Macroeconomic Theory II (Spring 2022)
Assignment 5 Solution

Consider the Aiyagari model we studied in the class. Suppose preferences are given by

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

Individuals are ex-ante identical, infinitely lived, but are subject to idiosyncratic productivity shocks. However, in the aggregate, by law of large numbers, these shocks wash out and there is no aggregate uncertainty. Suppose that the productivity process follows an AR(1) process:

$$\log \epsilon' = \rho \log \epsilon + \nu$$

where ν is normally-distributed with mean 0 and variance σ_ν^2. The budget constraint of the individual is as follows:

$$\begin{aligned} c + a' &= \omega\epsilon + (1+r)a \\ a' &\geq -b \end{aligned}$$

where c is consumption, a is asset holding, ω is wage per unit of efficiency, r is the rental rate of capital after depreciation. There are infinitely many firms, all identical, and hire capital and labor at competitive markets. There are infinitely many firms, all identical, and hire capital and labor at competitive markets. They have CRS technology to produce output: $Y = F(K,N) = K^\alpha N^{1-\alpha}$. Capital depreciates at the rate δ.

(a) Define a recursive competitive equilibrium for this economy.

A recursive competitive equilibrium for this economy is set of

- policy functions: $\{g_c(a,\epsilon;\lambda), g_a(a,\epsilon;\lambda)\}$ for HH
- policy functions: $\{g_N(\lambda), g_K(\lambda)\}$ for firm
- value function: $V(a,\epsilon;\lambda)$
- price functions: $\{\omega(\lambda), r(\lambda)\}$
- LOM: $G_\lambda(\lambda)$

such that

- Given price functions and LOM, policy functions for HH solve HH's problem

$$
\begin{aligned}
V(a,\epsilon;\lambda) &= \max_{c,a'} u(c) + \beta \mathbb{E} V(a',\epsilon';\lambda') \\
u(c) &= \frac{c^{1-\sigma}-1}{1-\sigma}
\end{aligned}
$$

subject to the budget constraint

$$
\begin{aligned}
c + a' &= \omega\epsilon + (1+r)a \\
a' &\geq -b
\end{aligned}
$$

- Given price functions and LOM, policy functions for Firm solve Firm's problem

$$
\max_{N,K} \quad F(K,N) - \delta K - \omega N - rK
$$

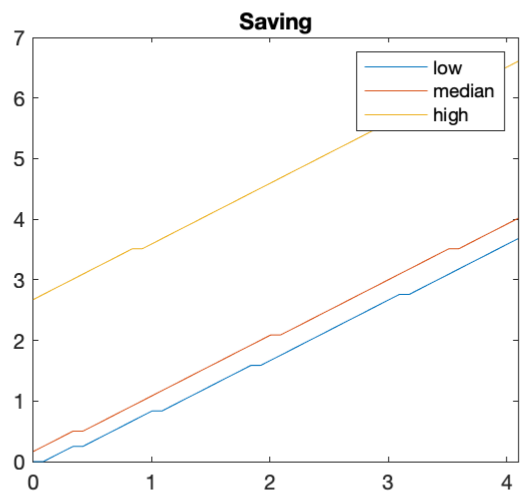
- Market clear

$$
\begin{aligned}
g_K(\lambda) &= \int_{A\times E} g_a(a,\epsilon;\lambda) d\lambda \\
g_N(\lambda) &= \int_{A\times E} \epsilon d\lambda
\end{aligned}
$$

- Consistency

$$
G_\lambda(\lambda)(A,E) = \int I\{g_a(a,\epsilon;\lambda) \in A\} \sum_{\epsilon' \in E} \pi(\epsilon'|\epsilon) d\lambda
$$

(b) Set $\beta = 0.96$, $\delta = 0.065$, $\sigma = 2$, $b = 0$, $\rho = 0.9$ and $\sigma_\nu = 0.2$. Use the Tauchen method I posted on Canvas to discretize the productivity process (you can start by using 5 grid points for the productivity shock and later check the robustness of the results when you increase the number of grid points for the productivity shock). Given the parameter values, solve the model. Plot the policy functions for labor supply and saving as a function of current asset holdings for three different productivity shock (lowest, median and highest productivity shock).



(c) Resolve the model by changing the persistency of the productivity shock $\rho = \{0, 0.5, 0.9\}$. Report the changes in equilibrium interest rate, wage, aggregate output, aggregate capital, average welfare, amount of precautionary saving, income inequality, wealth inequality and welfare inequality, where inequality is measured by the coefficient of variation of the variable of interest. Explain the reason for the changes in the results.

Macro variables:

Rho	r	Wage	Output	K	Welfare	Precautionary saving
0	0.087018	0.98146	1.4994	3.2549	6.2114	-2.141
0.5	0.042276	1.1653	1.7965	5.5264	7.3419	0.13048
0.9	0.021118	1.2985	2.3256	8.9114	6.8133	3.5155

Inequality:

Rho	Income	Wealth	Welfare
0	1.817	1.6218	0.5701
0.5	12.3188	11.1399	2.741
0.9	82.9748	70.0373	36.1446

HHs respond more to the more persistent shocks since they change more the expected future wage income.

(d) Resolve the model by changing the variance of the productivity shock: $\sigma_\nu = \{0.1, 0.2, 0.3\}$, and report the same statistics as in 1c. Explain the reason for the changes in the results.

Macro variables:

St	r	Wage	Output	K	Welfare	Precautionary saving
0.1	0.035361	1.2042	1.885	6.198	6.7271	0.80205
0.2	0.021118	1.2985	2.3256	8.9114	6.8133	3.5155
0.3	0.0085225	1.4036	3.1246	14.0243	6.7296	8.6284

Inequality:

St	Income	Wealth	Welfare
0.1	43.2837	38.62	14.2006
0.2	82.9748	70.0373	36.1446
0.3	188.8321	152.2522	79.0232

Higher shock variance increases the income uncertainty, so it increases precautionary saving.

(e) Resolve the model by changing risk aversion: $\sigma = \{1, 2, 5\}$, and report the same statistics as in 1c. Explain the reason for the changes in the results.

Macro variables:

Sigma	r	Wage	Output	K	Welfare	Precautionary saving
1	0.034544	1.209	2.1654	7.1785	9.9404	1.7826
2	0.021118	1.2985	2.3256	8.9114	6.8133	3.5155
5	-0.0042884	1.5424	2.7625	15.0158	2.7858	9.6199

Inequality:

Sigma	Income	Wealth	Welfare
1	75.0619	62.5929	64.4331
2	82.9748	70.0373	36.1446
5	97.4422	84.5062	41.1464

Higher risk aversion increases precautionary saving.

(f) Resolve the model by changing the borrowing limit: $\rho = \{1, 0.5, 0\}$, and report the same statistics as in 1c. Explain the reason for the changes in the results.

Macro variables:

B	r	Wage	Output	K	Welfare	Precautionary saving
-1	0.021955	1.2923	2.3145	8.7838	6.7125	3.3879
-0.5	0.021638	1.2946	2.3187	8.8317	6.7599	3.4358
0	0.021118	1.2985	2.3256	8.9114	6.8133	3.5155

Inequality:

B	Income	Wealth	Welfare
-1	90.3666	76.743	39.9963
-0.5	86.6854	73.416	38.0071
0	82.9748	70.0373	36.1446

Tighter borrowing limit increases precautionary saving, since the HHs are more likely to hit the borrowing limit.