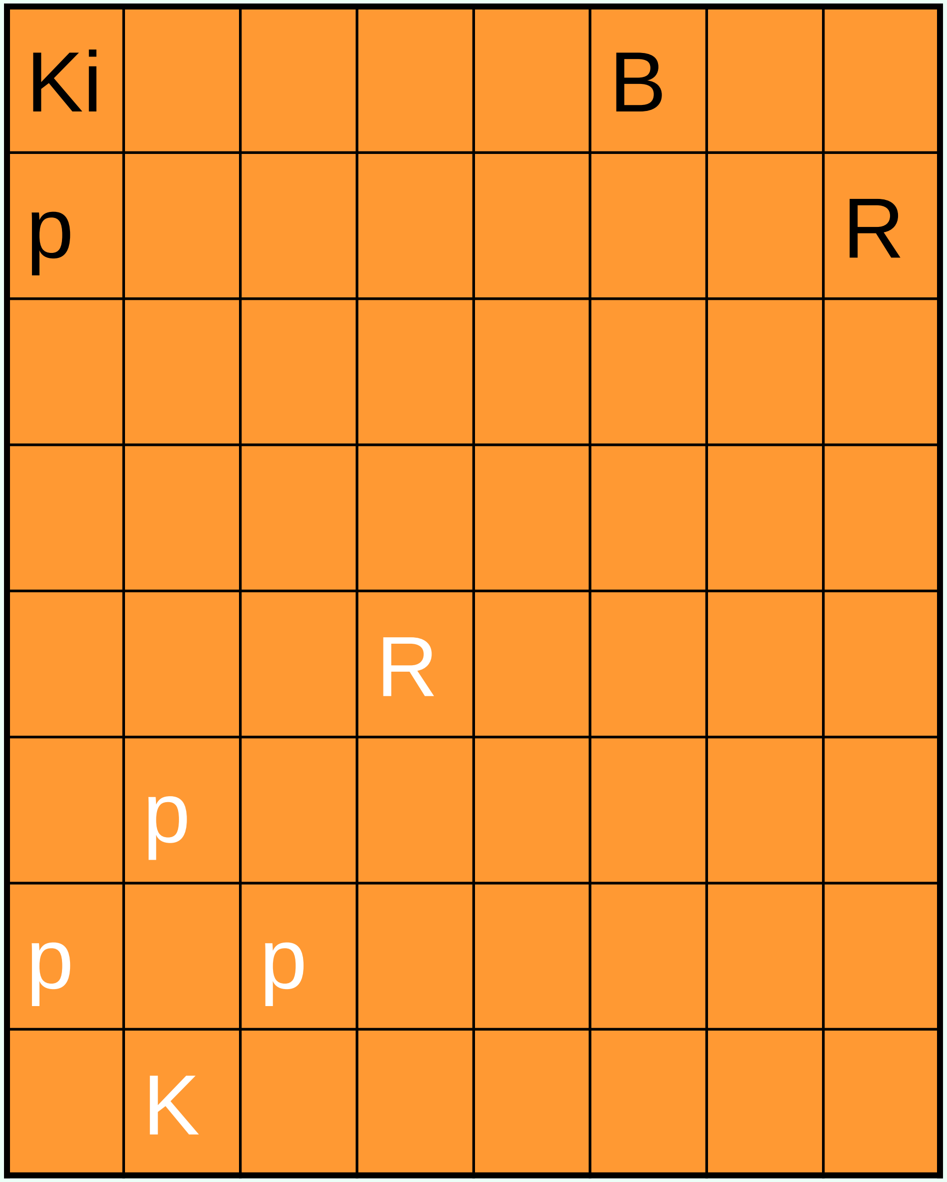


Initial position, *first (bottom) row:* rook, knight, bishop, queen, king, bishop, knight, and rook; *second row:* pawns

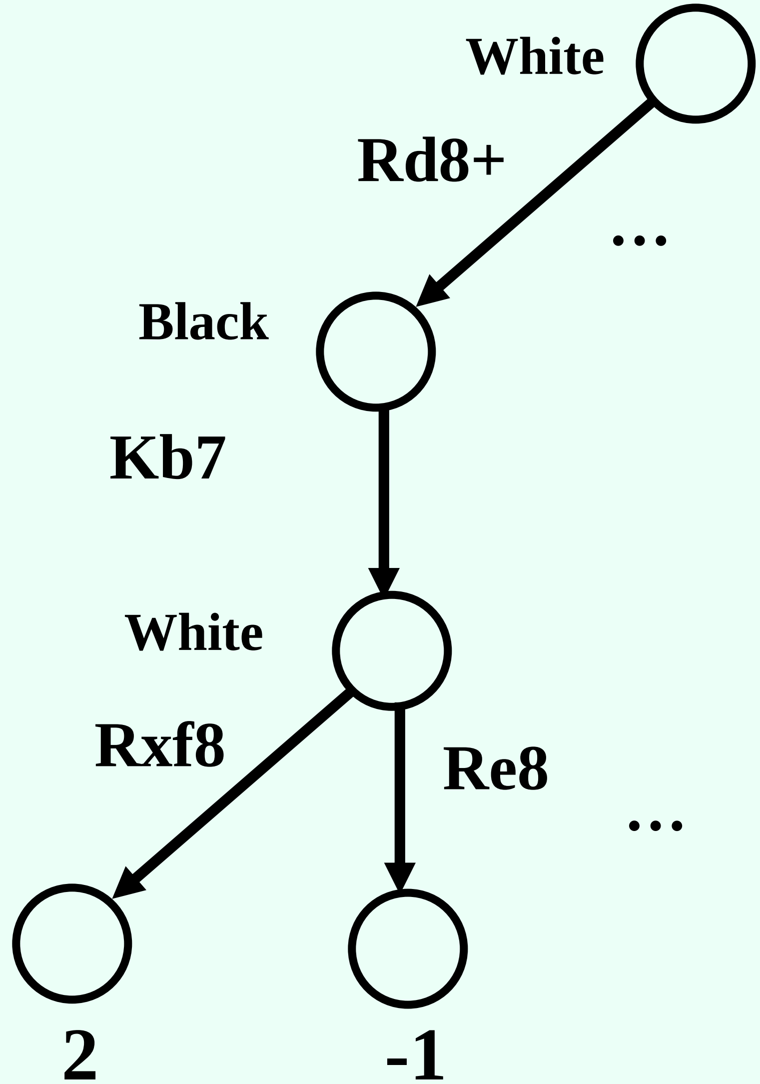
Chess example

Material value:
queen worth 9 points, rook 5, bishop 3, knight 3, pawn 1

- White to move

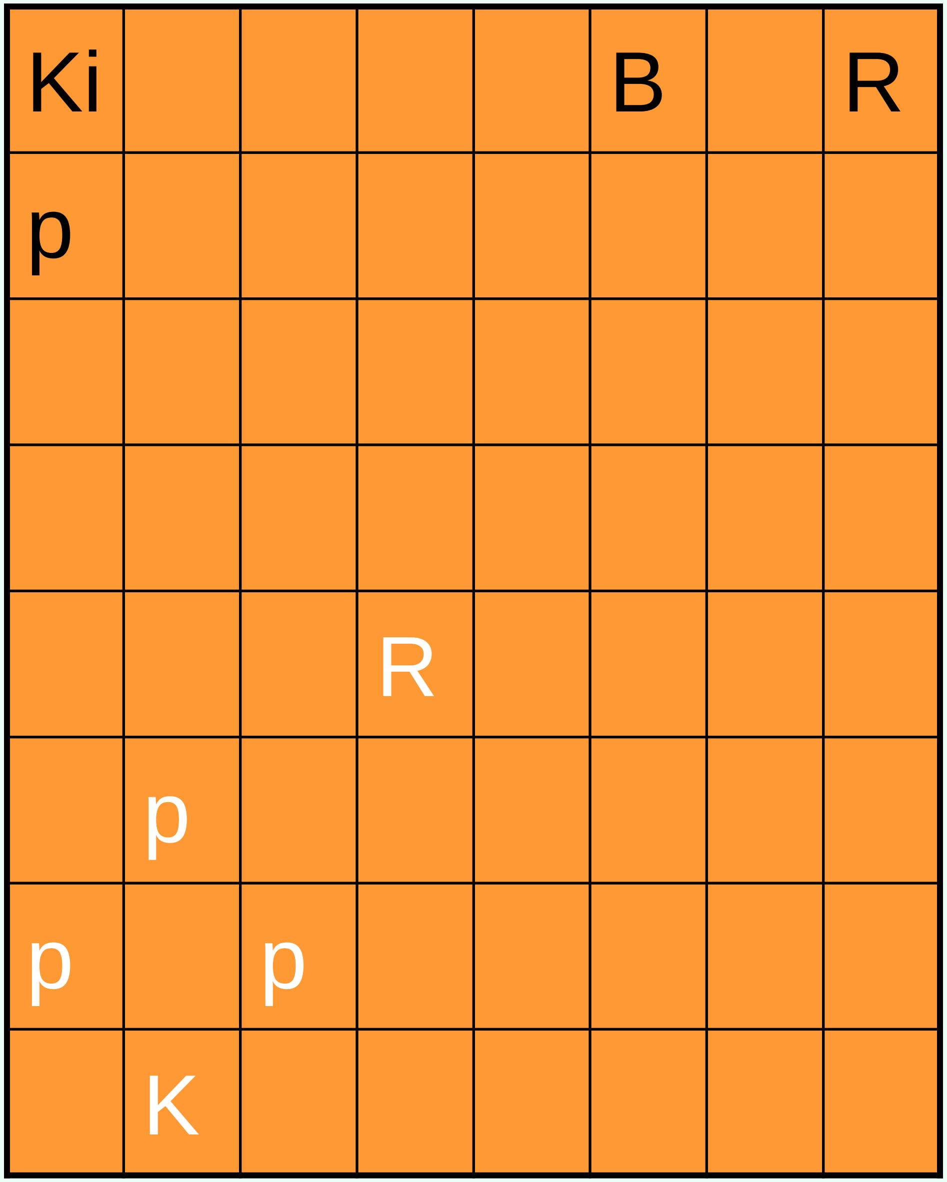


- Depth cutoff: 3 ply
  - Ply = move by one player

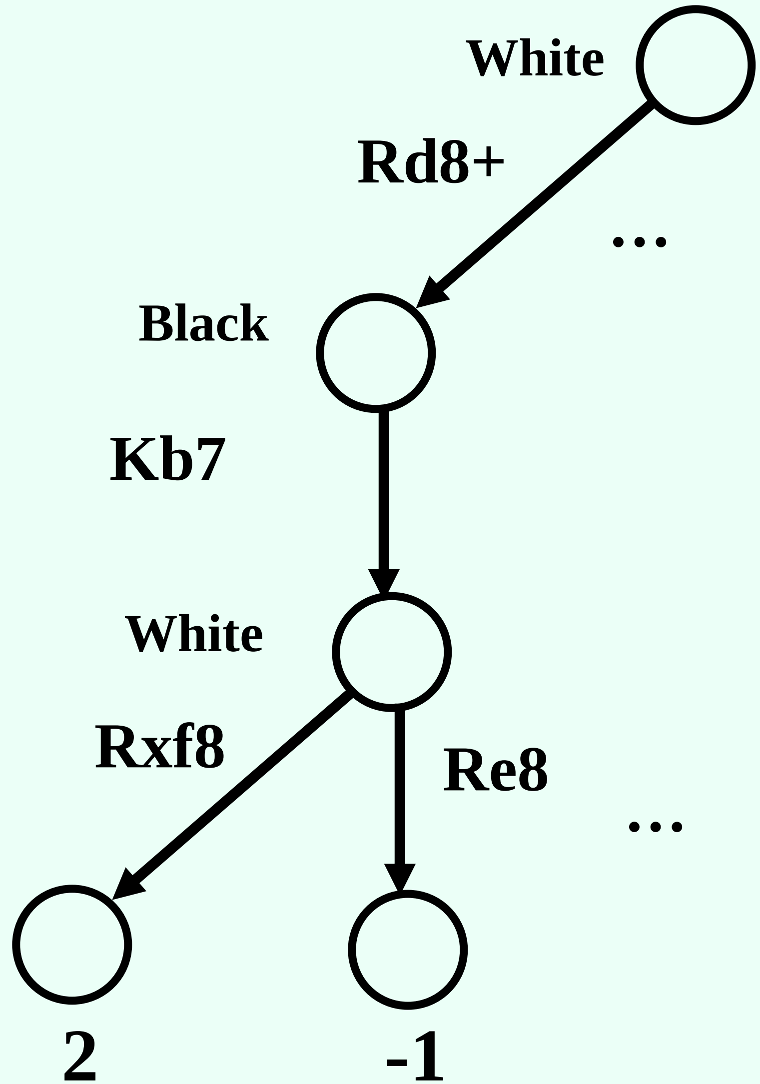


Chess (bad) example

- White to move



- Depth cutoff: 3 ply
  - Ply = move by one player



Depth cutoff obscures fact that white R will be captured

Addressing this problem

- Try to evaluate whether nodes are quiescent
  - Quiescent = evaluation function will not change rapidly in near future
  - Only apply evaluation function to quiescent nodes
- If there is an “obvious” move at a state, apply it before applying evaluation function

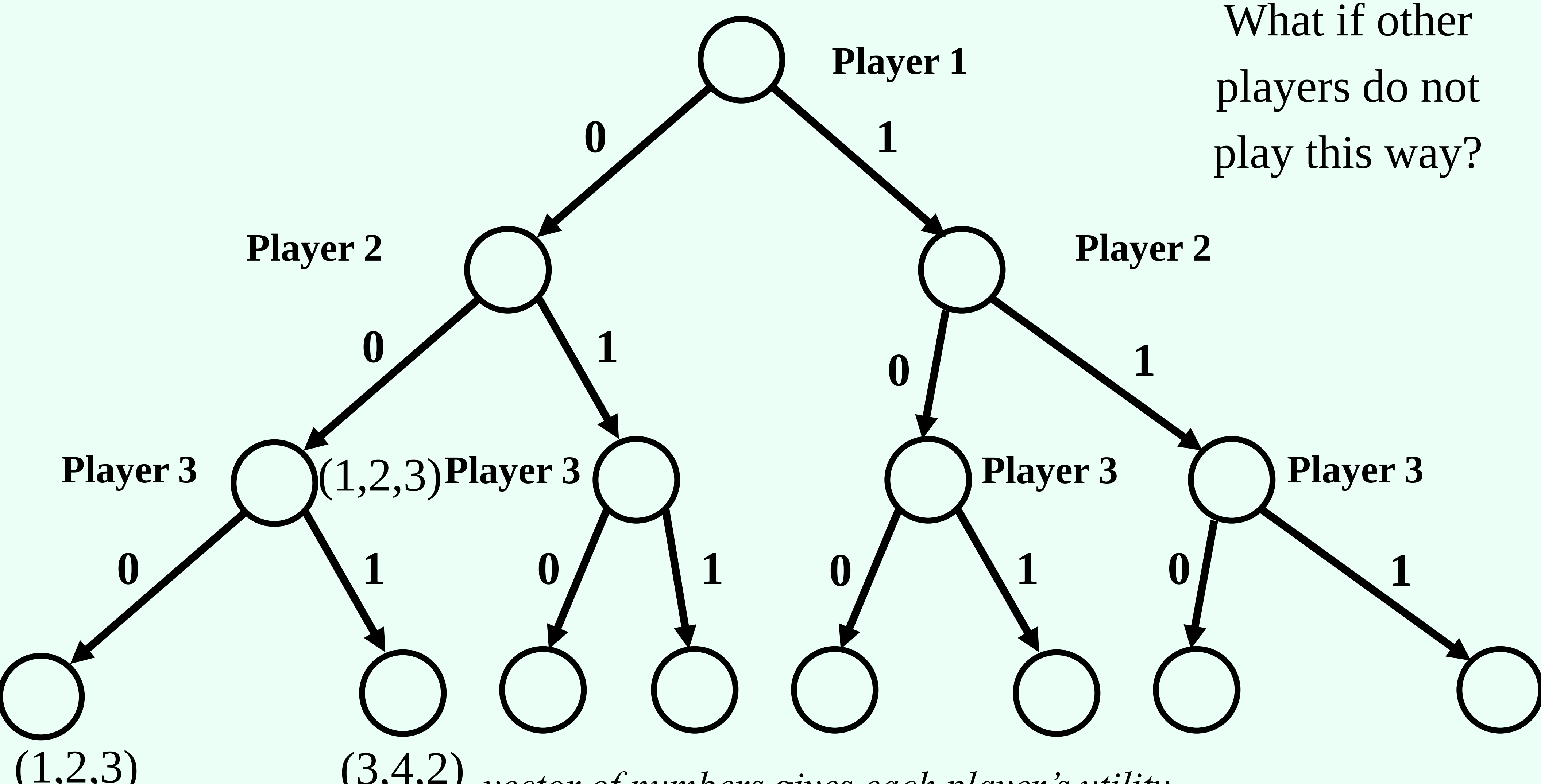
Playing against suboptimal players

- Minimax is optimal against other minimax players
- What about against players that play in some other way?

Many-player, general-sum games of perfect information

- Basic backward induction still works
  - No longer called minimax

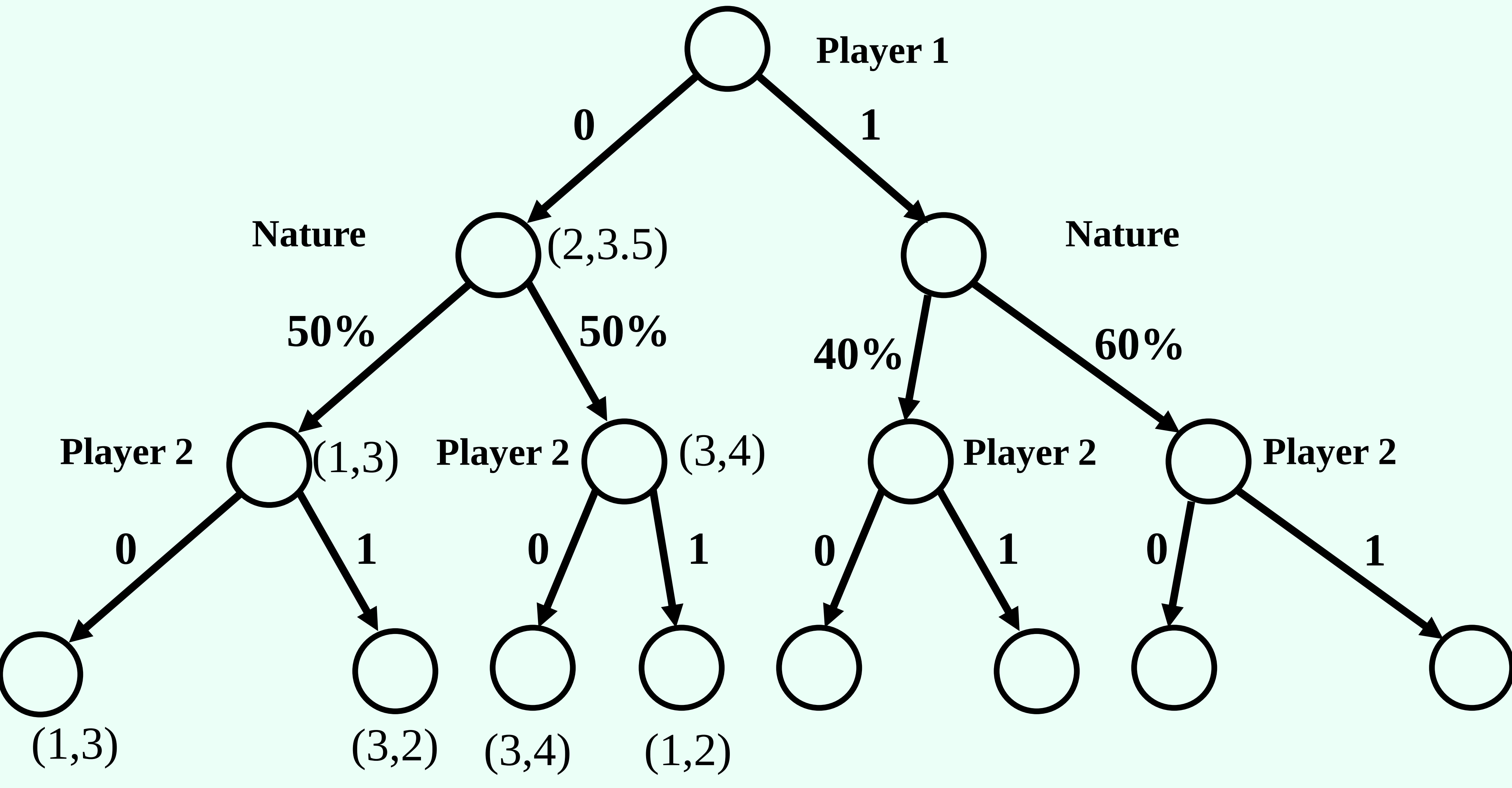
What if other players do not play this way?



vector of numbers gives each player's utility

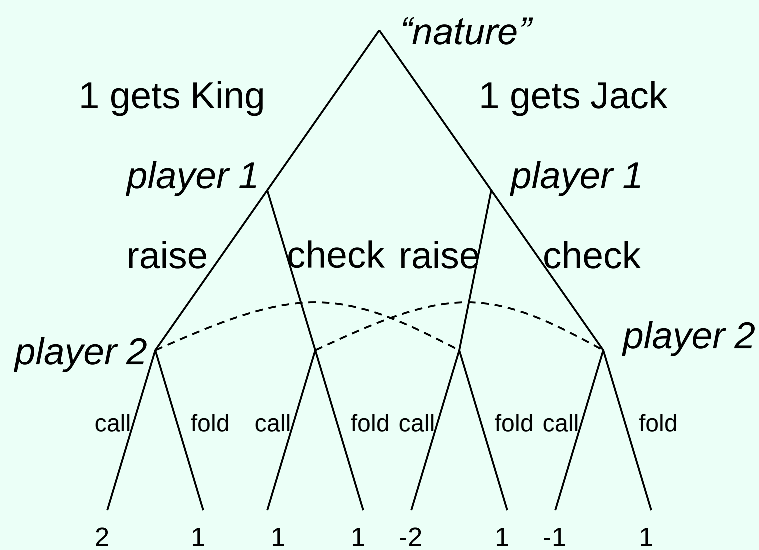
Games with random moves by "Nature"

- E.g., games with dice (Nature chooses dice roll)
- Backward induction still works…
  - Evaluation functions now need to be cardinally right (not just ordinally)



Games with imperfect information

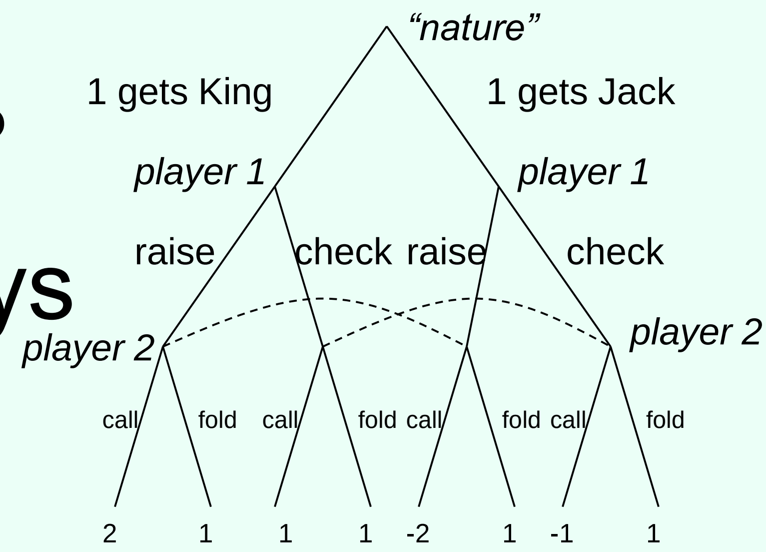
- Players cannot necessarily see the whole current state of the game
  - Card games
- Ridiculously simple poker game:
  - Player 1 receives King (winning) or Jack (losing),
  - Player 1 can raise or check,
  - Player 2 can call or fold
- Dashed lines indicate indistinguishable states



- Backward induction does **not** work, need random strategies for optimality! (more in Chapter 17)

Intuition for need of random strategies

- Suppose my strategy is “raise on King, check on Jack”
  – What will you do?
  – What is your expected utility?
- What if my strategy is “always raise”?
- What if my strategy is “always raise when given King, 10% of the time raise when given Jack”?



The state of the art for some games

- Chess:
  - 1997: IBM Deep Blue defeats Kasparov
  - … there is still some (very limited) debate about whether computers are really better…
  - … though now, if humans at a tournament come up with unexpectedly brilliant moves, it is often suspected that they secretly used a computer!
- Checkers:
  - Computer world champion since 1994
  - … there was still debate about whether computers are really better ("It wouldn't have beaten Tinsley if he were still around!" How to disprove that?) …
  - until 2007: checkers solved optimally by computer
- Go:
  - Branching factor really high, seemed like would stay out of reach for a while
  - Then AlphaGo came – superior to human players
- Poker:
  - AI now defeating top human players in 2-player ("heads-up") games
  - 3+ player case much less well-understood

Is this of any value to society?

- Some of the techniques developed for games have found applications in other domains
  - Especially “adversarial” settings
- Real-world strategic situations are usually not two-player, perfect-information, zero-sum, …
- But game theory does not need any of those
- Example application: security scheduling at airports