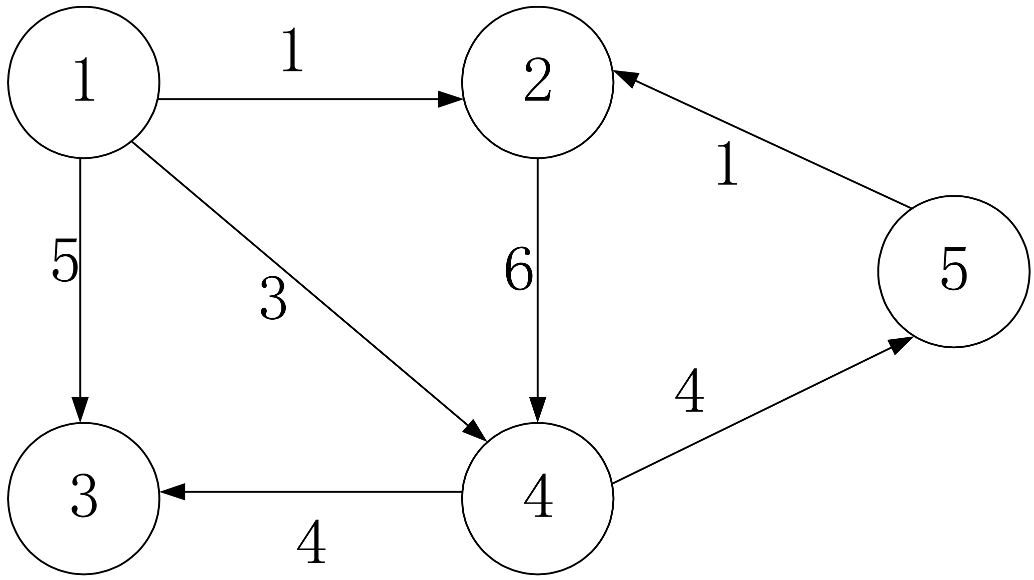
第十五章动态规划：最短路径一题（有向图见下面题）

第十六章回溯法：背包问题 n=4,w=[2,7,3,5],p=[1,2,4,6],c=11，用带限界函数的回溯法求解。

1 矩阵（回溯）

动态规划



解：先写出有向图对应的矩阵 A

$$A=\begin{pmatrix} \infty & 1 & 5 & 3 & \infty \\ \infty & \infty & \infty & 6 & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 4 & \infty & 4 \\ \infty & 1 & \infty & \infty & \infty \end{pmatrix}$$

$C^{(K)}=(C(i,j,k))$

$C(i, j, k)= \min\{C(i, j, k\text{-}1), C(i, k, k\text{-}1)+C(k, j,k\text{-}1)\}(i\neq j)$

其中$i \in \{i | C(i,k,k-1) \neq \infty, i \neq k\}$，

(也就是在矩阵的第 k 列不为∞的那些元素对应的行号)

$j \in \{j | C(k, j, k-1 \neq \infty, j \neq k\}$

(也就是在矩阵的第 k 行不为∞的那些元素对应的列号)

$$C^{(0)}=\begin{pmatrix}0 & 1 & 5 & 3 & \infty\\ \infty & 0 & \infty & 6 & \infty\\ \infty & \infty & 0 & \infty & \infty\\ \infty & \infty & 4 & 0 & 4\\ \infty & 1 & \infty & \infty & 0\end{pmatrix}$$

k=1 时：$i\in\phi, j\in\{2,3,4\}$

$$C^{(1)}=\begin{pmatrix}0 & 1 & 5 & 3 & \infty\\ \infty & 0 & \infty & 6 & \infty\\ \infty & \infty & 0 & \infty & \infty\\ \infty & \infty & 4 & 0 & 4\\ \infty & 1 & \infty & \infty & 0\end{pmatrix}$$

k=2 时：$i\in\{1,5\}, j\in\{4\}$

C(1,4,2) = min{C(1,4,1),C(1,2,1)+C(2,4,1)} = {3,1+6} = 3

C(5,4,2) = min{C(5,4,1),C(5,2,1)+C(2,4,1)} = min{∞,1+6} = 7

$$C^{(2)}=\begin{pmatrix}0 & 1 & 5 & 3 & \infty\\ \infty & 0 & \infty & 6 & \infty\\ \infty & \infty & 0 & \infty & \infty\\ \infty & \infty & 4 & 0 & 4\\ \infty & 1 & \infty & 7 & 0\end{pmatrix}$$

k=3 时：$i\in\{1,4\}, j\in\phi$

$$C^{(3)}=\begin{pmatrix}0 & 1 & 5 & 3 & \infty\\ \infty & 0 & \infty & 6 & \infty\\ \infty & \infty & 0 & \infty & \infty\\ \infty & \infty & 4 & 0 & 4\\ \infty & 1 & \infty & 7 & 0\end{pmatrix}$$

k=4 时：$i\in\{1,2,5\}, j\in\{3,5\}$

C(1,3,4) = min{C(1,3,4),C(1,4,3)+C(4,3,3)} = {5,3+4} = 5

C(1,5,4) = min{C(1,5,4),C(1,4,3)+C(4,5,3)} = {∞,3+4} = 7

C(2,3,4) = min{C(2,3,3),C(2,4,3)+C(4,3,3)} = {∞,6+4} = 10

C(2,5,4) = min{C(2,5,3),C(2,4,3)+C(4,5,3)} = {∞,6+4} = 10

$C(5,3,4) = \min\{C(5,3,3), C(5,4,3)+C(4,3,3)\}=\{\infty,7+4\} = 11$

$$C^{(4)} = \begin{pmatrix} 0 & 1 & 5 & 3 & 7 \\ \infty & 0 & 10 & 6 & 10 \\ \infty & \infty & 0 & \infty & \infty \\ \infty & \infty & 4 & 0 & 4 \\ \infty & 1 & 11 & 7 & 0 \end{pmatrix}$$

k=5 时：$i\in\{1,2,4\}$, $j\in\{2,3,4\}$

$C(1,2,5) = \min\{C(1,2,4), C(1,5,4)+C(5,2,4)\} = \{5,7+1\} = 5$

$C(1,3,5) = \min\{C(1,3,4), C(1,5,4)+C(5,3,4)\} = \{5,7+11\} = 5$

$C(1,4,5) = \min\{C(1,4,4), C(1,5,4)+C(5,4,4)\} = \{3,7+7\} = 3$

$C(2,3,5) = \min\{C(2,3,4), C(2,5,4)+C(5,3,4)\} = \{10,1+11\} = 10$

$C(2,4,5) = \min\{C(2,4,4), C(2,5,4)+C(5,4,4)\} = \{6, 1+7\} = 6$

$C(4,2,5) = \min\{C(4,2,4), C(4,5,4)+C(5,2,4)\} = \{\infty,4+1\} = 5$

$C(4,3,5) = \min\{C(4,3,4), C(4,5,4)+C(5,3,4)\} = \{4,4+11\} = 4$

$$C^{(5)} = \begin{pmatrix} 0 & 1 & 5 & 3 & \infty \\ \infty & 0 & 10 & 6 & 10 \\ \infty & \infty & 0 & \infty & \infty \\ \infty & 5 & 4 & 0 & 4 \\ \infty & 1 & 11 & 7 & 0 \end{pmatrix}$$

2 背包问题 n=4,w=[2,7,3,5],p=[1,2,4,6],c=11，用带限界函数的回溯法求解。

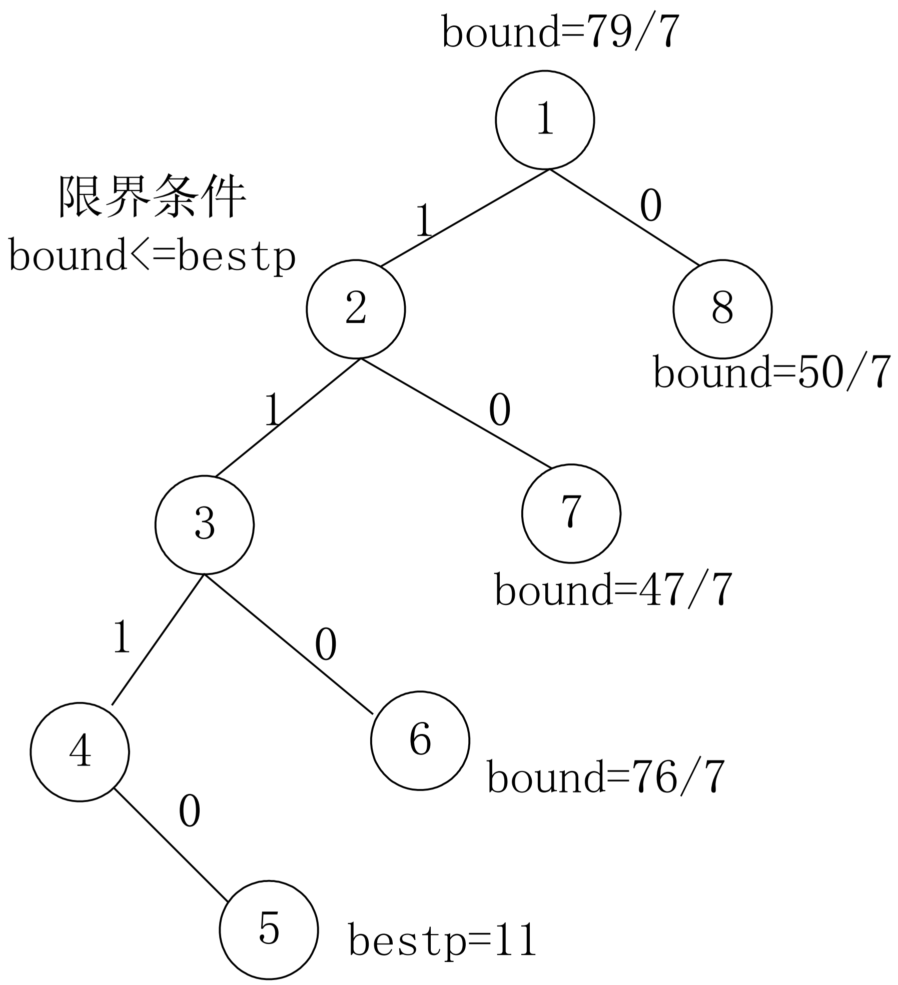
答：效益密度为[0.5,0.286,1.33,1.2]，按密度排列为[3,4,1,2]

w'=[3,5,2,7], p'=[4,6,1,2]

限界方法 1: cp+r<=bestp,则停止生成右子树。cp 为当前已得到的效益值，r 为尚为考虑的物品的效益值之和。

限界方法 2:定义 bound＝cp+对其余物品的贪心解效益值。如 bound<

＝bestp 则停止产生右子树



解 为 X ＝ [1,0,1,1]