



42 ARTIFICIAL INTELLIGENCE

# Machine Learning Bootcamp - Module 01

## Univariate Linear Regression

*Summary: Today you will implement a crucial method to improve your model's performance: **gradient descent**. Then you will discover the notion of **normalization**.*

# Notions covered and learning resources

## What notions will be covered by this module?

- Linear regression
- Gradient descent
- Learning rate
- Normalization

## Learning resources

You are recommended to use the following material: [Machine Learning MOOC - Stanford](#)

This series of videos is available at no cost: simply log in, select "Enroll for Free", and click "Audit" at the bottom of the pop-up window.

The following sections of the course are particularly relevant to today's exercises:

### Week 1: Introduction to Machine Learning

#### Train the model with Gradient Descent

- Gradient descent
- Implementing gradient descent
- Gradient descent intuition
- Learning rate
- Gradient descent for linear regression
- Running gradient descent

### Week 2: Regression with multiple input variables

#### Multiple linear Regression

- Multiple features

- Vectorization part1 (optional)
- Vectorization part2 (optional)

**Gradient descent in practice**

- Feature scaling part 1
- Feature scaling part 2

*All videos mentionned above are also available on this [Andrew Ng's YouTube playlist](#) from 15 to 21 included, 25 and 26*

# Chapter I

## Common Instructions

- The version of Python recommended to use is 3.7, you can check the version of Python with the following command: `python -V`
- The norm: during this bootcamp, it is recommended to follow the [PEP 8 standards](#), though it is not mandatory. You can install [pycodestyle](#) which is a tool to check your Python code.
- The function `eval` is never allowed.
- The exercises are ordered from the easiest to the hardest.
- Your exercises are going to be evaluated by someone else, so make sure that your variable names and function names are appropriate and civil.
- Your manual is the internet.
- If you are a student from 42, you can access our Discord server on [42 student's associations portal](#) and ask your questions to your peers in the dedicated Bootcamp channel.
- You can learn more about 42 Artificial Intelligence by visiting [our website](#).
- If you find any issue or mistake in the subject please create an issue on [42AI repository on Github](#).
- We encourage you to create test programs for your project even though this work **won't have to be submitted and won't be graded**. It will give you a chance to easily test your work and your peers' work. You will find those tests especially useful during your defence. Indeed, during defence, you are free to use your tests and/or the tests of the peer you are evaluating.
- We are constantly looking to improve these bootcamps, and your feedbacks are essential for us to do so !  
You can tell us more about your experience with this module by filling [this form](#).  
Thank you in advance and good luck for this bootcamp !

# Contents

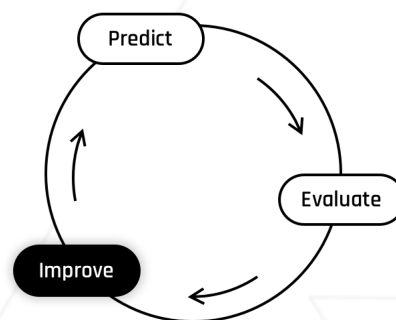
|             |  |           |
|-------------|--|-----------|
| <b>I</b>    | <b>Common Instructions</b>               | <b>3</b>  |
| <b>II</b>   | <b>Exercise 00</b>                       | <b>5</b>  |
| <b>III</b>  | <b>Exercise 01</b>                       | <b>10</b> |
| <b>IV</b>   | <b>Exercise 02</b>                       | <b>14</b> |
| <b>V</b>    | <b>Exercise 03</b>                       | <b>17</b> |
| <b>VI</b>   | <b>Exercise 04</b>                       | <b>20</b> |
| <b>VII</b>  | <b>Exercise 05</b>                       | <b>23</b> |
| <b>VIII</b> | <b>Exercise 06</b>                       | <b>26</b> |
| <b>IX</b>   | <b>Conclusion - What you have learnt</b> | <b>28</b> |

# Chapter II

## Exercise 00

### Improve

In the previous module, you have discovered the first two steps of the learning process: starting with a model that makes naive predictions and evaluating it. Now we are going to tackle the third part: **improving it!**



Lets take a new dataset:

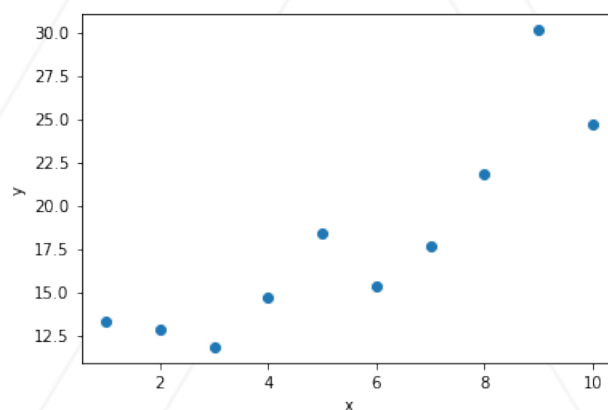


Figure II.1: Scatter plot of a given dataset

## Predict

Given our measure of performance, improvement entails **reducing the loss (or cost)** measured by the loss function. If we plot the loss of a model's predictions as a function of its  $\theta_1$  parameter (with a fixed value for  $\theta_0$ ), we obtain a curve like this one: On the graphs

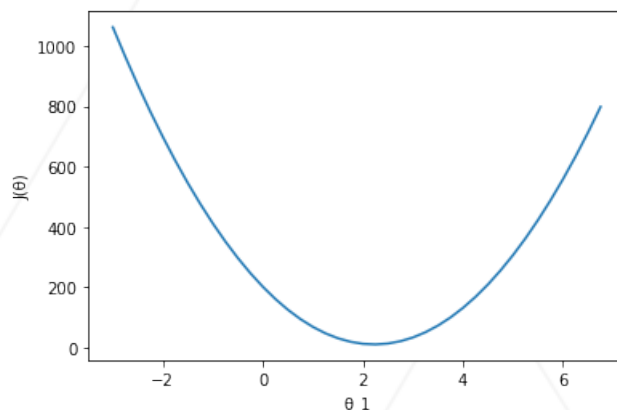


Figure II.2: Loss function given  $\theta_1$

below, you can see that extreme  $\theta_1$  values (which modify the slope of the hypothesis curve - in orange) correspond to a very high loss. On the other hand, as we get closer to the bottom of the curve, the loss gets lower.

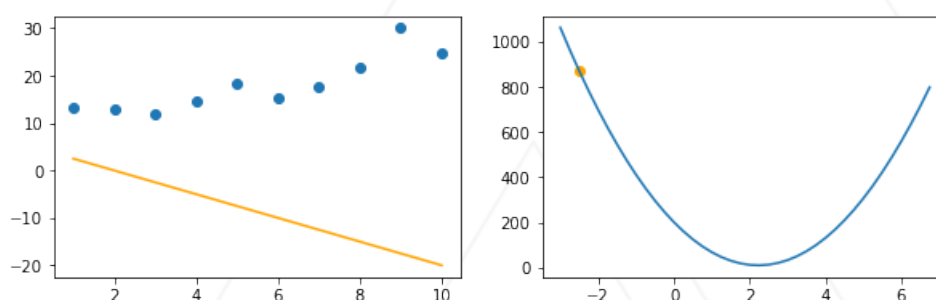


Figure II.3: A pretty bad model

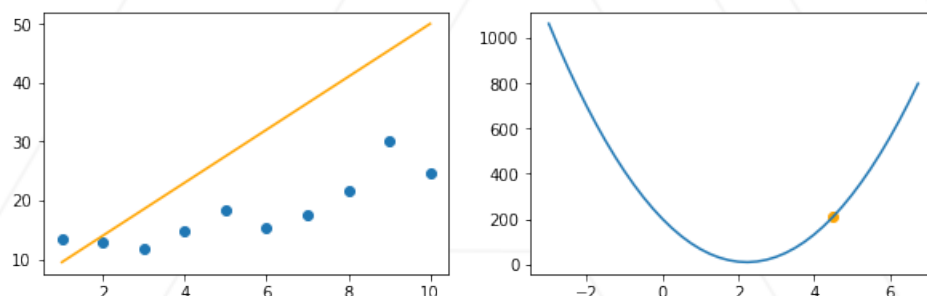


Figure II.4: A better (but still bad) model

The loss function's minimum corresponds to the bottom of the curve. We want  $\theta_1$  to get to this sweet spot. It means that wherever  $\theta_1$  starts at, as the training goes on, it needs to get closer to the value that matches  $J(\theta)$ 's minimum.

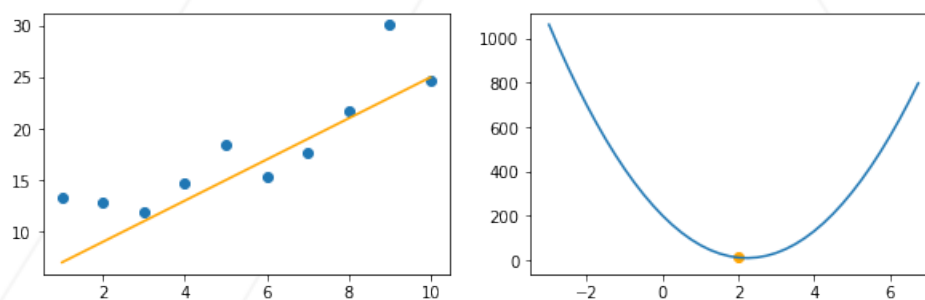


Figure II.5: A good model

## But how to get closer to the minimum?


Excellent question dear reader, we're glad you asked!

First, the algorithm needs to figure out in which direction  $\theta_1$  should be moved (i.e. increased or decreased). It does so by calculating the **slope** of the  $J(\theta)$  curve at the current position of  $\theta_1$ . If the slope is positive,  $\theta_1$  must be decreased. If the slope is negative, it must be increased. If you have studied calculus, you probably sense that all of this involves calculating the derivative of the loss function.

The story gets a little more complicated, however, because we have two parameters to adjust:  $\theta_0$  and  $\theta_1$ . Not just  $\theta_1$  (as we showed in our example to simplify). This means the  $J(\theta)$  function doesn't have only one derivative, but two **partial derivatives**. One that computes the slope of  $J$  with respect to  $\theta_0$ , and a second one for the slope of  $J$  with respect to  $\theta_1$ . Finally, we package those partial derivatives in a vector of dimension  $m$ , which is called **gradient** (noted  $\nabla$ ).

Don't worry if you don't master multivariate calculus yet, we have calculated the partial derivatives for you, all you will need to do is to write them in Python.



|   |               |
|---|---------------|
|  | Exercise : 00 |
| Linear Gradient - Iterative Version   |               |
| Turn-in directory : <code>ex00/</code>  |               |
| Files to turn in : <code>gradient.py</code>                                       |               |
| Forbidden functions : <code>None</code>   |               |

## Objective

Understand and manipulate the notion of gradient and gradient descent in machine learning.

You must write a function that computes the **gradient** of the loss function. It must compute a partial derivative with respect to each theta parameter separately, and return the vector gradient.

The partial derivatives can be calculated with the following formulas:

$$\nabla(J)_0 = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})$$

$$\nabla(J)_1 = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})x^{(i)}$$

Where:

- $\nabla(J)$  is the gradient vector of size  $2 \times 1$ , (this strange symbol :  $\nabla$  is called nabla)
- $x$  is a vector of dimension  $m$
- $y$  is a vector of dimension  $m$
- $x^{(i)}$  is the  $i^{\text{th}}$  component of vector  $x$
- $y^{(i)}$  is the  $i^{\text{th}}$  component of vector  $y$
- $\nabla(J)_j$  is the  $j^{\text{th}}$  component of  $\nabla(J)$
- $h_{\theta}(x^{(i)})$  corresponds to the model's prediction of  $y^{(i)}$

## Hypothesis Notation

$h_{\theta}(x^{(i)})$  is the same as what we previously noted  $\hat{y}^{(i)}$ . The two notations are equivalent. They represent the model's prediction (or estimation) of the  $y^{(i)}$  value. If you follow Andrew Ng's course material on Coursera, you will see him using the former notation. As a reminder:  $h_{\theta}(x^{(i)}) = \theta_0 + \theta_1 x^{(i)}$

## Instructions

In the `gradient.py` file create the following function as per the instructions given below:

```
def simple_gradient(x, y, theta):  
    """Computes a gradient vector from three non-empty numpy.array, with a for-loop.  
    The three arrays must have compatible shapes.  
    Args:  
        x: has to be a numpy.array, a vector of shape m * 1.  
        y: has to be a numpy.array, a vector of shape m * 1.  
        theta: has to be a numpy.array, a 2 * 1 vector.  
    Return:  
        The gradient as a numpy.array, a vector of shape 2 * 1.  
        None if x, y, or theta are empty numpy.array.  
        None if x, y and theta do not have compatible shapes.  
        None if x, y or theta is not of the expected type.  
    Raises:  
        This function should not raise any Exception.  
    """  
    ... Your code ...
```

## Examples

```
import numpy as np  
x = np.array([12.4956442, 21.5007972, 31.5527382, 48.9145838, 57.5088733]).reshape((-1, 1))  
y = np.array([37.4013816, 36.1473236, 45.7655287, 46.6793434, 59.5585554]).reshape((-1, 1))  
  
# Example 0:  
theta1 = np.array([2, 0.7]).reshape((-1, 1))  
simple_gradient(x, y, theta1)  
# Output:  
array([[ -19.0342574], [ -586.66875564]])  
  
# Example 1:  
theta2 = np.array([1, -0.4]).reshape((-1, 1))  
simple_gradient(x, y, theta2)  
# Output:  
array([[ -57.86823748], [ -2230.12297889]])
```

# Chapter III

## Exercise 01

### Linear Algebra Tricks part II

If you tried to run your code on a very large dataset, you would find that it sometimes takes a (very) long time to execute! That's because it doesn't use the power of Python libraries that are optimized for matrix operations.

Remember the linear algebra trick from the previous module? Let's use it again! If you concatenate a column of 1's to the left of the  $x$  vector, you get what we called matrix  $X'$ .

$$X' = \begin{bmatrix} 1 & x^{(1)} \\ \vdots & \vdots \\ 1 & x^{(m)} \end{bmatrix}$$

This transformation is very convenient because we can rewrite each 1 as  $x_0^{(i)}$ , and each  $x^{(i)}$  as  $x_1^{(i)}$ . So now the  $X'$  matrix looks like this:

$$X' = \begin{bmatrix} x_0^{(1)} & x_1^{(1)} \\ \vdots & \vdots \\ x_0^{(m)} & x_1^{(m)} \end{bmatrix}$$

Notice that each  $x^{(i)}$  example becomes a vector made of  $(x_0^{(i)}, x_1^{(i)})$ . The 0 and 1 indices on the  $x$  features correspond to the indices of the  $\theta$  parameters with which they will be multiplied.

Why does this matter? Well, if we take the equation from the previous exercise:

$$\nabla(J)_0 = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})$$

We can multiply it by 1 without changing its value:

$$\nabla(J)_0 = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot 1$$

And rewrite 1 as  $x_0^{(i)}$ :

$$\nabla(J)_0 = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

This means that the equation for  $\nabla(J)_0$  is now similar to the equation we had for  $\nabla(J)_1$ , so they can both be captured by ONE **generic equation**:

$$\nabla(J)_j = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)} \quad \text{for } j = 0, 1$$

And as you probably suspected, a generic equation opens the door to vectorization...

## Vectorizing the Gradient Calculation

Now it's time to learn how to calculate the entire gradient in one short, pretty, linear algebra equation!

- First, we'll use the  $X'$  matrix and our vectorized hypothesis equation  $h_{\theta}(x) = X'\theta$

$$\nabla(J)_j = \frac{1}{m} (X'\theta - y) X'_j \quad \text{for } j = 0, 1$$

- Second, we need to tweak the equation a bit so that it directly returns a  $\nabla(J)$  vector containing both  $\nabla(J)_0$  and  $\nabla(J)_1$ .


$$\nabla(J) = \frac{1}{m} X'^T (X'\theta - y)$$

If the equation does not seem obvious, play a bit with your vectors, on paper and in your code, until you get it.

### Notation Remark

$X'^T$ : You might wonder what the  $T$  is for. It means the  $X'$  matrix must be **transposed**.

Transposing a matrix flips it on its diagonal so that its rows become its columns and *vice-versa*. Here we need to make sure that matrix dimensions are appropriate and allow for multiplication, and to multiply the right items together.

|   |               |
|---|---------------|
|  | Exercise : 01 |
| Linear Gradient - Vectorized Version  |               |
| Turn-in directory : <i>ex01/</i>  |               |
| Files to turn in : <b>vec_gradient.py</b>   |               |
| Forbidden functions : <b>None</b>   |               |

## Objective

Understand and experiment with the notions of **gradient** and **gradient descent** in machine learning.

You must implement the following formula as a function:

$$\nabla(J) = \frac{1}{m} X'^T (X'\theta - y)$$

Where:

- $\nabla(J)$  is a vector of dimension  $2 \times 1$
- $X'$  is a **matrix** of dimensions  $(m \times 2)$
- $X'^T$  is the transpose of  $X'$ . Its dimensions are  $(2 \times m)$
- $y$  is a vector of dimension  $m$
- $\theta$  is a vector of dimension  $2 \times 1$

Be careful:

- the  $x$  you will get as an input is an  $m$  vector,
- $\theta$  is a  $2 \times 1$  vector. You have to transform  $x$  to fit the dimension of  $\theta$ !

## Instructions

In the `vec_gradient.py` file create the following function as per the instructions given below:

```
def simple_gradient(x, y, theta):  
    """Computes a gradient vector from three non-empty numpy.array, without any for loop.  
    The three arrays must have compatible shapes.  
    Args:  
        x: has to be a numpy.array, a vector of shape m * 1.  
        y: has to be a numpy.array, a vector of shape m * 1.  
        theta: has to be a numpy.array, a 2 * 1 vector.  
    Return:  
        The gradient as a numpy.ndarray, a vector of dimension 2 * 1.  
        None if x, y, or theta is an empty numpy.ndarray.  
        None if x, y and theta do not have compatible dimensions.  
    Raises:  
        This function should not raise any Exception.  
    """  
    ... Your code ...
```

## Examples

```
import numpy as np  
x = np.array([12.4956442, 21.5007972, 31.5527382, 48.9145838, 57.5088733]).reshape((-1, 1))  
y = np.array([37.4013816, 36.1473236, 45.7655287, 46.6793434, 59.5585554]).reshape((-1, 1))  
  
# Example 0:  
theta1 = np.array([2, 0.7]).reshape((-1, 1))  
gradient(x, y, theta1)  
# Output:  
array([[ -19.0342...], [ -586.6687...]])  
  
# Example 1:  
theta2 = np.array([1, -0.4]).reshape((-1, 1))  
gradient(x, y, theta2)  
# Output:  
array([[ -57.8682...], [ -2230.1229...]])
```

# Chapter IV

## Exercise 02

### Interlude - Gradient Descent

So far we've calculated the *gradient*, which indicates whether and by how much we should increase or decrease  $\theta_0$  and  $\theta_1$  in order to reduce the loss.

What we have to do next is to update the theta parameters accordingly, step by step, until we reach the minimum. This iterative process, called **Gradient Descent**, will progressively improve the performance of your regression model on the training data.


The gradient descent **algorithm** can be summed up to this: for a certain number of cycles, at each step, both  $\theta$  parameters are slightly moved in the opposite directions than what the gradient indicates.

The algorithm can be expressed in pseudocode as the following:

```
repeat until convergence: {  
    compute  $\nabla(J)$   
     $\theta_0 := \theta_0 - \alpha \nabla(J)_0$   
     $\theta_1 := \theta_1 - \alpha \nabla(J)_1$   
}
```

A few remarks on this algorithm:

- If you directly subtracted the gradient from  $\theta$ , your steps would be too big and you would quickly overshoot past the minimum. That's why we use  $\alpha$  (alpha), called the *learning rate*. It's a small float number (usually between 0 and 1) that decreases the magnitude of each update.
- The pseudocode says "repeat until convergence", but in your implementation, you will not actually check for convergence at each iteration. You will instead set a number of cycles that is sufficient for your gradient descent to converge.
- When training a linear regression model on a new dataset, you will have to choose appropriate alpha and the number of cycles through trial and error.

|   |               |
|---|---------------|
|  | Exercise : 02 |
| Gradient Descent  |               |
| Turn-in directory : <code>ex02/</code>  |               |
| Files to turn in : <code>fit.py</code>  |               |
| Forbidden functions : <b>any</b> function that calculates derivatives for you     |               |

## Objective

Understand and experiment with the notions of **gradient** and **gradient descent** in machine learning.

Be able to explain what it means to **fit** a Machine Learning model to a dataset. Implement a function that performs **Linear Gradient Descent** (LGD).

## Instructions

In this exercise, you will implement linear gradient descent to fit your model to the dataset.

The pseudocode for the algorithm is the following:

```
repeat until convergence: {
    compute  $\nabla(J)$ 
     $\theta_0 := \theta_0 - \alpha \nabla(J)_0$ 
     $\theta_1 := \theta_1 - \alpha \nabla(J)_1$ 
}
```

Where:

- $\alpha$  (alpha) is the *learning rate*. It's a small float number (usually between 0 and 1),
- For now, "repeat until convergence" will mean to simply repeat for `max_iter` (a number that you will choose wisely).

You are expected to write a function named `fit_` as per the instructions below:



```
def fit_(x, y, theta, alpha, max_iter):
    """
    Description:
        Fits the model to the training dataset contained in x and y.

    Args:
        x: has to be a numpy.ndarray, a vector of dimension m * 1: (number of training examples, 1).
        y: has to be a numpy.ndarray, a vector of dimension m * 1: (number of training examples, 1).
        theta: has to be a numpy.ndarray, a vector of dimension 2 * 1.
        alpha: has to be a float, the learning rate
        max_iter: has to be an int, the number of iterations done during the gradient descent

    Returns:
        new_theta: numpy.ndarray, a vector of dimension 2 * 1.
        None if there is a matching dimension problem.

    Raises:
        This function should not raise any Exception.

    """
    ... your code here ...
```

Hopefully, you have already written a function to calculate the linear gradient.

## Examples

```
import numpy as np
x = np.array([[12.4956442], [21.5007972], [31.5527382], [48.9145838], [57.5088733]])
y = np.array([[37.4013816], [36.1473236], [45.7655287], [46.6793434], [59.5585554]])
theta = np.array([1, 1]).reshape((-1, 1))

# Example 0:
theta1 = fit_(x, y, theta, alpha=5e-8, max_iter=1500000)
theta1
# Output:
array([[1.40709365],
       [1.1150909 ]])


# Example 1:
predict(x, theta1)
# Output:
array([[15.3408728 ],
       [25.38243697],
       [36.59126492],
       [55.95130097],
       [65.53471499]])
```



- You can create more training data by generating an  $x$  array with random values and computing the corresponding  $y$  vector as a linear expression of  $x$ . You can then fit a model on this artificial data and find out if it comes out with the same  $\theta$  coefficients that you first used.
- It is possible that  $\theta_0$  and  $\theta_1$  become "nan". In that case, it means you probably used a learning rate that is too large.

# Chapter V

## Exercise 03

|   |               |
|---|---------------|
|  | Exercise : 03 |
| Linear Regression with Class  |               |
| Turn-in directory : <i>ex03/</i>  |               |
| Files to turn in : <i>my_linear_regression.py</i>                                 |               |
| Forbidden functions : any functions from <i>sklearn</i>                           |               |

### Objective

Write a class that contains all the necessary methods to perform a linear regression.

### Instructions

In this exercise, you will not learn anything new but don't worry, it's for your own good.

You are expected to write your own `MyLinearRegression` class which looks similar to the class available in Scikit-learn: `sklearn.linear_model.LinearRegression`

```
class MyLinearRegression():
    """
    Description:
        My personal linear regression class to fit like a boss.
    """
    def __init__(self, thetas, alpha=0.001, max_iter=1000):
        self.alpha = alpha
        self.max_iter = max_iter
        self.thetas = thetas

    #... other methods ...
```

You will add the following methods:

- `fit_(self, x, y),`
- `predict_(self, x),`
- `loss_elem_(self, y, y_hat),`
- `loss_(self, y, y_hat).`

You have already implemented these functions, you just need a few adjustments so that they all work well within your `MyLinearRegression` class.

## Examples

```
import numpy as np
from my_linear_regression import MyLinearRegression as MyLR
x = np.array([[12.4956442], [21.5007972], [31.5527382], [48.9145838], [57.5088733]])
y = np.array([[37.4013816], [36.1473236], [45.7655287], [46.6793434], [59.5585554]])

lr1 = MyLR(np.array([[2], [0.7]]))

# Example 0.0:
y_hat = lr1.predict_(x)
# Output:
array([[10.74695094],
       [17.05055804],
       [24.08691674],
       [36.24020866],
       [42.25621131]])

# Example 0.1:
lr1.loss_elem_(y, y_hat)
# Output:
array([[710.45867381],
       [364.68645485],
       [469.96221651],
       [108.97553412],
       [299.37111101]])

# Example 0.2:
lr1.loss_(y, y_hat)
# Output:
195.34539903032385

# Example 1.0:
lr2 = MyLR(np.array([[1], [1]]), 5e-8, 1500000)
lr2.fit_(x, y)
lr2.thetas
# Output:
array([[1.40709365],
       [1.1150909 ]])


# Example 1.1:
y_hat = lr2.predict_(x)
# Output:
array([[15.3408728 ],
       [25.38243697],
       [36.59126492],
       [55.95130097],
       [65.53471499]])

# Example 1.2:
lr2.loss_elem_(y, y_hat)
# Output:
array([[486.66604863],
       [115.88278416],
       [ 84.16711596],
       [ 85.96919719],
       [ 35.71448348]])

# Example 1.3:
lr2.loss_(y, y_hat)
# Output:
80.83996294128525
```

# Chapter VI

## Exercise 04

|  |               |
|--|---------------|
|         | Exercise : 04 |
| Practicing Linear Regression   |               |
| Turn-in directory : <code>ex04/</code>   |               |
| Files to turn in : <code>linear_model.py</code> , <code>are_blue_pills_magics.csv</code> |               |
| Forbidden functions : <code>sklearn</code>   |               |

### Objective

Evaluate a linear regression model on a very small dataset, with a given hypothesis function  $h$ . Manipulate the loss function  $J$ , plot it, and briefly analyze the plot.

### Instructions

You can find in the `resources` folder a tiny dataset called `are_blue_pills_magics.csv` which gives you the driving performance of space pilots as a function of the quantity of the "blue pills" they took before the test.

You have a description of the data in the file named `are_blue_pills_magics.txt`. As your hypothesis function  $h$ , you will choose:

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

Where  $x$  is the variable, and  $\theta_0$  and  $\theta_1$  are the coefficients of the hypothesis. The hypothesis is a function of  $x$ .



You are strongly encouraged to use the class you have implemented in the previous exercise

Your program must:

- Read the dataset from the csv file
- Perform a linear regression

Then you will model the data and plot 2 different graphs:

- A graph with the data and the hypothesis you get for the spacecraft piloting score versus the quantity of "blue pills" (see figure VI.1)

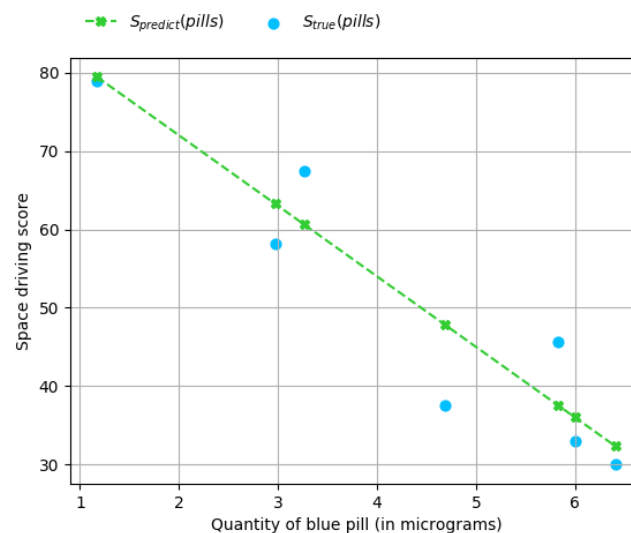


Figure VI.1: Space driving score as a function of the quantity of blue pill (in micrograms). In blue the real values and in green the predicted values.

- The loss function  $J(\theta)$  in function of the  $\theta$  values (see figure VI.2),

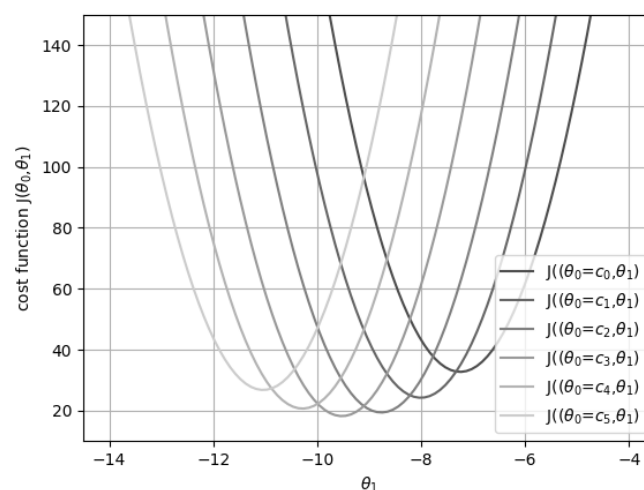


Figure VI.2: Evolution of the loss function  $J$  as a function of  $\theta_1$  for different values of  $\theta_0$ .

- You will calculate the MSE of the hypothesis you chose (you know how to do it already).

## Examples

```
import pandas as pd
import numpy as np
from sklearn.metrics import mean_squared_error
from my_linear_regression import MyLinearRegression as MyLR

data = pd.read_csv("are_blue_pills_magic.csv")
Xpill = np.array(data['Micrograms']).reshape(-1,1)
Yscore = np.array(data['Score']).reshape(-1,1)

linear_model1 = MyLR(np.array([[89.0], [-8]]))
linear_model2 = MyLR(np.array([[89.0], [-6]]))
Y_model1 = linear_model1.predict_(Xpill)
Y_model2 = linear_model2.predict_(Xpill)

print(MyLR.mse_(Yscore, Y_model1))
# 57.60304285714282
print(mean_squared_error(Yscore, Y_model1))
# 57.603042857142825
print(MyLR.mse_(Yscore, Y_model2))
# 232.16344285714285
print(mean_squared_error(Yscore, Y_model2))
# 232.16344285714285
```



Here, the use of scikit learn is to ensure that our code is performing as expected. The use of scikit learn is forbidden in the code you will turn-in.



There is no method named `.mse_` in sklearn's `LinearRegression` class, but there is also a method named `.score`. The `.score` method corresponds to the  $R^2$  score. The metric MSE is available in the `sklearn.metrics` module.

# Chapter VII

## Exercise 05

### Interlude - Normalization

The values inside the  $x$  vector can vary quite a lot in magnitude, depending on the type of data you are working with.

For example, if your dataset contains distances between planets in km, the numbers will be huge. On the other hand, if you are working with planet masses expressed as a fraction of the solar system's total mass, the numbers will be very small (between 0 and 1). Both cases may slow down convergence in Gradient Descent (or even sometimes prevent convergence at all). To avoid that kind of situation, **normalization** is a very effective way to proceed.


The idea behind this technique is quite straightforward: **scaling the data**.

With normalization, you can transform your  $x$  vector into a new  $x'$  vector whose values range between  $[-1, 1]$  more or less. Doing this allows you to see much more easily how a training example compares to the other ones:

- If an  $x'$  value is close to 1, you know it's among the largest in the dataset
- If an  $x'$  value is close to 0, you know it's close to the median
- If an  $x'$  value is close to  $-1$ , you know it's among the smallest

So with the upcoming normalization techniques, you'll be able to map your data to two different value ranges:  $[0, 1]$  or  $[-1, 1]$ . Your algorithm will like it and thank you for it.



|   |               |
|---|---------------|
|  | Exercise : 05 |
| Normalization I: Z-score Standardization  |               |
| Turn-in directory : <i>ex05/</i>  |               |
| Files to turn in : <b>z_score.py</b>  |               |
| Forbidden functions : <b>None</b>   |               |

## Objective

Introduction to standardization/normalization methods.

You must implement the following formula as a function:

$$x'^{(i)} = \frac{x^{(i)} - \frac{1}{m} \sum_{i=1}^m x^{(i)}}{\sqrt{\frac{1}{m-1} \sum_{i=1}^m (x^{(i)} - \frac{1}{m} \sum_{i=1}^m x^{(i)})^2}} \quad \text{for } i \text{ in } 1, \dots, m$$

Where:

- $x$  is a vector of dimension  $m$
- $x^{(i)}$  is the  $i^{\text{th}}$  component of the  $x$  vector
- $x'$  is the normalized version of the  $x$  vector

The equation is much easier to understand in the following form:

$$x'^{(i)} = \frac{x^{(i)} - \mu}{\sigma} \quad \text{for } i \text{ in } 1, \dots, m$$

This should remind you of something from **TinyStatistician**... doesn't it?!

Ok, let's do a quick recap !

- $\mu$  is the mean of  $x$
- $\sigma$  is the standard deviation of  $x$

## Instructions

In the `zscore.py` file, write the `zscore` function as per the instructions given below:

```
def zscore(x):
    """Computes the normalized version of a non-empty numpy.ndarray using the z-score standardization.
    Args:
        x: has to be a numpy.ndarray, a vector.
    Returns:
        x' as a numpy.ndarray.
        None if x is a non-empty numpy.ndarray or not a numpy.ndarray.
    Raises:
        This function shouldn't raise any Exception.
    """
    ... Your code ...
```


## Examples

```
# Example 1:
X = numpy.array([0, 15, -9, 7, 12, 3, -21])
zscore(X)
# Output:
array([-0.08620324,  1.2068453 , -0.86203236,  0.51721942,  0.94823559,
        0.17240647, -1.89647119])

# Example 2:
Y = np.array([2, 14, -13, 5, 12, 4, -19]).reshape((-1, 1))
zscore(Y)
# Output:
array([ 0.11267619,  1.16432067, -1.20187941,  0.37558731,  0.98904659,
        0.28795027, -1.72770165])
```

# Chapter VIII

## Exercise 06

|   |               |
|---|---------------|
|  | Exercise : 06 |
| Normalization II: Min-max Standardization   |               |
| Turn-in directory : <code>ex06/</code>  |               |
| Files to turn in : <code>minmax.py</code>   |               |
| Forbidden functions : <code>None</code>   |               |

### Objective

Introduction to standardization/normalization methods. Implement another normalization method.

You must implement the following formula as a function:

$$x'^{(i)} = \frac{x^{(i)} - \min(x)}{\max(x) - \min(x)} \quad \text{for } i = 1, \dots, m$$

Where:

- $x$  is a vector of dimension  $m$
- $x^{(i)}$  is the  $i^{\text{th}}$  component of vector  $x$
- $\min(x)$  is the minimum value found among the components of vector  $x$
- $\max(x)$  is the maximum value found among the components of vector  $x$

You will notice that this min-max standardization doesn't scale the values to the  $[-1, 1]$  range. What do you think the final range will be?

## Instructions

In the `minmax.py` file, create the `minmax` function as per the instructions given below:

```
def minmax(x):  
    """Computes the normalized version of a non-empty numpy.ndarray using the min-max standardization.  
    Args:  
        x: has to be an numpy.ndarray, a vector.  
    Returns:  
        x' as a numpy.ndarray.  
        None if x is a non-empty numpy.ndarray or not a numpy.ndarray.  
    Raises:  
        This function shouldn't raise any Exception.  
    """  
    ... Your code ...
```

## Examples

```
# Example 1:  
X = np.array([0, 15, -9, 7, 12, 3, -21]).reshape((-1, 1))  
minmax(X)  
# Output:  
array([0.58333333, 1.          , 0.33333333, 0.77777778, 0.91666667,  
       0.66666667, 0.          ])  
  
# Example 2:  
Y = np.array([2, 14, -13, 5, 12, 4, -19]).reshape((-1, 1))  
minmax(Y)  
# Output:  
array([0.63636364, 1.          , 0.18181818, 0.72727273, 0.93939394,  
       0.6969697 , 0.          ])
```

# Chapter IX

## Conclusion - What you have learnt

You are now done with module01, congratulations!

Based on all the notions and problems tackled today, you should be able to discuss and answer the following questions:

1. What is a hypothesis and what is its goal?
2. What is the loss function and what does it represent?
3. What is Linear Gradient Descent and what does it do? (hint: you have to talk about  $J$ , its gradient and the  $\theta$  parameters...)
4. What happens if you choose a learning rate that is too large?
5. What happens if you choose a very small learning rate, but still a sufficient number of cycles?
6. Can you explain MSE and what it measures?

## Contact

You can contact 42AI by email: [contact@42ai.fr](mailto:contact@42ai.fr)

Thank you for attending 42AI's Machine Learning Bootcamp module01 !

## Acknowledgements

The Python & ML bootcamps are the result of a collective effort. We would like to thank:

- Maxime Choulika (cmaxime),
- Pierre Peigné (ppeigne),
- Matthieu David (mdavid),
- Quentin Feuillade-Montixi (qfeuilla, [quentin@42ai.fr](mailto:quentin@42ai.fr))
- Mathieu Perez (maperez, [mathieu.perez@42ai.fr](mailto:mathieu.perez@42ai.fr))

who supervised the creation and enhancements of the present transcription.

- Louis Develle (ldevelle, [louis@42ai.fr](mailto:louis@42ai.fr))
- Owen Roberts (oroberts)
- Augustin Lopez (aulopez)
- Luc Lenotre (llenotre)
- Amric Trudel (amric@42ai.fr)
- Benjamin Carlier (bcarlier@student.42.fr)
- Pablo Clement (pclement@student.42.fr)
- Amir Mahla (amahla, [amahla@42ai.fr](mailto:amahla@42ai.fr))

for your investment in the creation and development of these modules.

- All prior participants who took a moment to provide their feedbacks, and help us improve these bootcamps !

This work is licensed under a [Creative Commons](https://creativecommons.org/licenses/by-nc-sa/4.0/) “Attribution-NonCommercial-ShareAlike 4.0 International” license.

