

Macroeconomic Theory II: Assignment 3

Due by Friday, February 18 @9am

Consider the stochastic neoclassical growth model we studied in class. Set the growth rates for population and technology to 0. Assume that $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$, $f(z, k, n) = zk^\alpha n^{1-\alpha}$ and $\log(z') = \rho \log(z) + \epsilon$ where ϵ is normally distributed with mean zero and variance σ_ϵ^2 . Set $\sigma = 2$, $\rho = 0.975$, $\sigma_\epsilon = 0.007$.

1. Assume a period in the model corresponds to a quarter in the data. Calibrate δ , β , and α to match investment-output ratio of 25%, share of labor cost in the production as 2/3, capital-to-quarterly output ratio of 12, and consumption-output ratio of 75%.
2. Compute the deterministic steady-state values for capital, consumption, output and investment.
3. Linearize the model around the steady-state (if you feel more comfortable you can use the log-linearization method). Show your equations.
4. Solve the policy function for capital and consumption using linearized (or log-linearized) model. The solution of linearized model will give you the rules for consumption and capital as deviations from steady-states. To find the policy functions in levels, convert these rules using the definitions for the deviations from steady-state.
5. Solve the model using value function iteration method.
 - (a) To do this you need to convert the AR(1) process into Markov process. I uploaded a code which does it for you. The method is called Tauchen method. It takes the inputs N , number of grid points for the shock, μ , mean value of the shock, ρ , persistency of the shock, σ , standard deviation of the shock, and m , the maximum distance to the mean in terms of multiples of the standard deviation. You can set $N = 7$ and $m = 3$. The other inputs are the parameters of the model. The output

of the code is Z , the discretized values for the shock, Z_{prob} , the transition matrix for the shock, where entry (i, j) of the matrix is showing the transition probability from state z_i to state z_j . Be careful that the levels you obtained using the matlab code, Z , are the log values of the shock. To covert them into actual levels, take the exponent of Z .

- (b) When discretizing the capital, center the grid points around k^* , steady-state capital level, and set the lower bound of capital to $\underline{k} = (1 - \kappa)k^*$ and upper bound to $\bar{k} = (1 + \kappa)k^*$. You can start by using $N_k = 200$ grid points for k and set $\kappa = 0.4$.
- (c) Solve the model using value function iteration method. Compute the associated policy functions and value functions. Plot them against capital for different values of the shock.
- (d) Report the robustness of your results with respect to N_k and κ . Try two other values, say $N_k \in \{100, 500\}$ and $\kappa \in \{0.3, 0.8\}$ and plot the policy function for capital for the mean value of the shock.
- (e) Compare the policy functions for capital obtained using linearized model and value function iteration method. Discuss the goodness of approximation in the linearized model.
- (f) Do the same comparison of policy functions for $\sigma = 5$ and $\sigma = 10$. Discuss the sensitivity of approximation with respect to σ .

HINT: To check the accuracy of your solutions for both methods, I suggest you to start with the model when $\sigma = 1$ and $\delta = 1$. In Assignment 2, you already obtained the policy functions and value functions for this model analytically. You can use that analytical solution to check the accuracy of your solutions. And then, you can change the parameters to the ones in this homework, and run your code.