EXPLORING CRYPTOGRAPHY PROTOCOLS

WITH LIMITED EMPHASIS ON MATHEMATICS ☺

Presented By
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ATTENTION

THESE SLIDES HAVE BEEN CRAFTED USING THE FOUNDATION OF MY MSC COURSE IN CRYPTOGRAPHY PROTOCOLS AT THE UNIVERSITY OF ISFAHAN.

I'VE MADE ADJUSTMENTS TO THE CONTENT TO ALIGN WITH THE SPECIFIC OBJECTIVES OF THIS PRESENTATION.

ALSO, MY INTENTION HAS BEEN TO MINIMIZE THE USE OF MATHEMATICAL CONCEPTS, WHICH MAY RESULT IN SOME CONCEPTS BEING SIMPLIFIED OR LESS PRECISE.

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Agenda

1. Identification and Entity Authentications Protocols
2. Zero Knowledge Protocols
3. **Key Establishment Protocols**
4. Threshold Cryptography and Secret Sharing Protocols
5. Types of Digital Signatures
6. Special Purpose Protocols (like simultaneous contract signing, mental poker, fair exchange)
7. Identity Based Cryptography
8. Secure Auctions and Elections Protocols
9. Secure Multiparty Computations
10. Cryptocurrency

Key Distribution

Types

- **Key Predistribution Schemes:**
  - TA passively delivers some unique keys to each entity through a secure channel in advance.
  - The majority, if not all, pairs of entities have a common key.
- **Session Key Distribution**
  - TA actively delivers session keys to the entities through an interactive protocol upon their request, ensuring secure encryption during the process.
  - Each entity has a common key with TA
- **Key Agreement**
  - The entities collaboratively generate the session key, utilizing either symmetric or asymmetric encryption.
  - No active involvement of the TA is necessary.

Key Distribution

Types of Keys

- **Master Key (Long-Lived Key)**
  - If it is a symmetric key, it is pre-shared between two entities or between an entity and the TA. A secure channel is required for its distribution.
  - An asymmetric key which is associated with a certified public key.

- **Session Key (Short-Lived Key)**
  - A asymmetric key is used for encryption or MAC generation.

Key Distribution

Some Terms

- **Known Session Key Security**
  - If an enemy discovers a session key, he must not be able to find out other session keys.
- **Prefect Forward Secrecy**
  - If the master key is revealed, the subsequent session keys must remain undisclosed.
- **Prefect Backward Secrecy**
  - If the master key is revealed, the previous session keys must remain undisclosed.

Key Predistribution Schemes

Diffie Hellman

$e_a = g^{da} \bmod n$

$1 \leq d_a \leq n-1$

Public key e_a
Private key d_a

No interaction is required.

Alice and Bob possess each other's certificates.

$e_b = g^{db} \bmod n$

$1 \leq d_b \leq n-1$

Public key e_b
Private key d_b

$K_{ab} = (e_b)^{da} = (g^{db})^{da} \bmod n$

$K_{ab} = (e_a)^{db} = (g^{da})^{db} \bmod n$

Key Predistribution Schemes

Unconditional Secure – Obvious Scheme

- TA generates a unique common key for each pair of users.
- The key is delivered through a secure channel.

Key Predistribution Schemes

Unconditional Secure – Blom Scheme

- K Is the maximum number of long-lived keys which if disclosed, secrecy of other keys would not be threaten.

- Symmetric Polynomials: if any of the variables in a polynomial are interchanged, then we get the same polynomial.
  - For example: `f(x,y) = f(y,x) = xy + 5(x+y) + 6`

Key Predistribution Schemes

Unconditional Secure – Blom Scheme (K=1)

TA

- chooses prime P
- chooses a random $1 \leq r_i \leq p-1$ and assign it to user i
- makes r_i public
- chooses random $1 \leq a,b,c \leq p-1$
- generates private $f(x,y) = a + b(x+y) + cxy \mod p$
- sends for user i through a secure channel:
  - $f(x,r_i) = a + b(x+r_i) + cxr_i = a + br_i + (b+cr_i)x$

Key Predistribution Schemes

Unconditional Secure – Blom Scheme (K=1)

User u,v want to generate a common key based on

$f(x,r_u) = a+b(x+r_u)+cxr_u = a+br_u+(b+cr_u)x$

$f(x,r_v) = a+b(x+r_v)+cxr_v = a+br_v+(b+cr_v)x$

User u

- $f(x,r_u) = g_u(x)$
- $g_u(r_v) = f(r_v,r_u)$

User v

- $f(x,r_v) = g_v(x)$
- $g_v(r_u) = f(r_u,r_v)$

Key Predistribution Schemes

Unconditional Secure – Blom Scheme (K=1)

- The scheme for K=1 is unconditionally secure against a single user, say w

Prove:

- User w want to find $K_{uv} = a + br_u + (b+cr_u)r_v$
- User w knows r_u and r_v but does not know a,b,c
- User w has: $f(x,r_w) = a + b(x+r_w) + cxr_w = \underbrace{a + br_w}_{a_w} + \underbrace{(b+cr_w)}_{b_w}x$

$$\begin{bmatrix} 1 & r_u+r_v & r_ur_v \\ 1 & r_w & 0 \\ 0 & 1 & r_w \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} k_{uv} \\ a_w \\ b_w \end{bmatrix}$$

Impossible!

3 equations, 4 variables

Key Predistribution Schemes

Unconditional Secure – Blom Scheme (K=1)

- The scheme for K=1 is NOT secure against coalition of 2 users
- Coalition of users x and w can calculate common key of each two others.
- User w:
  - $a_w = a+br_w$
  - $b_w = b+cr_w$
- User x:
  - $a_x = a+br_x$
  - $b_x = b+cr_x$

Possible!

4 equations, 3 variables

Key Predistribution Schemes

Unconditional Secure – Generalized Blom Scheme

TA

- chooses prime P
- Chooses a random $1 \le r_i \le p-1$ and assign it to user i
- makes r_i public
- Chooses a random $0 \le a_{i,j} \le p-1$ which $a_{ij}=a_{ji}$
- Creates $f(x,y) = \sum_{i=0}^{k} \sum_{j=0}^{k} a_{ij} x^i y^j$
- Sends a user, say u, through a secure channel:
  - $g_u(x) = f(x, r_u) \bmod p = \sum_{i=0}^{k} a_{ui} x^i$

Each two users can generates their common key by their r value

Key Predistribution Schemes

Unconditional Secure – Generalized Blom Scheme

The generalized scheme is unconditionally secure against K users.

But, each coalition of K+1 users can break the scheme.

Session Key Distribution

Scheme #1

TA

- n Users: $\{u_1, u_2, \ldots, u_n\}$
- v Keys: $\{k_1, k_2, \ldots, k_v\}$
- a v*n binary matrix
- (i,j)-entry is 1 if and only if k_i is assigned to u_j
- If f is a subset of users: keys(p)={indices of common keys between all of p users}
- Keys(p) ≠ ∅ => $\sum_{i \in keys(p)} k_i$

Session Key Distribution

Scheme #1

	u_1	u_2	u_3	u_4
k_1	1	1	0	0
k_2	1	0	1	0
k_3	1	0	0	1
k_4	0	1	1	0
k_5	0	1	0	1
k_6	0	0	1	1

$keys(u_1)$: {1,2,3}

$keys(u_2)$: {1,4,5}

=> $keys(u_1,u_2)$: {1}

- Only two 1-entry per row (secure against coalition of others)
- Each two users have exactly one common key
- Same as obvious scheme: a common key per two users
- Count of keys: $\binom{n}{2} = \binom{4}{2} = 6$

Session Key Distribution

Scheme #1

f as a subset of users is able to calculate k_p if and only if

$$Keys(p) \subseteq \bigcup_{u_j \in f} keys(uj)$$

Even if one user of keys(p) does not get involve in coalition, the key would not be found.

Session Key Distribution

Scheme #2

- Number of keys the user would be stored must be as less as possible
- Most of the time it is not needed to secure against coalition of the all users.
- Securing against a specific number (= less than n-2) of users is sufficient

Session Key Distribution

Scheme #2

	u_1	u_2	u_3	u_4	u_5	u_6	u_7
K_1	1	1	1	0	1	0	0
k_2	0	1	1	1	0	1	0
k_3	0	0	1	1	1	0	1
k_4	1	0	0	1	1	1	0
k_5	0	1	0	0	1	1	1
k_6	1	0	1	0	0	1	1
k_7	1	1	0	1	0	0	1

p

$keys(u_1,u_2)$: {1,7}

$K_p = k_1 \oplus k_7$

Secure against one, but Not against coalition of two

- Each two users can create a common key secured against others
- Number of keys (=7) s is much less than $\binom{n}{2} = 21$
- Each user stores 4 keys which is less than $n - 1 = 6$

Session Key Distribution

Fiat-Naor

For each arbitrary subset of users, the key could be calculated and secured against coalition of up to w users.

Choose w in range $[1,n]$

Number of keys: $V = \sum_{i=0}^{w} \binom{n}{i}$

Matrix M which each its row is a n-bit sequence with at least $n - w$ 1-bit.

Fiat-Naor W-KDP

Session Key Distribution

Fiat-Naor 1-KDP

$n = 6$, $w = 1$, $n - w = 5$

$$V = \sum_{i=0}^{w} \binom{n}{i} = \binom{6}{0} + \binom{6}{1} = 7$$

There is no other user has $\{k_1, k_4, k_5, k_6\}$

	u_1	u_2	u_3	u_4	u_5	u_6
K_1	1	1	1	1	1	1
k_2	1	1	1	1	1	0
k_3	1	1	1	1	0	1
k_4	1	1	1	0	1	1
k_5	1	1	0	1	1	1
k_6	1	0	1	1	1	1
k_7	0	1	1	1	1	1

p

keys(u_1,u_5,u_6): {1,4,5,6}

$$K_p = k_1 \oplus k_4 \oplus k_5 \oplus k_6$$

Session Key Distribution

Mitchel-Piper

There is a key per subset with exactly t users which is secured against up to w users

(X,A)-pair:

- X is a set contains v keys. $X:\{k_1,k_2,...,k_v\}$
- A is a subset of X, called a block. A_i is the key assigned to u_i specifies the column i of matrix M.

(X,A) is a (t,w)-CFF (Cover Free Family) if intersection of any t blocks must not cover union of any w blocks.

Matrix M is $V*N$ with (i,j)-entry: $\begin{cases} 1 & \text{if ki} \in \text{Aj} \\ 0 & otherwise \end{cases}$

Session Key Distribution

Example of Mitchel-Piper

Given N=7 and V=7

X: $\{k_1,k_2,\ldots,k_7\}$

A: { A_1: $\{k_1,k_4,k_6,k_7\}$,

...

A_7: $\{k_3,k_5,k_6,k_7\}$ }

(2,1)-CFF

Session Key Distribution

Mitchel-Piper

Objective: For given t, w and n, finding a (t,w)-CFF(v,n) with minimal value of v

✓ **Erdos's estimating algorithm**

Session Key Distribution

Needham–Shroeder (1978)

Session key with Alice: K_A
Session key with Bob: K_B
Session key: K
$t_B = e_{KB}(K|ID_A)$

r_A , ID_A , ID_B

$y_1 = e_{KA}(r_A|ID_B|K|t_B)$

t_B

$y_2 = e_K(r_B)$

Bob ensures Alice has the key

$y_3 = e_K(r_B-1)$

One way key confirmation

Session Key Distribution

Denning–Succo Attack

Known Session Attack

In role of Alice

Old t_B

$y_2 = e_K(r'_B)$

$y_3 = e_K(r'_B-1)$

Session Key Distribution

Simplified Kerberos (80s and 90s)

Session key with Alice: K_A
Session key with Bob: K_B
Session key: K
Validity period: L

$r_A \ , \ ID_A \ , \ ID_B$

$y_1 = e_{KA}(r_A|ID_B|K|L)$

$t_B = e_{KA}(K|ID_B|L)$

$t_B \ , \ y_2 = e_K(ID_A|time)$

$y_3 = e_K(time+1)$

Two way key confirmation

Session Key Distribution

Bellare–Rogaway (1995)

r_A , ID_A , ID_B

r_A , r_B , ID_A , ID_B

$e_{KB}(K)$, $MAC_B(ID_A|ID_B|r_B|e_{KB}(K))$

$e_{KA}(K)$, $MAC_A(ID_B|ID_A|r_A|e_{KA}(K))$

Session key with Alice: K_A
Session key with Bob: K_B
Session key: K

No key confirmation

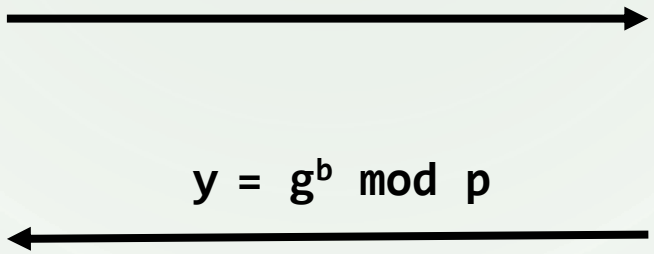
Key Agreement

Diffie Hellman

agreement on:
multiplicative
group G of order n,
with a generator g

a is random from
range [1,n-1]

$x = g^a \bmod p$



$y = g^b \bmod p$

agreement on:
multiplicative
group G of order n,
with a generator g

b is random from
range [1,n-1]

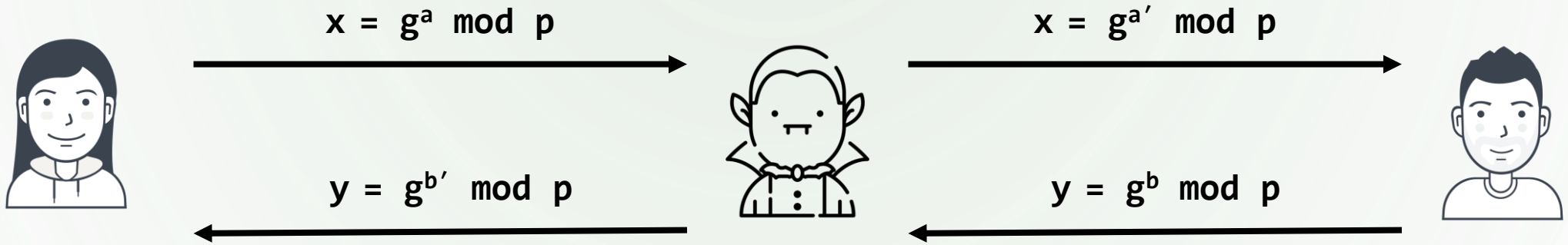
$y^a = g^{ba} \bmod n$

$x^b = g^{ab} \bmod n$

No Authentication

Key Agreement

Diffie Hellman – MITM Attack



So, we need to authenticate key agreement schemes

Key Agreement

Station By Station (STS) Scheme

agreement on:
multiplicative
group G of order n,
with a generator g

a is random from
range [1,n-1]

$Cert(A)\ ,\ x = g^a \bmod p$

$Cert(B)\ ,\ y = g^b \bmod p$

$Sig_B(ID_A|x|y)$

$Sig_A(ID_B|x|y)$

agreement on:
multiplicative
group G of order n,
with a generator g

b is random from
range [1,n-1]

Verify the sign, then

$y^a = g^{ba} \bmod n$

Verify the sign, then

$x^b = g^{ab} \bmod n$

Key Agreement

MTI Scheme

agreement on:
multiplicative
group G of order q,
with a generator g

$Cert(A)\ ,\ x = g^m \bmod p$

$Cert(B)\ ,\ y = g^n \bmod p$

agreement on:
multiplicative
group G of order n,
with a generator g

Public Key
$P_A = g^a \bmod p$

Private key a is from
range [1,q-1]

Private m is random
from range [1,q-1]

Public Key
$P_B = g^b \bmod p$

Private key b is from
range [1,q-1]

Private n is random
from range [1,q-1]

$K = y^a * P_B^m = g^{na+bm} \bmod n$

$K = x^b * P_a^n = g^{mb+an} \bmod n$

Key Agreement

MTI Scheme – Burmester – Known Session Key Attack

Eavesdrops

Cert(A) , $x = g^m \bmod p$

Cert(B) , $y = g^n \bmod p$

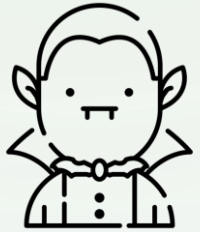
$K = g^{na+bm} \bmod n$

K= ?

Cert(T) , $y = g^n \bmod p$

Cert(A) , $x' = g^{m'} \bmod p$

$K_1 = g^{na+tm'} \bmod n$



Cert(T) , $x = g^m \bmod p$

Cert(B) , $y' = g^{n'} \bmod p$

$K_2 = g^{n't+bm} \bmod n$

$$\frac{K_1 * K_2}{(x' * y')^t} = \frac{(g^{na} * g^{tm'}) * (g^{n't} * g^{bm})}{g^{m't} * g^{n't}} = g^{na+bm} = K$$

Key Agreement

MTI Scheme – Burmester – Countermeasure

Break the symmetry

$Cert(A)\ ,\ x = g^m \bmod p$

$Cert(B)\ ,\ y = g^n \bmod p$

$K = y^a * P_B{}^m = g^{na+bm} \bmod n$

$K = h(y^a \mid P_B{}^m) = h(g^{na} \mid g^{bm})$

$K = x^b * P_A{}^n = g^{mb+an} \bmod n$

$K = h(P_A{}^n \mid x^b) = h(g^{an} \mid g^{mb})$

Key Agreement

$\phi(n)$ is the number of positive integers less than n that are coprime to n.
$\phi(10) = 4$ (1,3,7,9)
For prime p, $\phi(p) = p-1$

Self Certifying Sign – Givault Scheme

- No need to certification
- The Public key and ID authenticates each other implicitly.

$n = pq$
Public key e
Private key $d=e^{-1} \bmod \phi(n)$

Random a
Identification ID_A
Self Certifying P_A

$A = g^a \bmod n$

$P_A = (A - ID_A)^d \bmod n$

$B = g^b \bmod n$

$P_B = (B - ID_B)^d \bmod n$

Random b
Identification ID_B
Self Certifying P_B

$A = P_A^e + ID_A \bmod n$

$B = P_B^e + ID_B \bmod n$

Random u

$ID_A , P_A , x = g^u \bmod n$

$ID_B , P_B , y = g^v \bmod n$

Random v

$K = y^a * (P_B^e + ID_B)^u = g^{va+bu}$

$K = x^b * (P_A^e + ID_A)^v = g^{ub+av}$

Key Agreement

Group Key Agreement – burmester–desmedt (1994)

$x_1 , x_2 , \ldots , x_m$

$x_i = \left(\frac{b_{i+1}}{b_{i-1}}\right)^{a_i}$

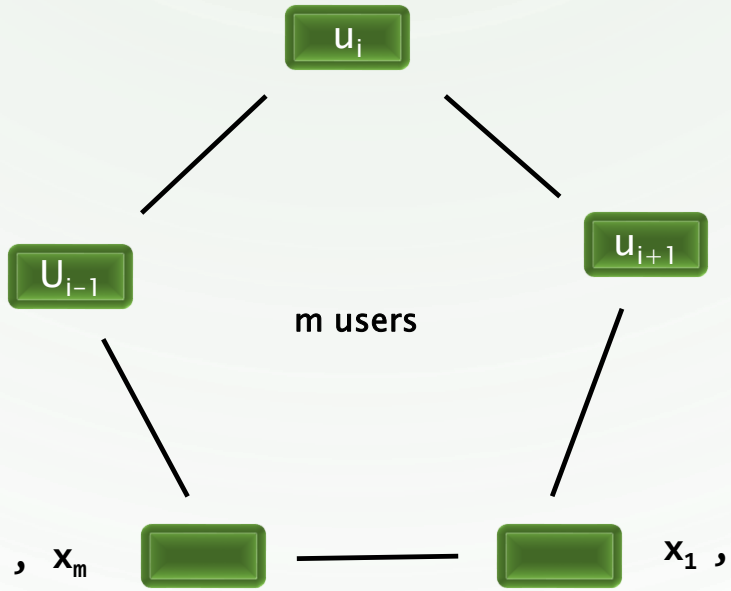
Random a_i

$b_i = g^{ai} \bmod n$

b_{i-1} , b_{i+1}

$K = (b_{i-1})^{m*ai}(x_i)^{m-1}(x_{i+1})^{m-2} \ldots (x_{i-2})^1$

$= g^{\sum_{i<j} a_i a_j}$



$x_{i-1} = \left(\frac{b_i}{b_{i-2}}\right)^{a_{i-1}}$

$x_1 , x_2 , \ldots , x_m$

Random a_{i-1}

$b_{i-1} = g^{ai-1} \bmod n$

b_{i-2} , b_i

Random a_{i+1}

$b_{i+1} = g^{ai+1} \bmod n$

b_i , b_{i+2}

$x_{i+1} = \left(\frac{b_{i+2}}{b_i}\right)^{a_{i+1}}$

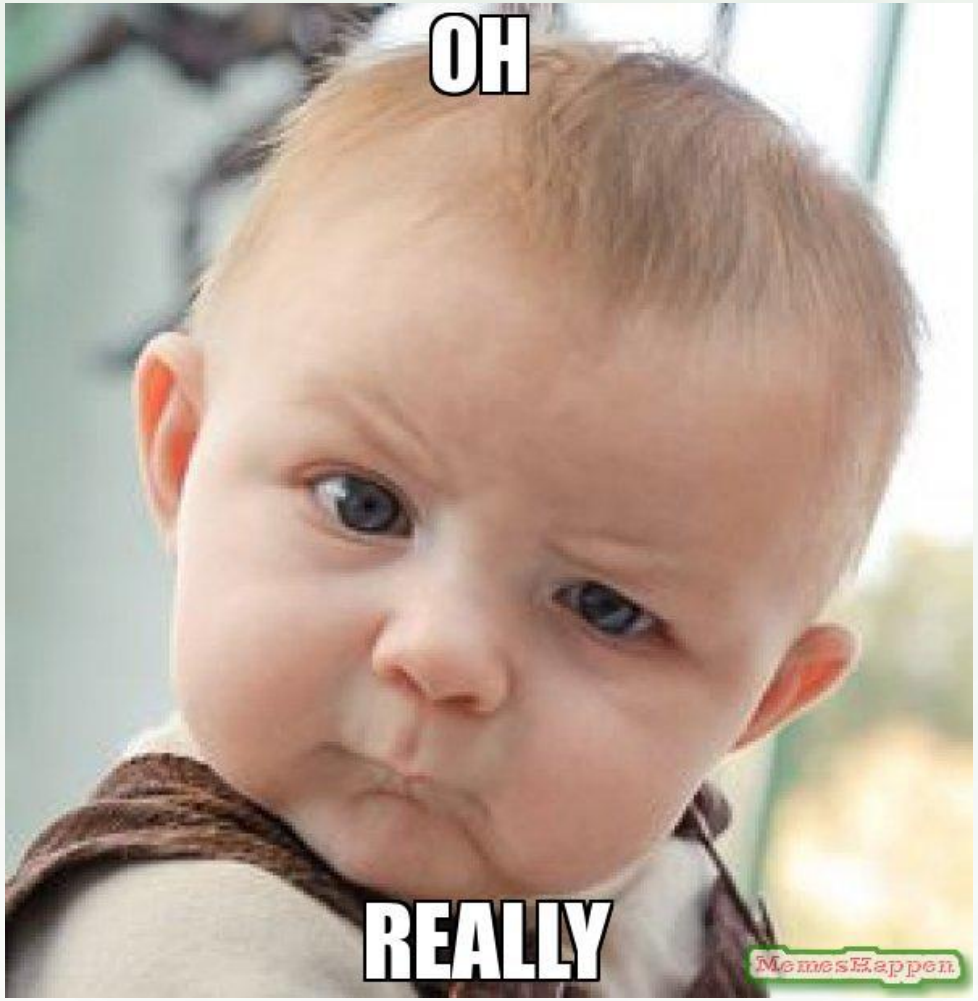
$x_1 , x_2 , \ldots , x_m$

$x_1 , x_2 , \ldots , x_m$

$x_1 , x_2 , \ldots , x_m$

Key Agreement

Group Key Agreement – burmester–desmedt (1994)



Key Agreement

Group Key Agreement – burmester–desmedt (1994)

$K = (b_{i-1})^{m*ai}(x_i)^{m-1}(x_{i+1})^{m-2} \dots (x_{i-2})^1$

$= g^{\sum_{i<j} a_i a_j}$

$K = g^{\sum_{i<j} a_i a_j} = g^{a_1a_2+a_2a_3+a_1a_3}$

x_1 , x_3

$x_2 = \left(\frac{b_3}{b_1}\right)\hat{}a_2$

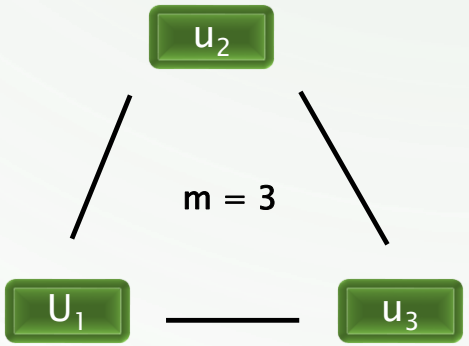
Random a_2

$b_2 = g^{a2} \bmod n$

b_1 , b_3

$K_1 = (b_3)^{3a1} (x_1)^2 (x_2)^1 = (b_3)^{3a1} \left(\frac{b_2}{b_3}\right)^{2a1} \left(\frac{b_3}{b_1}\right)^{1a2}$

$= (b_3)^{a1+a2} (b_2)^{2a1} (b_1)^{-a2} = (g^{a3})^{a1+a2} (g^{a2})^{2a1} (g^{a1})^{-a2}$



$x_1 = \left(\frac{b_2}{b_3}\right)\hat{}a_1$

x_2 , x_3

Random a_1

$b_1 = g^{a1} \bmod n$

b_2 , b_3

Random a_3

$b_3 = g^{a3} \bmod n$

b_1 , b_2

$x_3 = \left(\frac{b_1}{b_2}\right)\hat{}a_3$

x_1 , x_2

Key Agreement

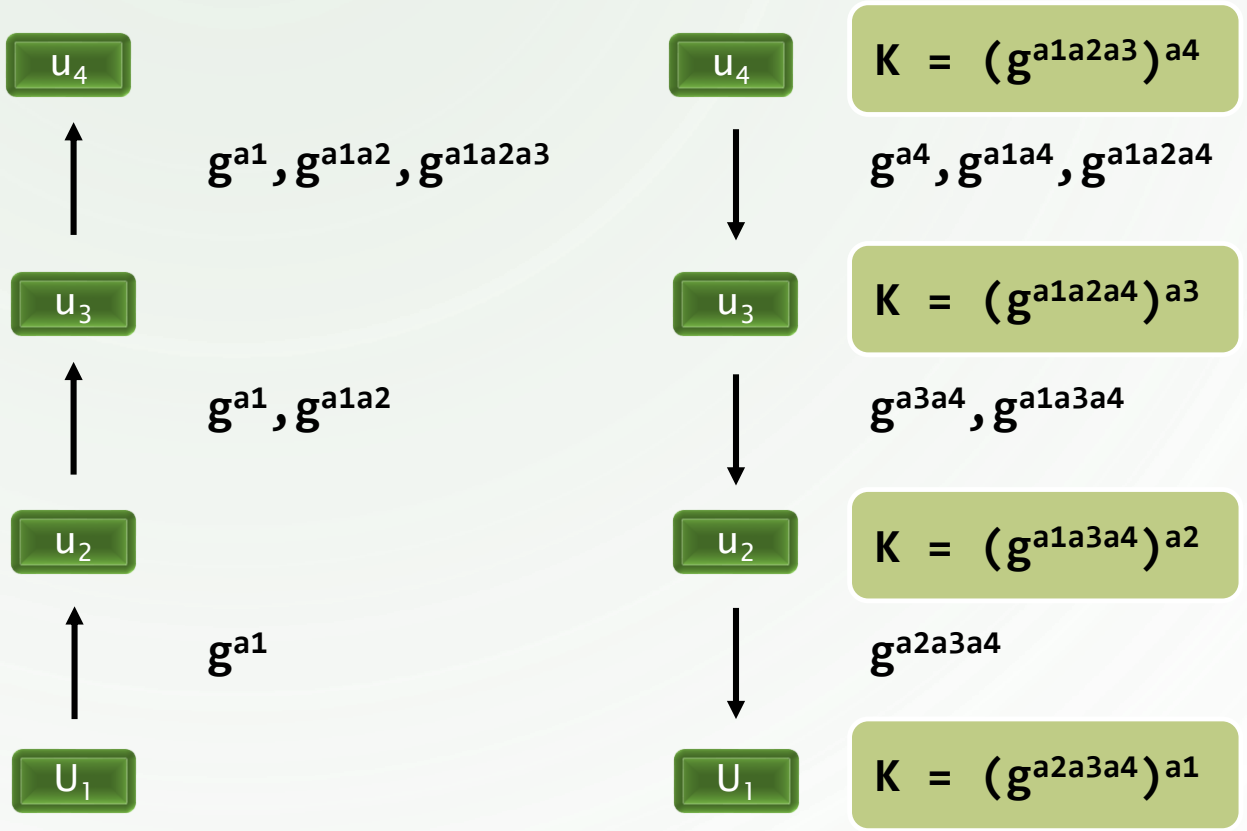
Group Key Agreement – Steiner

Description for
Number of users n = 4

Multiplicative group G
with a generator g
with order n
For n users

For authentication, each users must sign his message and pass his certificate

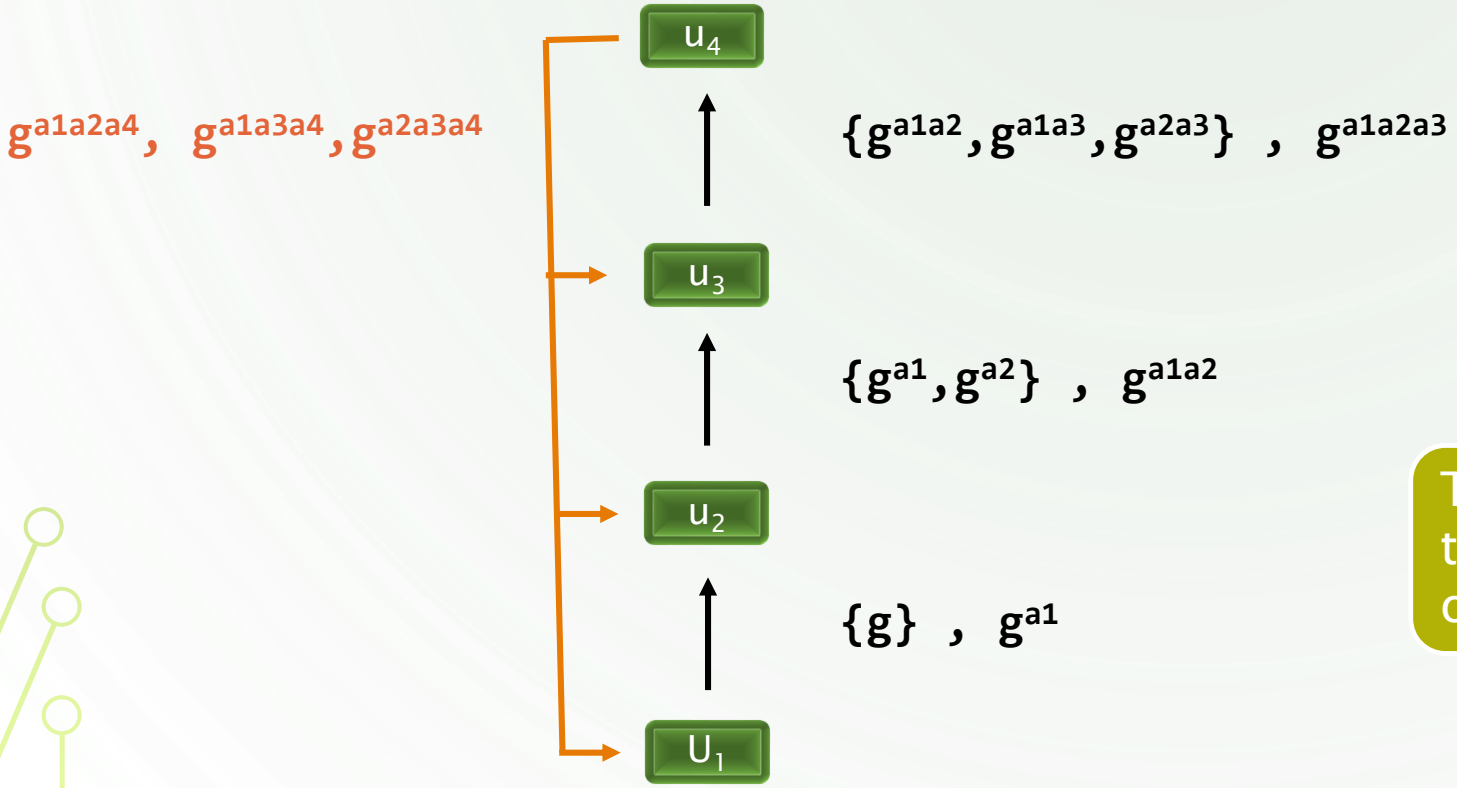
This scheme is generalized form of Diffie-Hellman



Key Agreement

Group Key Agreement – Steiner #2 (Cliques)

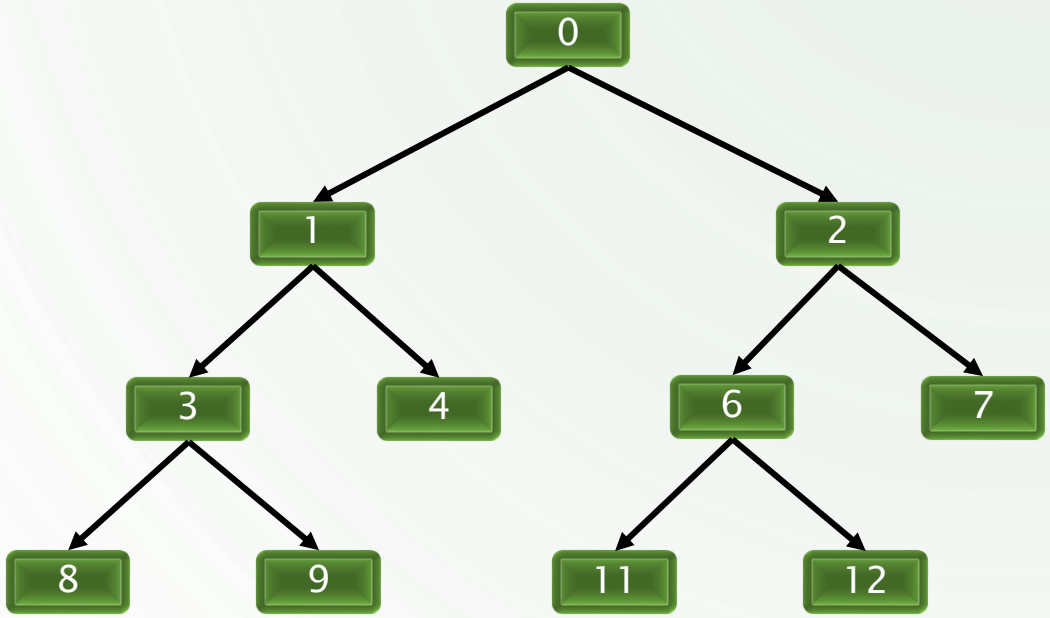
Description for
Number of users n = 4



This scheme supports the addition and removal of users from the group

Key Agreement

Group Key Agreement – Tree Based Group Diffie Hellman



V — Blinded Public Key (BK_V)
Private Key (K_V)

Each user knows private key of his parents

Each user knows public key of all others

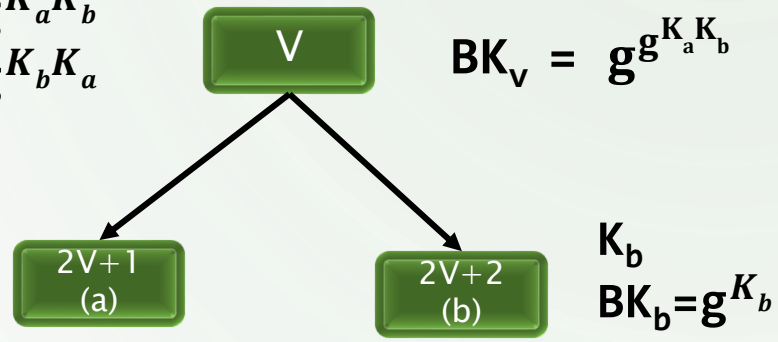
Common Key of All Users = Private Key of the Root

Key Agreement

Group Key Agreement – Tree Based Group Diffie Hellman

$K_v = BK_a^{K_b} = g^{K_a K_b}$

$K_v = BK_b^{K_a} = g^{K_b K_a}$



$BK_v = g^{g^{K_a K_b}}$

K_a

$BK_a = g^{K_a}$

K_b

$BK_b = g^{K_b}$

Private key of each non-leaf node would be generated by public key of one child and private key of another.

By continuing this process, each user can generate the private key of the root.

Ref

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