Bootcamp
Machine Learning

42 ARTIFICIAL INTELLIGENCE

Day01 - Linear Regression

During this day you will learn the first concepts constituing the field of machine learning.

Notions of the day

Matrix operations, gradient descent, cost function, normal equation, MSE, RMSE R-score and learning rate.

General rules

- The version of Python to use is 3.7, you can check the version of Python with the following command: python -V
- The norm: during this bootcamp you will follow the Pep8 standards https://www.python.org/dev/peps/pep-0008/
- The function eval is never allowed.
- The exercices are ordered from the easiest to the hardest.
- Your exercises are going to be evaluated by someone else, so make sure that your variable names and function names are appropriate and civil.
- Your manual is the internet.
- You can also ask question in the dedicated channel in Slack: 42-ai.slack.com.
- If you find any issue or mistakes in the subject please create an issue on our dedicated repository on Github: https://github.com/42-AI/bootcamp_machine-learning/issues.

Helper

Ensure that you have the right Python version.

```
> which python
/goinfre/miniconda/bin/python
> python -V
Python 3.7.*
> which pip
/goinfre/miniconda/bin/pip
```

Exercise 00 - Hypothesis Concept in Machine Learning

Exercise 01 - Cost Function Concept

Exercice 02 - Descent Gradient

Exercise 00 - Hypothesis Concept in Machine Learning

Turn in directory :	ex00
Files to turn in :	pred.py
Authorized modules :	numpy,
Forbidden functions :	None
Links :	https://www.coursera.org/learn/machine-learning/home/week/1
	https://www.coursera.org/learn/machine-learning/home/week/2
Hint :	Focus on the "Model and Cost Function" section of week 1
	and "Multivariate Linear Regression" section of week 2"

Objectives :

• Reinforce the mathematical skills tackled in **Mathematical Delights**, especially the **matrix-matrix operations**.
• Be able to explain what is a hypothesis in the machine learning context.
• Be able to implement a basic method based on the hypothesis function to obtain predicted output.

Instructions :
First of all, you have to get familiar with the concept of feature, training set, training example, output and hypothesis.
As hypothesis function h, you will choose:

$$y^{\hat{(i)}} = h(X^{(i)}) = \sum_{j=0}^{N} \theta_j . x_j^{(i)}$$

Where $X^{(i)}$ is the ith training example (line vector of N components), θ_j is the jth component of the θ vector. $\hat{y}^{(i)}$ is what we called the ith predicted output.

I would like to highlight that X is a matrix of dimension = (M,N) and theta is a matrix of dimension (N+1,1). Thus, I let you search and do what it is necessary to allow the matrix product..

Then you will code a function named **.predict_** as per the instructions bellow:

```
def predict_(theta, X):
    """
    Description:
        Prediction of output using the hypothesis function (linear model).
    Args:
        theta: has to be a numpy.ndarray, a vector of dimension (number of
features + 1, 1).
        X: has to be a numpy.ndarray, a matrix of dimension (number of
training examples, number of features).
    Returns:
        pred: numpy.ndarray, a vector of dimension (number of the training
examples,1).
        None if X does not match the dimension of theta.
    Raises:
        This function should not raise any Exception.
    """
        ... your code here ...
```

Examples :

```
>>>import numpy as np
>>>X1 = np.array([[0.], [1.], [2.], [3.], [4.]])
>>>theta1 = np.array([[2.], [4.]])
>>>predict_(theta1, X1)
array([[2], [6], [10], [14.], [18.]])
>>>X2 = np.array([[1], [2], [3], [5], [8]])
>>>theta2 = np.array([[2.]])
>>>predict_(theta2, X2)
Incompatible dimension match between X and theta.
None
>>>X3 = np.array([[0.2, 2., 20.], [0.4, 4., 40.], [0.6, 6., 60.], [0.8, 8.,
80.]])
>>>theta3 = np.array([[0.05], [1.], [1.], [1.]])
>>>predict_(theta3, X3)
array([[22.25], [44.45], [66.65], [88.85]])
```

Questions :
Be sure to understand the underlying concepts and be able to answer those questions during your evaluation:

- What is a hypothesis and what is it goal ?
- Considering a training set with 4242 examples and 3 features. How many components are there in the vector θ ?
- Considering the vector θ has a shape (6,1) and the output has the shape (7,1). What is the shape of the training set X ?

Exercise 01 - Cost function Concept

Turn in directory :	ex01
Files to turn in :	cost_function.py
Authorized modules :	numpy,
Forbidden functions :	None
Links :	https://www.coursera.org/learn/machine-learning/home/week/1
	https://www.coursera.org/learn/machine-learning/home/week/2
Hint :	Focus on the "Model and Cost Function" section of week 1
	and "Multivariate Linear Regression" section of week 2"

Objectives :

• Reinforce the mathematical skills tackled in **Mathematical Delights**, especially the **matrix-matrix operations**.
• Be able to explain what is the cost function in the machine learning context.
• Be able to implement a basic method based on the hypothesis function and output to obtain the value of the cost function.

Instructions :
In this exercise you will be interested with the concept of cost function usually noted J. The cost function is defined by the following formula:

$$J(\theta) = \frac{1}{2M}\sum_{i=1}^{M}\left(\hat{y^{(i)}} - y^{(i)}\right)^2 = \frac{1}{2M}\sum_{i=1}^{M}\left(h_\theta(X^{(i)}) - y^{(i)}\right)^2$$

Where :

• θ is the coefficient vector,
• M is the number of training examples,
• $\hat{y}^{(i)}$ is the ith predicted output.
• $y^{(i)}$ is the ith real output.
• $X^{(i)}$ is the ith training example (line vector of N components),

Then you will code the functions named **cost_elem_** and **cost_** as per the instructions below:

```
def cost_elem_(theta, X, Y):
    """
    Description:
        Calculates all the elements 0.5*M*(y_pred - y)^2 of the cost
function.
    Args:
        theta: has to be a numpy.ndarray, a vector of dimension (number of
features + 1, 1).
        X: has to be a numpy.ndarray, a matrix of dimension (number of
training examples, number of features).
    Returns:
        J_elem: numpy.ndarray, a vector of dimension (number of the training
examples,1).
        None if there is a dimension matching problem between X, Y or theta.
    Raises:
        This function should not raise any Exception.
    """
        ... your code here ...

def cost_(theta, X, Y):
    """
    Description:
        Calculates the value of cost function.
    Args:
        theta: has to be a numpy.ndarray, a vector of dimension (number of
features + 1, 1).
        X: has to be a numpy.ndarray, a vector of dimension (number of
training examples, number of features).
    Returns:
        J_value : has to be a float.
        None if X does not match the dimension of theta.
    Raises:
        This function should not raise any Exception.
    """
        ... your code here ...
```

Examples :

```
>>>import numpy as np
>>>X1 = np.array([[0.], [1.], [2.], [3.], [4.]])
>>>theta1 = np.array([[2.], [4.]])
>>>Y1 = np.array([[2.], [7.], [12.], [17.], [22.]])
>>>cost_elem_(theta1, X1, Y1)
array([[0.], [0.1], [0.4], [0.9], [1.6]])
>>>cost_(theta1, X1, Y1)
3.0
>>>X2 = np.array([[0.2, 2., 20.], [0.4, 4., 40.], [0.6, 6., 60.], [0.8, 8.,
80.]])
>>>theta2 = np.array([[0.05], [1.], [1.], [1.]])
>>>Y2 = np.array([[19.], [42.], [67.], [93.]])
>>>cost_elem_(theta2, X2, Y2)
array([[1.3203125], [0.7503125], [0.0153125], [2.1528125]])
>>>cost_(theta2, X2, Y2)
4.238750000000004
```

Remarks :
You may notice that the cost function is very similar to the MSE (see day00).

Questions :
Be sure to understand the underlying concepts and be able to answer those questions during your evaluation:

- What is the cost function and what is it goal ?
- What is the interest of the cost function derivative (you may look few more videos of the week 2 on coursera) ?
- **Bonus (hard) question :** Are there other forms of the cost function ? Cite at least 2 definitions (with formula) of the cost function and give a very short description.

Exercise 02 - Descent Gradient

Turn in directory :	ex02
Files to turn in :	fit.py
Authorized modules :	numpy
Forbidden functions :	all functions that perform derivative at your place
Links :	https://www.coursera.org/learn/machine-learning/home/week/1
	https://www.coursera.org/learn/machine-learning/home/week/2
Hint :	Focus on the "Parameter Learning" section of week 1
	and "Multivariate Linear Regression" section of week 2"

Objectives :

- Reinforce the mathematical skills tackled in **Mathematical Delights**, especially the **matrix-matrix operations**.
- Be able to explain what is a fit in the machine learning context.
- Be able to implement a funcion which will perform a linear gradient descent (LGD).

Instructions :

In this exercise you will be interested with the concept of linear gradient descent to perform fit of dataset. The linear gradient descent allows to correct the coefficients θ thanks to the gradient of the cost function.

You are expected to code a function named **fit_** as per the instructions bellow:

```
def fit_(theta, X, Y):
    """
    Description:
        Performs a fit of Y(output) with respect to X.
    Args:
        theta: has to be a numpy.ndarray, a vector of dimension (number of
features + 1, 1).
        X: has to be a numpy.ndarray, a matrix of dimension (number of
training examples, number of features).
        Y: has to be a numpy.ndarray, a vector of dimension (number of
training examples, 1).
    Returns:
        new_theta: numpy.ndarray, a vector of dimension (number of the
features +1,1).
        None if there is a matching dimension problem.
    Raises:
        This function should not raise any Exception.
    """
        ... your code here ...
```

Hopefully, you have already code a function to calculate the linear gradient (you might have forgot, but you really did this).

Examples :

```
>>>import numpy as np
>>>X1 = np.array([[0.], [1.], [2.], [3.], [4.]])
>>>Y1 = np.array([[2.], [6.], [10.], [14.], [18.]])
>>>theta1 = np.array([[1.], [1.]])
>>>theta1 = fit_(theta1, X1, Y1, alpha = 0.01, n_cycle=2000)
>>>theta1
array([[2.0023..],[3.9991..]])
>>>predict_(theta1, X1)
array([2.0023..], [6.002..], [10.0007..], [13.99988..], [17.9990..])
>>>
>>>X2 = np.array([[0.2, 2., 20.], [0.4, 4., 40.], [0.6, 6., 60.], [0.8, 8.,
80.]])
>>>Y2 = np.array([[19.6.], [-2.8], [-25.2], [-47.6]])
>>>theta2 = np.array([[42.], [1.], [1.], [1.]])
>>>theta2 = fit_(theta2, X2, Y2, alpha = 0.0005, n_cycle=42000)
>>>theta2
array([[41.99..],[0.97..], [0.77..], [-1.20..]])
>>>predict_(theta2, X2)
array([[19.5937..], [-2.8021..], [-25.1999..], [-47.5978..]])
```

Remarks :
You can generate other examples by choosing arbitrary X array and declare Y as linear expression of the X columns. Notice also that you can have the components of theta becoming "[nan]". In that case it means you probably used a too big learning rate.

Questions :
Be sure to understand the underlying concepts and be able to answer those questions during your evaluation:

- Can you explain the different steps in the fit method (hint: you have to talk about J, it gradient and the theta)?
- What happens if you choose a too big learning rate ?
- What can you say if you choose a very small learning rate and a reasonnable number of cycles ?

Exercise 03 - The Linear Regression with Class

Turn in directory :	ex03
Files to turn in :	mylinearregression.py
Authorized modules :	numpy
Forbidden modules :	sklearn

Objectives :

- Code your class containing all the methods to perform linear regression.

Instructions :
In this exercise you will not learn something new but do not worried it is for your own well-being (at least I think so).
You are expected to code your own class **MyLinearRegression** which looks similar to the class **sklearn.linear_model.LinearRegression**:

```
class MyLinearRegression():
    """
    Description:
        My personnal linear regression class to fit like a boss.
    """
    def __init__(self, theta):
        """
        Description:
            generator of the class, initialize self.
        Args:
            theta: has to be a list or a numpy array, it is a vector of
dimension (number of features + 1, 1).
        Raises:
            This method should noot raise any Exception.
        """
         ... your code here ...
    ... other methods ...
```

You will add the methods **predict_(self, X)**, **cost_elem_(self, X, Y)**, **cost_(self, X, Y)** and **fit_(self, X, Y)**.
You have already coded the functions, it will just need little modifications to put them in **MyLinearRegression**.

Examples :

```
>>>import numpy as np
>>>from mylinearregression import MyLinearRegression as MyLR
>>>X = np.array([[1., 1., 2., 3.], [5., 8., 13., 21.], [34., 55., 89.,
144.]])
>>>Y = np.array([[23.], [48.], [218.]])
>>>mylr = MyLR([[1.], [1.], [1.], [1.], [1]])
>>>mylr.predict_(X)
array([[8.], [48.], [323.]])
>>>mylr.cost_elem_(X,Y)
array([[37.5], [0.], [1837.5]])
>>>mylr.cost_(X,Y)
1875.0
>>> mylr.fit_(X, Y, alpha = 1.6e-4, n_cycle=200000)
>>>mylr.theta
array([[18.023..], [3.323..], [-0.711..], [1.605..], [-0.1113..]])
>>>mylr.predict_(X)
array([[23.499..], [47.385..], [218.079...]])
>>>mylr.cost_elem_(X,Y)
array([[0.041..], [0.062..], [0.001..]])
>>>mylr.cost_(X,Y)
0.1056..
```

Exercise 04 - Linear Regression

Turn in directory :	ex04
Files to turn in :	linear_m
Authorized modules :	numpy, r
Forbidden modules :	sklearn
Remarks :	Read the

Objectives:

• Reinforce the mathematical skills tackled in **Mathemati**
matrix-matrix operations.
• Be able to perform a fit with a very small dataset, know
• Manipulate the cost function J, plot it and briefly analyz

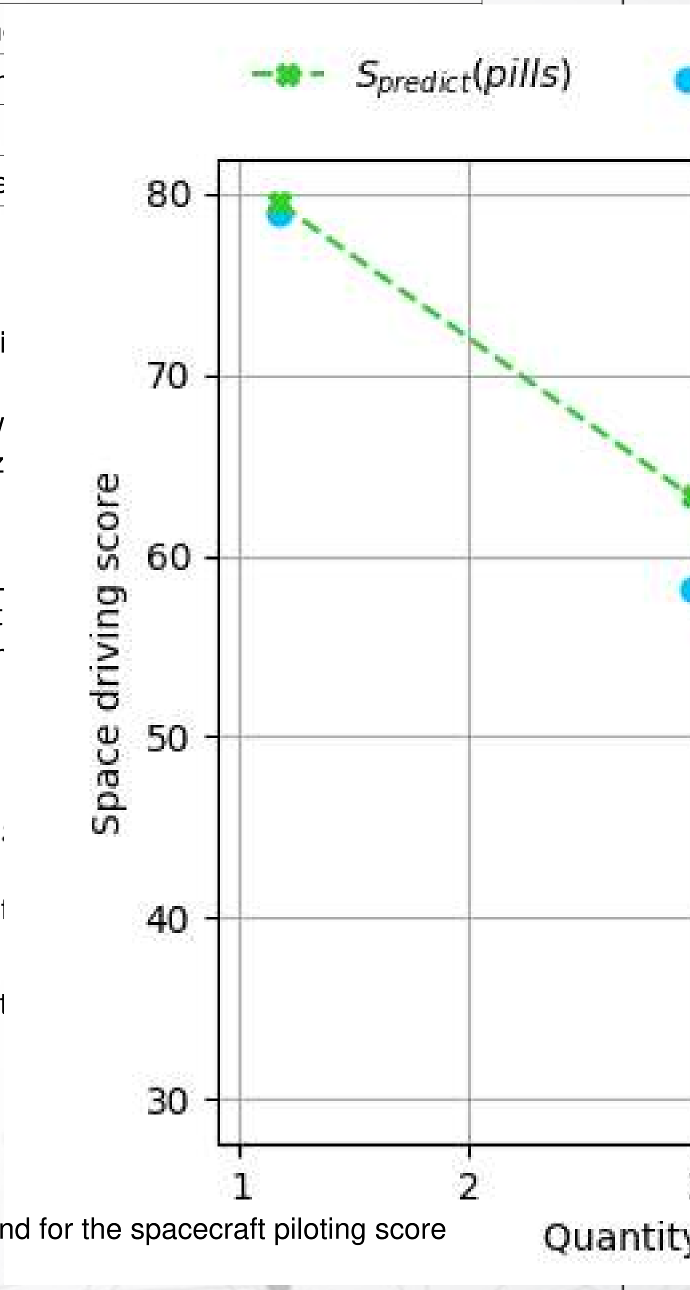
Instructions:
You can find in the ressources a tiny dataset called **"are_**
you mathematical performance of patients in function of t
took before the test. You have a description of the data ir
"are_blue_pills_magics.txt".
As hypothesis function h, you will choose:

$$h(x) = \theta_0 + \theta_1$$

Where x is the variable, θ_0 and θ_1 are the coefficients o

function of x. It gives you the predicted values usually no

You will model the data and plot 2 differents graphs:



• The graph with the data and the best hypothesis you find for the spacecraft piloting score versus the quantity of "blue pills" (see example figure 1),
driving score in function of the quantity of blue pill (in micrograms). In blue the real values and in green the predicted values.
• The cost function $J(\theta)$ in function of the θ values (see example in figure 2),

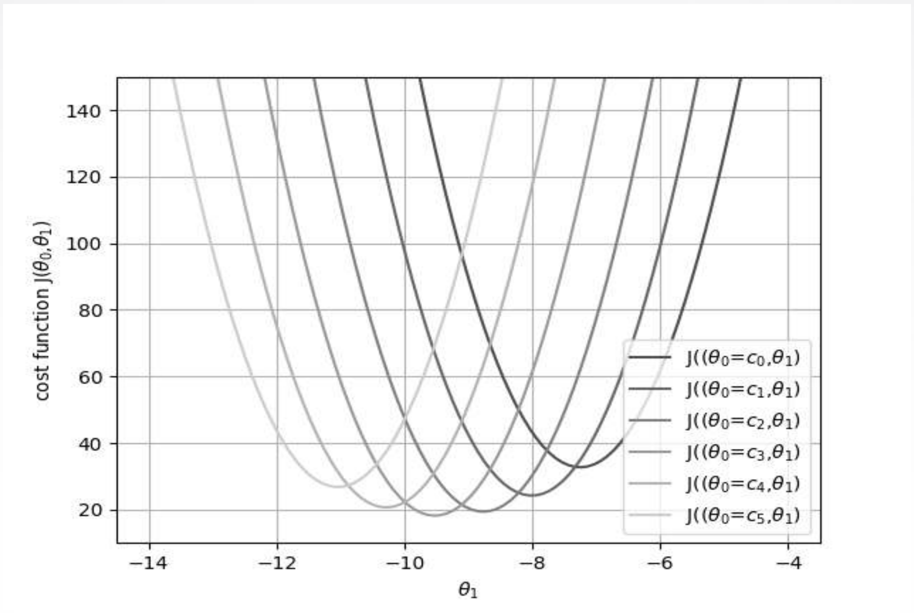


Figure 2: Evolution of the cost function J in fuction of θ_1 for different values of θ_0.

- You will calculate the MSE of the hypothesis you chose (you know how to do it already) add it to your class **MyLinearRegression**.

Examples

```
>>>import pandas as pd
>>>import numpy as np
>>>from sklearn.metrics import mean_squared_error
>>>from mylinearregression import MyLinearRegression as MyLR
>>>
>>>data = pd.read_csv("are_blue_pills_magics.csv")
>>>Xpill = np.array(data[Micrograms]).reshape(-1,1)
>>>Yscore = np.array(data[Score]).reshape(-1,1)
>>>
>>>linear_model1 = MyLR(np.array([[89.0], [-8]]))
>>>linear_model2 = MyLR(np.array([[89.0], [-6]]))
>>>Y_model1 = linear_model1.predict_(Xpill)
>>>Y_model2 = linear_model2.predict_(Xpill)
>>>
>>>print(linear_model1.mse_(Xpill, Yscore))
# 57.60304285714282
>>>print(mean_squared_error(Yscore, Y_model1))
# 57.603042857142825
>>>print(linear_model2.mse_(Xpill, Yscore))
# 232.16344285714285
>>>print(mean_squared_error(Yscore, Y_model1))
# 232.16344285714285
```

Clarifications and hints

There is no method named **.mse_** in the class LinearRegression of the module sklearn.linear_model but there is a method named **.score**. The **.score** method correspond to the R^2 score. The metric MSE is available in the module sklearn.metrics.

Be sure to understand the underlying concepts and be able to answer those questions during the evaluation:

- What is a hypothesis and what is it goal ? (It is a second chance to let you say something intelligible, no need to thanks me)
- What is the cost function and what is it representing ?
- What is the linear gradient descent and what is it doing ?
- Can you explain the MSE and what is it spotting ?

Exercise 05 - Mutiples features and Linear Gradient Descent

Turn in directory :	ex05
Files to turn in :	multi_linear_model.py
Authorized modules :	numpy, matplotlib
Forbidden modules :	sklearn
Forbidden functions :	LinearRegression
Links :	https://www.coursera.org/learn/machine-learning/home/week/2
Hint :	Really, spend time on "Multivariate Linear Regression" section until it is clear

Objectives :

• Reinforce the mathematical skills tackled in **Mathematical Delights**, especially the vectorized form of **linear cost function** and **gradient descent**.
• Be able to manipulate the **linear cost function** and the **linear gradient descent** for a multiple features problem (and knowing what you are doing of course).
• Be able to visualize the different objects via graphics and extract basic informations based on them.

Instructions :
As you are able to perform a simple linear regression with one feature (well done !) it is time to dream bigger.
Lucky you are, we give you a new dataset with multiple features that you will find in the ressources attached.
The dataset is called **"spacecraft_data.csv"** which contains the prices of spacecrafts in function of multiple features (multiple features means you will need a multi-linear model but hold on for the moment). A description of the dataset is provided in the file **"spacecraft_data_description.txt"**.

Part One: single linear regression

As a starter, you will try to fit the data with a single linear regression and see what we get. Thus, your hypothesis h(X) would be given by:

$$h(X) = X \cdot \theta = \begin{bmatrix} 1 & x_1^{(1)} \\ \vdots & \vdots \\ 1 & x_1^{(M)} \end{bmatrix} \cdot \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} \theta_0 + \theta_1 x_1^{(1)} \\ \vdots \\ \theta_0 + \theta_1 x_1^{(M)} \end{bmatrix}$$

assuming X is an array of dimension (M, N+1) (M corresponding to the number of training examples and N to the number of features). Here in this part 1, N is equal to 1.

As you already noticed, an extra column of 1 is add at the beginning of the X matrix such as the matrix product gives a vector of dimension (M, 1) where the ith component is $\theta_0 + \theta_1 x_1^{(i)}$. This is just a trick to perform the calculation of h(X) in the vectorized way.

You are expected to :

• The graph with the data, the hypothesis $h^{LGD}_{\theta_0,\theta_{age}}(age)$ obtained via linear gradient descent versus age (see example figure 1),

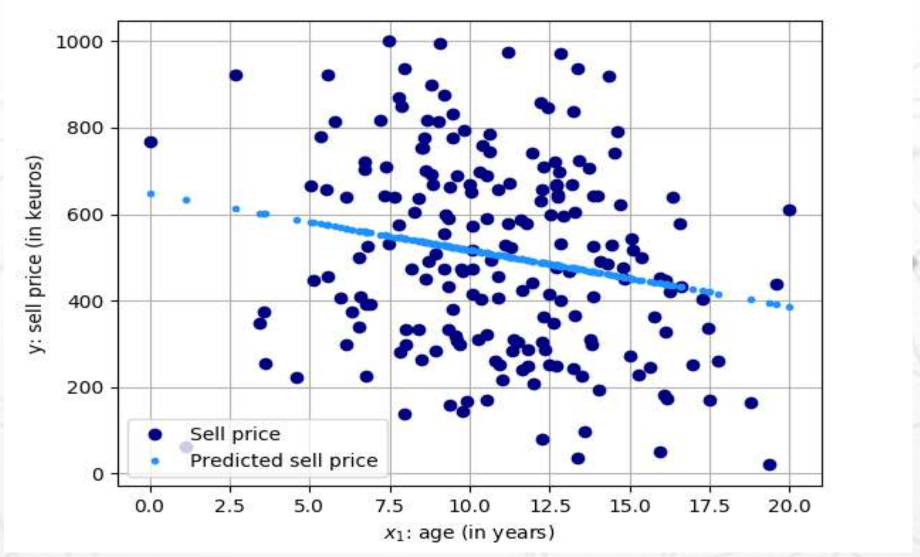


Figure 1: Evolution of the sell price of spacecrafts with respect to the age of the spacecraft and representation of the predicted values of our first model.

- The graph with the data, the hypothesis $h^{LGD}_{\theta_0,\theta_{thrust}}(thrust)$ obtained via linear gradient descent versus thrust (see example figure 2),

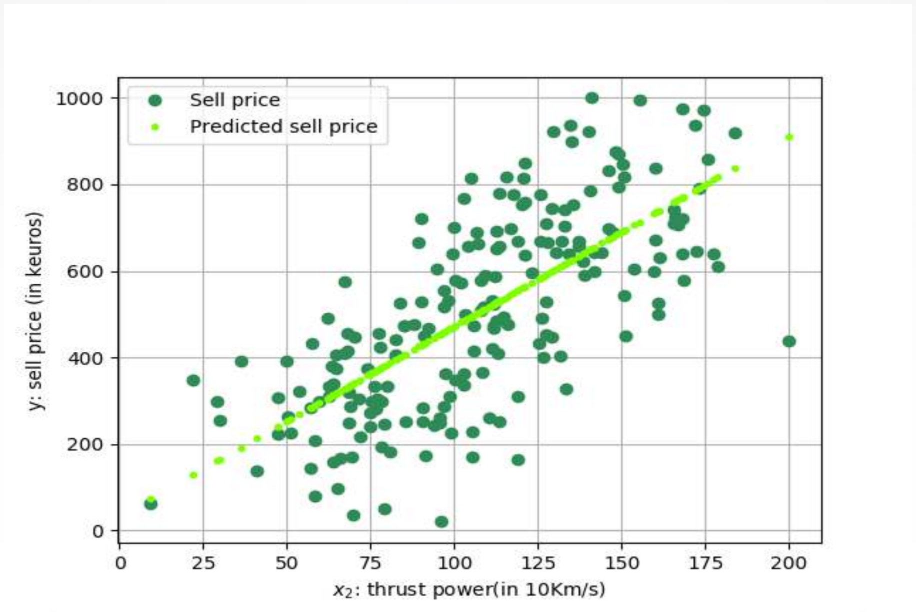


Figure 2: Evolution of the sell price of spacecrafts with respect to the thrust power of the spacecraft engines and representation of the predicted values of our second model.

- The graph with the data, the hypothesis $h^{LGD}_{\theta_0,\theta_{Tmeters}}(Tmeters)$ obtained via linear gradient descent versus Tmeters (see example figure 3),

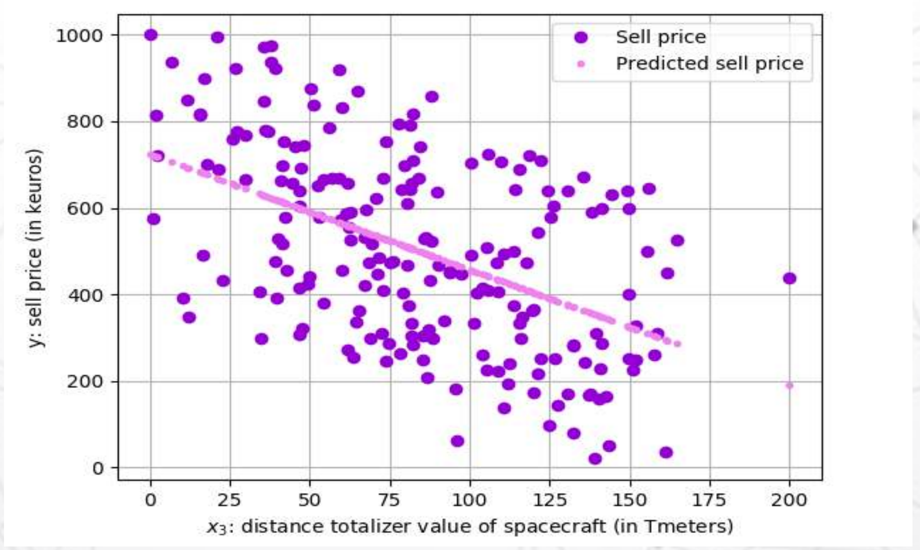


Figure 3: Evolution of the sell price of spacecrafts with respect to the terameters driven and the predicted values of our third model.

Reminder :

• You may obtain after the fit θ=array([[nan, nan]]). It may come from a too large learning rate.
• Also, be aware that you set the number of cycles of the fitting process, but it does not guarantee that the fit is the best or a good one. For this purpose, a tolerance parameter might be used (not see here, so do not worried).
Here, you can increase the number of cycle and play around with the learning rate to try to get a good fit.

Hint :

• Plot the data in function of the feature you are working with, allows you to guess initial values of theta that are not too bad, which will help to converge to a good fit.

Examples:

```
>>>import pandas as pd
>>>import numpy as np
>>>form mylinearregression import MyLinearRegression as MyLR
>>>
>>>data = pd.read_csv("spacecraft_data.csv")
>>>[...]
>>>myLR_age = MyLR([[1000.0], [-1.0]])
>>>myLR_age.fit_(X[:,0].reshape(-1,1), Y, alpha = 2.5e-5, n_cycle = 100000)
>>>
>>>RMSE_age = myLR_age.mse_(X[:,0].reshape(-1,1),Y)
>>> print(RMSE_age)
57636.77729...
```

Are the fits with a single variable precised ? Why ? (What did I say a the beginning ?)

Part Two: Multilinear Regression (A New Hope)

Now, it is time for your first multilenear regression !
As you might expected, the formula of the hyphothesis change a little and is given by:

$$h(X) = X\theta = \begin{bmatrix} x_0^{(1)} & \cdots & x_N^{(1)} \\ \vdots & \ddots & \vdots \\ x_0^{(M)} & \cdots & x_N^{(M)} \end{bmatrix} \cdot \begin{bmatrix} \theta_0 \\ \vdots \\ \theta_N \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} \theta_i x_i^{(1)} \\ \vdots \\ \sum_{i=1}^{N} \theta_i x_i^{(M)} \end{bmatrix}$$

where X is the training dataset matrix, θ the coefficients vector, M is the number of training samples and N the number of features.

But It should not change the methods you coded, hopefully !

Examples :

```
>>>import pandas as pd
>>>import numpy as np
>>>form mylinearregression import MyLinearRegression as MyLR
>>>
>>>data = pd.read_csv("spacecraft_data.csv")
>>>X = np.array(data[['Age','Thrust_power','Terameters']])
>>>Y = np.array(data[['Sell_price']])
>>>my_lreg = MyLR([1.0, 1.0, 1.0, 1.0])
>>>my_lreg.mse_(X,Y)
144044.877...
>>>my_lreg.fit_(X,Y, alpha = 1e-4, n_cycle = 600000)
>>>my_lreg.theta
array([[334.994...],[-22.535...],[5.857...],[-2.586...]])
>>>my_lreg.mse_(X,Y)
586.896999...
```

Remarks :
You can obtain a better fit, if you increase the number of cycles.

You are expected to:

• Plot the output and predicted output on the same graph, in function of the age(see figure below),

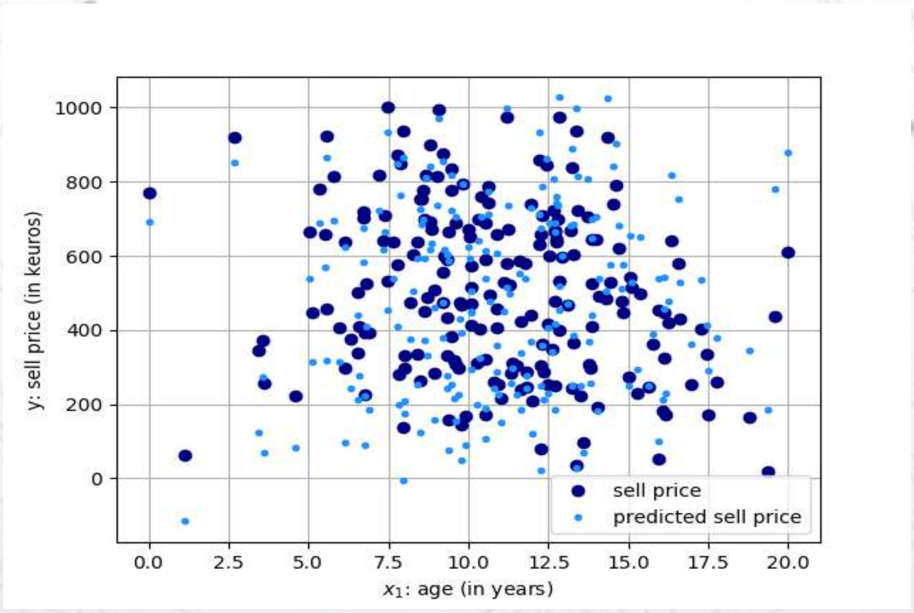


Figure 1: Evolution of the sell prices of spacecrafts and evolution of predicted sell prices of spacecrafts with the multi-variables hypothesis, with respect to the age.

• Plot the output and predicted output on the same graph, in function of the thrust power (see figure below),

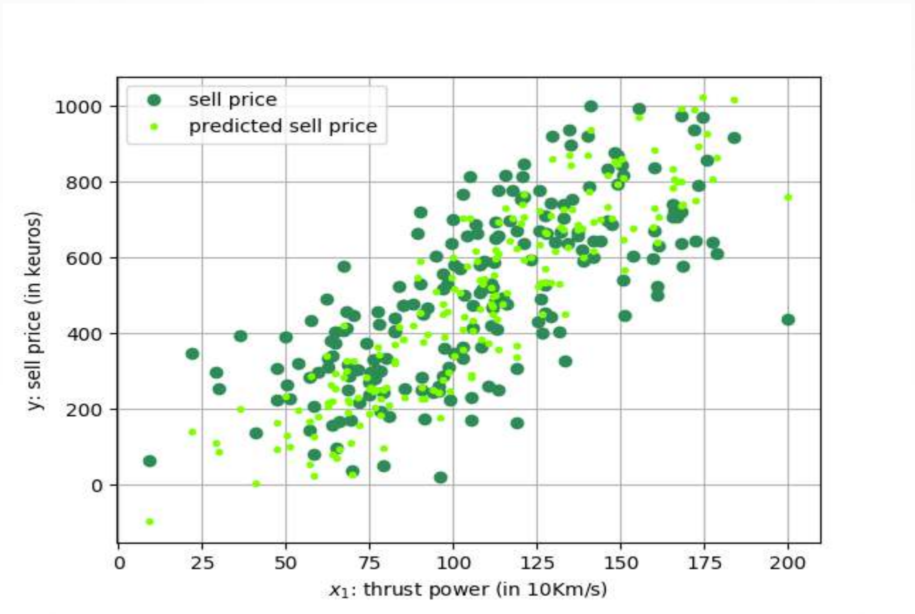


Figure 2: Evolution of the sell prices of spacecrafts and the evolution of predicted sell prices of spacecrafts with the multi-variables hypothesis, with respect to the thrust power of the engines.

• Plot the output and predicted output on the same graph, in function of the distance (see figure below),

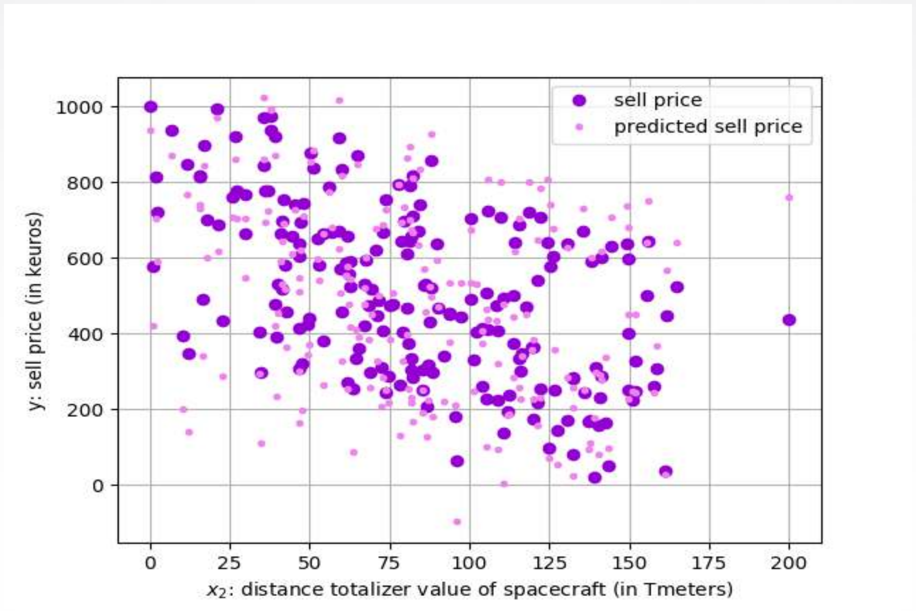


Figure 3: Evolution of the sell prices of spacecrafts and evolution of the predicted sell prices of spacecrafts with multi-variables hypothesis, with respect to the terameters driven.

What conclusion can you make in regards of the results of both parts ?

Questions:

Be sure to understand the underlying concepts and be able to answer those questions during your evaluation:

- What can you say on the influence of the choice of the hypothesis ?

Exercise 06 - Mutiples features and Normal Equation

Turn in directory :	ex06
Files to turn in :	normal_equation_model.py
Authorized modules :	numpy, matplotlib
Forbidden modules :	sklearn
Forbidden functions :	LinearRegression
Links	https://www.coursera.org/learn/machine-learning/home/week/2
Remarks :	Take a look to the "Computing Parameters Analytically" section

Objectives :

• Reinforce the mathematical skills tackled in **Mathematical Delights**, especially the vectorized form of **matrix-matrix oparations** (intermediate manipulation).
• Be able to perform **linear cost function** and what one names **normal equation** for a multiple features problem (and still knowing what you are doing of course obviously).
• Be able to implement a vectorized method **normalequation** in order to perform a full linear regression with the normal equation method.

Instructions :
We continue to play with the same dataset (**spacraft_data.csv**) given in the previous exercise.
You will code a new method named **normalequation** which will allows you to perform a fit of the dataset based on the method called **normalequation.**
The normal equation is:

$$\theta_{NE} = \left(X^T X\right)^{-1} \cdot X^T Y$$

where X is the training dataset matrix, θ_{NE} the coefficients vector obtained with the normal equation, Y are the output vector.

For folks who do not understand what the meaning of the superscripts "T" and "-1" over a matrix, they respectively tell you we take the transpose and the inverse of the concerned matrix. Do not worry, for those who do not know what it means or do not remember how to do, well you can still do the exercise because numpy will run the

maths for you, but you will not understand what happens behind the scene for the moment. Fortunaly, the mathematic workshops will be coming soon, what a wonderful opportunity to learn to do transposition and inversion of matrices. Lucky you !

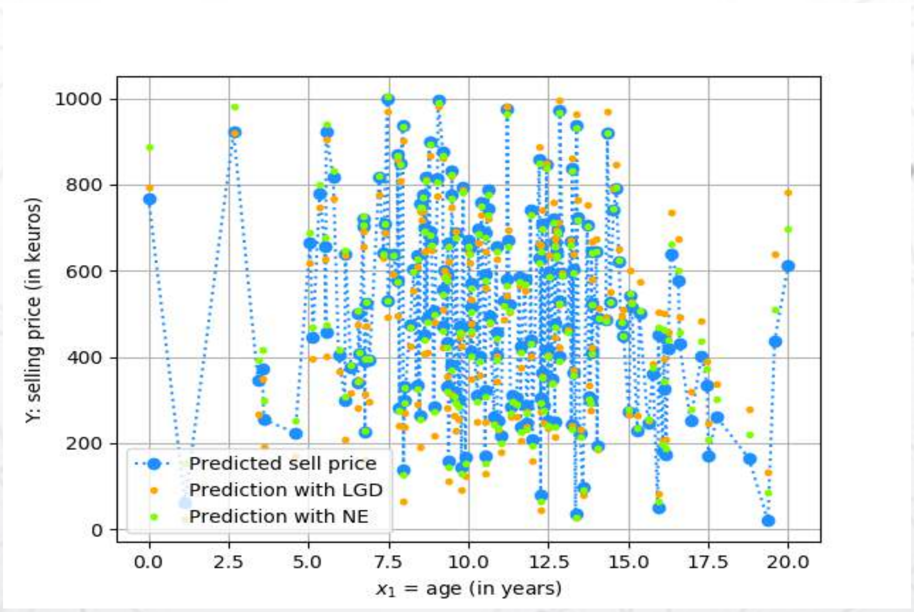
You are expected to:

- Add the method **normalequation_** in the class **MyLinearRegression** as per the instructions given below:

```
def normalequation_(self, X, Y):
    """
    Description:
        Perform the normal equation to get the theta parameters of the
hypothesis h and stock them in self.theta.
    Args:
        X: has to be a numpy.ndarray, a matrix of dimension (number of
training examples, number of features)
        Y: has to be a numpy.ndarray, a vector of dimension (number of
training examples,1)
    Returns:
        No return expected.
    Raises:
        This method should not raise any Exceptions.
    """
         ... your code ...
```

You will model the data and plot differents graphs (yes again):

- The graph with the data, the hypothesis $h_\theta^{LGD}(X)$ obtained via linear gradient descent and $h_\theta^{NE}(X)$ (see example figure below) with respect to the feature of your choice,



• Calculate the MSE between the data and the predicted data generate with the model train with the linear gradient descent, then with normal equation technique.

Example :

```
>>>import pandas as pd
>>>import numpy as np
>>>from mylinearregression import MyLinearRegression as MyLR
>>>data = pd.read_csv("spacecraft_data.csv")
>>>[...]
>>>myLR_ne = MyLR([1., 1., 1., 1.])
>>>myLR_lgd = MyLR([1., 1., 1., 1.])
>>>myLR_lgd.fit_(X,Y, alpha = 5e-5, n_cycle = 10000)
>>>myLR_ne.normalequation_(X,Y)
>>>myLR_lgd.mse_(X,Y)
2265.1048...
>>>myLR_ne.mse_(X,Y)
413.05299...
```

Be sure to understand the underlying concepts and be able to answer those questions during your evaluation:

• Between the LGD and the NE fits which one is the best model ? (quantitative arguments are welcome)
• What are the advantages and drawbacks of the linear gradient descent and normal equation ?
• In which case, the method normal equation cannot be used ?

Bonus Exercise - Learning Rate and Quadratic Hypothesis

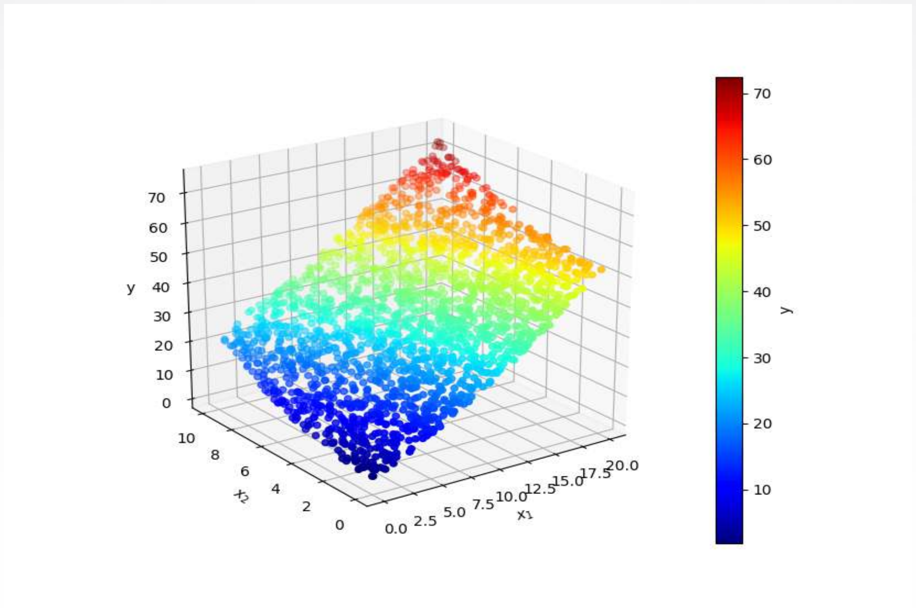
Turn in directory :	ex07
Files to turn in :	so_much_hyp.py
	alpha.py
Authorized modules :	numpy, matplotlib
Forbidden modules :	sklearn
Forbidden functions :	LinearRegression
Links :	Did I already give you a link to coursera Machine Learning MOOC ?
Remarks :	You know what to look at, I think...

Objectives :

- Learn other metrics such as RMSE and R2score, and understand there limits.
- Discover the non-linear dependency to elaborate more complex hypothesis and reinforce mathematical skills at the same time.
- Gain a deeper understanding of the learning rate.

Instructions :
__ With great power comes great responsability __ (Uncle Ben).
Uncle Ben was right, with a large power thrust comes a risk of spacecraft crash.
With the apparition of individual spacecraft, the number of wild rides increases a lot, especially in the asteroid Saturn Belt.
Experts are concerned with the impact of these rides and worried about the consequence on the number of asteroids orbiting and the number of asteroids leaving the orbit of Saturn !

For this last exercise you have to find a model to describe the number of asteroids leaving the orbit of Saturn due to wild rides.
The dataset related to this important problem is named **"saturn_asteroids.csv"**. The data description is in the file **"saturn_asteroids_description.txt"**.
Also, you have a really nice graphical representations just below.



You will use the methods you code during the day to fit the data.
The hypothesis function you will need is a more complex one. This time you have to find out the best model to fit the data.
Do not worry, it is not too complicated.
You have to justify your hypothesis (know what and why you are doing).

Part Zero - Nothing Else (but) Metrics

This part will focus on the metrics. You already know about one of them : MSE. There are several more which are quite usual : RMSE, MAE and R2score.
In this part you will code 2 more methods you will add to **MyLinearRegression** class : RMSE and R2score, as per the instructions below:

```
def rmse_(self, X, Y):
    """
    Description:
        Calculate the RMSE between the predicted output and the real output.
    Args:
        X: has to be a numpy.ndarray, a matrix of dimension (number of
training examples, number of features).
        Y: has to be a numpy.ndarray, a vector of dimension (number of
training examples, 1).
    Returns:
        rmse: has to be a float.
        None if there is a matching dimension problem.
    Raises:
        This function should not raise any Exception.
    """
        ... your code here ...
def r2score_(self, X, Y):
    """
    Description:
        Calculate the R2score between the predicted output and the output.
    Args:
        X: has to be a numpy.ndarray, a matrix of dimension (number of
training examples, number of features).
        Y: has to be a numpy.ndarray, a vector of dimension (number of
training examples, 1).
    Returns:
        r2score: has to be a float.
        None if there is a matching dimension problem.
    Raises:
        This function should not raise any Exception.
    """
        ... your code here ...
```

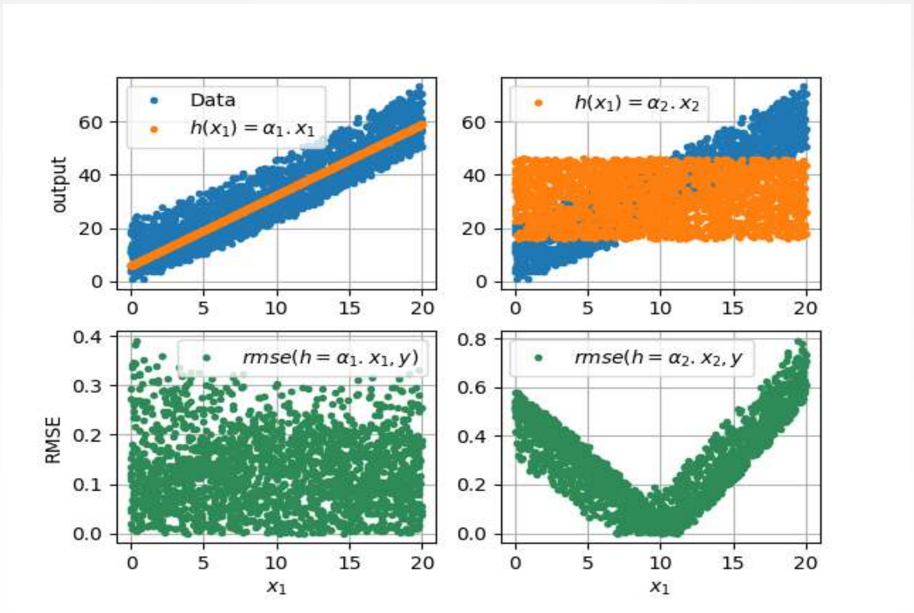
Remarks:
You might consider to code 2 more methods **rmse_elem_()** and **r2score_()** similar to what you did for the cost function in the exercise 01.

Part One - Find the best hypothesis

For this part, you will write your code in the file **"so_much_hyp.py"**.
You can code new methods to help you to justify the choice of the hypothesis but it is not compulsory.
The major work for this exercise concerns the data.

You are expected to:

- Find a good hypothesis and justify that your model is a good one. So you have to imagine different hypothesis and try them (see figure below).



Hints :

Take a look to the metrics (the RMSE for example) with respect to the features.

Notice that the graphs $\sqrt{1/M(\widehat{y^{(i)}} - y^{(i)})^2}$ is not the rmse that we plot precisely, but

the term in the sum (**rmse_elem_** ...).

Here, what you can get:

```
>>>import pandas as pd
>>>import numpy as np
from mylinearregression import MyLinearRegression as MyLR
>>>data = pd.read_csv("saturn_asteroids.csv")
>>>X = np.array(data[['Mean_speed','N_spacecraft']])
>>>Y = np.array(data[['ALSB']])
>>>hypo1 = MyLR([1., 1.])
>>>hypo2 = MyLR([1., 1.])
>>>hypo1.fit_(X[:,0], Y, alpha = 1e-4, n_cycle = 1e5)
>>>hypo2.fit_(X[:,1], Y, alpha = 1e-4, n_cycle = 1e5)
>>>hypo1.rmse_(X[:,0],Y)
6.439...
>>>hypo2.rmse_(X[:,0],Y)
15.089...
```

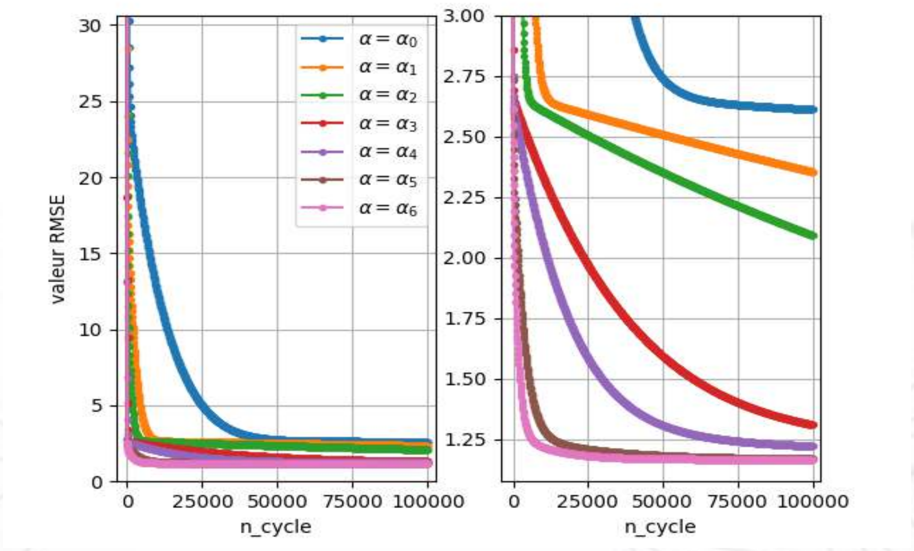
Best fit to bet: RMSE = 0.05977051791842336, good luck. Of course, if you are using normal equation, You cannot consider you compete fairly (boo !!).

Part Two - The learning rate α

For this part you will write your code in the file **alpha.py**.
We will focus on the concept of learning rate, in consequence, you will have to fit with the method **MyLinearRegression.fit** you coded previously.

You are expected to:

- Find a good learning rate α to fit the data with the hypothesis you have chosen in the previous part, you choice will be motivated by the convergence reached in the minimum cycle (see figure below).



Be sure to understand the underlying concept and be able to answer those questions evaluation:

- What happens if you choose a learning rate too big ?
- How the metric and the θ's evolve with a slighly too big learning rate ?
- How the convergence evolves in function of the cycle ? Do you have any idea to speed up the convergence as the numbre of cycle increase ? (It is an open question, be creative and smart).