

Formal Methods (形式化方法)

Lecture 11. Bag (包)

智能与计算学部 章衡

2021年上学期



1 Definition and Representation

2 Bag Operators



Outline

1 Definition and Representation

2 Bag Operators



Why need to use bags?

- The multiple occurrences of elements are unable to be expressed in a set.



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$$\langle \text{Alice}, \text{Bob}, \text{Jone}, \text{Bob} \rangle$$



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- The multiple occurrences of elements can be expressed in a sequence, but accompanied with an order among the elements

$$\langle \text{Alice}, \text{Bob}, \text{Jone}, \text{Bob} \rangle$$

- In many applications, however, we need a structure to record the multiple occurrences without an order among the elements; such a structure is called a **bag**(包), a.k.a. **multiset**



$[[\cdot]]$ -representation of a bag

Definition ($[[\cdot]]$ -representation)

Given a set X , a **bag** on X can be represented by an expression of the following form:

$$[[a_1, a_2, \dots, a_n]]$$

where $a_1, a_2, \dots, a_n \in X$.



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Example

$$[[\text{Alice}, \text{Bob}, \text{Jone}, \text{Bob}]]$$


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Example

$$[[\text{Alice}, \text{Bob}, \text{Jone}, \text{Bob}]] = [[\text{Alice}, \text{Bob}, \text{Bob}, \text{Jone}]]$$



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- Elements in a bag must be of the **same type**
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Example

$$\begin{aligned} [[\text{Alice}, \text{Bob}, \text{Jones}, \text{Bob}]] &= [[\text{Alice}, \text{Bob}, \text{Bob}, \text{Jones}]] \\ &\neq [[\text{Alice}, \text{Bob}, \text{Jones}]] \end{aligned}$$



$\llbracket \cdot \rrbracket$ -representation of a bag

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Given a set X , a **bag** on X can be represented by an expression of the following form:

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- Elements in a bag must be of the **same type**
- The order among elements **plays no role** in the representation

Example

$$\begin{aligned} \llbracket \text{Alice}, \text{Bob}, \text{Jone}, \text{Bob} \rrbracket &= \llbracket \text{Alice}, \text{Bob}, \text{Bob}, \text{Jone} \rrbracket \\ &\neq \llbracket \text{Alice}, \text{Bob}, \text{Jone} \rrbracket \end{aligned}$$

Limitation of $\llbracket \cdot \rrbracket$ -representation

- A bag might have an **infinite number of elements** (noting that a set is also a bag), however, the $\llbracket \cdot \rrbracket$ -representation is not able to express an infinite bag.

Function-representation of a bag

Example

In order to evaluate the quality of online courses, we not only care about which students participate in the learning activities, but also how often a student participates in the learning activities:

Alice : 1, Bob : 2, Jone : 0, Mike : 3



Function-representation of a bag

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Definition

Given a set X , let

$$\text{bag } X == X \rightarrow \mathbb{N}_1,$$

and call each function in $\text{bag } X$ a **bag** on X .

Moreover, we use $\mathbf{b} : \text{bag } X$ to declare that \mathbf{b} is a bag on X .



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Example

`studyRecord : bag Student`

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Operators: Bag count (包计数)

Definition

Suppose $B : \text{bag } X$. For all $a \in X$, let

$$\text{count } B \, a == \begin{cases} n & \text{if } a \mapsto n \in B \\ 0 & \text{otherwise} \end{cases}$$



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Example

$L = \llbracket \text{Alice, Alice, Jone, Bob, Bob, Bob} \rrbracket$.



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$L = \llbracket \text{Alice, Alice, Jone, Bob, Bob, Bob} \rrbracket$.
 $\{\text{Alice} \mapsto 2, \text{Jone} \mapsto 1, \text{Bob} \mapsto 3\}$.



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Example

$L = \llbracket \text{Alice, Alice, Jone, Bob, Bob, Bob} \rrbracket$.
 $\{\text{Alice} \mapsto 2, \text{Jone} \mapsto 1, \text{Bob} \mapsto 3\}$.
 $\text{count } L \ \text{Alice} = 2, \text{count } L \ \text{Bob} = 3,$
 $\text{count } L \ \text{Jone} = 1, \text{count } L \ \text{Mike} = 0.$



Operators: Bag scaling (包扩大)

Definition

Suppose $B : \text{bag } X$. For all $k \in \mathbb{N}$, let

$$k \otimes B == \{a : X; n : \mathbb{N}_1 \mid a \mapsto n \in B \wedge k > 0 \bullet a \mapsto k * n\},$$

and call it the bag **scaled from B by k**.



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Example

$$L = \llbracket \text{Alice}, \text{Alice}, \text{Jone}, \text{Bob}, \text{Bob}, \text{Bob} \rrbracket. \\ \{ \text{Alice} \mapsto 2, \text{Jone} \mapsto 1, \text{Bob} \mapsto 3 \}.$$



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Example

$$L = \llbracket \text{Alice}, \text{Alice}, \text{Jone}, \text{Bob}, \text{Bob}, \text{Bob} \rrbracket.$$

$$\{\text{Alice} \mapsto 2, \text{Jone} \mapsto 1, \text{Bob} \mapsto 3\}.$$

$$2 \otimes L = \{\text{Alice} \mapsto 4, \text{Jone} \mapsto 2, \text{Bob} \mapsto 6\}.$$



count and \otimes : Generic definitions

count: Bag count

\otimes : Bag scaling



count and \otimes : Generic definitions

count: Bag count

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Definition

$[X]$

$\text{count} : \text{bag } X \rightarrow \mathbb{N}$

$- \otimes - : \mathbb{N} \times \text{bag } X \rightarrow \text{bag } X$

$\forall B : \text{bag } X \bullet$

$\text{count } B = (\lambda x : X \bullet 0) \oplus B$

$\forall n : \mathbb{N}; B : \text{bag } X; x : X \bullet$

$\text{count } (n \otimes B) x = n * \text{count } B x$

Recall that $f \oplus g == ((\text{dom } g) \triangleleft f) \cup g$.



Properties of bags and \otimes

Let $B : \text{bag } X$ and $a_1, \dots, a_n : X$. Then we have

- ① $\text{dom } \llbracket a_1, \dots, a_n \rrbracket = \{a_1, \dots, a_n\}$
- ② $n \otimes \llbracket \rrbracket = 0 \otimes B = \llbracket \rrbracket$
- ③ $1 \otimes B = B$
- ④ $(n * m) \otimes B = n \otimes (m \otimes B)$



Operators: Bag membership and sub-bag relation

Definition

Suppose $B : \text{bag } X$. For all $c \in X$, we say that c is a **member** of B , denoted **c in B** , if $c \in \text{dom } B$.



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Definition

Suppose $A, B : \text{bag } X$. We say that B is a **sub-bag** of A , denoted **$B \sqsubseteq A$** , if for all $c \in X$, we have $\text{count } B c \leq \text{count } A c$.



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Example

$$L = \llbracket \text{Alice, Alice, Jone, Bob, Bob, Bob} \rrbracket.$$
$$T = \llbracket \text{Alice, Alice, Alice, Jone, Bob, Bob, Bob, Mike} \rrbracket.$$

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$L = \llbracket \text{Alice, Alice, Jone, Bob, Bob, Bob} \rrbracket$.

$T = \llbracket \text{Alice, Alice, Alice, Jone, Bob, Bob, Bob, Mike} \rrbracket$.

Alice in L , Bob in L , $\neg(\text{Mike in } L)$, Mike in T .

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Suppose $A, B : \text{bag } X$. We say that B is a **sub-bag** of A , denoted $B \sqsubseteq A$, if for all $c \in X$, we have $\text{count } B c \leq \text{count } A c$.

Example

$$L = \llbracket \text{Alice, Alice, Jone, Bob, Bob, Bob} \rrbracket.$$

$$T = \llbracket \text{Alice, Alice, Alice, Jone, Bob, Bob, Bob, Mike} \rrbracket.$$

$$\text{Alice in } L, \text{Bob in } L, \neg(\text{Mike in } L), \text{Mike in } T.$$

$$L \sqsubseteq T, T \not\sqsubseteq L.$$

in and \sqsubseteq : Generic definitions

in: Bag member

\sqsubseteq : Sub-bag relation



in and \sqsubseteq : Generic definitions

in: Bag member

 \sqsubseteq : Sub-bag relation

Definition

 $[X]$ $_ \text{ in } _ : X \leftrightarrow \text{bag } X$ $_ \sqsubseteq _ : \text{bag } X \leftrightarrow \text{bag } X$ $\forall x : X; B : \text{bag } X \bullet$ $(x \text{ in } B \Leftrightarrow x \in \text{dom } B)$ $\forall A, B : \text{bag } X \bullet$ $B \sqsubseteq A \Leftrightarrow (\forall x : X \bullet \text{count } B \ x \leq \text{count } A \ x)$

Properties of in and \sqsubseteq

Suppose $B, C, D : \text{bag } X$. Then we have

- ① $x \text{ in } B \Leftrightarrow \text{count } B \ x > 0$
- ② $B \sqsubseteq C \Rightarrow \text{dom } B \sqsubseteq \text{dom } C$
- ③ $[\] \sqsubseteq B$
- ④ $B \sqsubseteq B$
- ⑤ $B \sqsubseteq C \wedge C \sqsubseteq B \Rightarrow B = C$
- ⑥ $B \sqsubseteq C \wedge C \sqsubseteq D \Rightarrow B \sqsubseteq D$



Operators: Bag union and bag difference

Example

Suppose $B, C : \text{bag } X$, and

$$B = \{a \mapsto 2, b \mapsto 3\}$$

$$C = \{a \mapsto 1, c \mapsto 2\}$$



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Example

Suppose $B, C : \text{bag } X$, and

$$B = \{a \mapsto 2, b \mapsto 3\}$$

$$C = \{a \mapsto 1, c \mapsto 2\}$$

$$\text{count } B \ a = 2,$$

$$\text{count } B \ b = 3,$$

$$\text{count } B \ c = 0$$

$$\text{count } C \ a = 1,$$

$$\text{count } C \ b = 0,$$

$$\text{count } C \ c = 2$$



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Example

Suppose $B, C : \text{bag } X$, and

$B = \{a \mapsto 2, b \mapsto 3\}$	$\text{count } B \ a = 2,$	$\text{count } B \ b = 3,$	$\text{count } B \ c = 0$
$C = \{a \mapsto 1, c \mapsto 2\}$	$\text{count } C \ a = 1,$	$\text{count } C \ b = 0,$	$\text{count } C \ c = 2$

How to define the bag union of B and C ?



Operators: Bag union and bag difference

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$C = \{a \mapsto 1, c \mapsto 2\}$	$\text{count } C \ a = 1,$	$\text{count } C \ b = 0,$	$\text{count } C \ c = 2$

How to define the bag union of B and C ?

$$B \uplus C = \{a \mapsto 3, b \mapsto 3, c \mapsto 2\}$$



Operators: Bag union and bag difference

Example

Suppose $B, C : \text{bag } X$, and

$$\begin{array}{llll} B = \{a \mapsto 2, b \mapsto 3\} & \text{count } B \ a = 2, & \text{count } B \ b = 3, & \text{count } B \ c = 0 \\ C = \{a \mapsto 1, c \mapsto 2\} & \text{count } C \ a = 1, & \text{count } C \ b = 0, & \text{count } C \ c = 2 \end{array}$$

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How to define the bag difference of B and C ?



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How to define the bag union of B and C ?

$$B \uplus C = \{a \mapsto 3, b \mapsto 3, c \mapsto 2\}$$

How to define the bag difference of B and C ?

$$B \ominus C = \{a \mapsto 1, b \mapsto 3\}$$



Operators: Bag union and bag difference

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Suppose $B, C : \text{bag } X$, and

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How to define the bag union of B and C ?

$$B \uplus C = \{a \mapsto 3, b \mapsto 3, c \mapsto 2\}$$

How to define the bag difference of B and C ?

$$B \ominus C = \{a \mapsto 1, b \mapsto 3\}$$

- Bag union: $\text{count } B \uplus C \ a = \text{count } B \ a + \text{count } C \ a$

Operators: Bag union and bag difference

Example

Suppose $B, C : \text{bag } X$, and

$$\begin{array}{llll} B = \{a \mapsto 2, b \mapsto 3\} & \text{count } B \ a = 2, & \text{count } B \ b = 3, & \text{count } B \ c = 0 \\ C = \{a \mapsto 1, c \mapsto 2\} & \text{count } C \ a = 1, & \text{count } C \ b = 0, & \text{count } C \ c = 2 \end{array}$$

How to define the bag union of B and C ?

$$B \uplus C = \{a \mapsto 3, b \mapsto 3, c \mapsto 2\}$$

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- Bag union: $\text{count } B \uplus C \ a = \text{count } B \ a + \text{count } C \ a$
- Bag difference: $\text{count } B \ominus C \ a = \max\{\text{count } B \ a - \text{count } C \ a, 0\}$

\uplus and \setminus : Generic definitions

\uplus : Bag union
 \setminus : Bag difference



\uplus and $\dot{\cup}$: Generic definitions

\uplus : Bag union
 $\dot{\cup}$: Bag difference

Definition

$$\begin{aligned} & \text{---} [X] \text{---} \\ & \text{---} \uplus \text{---}, \text{---} \dot{\cup} \text{---} : \text{bag } X \times \text{bag } X \rightarrow \text{bag } X \\ & \text{---} \\ & \forall B, C : \text{bag } X; x : X \bullet \\ & \quad \text{count } (B \uplus C) x = \text{count } B x + \text{count } C x \wedge \\ & \quad \text{count } (B \dot{\cup} C) x = \max \{ \text{count } B x - \text{count } C x, 0 \} \end{aligned}$$



Properties of bag union and difference

Suppose $B, C, D : \text{bag } X; m, n : \mathbb{N}$. Then we have

- ① $\text{dom}(B \uplus C) = \text{dom } B \cup \text{dom } C$
- ② $[\] \uplus B = B \uplus [\] = B$
- ③ $B \uplus C = C \uplus B$
- ④ $(B \uplus C) \uplus D = B \uplus (C \uplus D)$
- ⑤ $B \uplus [\] = B$
- ⑥ $[\] \uplus B = [\]$
- ⑦ $(B \uplus C) \uplus C = B$
- ⑧ $(n + m) \otimes B = n \otimes B \uplus m \otimes B$
- ⑨ $n \geq m \Rightarrow (n - m) \otimes B = n \otimes B \uplus m \otimes B$
- ⑩ $n \otimes (B \uplus C) = n \otimes B \uplus n \otimes C$
- ⑪ $n \otimes (B \uplus C) = n \otimes B \uplus n \otimes C$



Operators: function items

Example

$\langle a, a, b, b, b, c \rangle$



Operators: function items

Example

$\langle a, a, b, b, b, c \rangle$

$[[a, a, b, b, b, c]]$



Operators: function items

Example

$$\langle a, a, b, b, b, c \rangle$$

$$[[a, a, b, b, b, c]]$$

$$\text{items}\langle a, a, b, b, b, c \rangle = \{a \mapsto 2, b \mapsto 3, c \mapsto 1\}.$$


items: Generic definition

Definition

$$[X]$$

$$\text{items} : \text{seq } X \rightarrow \text{bag } X$$

$$\forall s : \text{seq } X; x : X \bullet$$

$$\text{count}(\text{items } s) x = \#(s \triangleright \{x\})$$


Properties of items

Suppose $s, t : \text{seq } X$. Then we have

- ① $\text{dom}(\text{items } s) = \text{ran } s$
- ② $\text{items } \langle a_1, \dots, a_n \rangle = \llbracket a_1, \dots, a_n \rrbracket$
- ③ $\text{items}(s \frown t) = \text{items } s \uplus \text{items } t$
- ④ $\text{items } s = \text{items } t \Leftrightarrow (\exists f : \text{dom } s \rightarrow \text{dom } t \bullet s = f \circ t)$



Exercises

Ex. 1

Let

$$\text{balls} == \llbracket \text{blue, green, white, green, blue, red, blue} \rrbracket$$

$$\text{balls}_1 == \{\text{white} \mapsto 2, \text{red} \mapsto 1, \text{blue} \mapsto 2, \text{green} \mapsto 2\}$$

$$\text{balls}_2 == \{\text{green} \mapsto 2, \text{blue} \mapsto 2, \text{white} \mapsto 1\}$$

What are the bags represented by the following expressions?

- 1 $\text{balls} \uplus \text{balls}_1$;
- 2 $\text{balls} \uplus \text{balls}_2$;
- 3 $\text{balls} \cup \text{balls}_1$;
- 4 $\text{balls} \cup \text{balls}_2$.

Ex. 2

Let $S == \langle \text{red, white, red, green, green, red, blue} \rangle$. What is the bag represented by items S ?

Ex. 3

Let $s, t : \text{seq } X$. Prove $\text{items}(s \frown t) = \text{items } s \uplus \text{items } t$.