Summer School in Structural Estimation
Two Extra Speed Tricks

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Introduction

- There are two places one can use interpolation to make one's solutions or simulations or both more accurate.
- I will start with simulation interpolation.
- Then I will move on to interpolation when one is actually solving the model.

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Simulation Interpolation

- Let's say we have solved the basic model from the first section, but we would like to simulate it using more values for the z shock than are in the grid used to solve the model.
- Make a set of z shocks (and the accompanying transition matrix), using many more points than you did for the model solution. It's easiest if the endpoints of the grid are the same.
- Then start simulating. If a simulated shock lands between the two original shocks, then collect the two possible new capital stocks, and linearly interpolate between them.
- But now you are off of the original capital stock grid, so you will have to do the next interpolation in more both the k and the z dimension.
- If you have more than one choice variable, you will have to interpolate in more than one dimension, which is ugly but easy.

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Value Function Interpolation

- In our first simple model we used a finite number points in our capital stock grid.
- However, who is to say that those are the only those particular points that might be *optimal*?
- It might be optimal to be somewhere in between two of the points.
- The solution is to have more points in the *policy* grid than in the *state* grid.

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Value Function Interpolation

- Let's suppose that we have our original nk capital stock points, but that we have expanded the policy choices to, say, $np > nk$ choices.

- To see how to deal with this, let's go back to the original Bellman equation.

$$\Pi(K, z) = \max_{k'} \{zK^{\alpha} - (K' - (1-\delta)K) + \beta \mathbb{E}\Pi(K', z')\}$$

- **For each value of** z, the continuation value matrix in blue is $nk \times nk$, but the profit matrix in red is $np \times nk$.

- So you "fill in" the np points that are between each of the rows of the blue matrix by interpolation.

Value Function Interpolation

- You should always use value function interpolation.
- In terms of computation time, it is a fairly "cheap" operation.
- In contrast, maximization is a fairly time consuming operation.
- So interpolation lets you economize on the maximization step (i.e. go faster) **and** get more accurate solutions.
- Of course, if you do interpolation here, you will also have to do interpolation in the simulation step.

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Limiting the policy space

- Suppose in iteration 1, you find that the maximized Bellman equation occurs at policy point $\#j$.
- Then why not limit the range of the policy space to $[j - j', j + j']$, where j' is an integer that is less than half the dimension of the policy space.
- If you pick j' too small, you might not get convergence.
- If you pick j' too large, you might not get much of a speed boost.

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