Machine Learning Bootcamp - Module 00

Stepping into Machine Learning

Summary: You will start by reviewing some linear algebra and statistics. Then, you will implement your first model and learn how to evaluate its performances.

Notions covered and learning resources

What notions will be covered by this module?

- Sum
- Mean
- Variance
- Standard deviation
- Operations on vectors and matrices
- Hypothesis
- Regression
- Loss function

Learning resources

You are recommended to use the following material: Machine Learning MOOC - Stanford

This series of videos is available at no cost: simply log in, select "Enroll for Free", and click "Audit" at the bottom of the pop-up window.

The following sections of the course are particularly relevant to today's exercises:

Week 1: Introduction to Machine Learning

Supervised vs. Unsupervised Machine Learning

- What is Machine Learning?
- Supervised Learning Part 1
- Supervised Learning Part 2

- Unsupervised Learning Part 1
- Unsupervised Learning Part 2

Regression Model

- Regression Model Part 1
- Regression Model Part 2
- Cost Function Formula
- Cost Function Intuition
- Visualizing the cost function
- Visualizing Example
- *Keep the rest for tommorow ;-)*

All videos above are available also on this Andrew Ng's YouTube playlist from 3 to 14 includes

Linear Algebra reminders

- Matrices and Vectors
- Addition and Scalar Multiplication
- Matrix - Vector Multiplication
- Matrix - Matrix Multiplication
- Matrix - Multiplication Properties
- Inverse and Transpose

Chapter I

Common Instructions

- The version of Python recommended to use is 3.7. You can check your Python's version with the following command: `python -V`
- The norm: during this bootcamp, it is recommended to follow the PEP 8 standards, though it is not mandatory. You can install pycodestyle or Black, which are convenient packages to check your code.
- The function `eval` is never allowed.
- The exercises are ordered from the easiest to the hardest.
- Your exercises are going to be evaluated by someone else, so make sure that your variable names and function names are appropriate and civil.
- Your manual is the internet.
- If you're planning on using an AI assistant such as a LLM, make sure it is helpful for you to **learn and practice**, not to provide you with hands-on solution ! Own your tool, don't let it own you.
- If you are a student from 42, you can access our Discord server on 42 student's associations portal and ask your questions to your peers in the dedicated Bootcamp channel.
- You can learn more about 42 Artificial Intelligence by visiting our website.
- If you find any issue or mistake in the subject please create an issue on 42AI repository on Github.
- We encourage you to create test programs for your project even though this work **won't have to be submitted and won't be graded**. It will give you a chance to easily test your work and your peers' work. You will find those tests especially useful during your defence. Indeed, during defence, you are free to use your tests and/or the tests of the peer you are evaluating.

Contents

Chapter II

Exercise 00

	Exercise : 00
The Matrix	
Turn-in directory : *ex*00/	
Files to turn in : `matrix.py, test.py`	
Forbidden functions : `Numpy`	

Objective

Basic understanding and manipulation of elementary matrix operations.

In this exercise, you have to create a `Matrix` and a `Vector` class.

The goal is to have matrices and to be able to perform both matrix-matrix operations and matrix-vector operations with them.

Instructions

You will provide a test file to prove that your classes work as expected.

Matrix class

Your `Matrix` class must have the 2 following attributes:

- `data`: list of lists
- `shape`: the dimensions of the matrix as a tuple (rows, columns)

You should be able to initialize the object with either:

- the elements of the matrix as a list of lists: `Matrix([[1.0, 2.0], [3.0, 4.0]])`
- a shape: `Matrix((3, 3))` (the matrix will be filled with zeros by default)

You will implement all of the following built-in functions (called `magic/special methods`) for your `Matrix` class:

```
# add : only matrices of same dimensions.
__add__
__radd__
# sub : only matrices of same dimensions.
__sub__
__rsub__
# div : only scalars.
__truediv__
__rtruediv__
# mul : scalars, vectors and matrices , can have errors with vectors and matrices,
# returns a Vector if we perform Matrix * Vector mutliplication.
__mul__
__rmul__
__str__
__repr__
```

You will also implement:

- a `.T()` method which returns the transpose of the matrix (see examples below)

Vector class

Then, you must create a `Vector` class that inherits from the Matrix class.

At initialization, you must check that a column or a row vector is passed as the data argument. If not, you must send an error message :

```
v1 = Vector([[1, 2, 3]]) # create a row vector
v2 = Vector([[1], [2], [3]]) # create a column vector
v3 = Vector([[1, 2], [3, 4]]) # return an error
```

For `Vector`, you must implement:

- a `.dot(self, v: Vector)` method which returns the dot product between the current vector and v. If shapes don't match, you must properly handle errors.

`Caution: when you do operations between Vector, it must return a Vector and not a Matrix`

`type(self)`

Examples

```
m1 = Matrix([[0.0, 1.0], [2.0, 3.0], [4.0, 5.0]])
m1.shape
# Output:
(3, 2)

m1.T()
# Output:
Matrix([[0., 2., 4.], [1., 3., 5.]])

m1.T().shape
# Output:
(2, 3)
```

```
m1 = Matrix([[0., 2., 4.], [1., 3., 5.]])
m1.shape
# Output:
(2, 3)

m1.T()
# Output:
Matrix([[0.0, 1.0], [2.0, 3.0], [4.0, 5.0]])

m1.T().shape
# Output:
(3, 2)
```

```
m1 = Matrix([[0.0, 1.0, 2.0, 3.0],
             [0.0, 2.0, 4.0, 6.0]])

m2 = Matrix([[0.0, 1.0],
             [2.0, 3.0],
             [4.0, 5.0],
             [6.0, 7.0]])

m1 * m2
# Output:
Matrix([[28., 34.], [56., 68.]])
```

```
m1 = Matrix([[0.0, 1.0, 2.0],
             [0.0, 2.0, 4.0]])
v1 = Vector([[1], [2], [3]])

m1 * v1
# Output:
Matrix([[8], [16]])
# Or: Vector([[8], [16]
```

```
v1 = Vector([[1], [2], [3]])
v2 = Vector([[2], [4], [8]])

v1 + v2
# Output:
Vector([[3],[6],[11]])
```

Mathematical notions

Matrix - vector operations

- Multiplication between a $(m \times n)$ matrix and a vector of dimension n

$$Xy = \begin{bmatrix} x_1^{(1)} & \dots & x_n^{(1)} \\ \vdots & \ddots & \vdots \\ x_1^{(m)} & \dots & x_n^{(m)} \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x^{(1)} \cdot y \\ \vdots \\ x^{(m)} \cdot y \end{bmatrix}$$

In other words:

$$Xy = \begin{bmatrix} \sum_{i=1}^{n} x_i^{(1)} \cdot y_i \\ \vdots \\ \sum_{i=1}^{n} x_i^{(m)} \cdot y_i \end{bmatrix}$$

Matrix - matrix operations

- Addition between two matrices of same dimension $(m \times n)$,

$$X + Y = \begin{bmatrix} x_1^{(1)} & \dots & x_n^{(1)} \\ \vdots & \ddots & \vdots \\ x_1^{(m)} & \dots & x_n^{(m)} \end{bmatrix} + \begin{bmatrix} y_1^{(1)} & \dots & y_n^{(1)} \\ \vdots & \ddots & \vdots \\ y_1^{(m)} & \dots & y_n^{(m)} \end{bmatrix} = \begin{bmatrix} x_1^{(1)} + y_1^{(1)} & \dots & x_n^{(1)} + y_n^{(1)} \\ \vdots & \ddots & \vdots \\ x_1^{(m)} + y_1^{(m)} & \dots & x_n^{(m)} + y_n^{(m)} \end{bmatrix}$$

- Substraction between two matrices of same dimension $(m \times n)$,

$$X - Y = \begin{bmatrix} x_1^{(1)} & \dots & x_n^{(1)} \\ \vdots & \ddots & \vdots \\ x_1^{(m)} & \dots & x_n^{(m)} \end{bmatrix} - \begin{bmatrix} y_1^{(1)} & \dots & y_n^{(1)} \\ \vdots & \ddots & \vdots \\ y_1^{(m)} & \dots & y_n^{(m)} \end{bmatrix} = \begin{bmatrix} x_1^{(1)} - y_1^{(1)} & \dots & x_n^{(1)} - y_n^{(1)} \\ \vdots & \ddots & \vdots \\ x_1^{(m)} - y_1^{(m)} & \dots & x_n^{(m)} - y_n^{(m)} \end{bmatrix}$$

- Multiplication or division between one matrix $(m \times n)$ and one scalar,

$$\alpha X = \alpha \begin{bmatrix} x_1^{(1)} & \dots & x_n^{(1)} \\ \vdots & \ddots & \vdots \\ x_1^{(m)} & \dots & x_n^{(m)} \end{bmatrix} = \begin{bmatrix} \alpha x_1^{(1)} & \dots & \alpha x_n^{(1)} \\ \vdots & \ddots & \vdots \\ \alpha x_1^{(m)} & \dots & \alpha x_n^{(m)} \end{bmatrix}$$

- Mutiplication between two matrices of compatible dimension: $(m \times n)$ and $(n \times p)$,

$$XY = \begin{bmatrix} x_1^{(1)} & \dots & x_n^{(1)} \\ \vdots & \ddots & \vdots \\ x_1^{(m)} & \dots & x_n^{(m)} \end{bmatrix} \begin{bmatrix} y_1^{(1)} & \dots & y_p^{(1)} \\ \vdots & \ddots & \vdots \\ y_1^{(n)} & \dots & y_p^{(n)} \end{bmatrix} = \begin{bmatrix} x^{(1)} \cdot y_1 & \dots & x^{(1)} \cdot y_p \\ \vdots & \ddots & \vdots \\ x^{(m)} \cdot y_1 & \dots & x^{(m)} \cdot y_p \end{bmatrix}$$

In other words:

$$XY = \begin{bmatrix} \sum_{i=1}^{n} x_i^{(1)} \cdot y_1^{(i)} & \dots & \sum_{i=1}^{n} x_i^{(1)} \cdot y_p^{(i)} \\ \vdots & \ddots & \vdots \\ \sum_{i=1}^{n} x_i^{(m)} \cdot y_1^{(i)} & \dots & \sum_{i=1}^{n} x_i^{(m)} \cdot y_p^{(i)} \end{bmatrix}$$

Chapter III

Exercise 01

	Exercise : 01
TinyStatistician	
Turn-in directory : *ex*01/	
Files to turn in : `TinyStatistician.py`	
Forbidden functions : `Any functions which calculates mean, median, quartiles, percentiles, variance or standard deviation`	

Objective

These exercises are key assignments from the last bootcamp. If you haven't completed them yet, now is the time, as they will be essential to your success in this module !

Instructions

Create a class named `TinyStatistician` with the following methods.

All methods take a list or a numpy.array as first parameter. You have to protect your functions against input errors.

- `mean(x)`: computes the mean of a given non-empty list or array x, using a for-loop. The method returns the mean as a float, otherwise None if x is an empty list or array, or a non-expected type object.
  This method should not raise any Exception.

  Given a vector x of dimension m * 1, the mathematical formula of its mean is:
  $$\bar{x} = \frac{\sum_{i=1}^{m} x_i}{m}$$
- `median(x)`: computes the median, which is also the 50th percentile, of a given non-empty list or array x .

This method returns the median as a float, otherwise None if x is an empty list or array or a non-expected type object.
This method should not raise any Exception.

- `quartile(x)`: computes the 1st and 3rd quartiles, also called the 25th percentile and the 75th percentile, of a given non-empty list or array x.
  This method returns the quartiles as a list of 2 floats, otherwise None if x is an empty list or array or a non-expected type object.
  This method should not raise any Exception.

- `percentile(x, p)`: computes the expected percentile of a given non-empty list or array x.
  This method returns the percentile as a float, otherwise None if x is an empty list or array or a non-expected type object.
  The second parameter is the demanded percentile.
  This method should not raise any Exception.

- `var(x)`: computes the sample variance of a given non-empty list or array x.
  This method returns the sample variance as a float, otherwise None if x is an empty list or array or a non-expected type object.
  This method should not raise any Exception.

  Given a vector x of dimension m * 1 representing a population sample, the mathematical formula of its variance is:

$$\sigma^2 = \frac{\sum_{i=1}^{m}(x_i - \bar{x})^2}{m-1} = \frac{\sum_{i=1}^{m}[x_i - (\frac{1}{m}\sum_{j=1}^{m}x_j)]^2}{m-1}$$

- `std(x)`: computes the sample standard deviation of a given non-empty list or array x.
  The method returns the sample standard deviation as a float, otherwise None if x is an empty list or array or a non-expected type object.
  This method should not raise any Exception.

  Given a vector x of dimension m * 1, the mathematical formula of the sample's standard deviation is:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{m}(x_i - \bar{x})^2}{m-1}} = \sqrt{\frac{\sum_{i=1}^{m}[x_i - (\frac{1}{m}\sum_{j=1}^{m}x_j)]^2}{m-1}}$$

Examples

```
a = [1, 42, 300, 10, 59]
TinyStatistician().mean(a)
# Output:
82.4

TinyStatistician().median(a)
# Output:
42.0

TinyStatistician().quartile(a)
# Output:
[10.0, 59.0]

TinyStatistician().percentile(a, 10)
# Output:
4.6

TinyStatistician().percentile(a, 15)
# Output:
6.4

TinyStatistician().percentile(a, 20)
# Output:
8.2

TinyStatistician().var(a)
# Output:
12279.439999999999

TinyStatistician().std(a)
# Output:
110.81263465868862
```

Numpy uses a different definition of percentile, it does linear interpolation between the two closest list element to the percentile. Make sure to understand the difference between the population and the sample definition for the statistic metrics.

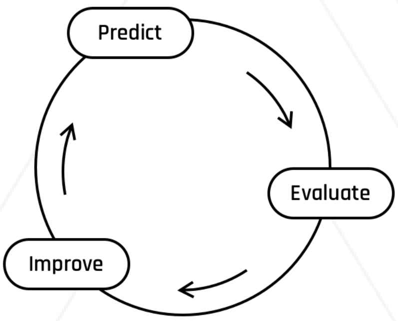
Chapter IV

Exercise 02

Interlude - Predict, Evaluate, Improve

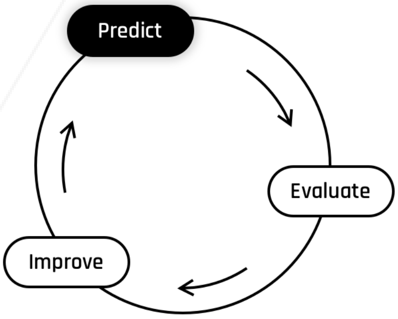
> A computer program is said to learn from experience E, with respect to some class of tasks T, and performance measure P, if its performance at tasks in T, as measured by P, improves with experience E.
>
> *Tom Mitchell,*
> *Machine Learning, 1997*

In other words, to learn you have to improve.
To improve, you have to evaluate your performances.
To evaluate your performances, you need to start performing on the task you want to be good at...
Repeat until convergence ...



One of the most common tasks in Machine Learning is **prediction**.
This will be your algorithm's task.
This will be your task.

Predict



A very simple model

We have some data. We want to model it.

- First we need to *make an assumption*, or hypothesis, *about the structure of the data and the relationship between the variables.*

- Then we can *apply that hypothesis to our data to make predictions.*

$$hypothesis(data) = predictions$$

Hypothesis

Let's start with a very simple and intuitive **hypothesis** on how the price of a spaceship can be predicted based on the power of its engines.
We will consider that *the more powerful the engines are, the more expensive the spaceship is.*
Furthermore, we will assume that the price increase is **proportional** to the power increase.

In other words, we will look for a **linear relationship** between the two variables.

This means that we will formulate the price prediction with a **linear equation**, that you might be already familiar with:

$$\hat{y} = ax + b$$

We add the ˆ symbol over the y to specify that $\hat{y}$ *(pronounced y-hat)* is a **prediction** (or estimation) of the real value of y. The prediction is calculated with the **parameters** a and b and the input value x.

For example, if $a = 5$ and $b = 33$, then $\hat{y} = 5x + 33$.

But in Machine Learning, we don't like using the letters a and b. Instead we will use the following notation:

$$\hat{y} = \theta_0 + \theta_1 x$$

So if $\theta_0 = 33$ and $\theta_1 = 5$, then $\hat{y} = 33 + 5x$.

To recap, this linear equation is our **hypothesis**. Then, all we will need to do is find the right values for our parameters θ_0 and θ_1 and we will get a fully-functional predictive **model**.

Predictions

Now, how can we generate a set of predictions on an entire dataset?
Let's consider a dataset containing m data points (or space ships), called **examples**.

What we do is stack the x and $\hat{y}$ values of all examples in vectors of length m. The relation between the elements in our vectors can then be represented with the following formula:

$$\hat{y}^{(i)} = \theta_0 + \theta_1 x^{(i)} \qquad \text{for i = 1, ..., m}$$

Where:

- $\hat{y}^{(i)}$ is the i^{th} component of vector y
- $x^{(i)}$ is the i^{th} component of vector x

Which can be experessed as:

$$\hat{y} = \begin{bmatrix} \theta_0 + \theta_1 \times x^{(1)} \\ \vdots \\ \theta_0 + \theta_1 \times x^{(m)} \end{bmatrix}$$

For example,

$$\text{given } \theta = \begin{bmatrix} 33 \\ 5 \end{bmatrix} \text{ and } x = \begin{bmatrix} 1 \\ 3 \end{bmatrix}:$$

$$\hat{y} = h_\theta(x) = \begin{bmatrix} 33 + 5 \times 1 \\ 33 + 5 \times 3 \end{bmatrix} = \begin{bmatrix} 38 \\ 48 \end{bmatrix}$$

More information

Why the θ notation?

You might have two questions at the moment:

- **WTF is that weird symbol?**

  This strange symbol, θ, is called "theta".

- **Why use this notation instead of a and b, like we're used to?**

  Despite its seeming more complicated at first, the theta notation is actually meant to simplify your equations later on.Why?

  a and b are good for a model with two parameters, but you will soon need to build more complex models that take into account more variables than just x.
  You could add more letters like this: $\hat{y} = ax_1 + bx_2 + cx_3 + ... + yx_{25} + z$

  But how do you go beyond 26 parameters? And how easily can you tell what parameter is associated with, let's say, x_{19}? That's why it becomes more handy to describe all your parameters using the theta notation and indices.

  With θ, you just have to increment the number to name the parameter:

  $\hat{y} = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + ... + \theta_{2468} x_{2468}$ ... Easy right?

Another common notation

$$\hat{y} \quad = \quad h_\theta(x)$$

Because $\hat{y}$ is calculated with our linear hypothesis using θ and x, it is sometimes written as $h_\theta(x)$. The h stands for *hypothesis*, and can be read as *"the result of our hypothesis h given x and theta"*.

Then if $x = 7$, we can calculate: $\hat{y} = h_\theta(x) = 33 + 5 \times 7 = 68$ We can now say that according to our linear model, the **predicted value** of y given $(x = 7)$ is 68.

	Exercise : 02
Simple Prediction	
Turn-in directory : *ex*02/	
Files to turn in : `prediction.py`	
Forbidden functions : `any functions which performs prediction`	

Objective

Understand and manipulate the notion of hypothesis in machine learning.

You must implement the following formula as a function:

$$\hat{y}^{(i)} = \theta_0 + \theta_1 x^{(i)} \qquad \text{for i = 1, ..., m}$$

Where:

- x is a vector of dimension m, the vector of examples/features (without the y values)
- $\hat{y}$ is a vector of dimension m * 1, the vector of predicted values
- θ is a vector of dimension 2 * 1, the vector of parameters
- $y^{(i)}$ is the i^{th} component of vector y
- $x^{(i)}$ is the i^{th} component of vector x

Instructions

In the prediction.py file, write the following function as per the instructions given below:

```
def simple_predict(x, theta):
    """Computes the vector of prediction y_hat from two non-empty numpy.ndarray.
    Args:
      x: has to be an numpy.ndarray, a one-dimensional array of size m.
      theta: has to be an numpy.ndarray, a one-dimensional array of size 2.
    Returns:
      y_hat as a numpy.ndarray, a one-dimensional array of size m.
      None if x or theta are empty numpy.ndarray.
      None if x or theta dimensions are not appropriate.
    Raises:
      This function should not raise any Exception.
    """
    ... Your code ...
```

Examples

```
import numpy as np
x = np.arange(1,6)

# Example 1:
theta1 = np.array([5, 0])
simple_predict(x, theta1)
# Ouput:
array([5., 5., 5., 5., 5.])
# Do you understand why y_hat contains only 5s here?


# Example 2:
theta2 = np.array([0, 1])
simple_predict(x, theta2)
# Output:
array([1., 2., 3., 4., 5.])
# Do you understand why y_hat == x here?


# Example 3:
theta3 = np.array([5, 3])
simple_predict(x, theta3)
# Output:
array([ 8., 11., 14., 17., 20.])


# Example 4:
theta4 = np.array([-3, 1])
simple_predict(x, theta4)
# Output:
array([-2., -1.,  0.,  1.,  2.])
```

Chapter V

Exercise 03

Interlude - A Simple Linear Algebra Trick

As you know, vectors and matrices can be multiplied to perform linear combinations. Let's do a little linear algebra trick to optimize our calculation and use matrix multiplication. If we add a column full of 1's to our vector of examples x, we can create the following matrix:

$$X' = \begin{bmatrix} 1 & x^{(1)} \\ \vdots & \vdots \\ 1 & x^{(m)} \end{bmatrix}$$

We can then rewrite our hypothesis as:

$$\hat{y}^{(i)} = \theta \cdot x'^{(i)} = \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} \cdot \begin{bmatrix} 1 & x^{(i)} \end{bmatrix} = \theta_0 + \theta_1 x^{(i)}$$

Therefore, the calculation of each $\hat{y}^{(i)}$ can be done with only one vector multiplication.

But we can even go further, by calculating the whole $\hat{y}$ vector in one operation:

$$\hat{y} = X' \cdot \theta = \begin{bmatrix} 1 & x^{(1)} \\ \vdots & \vdots \\ 1 & x^{(m)} \end{bmatrix} \cdot \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} \theta_0 + \theta_1 x^{(1)} \\ \vdots \\ \theta_0 + \theta_1 x^{(m)} \end{bmatrix}$$

We can now get to the same result as in the previous exercise with just a single multiplication between our brand new X' matrix and the θ vector!

A Note on Notation

In further Interludes, we will use the following convention:

- Capital letters represent matrices (e.g.: X)
- Lower-case letters represent vectors and scalars (e.g.: $x^{(i)}$, y)

	Exercise : 03
Add Intercept	
Turn-in directory : *ex*03/	
Files to turn in : `tools.py`	
Forbidden functions : `None`	

Objective

Understand and manipulate the notion of hypothesis in machine learning.
You must implement a function which adds an extra column of 1's on the left side of a given vector or matrix.

Instructions

In the tools.py file create the following function as per the instructions given below:

```
def add_intercept(x):
    """Adds a column of 1's to the non-empty numpy.array x.
    Args:
      x: has to be a numpy.array. x can be a one-dimensional (m * 1) or two-dimensional (m * n) array.
    Returns:
      X, a numpy.array of dimension m * (n + 1).
      None if x is not a numpy.array.
      None if x is an empty numpy.array.
    Raises:
      This function should not raise any Exception.
    """
    ... Your code ...
```

Examples

```
import numpy as np

# Example 1:
x = np.arange(1,6)
add_intercept(x)
# Output:
array([[1., 1.],
       [1., 2.],
       [1., 3.],
       [1., 4.],
       [1., 5.]])


# Example 2:
y = np.arange(1,10).reshape((3,3))
add_intercept(y)
# Output:
array([[1., 1., 2., 3.],
       [1., 4., 5., 6.],
       [1., 7., 8., 9.]])
```

Chapter VI

Exercise 04

	Exercise : 04
Prediction	
Turn-in directory : *ex*04/	
Files to turn in : `prediction.py`	
Forbidden functions : `None`	

Objective

Understand and manipulate the notion of hypothesis in machine learning.

You must implement the following formula as a function:

$$\hat{y}^{(i)} = \theta_0 + \theta_1 x^{(i)} \qquad \text{for i = 1, ..., m}$$

Where:

- $\hat{y}^{(i)}$ is the i^{th} component of vector $\hat{y}$
- $\hat{y}$ is a vector of dimension m, the vector of predicted values
- θ is a vector of dimension 2×1, the vector of parameters
- $x^{(i)}$ is the i^{th} component of vector x
- x is a vector of dimension m, the vector of examples

But this time you have to do it with the linear algebra trick!

$$\hat{y} = X' \cdot \theta = \begin{bmatrix} 1 & x^{(1)} \\ \vdots & \vdots \\ 1 & x^{(m)} \end{bmatrix} \cdot \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} \theta_0 + \theta_1 x^{(1)} \\ \vdots \\ \theta_0 + \theta_1 x^{(m)} \end{bmatrix}$$

- the argument x is an m vector
- θ is a 2×1 vector.

You have to transform x into X' to fit the dimension of θ!

Instructions

In the prediction.py file create the following function as per the instructions given below:

```
def predict_(x, theta):
    """Computes the vector of prediction y_hat from two non-empty numpy.array.
    Args:
      x: has to be an numpy.array, a one-dimensional array of size m.
      theta: has to be an numpy.array, a two-dimensional array of shape 2 * 1.
    Returns:
      y_hat as a numpy.array, a two-dimensional array of shape m * 1.
      None if x and/or theta are not numpy.array.
      None if x or theta are empty numpy.array.
      None if x or theta dimensions are not appropriate.
    Raises:
      This function should not raise any Exceptions.
    """
    ... Your code ...
```

Examples

```
import numpy as np
x = np.arange(1,6)

# Example 1:
theta1 = np.array([[5], [0]])
predict_(x, theta1)
# Ouput:
array([[5.], [5.], [5.], [5.], [5.]])
# Do you remember why y_hat contains only 5's here?

# Example 2:
theta2 = np.array([[0], [1]])
predict_(x, theta2)
# Output:
array([[1.], [2.], [3.], [4.], [5.]])
# Do you remember why y_hat == x here?

# Example 3:
theta3 = np.array([[5], [3]])
predict_(x, theta3)
# Output:
array([[ 8.], [11.], [14.], [17.], [20.]])


# Example 4:
theta4 = np.array([[-3], [1]])
predict_(x, theta4)
# Output:
array([[-2.], [-1.], [ 0.], [ 1.], [ 2.]])
```

Chapter VII

Exercise 05

	Exercise : 05
Let's Make Nice Plots	
Turn-in directory : *ex*05/	
Files to turn in : `plot.py`	
Forbidden functions : `None`	

> For your information, the task we are performing here is called **regression**. It means that we are trying to predict a continuous numerical attribute for all examples (like a price, for instance). Later in the bootcamp, you will see that we can predict other things such as categories.

Objective

You must implement a function to plot the data and the prediction line (or regression line).

You will plot the data points (with their x and y values), and the prediction line that represents your hypothesis (h_θ).

Instructions

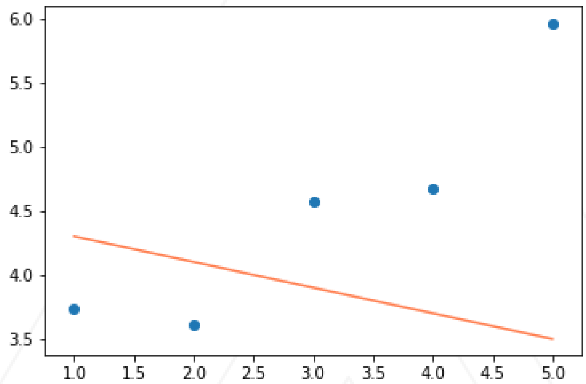
In the plot.py file, create the following function as per the instructions given below:

```
def plot(x, y, theta):
    """Plot the data and prediction line from three non-empty numpy.array.
    Args:
      x: has to be an numpy.array, a one-dimensional array of size m.
      y: has to be an numpy.array, a one-dimensional array of size m.
      theta: has to be an numpy.array, a two-dimensional array of shape 2 * 1.
    Returns:
        Nothing.
    Raises:
      This function should not raise any Exceptions.
    """
    ... Your code ...
```

Examples

```
import numpy as np
x = np.arange(1,6)
y = np.array([3.74013816, 3.61473236, 4.57655287, 4.66793434, 5.95585554])

# Example 1:
theta1 = np.array([[4.5],[-0.2]])
plot(x, y, theta1)
# Output:
```



```
# Example 2:
theta2 = np.array([[-1.5],[2]])
plot(x, y, theta2)
# Output:
```

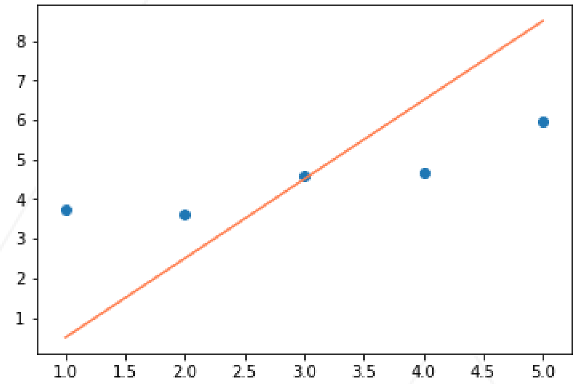


Figure VII.1: Example 2

```
# Example 3:
theta3 = np.array([[3],[0.3]])
plot(x, y, theta3)
# Output:
```

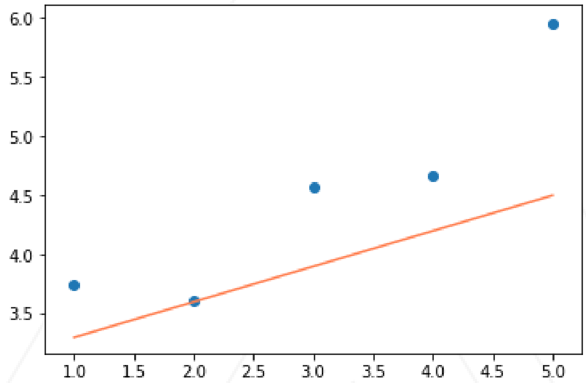
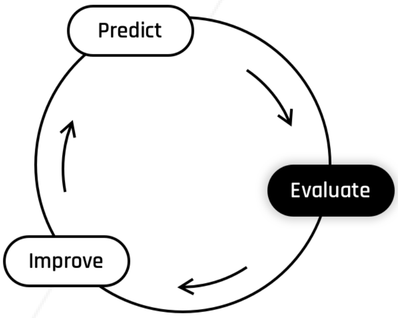


Figure VII.2: Example 3

Chapter VIII

Exercise 06

Interlude - Evaluate



Introducing the loss function

How good is our model? It is hard to say just by simply looking at the plots! We can clearly observe that certain regression lines seem to fit the data better than others, but it would be convenient to find a way to measure it.

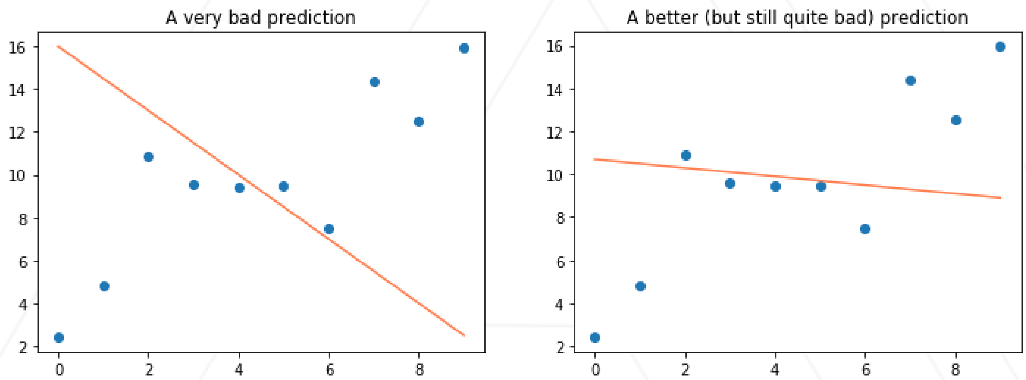


Figure VIII.1: bad prediction

To evaluate our model, we are going to use a **metric** called **the loss function** (sometimes called **cost function**).

The loss function tells us how bad our model is performing, how much it *costs* us to use it, how much information we *lose* when we use it. If the model is good, we won't lose that much; if it's terrible instead, we will have a high loss!

The metric you choose will deeply impact the evaluation (and therefore also the training) of your model.

A frequent way to evaluate the performance of a regression model is to measure the distance between each predicted value ($\hat{y}^{(i)}$) and the real value it tries to predict ($y^{(i)}$). The distances are then squared, and averaged to get one single metric, denoted J:

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(\hat{y}^{(i)} - y^{(i)})^2$$

The smaller, the better!

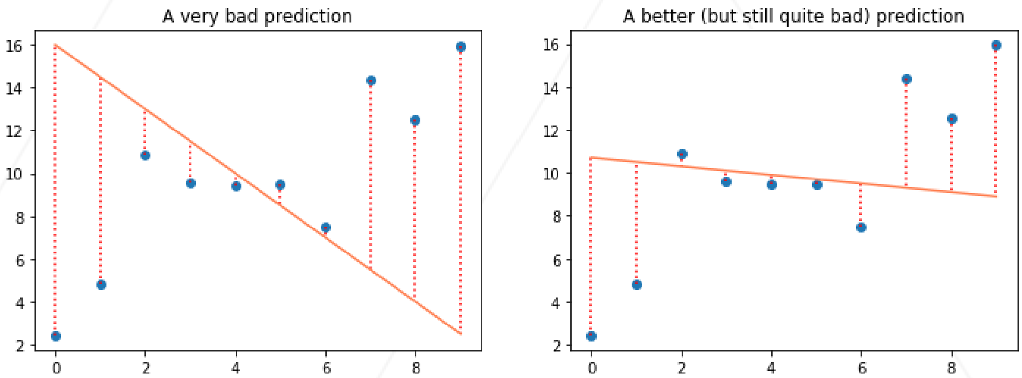


Figure VIII.2: bad prediction with distance

	Exercise : 06
Loss function	
Turn-in directory : *ex*06/	
Files to turn in : `loss.py`	
Forbidden functions : `None`	

Objective

Understand and experiment with the **loss function** in machine learning.

You must implement the following formula as a function (and another one very close to it):

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(\hat{y}^{(i)} - y^{(i)})^2$$

Where:

- $\hat{y}$ is a vector of dimension $m \times 1$, the vector of predicted values
- y is a vector of dimension $m \times 1$, the vector of expected values
- $\hat{y}^{(i)}$ is the ith component of vector $\hat{y}$
- $y^{(i)}$ is the ith component of vector y

Instructions

The implementation of the loss function has been split in two functions:

- `loss_elem_()`, which computes the squared distances for all examples $(\hat{y}^{(i)} - y^{(i)})^2$,
- `loss_()`, which averages the squared distances of all examples (the $J_(\theta)$ above).

In the loss.py file create the following functions as per the instructions given below:

```
def loss_elem_(y, y_hat):
        """
        Description:
                Calculates all the elements (y_pred - y)^2 of the loss function.
        Args:
      y: has to be an numpy.array, a two-dimensional array of shape m * 1.
      y_hat: has to be an numpy.array, a two-dimensional array of shape m * 1.
        Returns:
                J_elem: numpy.array, a array of dimension (number of the training examples, 1).
                None if there is a dimension matching problem.
                None if any argument is not of the expected type.
        Raises:
                This function should not raise any Exception.
        """
        ... your code here ...

def loss_(y, y_hat):
        """
        Description:
                Calculates the value of loss function.
        Args:
      y: has to be an numpy.array, a two-dimensional array of shape m * 1.
      y_hat: has to be an numpy.array, a two-dimensional array of shape m * 1.
        Returns:
                J_value : has to be a float.
                None if there is a dimension matching problem.
                None if any argument is not of the expected type.
        Raises:
                This function should not raise any Exception.
        """
        ... your code here ...
```

Examples

```
import numpy as np

x1 = np.array([[0.], [1.], [2.], [3.], [4.]])
theta1 = np.array([[2.], [4.]])
y_hat1 = predict_(x1, theta1)
y1 = np.array([[2.], [7.], [12.], [17.], [22.]])

# Example 1:
loss_elem_(y1, y_hat1)

# Output:
array([[0.], [1], [4], [9], [16]])

# Example 2:
loss_(y1, y_hat1)

# Output:
3.0

x2 = np.array([0, 15, -9, 7, 12, 3, -21]).reshape(-1, 1)
theta2 = np.array(np.array([[0.], [1.]]))
y_hat2 = predict_(x2, theta2)
y2 = np.array([2, 14, -13, 5, 12, 4, -19]).reshape(-1, 1)

# Example 3:
loss_(y2, y_hat2)

# Output:
2.142857142857143

# Example 4:
loss_(y2, y2)

# Output:
0.0
```

This loss function is very close to the one called **"Mean Squared Error"**, which is frequently mentioned in Machine Learning resources. The difference is in the denominator as you can see in the formula of the $MSE = \frac{1}{m}\sum_{i=1}^{m}(\hat{y}^{(i)} - y^{(i)})^2$.
Except for the division by $2m$ instead of m, these functions are rigourously identical: $J(\theta) = \frac{MSE}{2}$.
MSE is called like that because it represents the mean of the errors (i.e.: the differences between the predicted values and the true values), squared.
You might wonder why we choose to divide by two instead of simply using the MSE? **(It's a good question, by the way.)**

- First, it does not change the overall model evaluation: if all performance measures are divided by two, we can still compare different models and their performance ranking will remain the same.
- Second, it will be very convenient when we calculate the gradient tommorow. Be patient, and trust us ;)

Chapter IX

Exercise 07

	Exercise : 07
Vectorized loss function	
Turn-in directory : *ex*07/	
Files to turn in : `vec_loss.py`	
Forbidden functions : `None`	

Objective

Understand and experiment with the **loss function** in machine learning.

You must implement the following formula as a function:

$$J(\theta) \quad = \quad \frac{1}{2m}(\hat{y} - y) \cdot (\hat{y} - y)$$

Where:

- $\hat{y}$ is a vector of dimension m, the vector of predicted values
- y is a vector of dimension m, the vector of expected values

Instructions

In the `vec_loss.py` file, create the following function as per the instructions given below:

```
def loss_(y, y_hat):
    """Computes the half mean-squared-error of two non-empty numpy.arrays, without any for loop.
    The two arrays must have the same dimensions.
    Args:
      y: has to be an numpy.array, a one-dimensional array of size m.
      y_hat: has to be an numpy.array, a one-dimensional array of size m.
    Returns:
      The half mean-squared-error of the two vectors as a float.
      None if y or y_hat are empty numpy.array.
      None if y and y_hat does not share the same dimensions.
    Raises:
      This function should not raise any Exceptions.
    """
    ... Your code ...
```

Examples

```
import numpy as np
X = np.array([0, 15, -9, 7, 12, 3, -21])
Y = np.array([2, 14, -13, 5, 12, 4, -19])

# Example 1:
loss_(X, Y)
# Output:
2.142857142857143

# Example 2:
loss_(X, X)
# Output:
0.0
```

Chapter X

Exercise 08

Interlude - Fifty Shades of Linear Algebra

In the last exercise, we implemented the **loss function** in two subfunctions. It worked, but it's not very pretty. What if we could do it all in one step, with linear algebra?

As we did with the hypothesis, we can use a vectorized equation to improve the calculations of the loss function.

So now let's take a look at how squaring and averaging can be performed (more or less) in a single matrix multiplication!

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(\hat{y}^{(i)} - y^{(i)})^2$$

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}[(\hat{y}^{(i)} - y^{(i)})(\hat{y}^{(i)} - y^{(i)})]$$

Now, if we apply the definition of the dot product:

$$J(\theta) = \frac{1}{2m}(\hat{y} - y) \cdot (\hat{y} - y)$$

	Exercise : 08
Lets Make Nice Plots Again	
Turn-in directory : *ex*08/	
Files to turn in : `plot.py`	
Forbidden functions : `None`	

Objective

You must implement a function which plots the data, the prediction line, and the loss.

You will plot the x and y coordinates of all data points as well as the prediction line generated by your theta parameters.

Your function must also display the overall loss (J) in the title, and draw small lines marking the distance between each data point and its predicted value.

Instructions

In the plot.py file create the following function as per the instructions given below:

```
def plot_with_loss(x, y, theta):
"""Plot the data and prediction line from three non-empty numpy.ndarray.
    Args:
      x: has to be an numpy.ndarray, one-dimensional array of size m.
      y: has to be an numpy.ndarray, one-dimensional array of size m.
      theta: has to be an numpy.ndarray, one-dimensional array of size 2.
    Returns:
        Nothing.
    Raises:
      This function should not raise any Exception.
    """
    ... Your code ...
```

Examples

```
import numpy as np
x = np.arange(1,6)
y = np.array([11.52434424, 10.62589482, 13.14755699, 18.60682298, 14.14329568])

# Example 1:
theta1= np.array([18,-1])
plot_with_loss(x, y, theta1)
# Output:
```

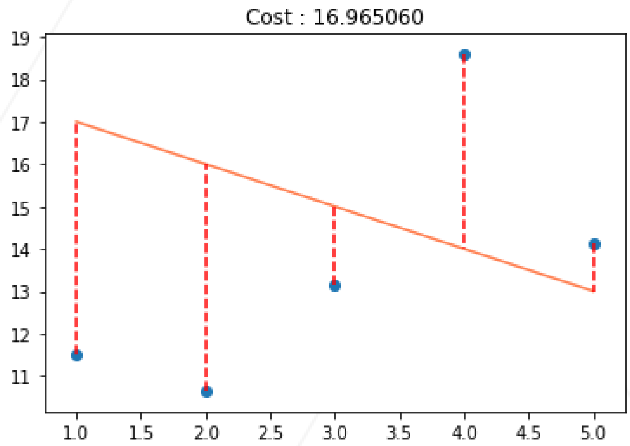


Figure X.1: Example 1

```
# Example 2:
theta2 = np.array([14, 0])
plot_with_loss(x, y, theta2)
# Output:
```

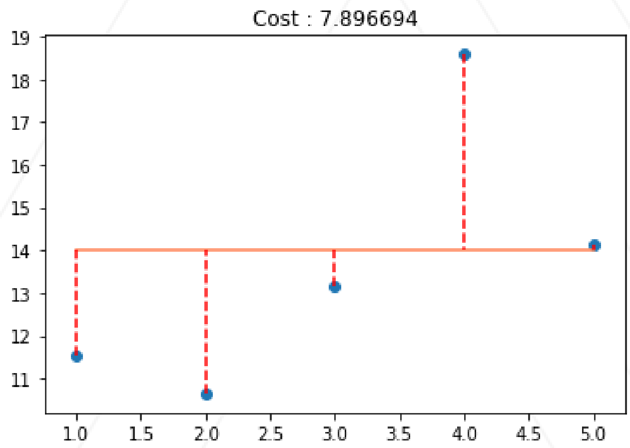


Figure X.2: Example 2

```
# Example 3:
theta3 = np.array([12, 0.8])
plot_with_loss(x, y, theta3)
# Output:
```

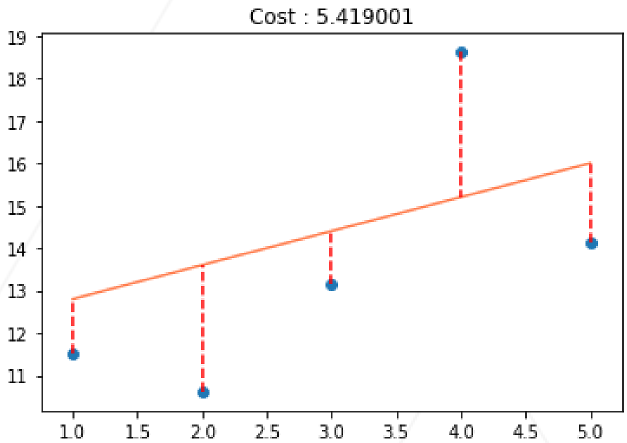


Figure X.3: Example 3

Chapter XI

Exercise 09

	Exercise : 09
Other loss functions	
Turn-in directory : *ex*09/	
Files to turn in : `other_losses.py`	
Forbidden functions : `None`	

Deepen the notion of loss function in machine learning.

You certainly had a lot of fun implementing your loss function! Remember we told you it was **one among many possible ways of measuring the loss**.

Now, you will get to implement other metrics. You already know about one of them: **MSE**. There are several more which are quite common: **RMSE**, **MAE** and **R2score**.

Objective

You must implement the following formulas as functions:

$$MSE(y, \hat{y}) = \frac{1}{m}\sum_{i=1}^{m}(\hat{y}^{(i)} - y^{(i)})^2$$

$$RMSE(y, \hat{y}) = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(\hat{y}^{(i)} - y^{(i)})^2}$$

$$MAE(y, \hat{y}) = \frac{1}{m}\sum_{i=1}^{m}|\hat{y}^{(i)} - y^{(i)}|$$

$$R^2(y, \hat{y}) = 1 - \frac{\sum_{i=1}^{m}(\hat{y}^{(i)} - y^{(i)})^2}{\sum_{i=1}^{m}(y^{(i)} - \bar{y})^2}$$

Where:

- y is a vector of dimension m
- $\hat{y}$ is a vector of dimension m
- $y^{(i)}$ is the i^{th} component of vector y
- $\hat{y}^{(i)}$ is the i^{th} component of $\hat{y}$
- $\bar{y}$ is the mean of the y vector

Instructions

In the `other_losses.py` file, create the following functions as per the instructions given below:

```
def mse_(y, y_hat):
        """
        Description:
                Calculate the MSE between the predicted output and the real output.
        Args:
        y: has to be a numpy.array, a two-dimensional array of shape m * 1.
        y_hat: has to be a numpy.array, a two-dimensional vector of shape m * 1.
        Returns:
                mse: has to be a float.
                None if there is a matching dimension problem.
        Raises:
                This function should not raise any Exceptions.
        """
                ... your code here ...
```

```
def rmse_(y, y_hat):
        """
        Description:
                Calculate the RMSE between the predicted output and the real output.
        Args:
              y: has to be a numpy.array, a two-dimensional array of shape m * 1.
        y_hat: has to be a numpy.array, a two-dimensional array of shape m * 1.
        Returns:
                rmse: has to be a float.
                None if there is a matching dimension problem.
        Raises:
                This function should not raise any Exceptions.
        """
                ... your code here ...
```

```
def mae_(y, y_hat):
        """
        Description:
                Calculate the MAE between the predicted output and the real output.
        Args:
        y: has to be a numpy.array, a two-dimensional array of shape m * 1.
        y_hat: has to be a numpy.array, a two-dimensional array of shape m * 1.
        Returns:
                mae: has to be a float.
                None if there is a matching dimension problem.
        Raises:
                This function should not raise any Exceptions.
        """
                ... your code here ...
```

```
def r2score_(y, y_hat):
        """
        Description:
                Calculate the R2score between the predicted output and the output.
        Args:
        y: has to be a numpy.array, a two-dimensional array of shape m * 1.
        y_hat: has to be a numpy.array, a two-dimensional array of shape m * 1.
        Returns:
                r2score: has to be a float.
                None if there is a matching dimension problem.
        Raises:
                This function should not raise any Exceptions.
        """
                ... your code here ...
```

You might consider implementing four more methods, similar to what you did for the loss function in exercise 07:

- mse_elem(),
- rmse_elem(),
- mae_elem(),
- r2score_elem().

Examples

```
import numpy as np
from sklearn.metrics import mean_squared_error, mean_absolute_error, r2_score
from math import sqrt

# Example 1:
x = np.array([[0], [15], [-9], [7], [12], [3], [-21]])
y = np.array([[2], [14], [-13], [5], [12], [4], [-19]])

# Mean-squared-error
## your implementation
mse_(x,y)
## Output:
4.285714285714286
## sklearn implementation
mean_squared_error(x,y)
## Output:
4.285714285714286

# Root mean-squared-error
## your implementation
rmse_(x,y)
## Output:
2.0701966780270626
## sklearn implementation not available: take the square root of MSE
sqrt(mean_squared_error(x,y))
## Output:
2.0701966780270626

# Mean absolute error
## your implementation
mae_(x,y)
# Output:
1.7142857142857142
## sklearn implementation
mean_absolute_error(x,y)
# Output:
1.7142857142857142

# R2-score
## your implementation
r2score_(x,y)
## Output:
0.9681721733858745
## sklearn implementation
r2_score(x,y)
## Output:
0.9681721733858745
```

Chapter XII

Conclusion - What you have learnt

This first series of exercises is finished, well done!

Based on all the notions and problems tackled today, you should be able to discuss and answer the following questions:

1. Why do we concatenate a column of ones to the left of the x vector when we use the linear algebra trick?

2. Why does the loss function square the distances between the data points and their predicted values?

3. What does the loss function's output represent?

4. Towards which value do we want the loss function to tend? What would that mean?

5. Do you understand why matrix multiplications are not commutative?

These questions are an opportunity for discussion among your peers, or to simply reflect on your own acquired understanding during this day !

`Your feedbacks are essential for us to improve these bootcamps ! Please take a few minutes to tell us about your experience in this module by filling this form. Thank you in advance !`

Contact

You can contact 42AI by email: contact@42ai.fr

Thank you for attending 42AI's Machine Learning Bootcamp !

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