The finite difference equations governing the motion of the baroclinic (internal) modes, governing equations are expressed as

$$\delta_c^t\eta + \delta_f^x(\overline{D}_b^x U) + \delta_f^y(\overline{D}_b^y V) + \delta_f^\sigma(W) = 0 \tag{1}$$

$$\delta^t(\overline{\overline{D}_b^x U}^t) + \delta_b^x[\overline{(\overline{D}_b^x U)_f}^x \overline{U}_f^x] + \delta_f^y[\overline{(\overline{D}_b^y V)_b}^x \overline{U}_b^y] + \delta_f^\sigma(\overline{W}_b^x \overline{U}_b^\sigma) - \overline{\tilde{f}\overline{V}_f^y D_b}^x - \overline{(f\overline{V}_f^y D)_b}^x + g\overline{D}_b^x \delta_b^x \eta = \delta_b^\sigma\left[\overline{K_M}_b^x(\frac{\delta_f^\sigma U}{\overline{D}_b^x})^{n+1}\right] - \frac{g(\overline{D}_b^x)^2}{\rho_0}\delta_b^x\left[\Sigma_{zz=1}^k(\overline{\rho'_{zz}}_b^\sigma \Delta_b^\sigma \sigma_{zz})\right] + \frac{g\overline{D}_b^x\delta_b^x D}{\rho_0}\left[\Sigma_{zz=1}^k \overline{\sigma_{zz}}_b^\sigma \delta_b^\sigma(\overline{\rho'_{zz}})_b^x\right] + \delta_b^x(2A_M D\delta_f^x U^{n-1}) + \delta_f^y\left[\overline{\overline{A_M}_b^x}_b^y \overline{\overline{D}_b^x}_b^y(\delta_b^x V + \delta_b^y U)^{n-1}\right] \tag{2}$$

$$\delta^t(\overline{\overline{D}_b^y V}^t) + \delta_f^x[\overline{(\overline{D}_b^x U)_b}^y \overline{V}_b^x] + \delta_b^y[\overline{(\overline{D}_b^y V)_f}^y \overline{V}_f^y] + \delta_f^\sigma(\overline{W}_b^y \overline{V}_b^\sigma) + \overline{\tilde{f}\overline{U}_f^x D_b}^y + \overline{(f\overline{U}_f^x D)_b}^y + g\overline{D}_b^y \delta_b^y \eta = \delta_b^\sigma\left[\overline{K_M}_b^y(\frac{\delta_f^\sigma V}{\overline{D}_b^y})^{n+1}\right] - \frac{g(\overline{D}_b^y)^2}{\rho_0}\delta_b^y\left[\Sigma_{zz=1}^k(\overline{\rho'_{zz}}_b^\sigma \Delta_b^\sigma \sigma_{zz})\right] + \frac{g\overline{D}_b^y\delta_b^y D}{\rho_0}\left[\Sigma_{zz=1}^k \overline{\sigma_{zz}}_b^\sigma \delta_b^\sigma(\overline{\rho'_{zz}})_b^y\right] + \delta_b^y(2A_M D\delta_f^y V^{n-1}) + \delta_f^x\left[\overline{\overline{A_M}_b^x}_b^y \overline{\overline{D}_b^x}_b^y(\delta_b^x V + \delta_b^y U)^{n-1}\right] \tag{3}$$

$$\delta^t(\overline{TD})^t + \delta_f^x(\overline{T}_b^x U\overline{D}_b^x) + \delta_f^y(\overline{T}_b^y V\overline{D}_b^y) + \delta_f^\sigma(\overline{T}_b^\sigma W) = \delta_b^\sigma(\frac{K_H}{D^{n+1}}\delta_f^\sigma T^{n+1}) + \delta_f^x\left[\overline{A_H}_b^x \overline{H}_b^x \delta_b^x(T^{n-1} - T_{CLIM})\right] + \delta_f^y\left[\overline{A_H}_b^y \overline{H}_b^y \delta_b^y(T^{n-1} - T_{CLIM})\right] + \delta_f^\sigma R \tag{4}$$

$$\delta^t(\overline{SD})^t + \delta_f^x(\overline{S}_b^x U\overline{D}_b^x) + \delta_f^y(\overline{S}_b^y V\overline{D}_b^y) + \delta_f^\sigma(\overline{S}_b^\sigma W) = \delta_b^\sigma(\frac{K_H}{D^{n+1}}\delta_f^\sigma S^{n+1}) + \delta_f^x\left[(\overline{A_H}_b^x \overline{H}_b^x \delta_b^x(S^{n-1} - S_{CLIM})\right] + \delta_f^y\left[\overline{A_H}_b^y \overline{H}_b^y \delta_b^y(S^{n-1} - S_{CLIM})\right] \tag{5}$$

$$\overline{\delta^t(q^2D)}^t + \delta_f^x(\overline{U}_b^\sigma \overline{q^2}_b^x \overline{D}_b^x) + \delta_f^y(\overline{V}_b^\sigma \overline{q^2}_b^y \overline{D}_b^y) + \delta_c^\sigma(Wq^2) = \delta_b^\sigma\left[\overline{K_q}_f^\sigma(\frac{\delta_f^\sigma q^2}{D})^{n+1}\right] + \frac{2K_M}{D}\left[(\delta_b^\sigma \overline{U}_f^x)^2 + (\delta_b^\sigma \overline{V}_f^y)^2\right] + \frac{2g}{\rho_0}K_H\delta_b^\sigma\rho - \frac{2Dq^3}{B_1 l} + \delta_f^x\left[\overline{\overline{A_M}_b^x}_b^\sigma \overline{H}_b^x \delta_b^x(q^2)^{n-1}\right] + \delta_f^y\left[\overline{\overline{A_M}_b^y}_b^\sigma \overline{H}_b^y \delta_b^y(q^2)^{n-1}\right] \tag{6}$$

$$\overline{\delta^t(q^2lD)}^t + \delta_f^x(\overline{U}_b^\sigma \overline{q^2l}_b^x \overline{D}_b^x) + \delta_f^y(\overline{V}_b^\sigma \overline{q^2l}_b^y \overline{D}_b^y) + \delta_c^\sigma(Wq^2l) = \delta_b^\sigma\left[\overline{K_q}_f^\sigma(\frac{\delta_f^\sigma q^2 l}{D})^{n+1}\right] + lE_1\frac{K_M}{D}\left[(\delta_b^\sigma \overline{U}_f^x)^2 + (\delta_b^\sigma \overline{V}_f^y)^2\right] + \frac{lE_1 g}{\rho_0}K_H\delta_b^\sigma\rho - \frac{Dq^3}{B_1}\left\{1 + E_2\left[\frac{l}{\kappa D}\left(\frac{-1}{\sigma} + \frac{1}{1+\sigma}\right)\right]^2\right\} + \delta_f^x\left[\overline{\overline{A_M}_b^x}_b^\sigma \overline{H}_b^x \delta_b^x(q^2l)^{n-1}\right] + \delta_f^y\left[\overline{\overline{A_M}_b^y}_b^\sigma \overline{H}_b^y \delta_b^y(q^2l)^{n-1}\right] \tag{7}$$

$$\frac{\partial p}{\partial \sigma} = -\rho g D \tag{8}$$

$$\rho = \rho(T, S, p) \tag{9}$$

The finite difference equations governing the motion of the external modes, governing equations are expressed as

$$\delta^t\overline{\eta}^t + \delta_f^x(\overline{D}_b^x \overline{U}) + \delta_f^y(\overline{D}_b^y \overline{V}) = 0 \tag{10}$$

$$\delta^t\overline{(\overline{D}_b^x \overline{U})}^t + \delta_b^x(\overline{\overline{D}_b^x \overline{U}_f}^x \overline{\overline{U}}_f^x) + \delta_f^y(\overline{\overline{D}_b^y \overline{V}_b}^x \overline{\overline{U}}_b^y) - \overline{\tilde{\overline{f}}\overline{\overline{V}}_f^y D_b}^x - \overline{f\overline{\overline{V}}_f^y D_b}^x + g\overline{D}_b^x \delta_b^x\eta = \delta_b^x(2\overline{A_M}D\delta_f^x\overline{U}^{n-1}) + \delta_f^y\left[\overline{\overline{\overline{A_M}}_b^x}_b^y \overline{\overline{D}_b^x}_b^y(\delta_b^x\overline{V} + \delta_b^y\overline{U})^{n-1}\right] + \phi_x \tag{11}$$

$$\delta^t\overline{(\overline{D}_b^y \overline{V})}^t + \delta_f^x(\overline{\overline{D}_b^x \overline{U}_b}^y \overline{\overline{V}}_b^x) + \delta_b^y(\overline{\overline{D}_b^y \overline{V}_f}^y \overline{\overline{V}}_f^y) + \overline{\tilde{\overline{f}}\overline{\overline{U}}_f^x D_b}^y + \overline{f\overline{\overline{U}}_f^x D_b}^y + g\overline{D}_b^y \delta_b^y\eta = \delta_b^y(2\overline{A_M}D\delta_f^y\overline{V}^{n-1}) + \delta_f^x\left[\overline{\overline{\overline{A_M}}_b^x}_b^y \overline{\overline{D}_b^x}_b^y(\delta_b^x\overline{V} + \delta_b^y\overline{U})^{n-1}\right] + \phi_y \tag{12}$$

where

$$\tilde{f} = \frac{1}{\Delta x\Delta y}[\overline{V}_f^y\delta_c^x(\Delta y) - \overline{U}_f^x\delta_c^y(\Delta x)] \tag{13}$$

$$A_M = \frac{1}{2}C\Delta x\Delta y\sqrt{(\delta_f^x U)^2 + \frac{1}{2}[\delta_c^x(\overline{U}_f^x)_c + \delta_c^x(\overline{V}_f^y)]^2 + (\delta_f^y V)^2} \tag{14}$$

$$A_H = A_M * TPRNI \tag{15}$$

$$K_M = qlS_M \tag{16}$$

$$K_H = qlS_H \tag{17}$$

$$K_q = qlS_q \tag{18}$$

$$\tilde{\overline{f}} = \frac{1}{\Delta x\Delta y}\left[\overline{\overline{V}}_f^y\delta_c^x(\Delta y) - \overline{\overline{U}}_f^x\delta_c^y(\Delta x)\right] \tag{19}$$

$$\phi_x = -WU(0) + WU(-1) - \frac{g(\overline{D}_b^x)^2}{\rho_0}\Sigma_{z=1}^k\left\{\left[\Sigma_{zz=1}^k\delta_b^x\overline{\rho'_{zz}}_b^\sigma\Delta_b^\sigma\sigma_{zz}\right]_z\Delta\sigma_z\right\} + \frac{g\overline{D}^x\delta_b^x D}{\rho_0}\Sigma_{z=1}^k\left\{\left[\Sigma_{zz=1}^k\overline{\sigma_{zz}}_b^\sigma\delta_b^\sigma(\overline{\rho'_{zz}})_b^x\right]_z\Delta\sigma_z\right\} + G_x \tag{20}$$

$$\phi_y = -WV(0) + WV(-1) - \frac{g(\overline{D}_b^y)^2}{\rho_0}\Sigma_{z=1}^k\left\{\left[\Sigma_{zz=1}^k\delta_b^y\overline{\rho'_{zz}}_b^\sigma\Delta_b^\sigma\sigma_{zz}\right]_z\Delta\sigma_z\right\} + \frac{g\overline{D}^y\delta_b^y D}{\rho_0}\Sigma_{z=1}^k\left\{\left[\Sigma_{zz=1}^k\overline{\sigma_{zz}}_b^\sigma\delta_b^\sigma(\overline{\rho'_{zz}})_b^y\right]_z\Delta\sigma_z\right\} + G_y \tag{21}$$