# Simulation Estimators – Part II

## Summer School in Structural Estimation

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# Outline

1. Identification Again
2. Indirect Inference
3. Tips

# Identification checks

- There are several ways to check whether your model is well identified
- Check #1: Check the standard errors
  - Huge standard errors are usually a symptom of poor identification
  - The parameters' standard errors depend on $\partial g\left(y_{is}\left(\theta\right)\right)/\partial\theta$, the sensitivity of moments to parameters
  - If the sensitivity is low, the derivative will be near zero, which will produce huge standard errors for the structural parameters
- Check #2: Does the Jacobian matrix $\partial g\left(y_{is}\left(\theta\right)\right)/\partial\theta$ have full rank?
  - If not, your model is not locally identified
  - This is almost the same as a standard error check, but not quite.

# Computer output from an estimation gone wrong

```
 Parameter estimates:
&        0.100 &        0.231 &        0.490 &        0.756 &        0.518 &        0.128 &        0.000 &        0.008 &
&(       2.033)&(       0.013)&(       0.000)&(       0.073)&(       0.264)&(       0.000)&(       0.004)&(       0.000)&(

 Gradient matrix:

      0.00000000      0.00000000      1.02758239      0.00000000      0.00000000     38.25073505      0.00000000      0.00000000
      0.00000000      0.00000000      0.28512603      0.00000000      0.00000000      4.82100673      0.00000000      0.00000000
      0.00000000      0.00000000     -9.97948112      0.00000000      0.00000000    370.80545008      0.00000000      0.00000000
      0.00000000      0.00000000      0.40639694      0.00000000      0.00000000      1.02101543      0.00000000      0.00000000
      0.00000000      0.00000000      0.84485716      0.00000000      0.00000000      0.04210265      0.00000000      0.00000000
      0.00000000      0.00000000   -138.58951690      0.00000000      0.00000000      0.00889156     -0.00000004      0.00000000
      0.00000000      0.05056345      0.03244400     -0.16356671      0.00000000     -5.85773601     -2.79010277     -2.79010369
      0.00000000      0.01667884      0.00342592     -0.00333117      0.00000000     -0.09697136     -0.07479289     -0.07479291
      0.00000000     -0.29607166      1.09787363      0.14226033      0.00000000    111.43914416      0.00006803      0.00000173
     -2.06371354      3.34618706     10.68850788    -11.02873288     -0.77796138    -74.85216998    736.96355852   -153.90999884
     -0.01039442      0.02335486     -0.62675135     -0.05792594     -0.00419307     -6.10626047      3.66442260     -0.78400489
      0.00000000      0.04861350     -3.09825605      0.11134717      0.00898020    -25.21239839      2.03722313      1.89152864
      0.00000000      0.00122617     -0.74021211     -0.00036989      0.00009098     -0.66126219     -0.00761401     -0.00819525
      0.00000000      0.08164404     -1.21010451     -0.13848512     -0.00552660     -3.77971077     -2.64720025     -2.53142955
      0.00000000      0.00444937      0.02217510     -0.00057544      0.00000972     -0.26535678     -0.01661087     -0.01716714
      0.00000000      0.00038676      0.00024816     -0.00125113      0.00000000     -0.04480597     -0.02134156      0.11961819
```

# Identification checks

- Check # 3: Can estimation recover the true parameter values?
  - Simulate a “fake” dataset off the model
  - Estimate the model, treating the fake data as if it were real data
  - Does the estimator recover the true, known parameter values?
- Check #4: Start searching from different initial parameter guesses
  - Should reach same final estimate regardless of initial guess
  - If not: Coding errors? Stuck in local min? Model not well identified?
- Check #5: Babysit your code
  - Some parameters converge faster than other. Keep track of this.
  - If one or two parameters are changing drastically during the estimation, you have a problem.

# Local identification diagnostic

- Andrews, Gentzkow and Shapiro (2017) provide a local diagnostic that measures the sensitivity of $\hat{\boldsymbol{\theta}}$ to the estimated moments, $\hat{\boldsymbol{g}}$.
- Let $\boldsymbol{G}$ be the Jacobian of $g$ w.r.t $\theta$.
- The diagnostic is (and it should look familiar):

$$\boldsymbol{S} = -(\boldsymbol{G}'\boldsymbol{\Xi}\boldsymbol{G})^{-1}\boldsymbol{G}'\boldsymbol{\Xi}.$$

- This measure is not scale invariant, so they propose the following normalization:

$$\boldsymbol{S}_{l,p}\sqrt{\frac{\boldsymbol{\Lambda}_{l,l}}{\boldsymbol{V}_{p,p}}},$$

  where $\boldsymbol{V}_{p,p}$ a moment variance and $\boldsymbol{\Lambda}_{l,l}$ is a parameter variance.
- This diagnostic captures both
  - the steepness of the gradients
  - the precision of the estimation of the moments that identify the parameters

# Identification has more than one meaning

- A parameter can be identified (in the formal statistical sense) without being economically interesting
- Two examples:
  - Regression of endogenous $Y$ on endogenous $X$
  - Regression of endogenous $Y$ on exogenous $X$ (without a clear and interesting economic mechanism)
- In both cases, the econometric objective function has a unique minimum.
- Your ultimate goal: Identify **economically interesting** parameters
  - The parameters may be elasticities defining causal effects
  - But they need not be!
  - **Not all economically interesting parameters are causal elasticities**

# “Identification is not causality, and vice versa”

- (Kahn and Whited 2018), Review of Corporate Finance Studies
- Exogenous variation is:
  - Always necessary to identify a causal relation
  - Never sufficient for identifying an economically interesting parameter
    - You also need an economic model (either mathematical or verbal)
  - Only sometimes necessary to identify an economically interesting parameter
- Interesting parameters can sometimes be identified without exogenous variation
  - This is often what’s going on in structural corporate finance.

# Do you need exogenous variation to do SMM?

- Causal elasticities are not always helpful in structural estimation! (Kahn and Whited 2018)
- Compare SMM estimators of a simple investment model
  - Approach #1: Target moments that measure a causal elasticity
  - Approach #2: Target moments that measure an endogenous elasticity
- Result: The performance of the two approaches is nearly identical!
- In both cases,
  - Multiple moments and multiple parameters interact
  - Identification leans on the structure of the model
- Identification **always** leans on assumptions
  - Even in reduced-form papers with exogenous variation
  - Those papers often lean on a “verbal” economic model

# Causal elasticities can be helpful moments to target . . .

- **If** the “exogenous variation” is the same in the model and the data.
  - For example, do not call an endogenous price in the real data “exogenous,” and then model an exogenous price.
- **If** you want to understand what fundamental forces affect the causal elasticity.

# Outline

# Economic Models and Auxiliary Models

- If a model does not provide a closed form likelihood, or the simulated likelihood needs to be calculated for more than two variables (and two is a stretch), then you can use an auxiliary model to estimate the model parameters.
- A likelihood is a description of the true data generating process.
- An auxiliary model is an approximation of the true data generating process.
  - What if you have a DSGE model of the macroeconomy. The likelihood is impossible to solve for, but a VAR might describe the data approximately.
  - What if you have a model of investment spikes or infrequent price adjustments. The auxiliary model could be a duration model.

# Auxiliary Models

- In practice, the auxiliary model is itself characterized by a set of parameters.
- These parameters can be estimated using either the observed data or the simulated data.
- Indirect inference chooses the parameters of the underlying economic model so that these two sets of estimates of the parameters of the auxiliary model are as close as possible.

# Auxiliary Models

- You should be able to match exactly if you have as many parameters in the auxiliary model as you do in the economic model.
- But the number of auxiliary parameters can be greater than the number of economic parameters.
- To the extent that the parameters of the auxiliary model are functions of moments of the data, indirect inference can be thought of as a superset of SMM.
- It falls under the category of a simulated minimum distance (SMD) estimator.

# Estimation: Minimum Distance Style

- Notation:
  - $x_N$ is a data matrix of length $N$
  - $x_N^s$ is a simulated data matrix of length $N$ from simulation $s$, $s = 1, \dots, S$.
  - $\theta$ a vector of **auxiliary** model parameters estimated with **real** data.
  - $\theta^s$ a vector of **auxiliary** model parameters estimated with **simulated** data.
  - $b$ is the vector of parameters from the **economic** model.

- Without loss of generality, the parameters of the auxiliary model can be represented as the solution to the maximization of a criterion function

$$\theta_N = \arg\max_{\theta} J_N \left( x_N, \theta \right),$$

- Examples?

# Estimation: Minimum Distance Style

- Construct $S$ simulated data sets based on a given parameter vector, $b$.
- For each of these data sets, estimate $\theta^s$ by maximizing an analogous criterion function

$$\theta_N^s\left(b\right) = \arg\max_{\theta} J_N\left(x_N^s, \theta^s\left(b\right)\right),$$

- Note that the $\theta_N^s\left(b\right)$, as explicit functions of the structural parameters, $b$.
- The inverse of this function is what Gourieroux and Monfort call a “binding” function.

# Estimation: Minimum Distance Style

- The indirect estimator of $b$ is then defined as the solution to the minimization of

$$\begin{aligned}\hat{b} &= \arg\min_{b} \left[\theta_N - \frac{1}{S}\sum_{h=1}^{S}\theta_N^s(b)\right]' \hat{W}_N \left[\theta_N - \frac{1}{S}\sum_{h=1}^{S}\theta_N^s(b)\right] \\ &\equiv \arg\min_{b} \hat{G}_N' \hat{W}_N \hat{G}_N\end{aligned}$$

- $\hat{W}_N$ is a positive definite matrix that converges in probability to a deterministic positive definite matrix $W$.
- As in GMM the optimal weight matrix is the inverse of the covariance matrix of $\theta$.
- The main difference between SMM and this flavor of indirect inference is that the former uses **moments** and the latter uses **functions of moments**.

# Inference: Minimum Distance Style

- The indirect estimator is asymptotically normal for fixed $S$. Define $J \equiv \text{plim}_{N\to\infty}\left(J_N\right)$. Then

$$\sqrt{N}\left(\hat{b}-b\right) \xrightarrow{d} \mathcal{N}\left(0, \text{avar}(\hat{b})\right)$$

  where

$$\text{avar}(\hat{b}) \equiv \left(1+\frac{1}{S}\right)\left[\frac{\partial J}{\partial b \partial \theta'}\left(\frac{\partial J}{\partial \theta}\frac{\partial J'}{\partial \theta}\right)^{-1}\frac{\partial J}{\partial \theta \partial b'}\right]^{-1}.$$

- The technique provides a test of the overidentifying restrictions of the model, with

$$\frac{NS}{1+S}\hat{G}_N'\hat{W}_N\hat{G}_N$$

  converging in distribution to a $\chi^2$, with degrees of freedom equal to the dimension of $\theta$ minus the dimension of $b$.

# Outline

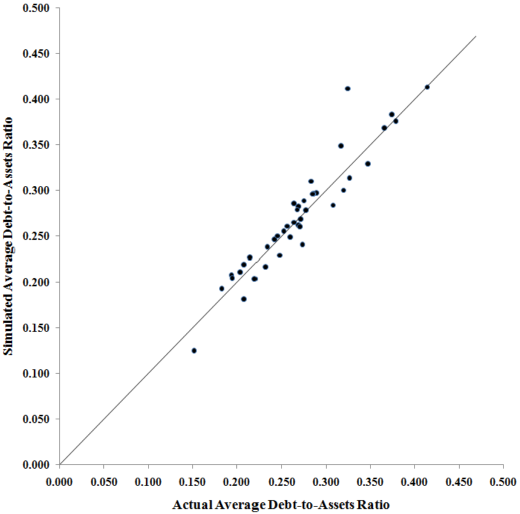
# Miscellaneous Pieces of Advice from the Trenches

- Heterogeneity
- The black box problem
- Choosing the weight matrix
- Computation

# Firm Heterogeneity

- It is really hard to address the issue of firm heterogeneity using SMM.
- This is perhaps the biggest drawback of this technique.
- The models we simulate are usually of a single firm or at best an industry equilibrium with very limited heterogeneity.
- So you have to suck as much heterogeneity out of your data as you can before you can have any hope of fitting the model to the data.
  - Firm and time fixed effects
- You can also do sample splits. This used to be computationally infeasible, but . . .

# Sample Splits[1]




[1] DeAngelo, DeAngelo and Whited (2011)

# Do Not Construct a Black Box

- More parameters $\neq$ a better model!!!!!!
- Different features of the data should change when underlying parameters change.
- This is less likely to happen in a model with many extraneous parameters.
- If the author cannot clearly explain which features of the data identify each parameter, the paper / job market candidate is a “reject”
- Structural estimation should not be a black box.

# How Not to Construct a Black Box

- The following is eerily the same as, but not taken from, the material in this video by Michael Keane:

  `https://www.youtube.com/watch?v=0hazaPBAYWE`

- If you don't believe me, maybe you will believe him.

# How Not to Construct a Black Box

- Start simple!!!
- Make sure the simple model is **right**.
- Figure out where the simple model succeeds and fails.
- Add a feature that will help you answer the question you want to answer.
- Make sure the slightly more complicated model is right.
- Figure out where the more complicated model succeeds and fails.
- Converge.

# The question comes first — Not the model

- Before going structural, convince yourself that a structural approach is absolutely necessary.
- Are there data limitations?
- Are you interested in something besides a causal elasticity?

# Use the right weighting matrix

- Many structural estimation papers use odd weighting matrices.
- Identity weight matrix
  - This choice mechanically puts more weight on moments that are larger in absolute value.
  - Not an economically sensible choice.
  - Bazdresch, Kahn and Whited (2018) note poor finite sample performance.

# Use the right weighting matrix

- Inverse of the squared moments on the diagonal
  - This choice minimizes the percentage differences in the moments.
  - This choice mechanically puts more weight on moments that are smaller in absolute value.
  - Not an economically sensible choice.

# Use the right weighting matrix

- Inverse moment variance along the diagonal
  - This choice puts the least weight on moments that are estimated with the least precision.
  - A statistically more or less sensible choice.
  - The old version of Bazdresch et al. (2018) (before the referees got to it) note poor finite sample performance.

# Use the right weighting matrix

- Inverse clustered moment covariance matrix
  - This choice minimizes overall model error.
  - Takes into account moment covariances.
  - Bazdresch et al. (2018) note good finite sample performance.
- If at all possible, do this.

# Do not treat the process of estimating the model mechanically!

- First, try to calibrate the model by hand.
- You will learn about which parts of the model are useful for understanding which parts of the data.
- You will learn a lot about the economics of the model.
- You will also learn a lot about identification.
- You will end up with a good starting value for your estimation.
- Only after you have done this should you start your estimation running.

# You are going to have to minimize an objective function:

- You can't use a gradient-based method unless you have a closed-form GMM or MLE problem
- Multi-start algorithms, combined with Nelder Meade tend to work very poorly, except, maybe.
- Tiktak algorithm (Arnoud, Guvenen and Kleineberg 2017), but it uses Dropbox for parallelization . . .
- Use the simulated annealing (SA), particle swarm (PS), or differential evolution algorithm (DE) to avoid local minima
- DE and PS can be parallelized, but I have found that they can converge way too fast or not at all, depending on technical parameter settings. In technical terms, they're downright squirrelly.
- Use the same seed for the random-number generator each time you simulate data off the model. Do not mess up this step.

# Software

- Do not use Matlab, R, Python, Octave, Gauss, or any other *interpreted* language.
- They are too slow!
- To estimate a model, you usually have to solve it $\sim$50,000 times.
- Use a compiled language: C, C++, Fortran
- People tell me that Numba $+$ Python is fast but unstable for large problems.
- My initial experimentation with Julia has not been bad. Still not as fast as Fortran.
- Learn how to exploit multiple processors, a graphics card, a supercomputer, . . . .

# Remember that you are actually doing estimation

- Get the standard errors right.
- The actual data are usually not i.i.d.
- When estimating the covariance matrix for empirical moments, you must take into account
  - Heteroskedasticity
  - Time-series autocorrelation
  - Cross-sectional correlation
  - Serial correlation, including correlation across moments.
- You know what to do!

Andrews, I., Gentzkow, M.„ Shapiro, J.M., 2017. Measuring the sensitivity of parameter estimates to sample statistics. Quarterly Journal of Economics 132, 1553–1592.

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