Macroeconomic Theory II: Assignment 5

Due by Friday, April 1 @9am

1. Consider the Aiyagari model we studied in the class. Suppose preferences are given by:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

Individuals are ex-ante identical, infinitely lived, but are subject to idiosyncratic productivity shocks. However, in the aggregate, by law of large numbers, these shocks wash out and there is no aggregate uncertainty. Suppose that the productivity process follows an AR(1) process:

$$\log \epsilon' = \rho \log \epsilon + v$$

where v is normally-distributed with mean 0 and variance σ_v^2. The budget constraint of the individual is as follows:

$$\begin{aligned} c + a' &= w\epsilon + a(1+r) \\ a' &\geq -b \end{aligned}$$

where c is consumption, a is asset holding, w is wage per unit of efficiency, and r is the rental rate of capital after depreciation. There are infinitely many firms, all identical, and hire capital and labor at competitive markets. w is the wage rate and r is the rental rate of capital. They have CRS technology to produce output: $Y = F(K,N) = K^{\alpha} N^{1-\alpha}$. Capital depreciates at the rate δ.

   (a) Define a recursive competitive equilibrium for this economy.

   (b) Set $\beta = 0.96$, $\delta = 0.065$, $\sigma = 2$, $b = 0$, $\rho = 0.9$ and $\sigma_v = 0.2$. Use the Tauchen method I posted on Canvas to discretize the productivity process (you can start by using 5 grid points for the productivity shock and later check the robustness of the results when you increase the number of grid points for the productivity shock). Given the parameter values, solve the model. Plot the policy functions for consumption and saving as a function of current asset holdings for three different productivity shock (lowest, median and highest productivity shock).

   (c) Resolve the model by changing the persistency of the productivity shock: $\rho = \{0, 0.5, 0.9\}$. Report the changes in equilibrium interest rate, wage, aggregate output, aggregate capital, average welfare, amount of precautionary saving, income inequality, wealth inequality and welfare inequality, where inequality is measured by the coefficient of variation of the variable of interest. Explain the reason for the changes in the results.

   (d) Resolve the model by changing the variance of the productivity shock: $\sigma_v = \{0.1, 0.2, 0.3\}$ and report the same statistics as in 1c. Explain the reason for the changes in the results.

(e) Resolve the model by changing risk aversion: $\sigma = \{1, 2, 5\}$, and report the same statistics as in 1c. Explain the reason for the changes in the results.

(f) Resolve the model by changing the borrowing limit: $b = \{1.0, 0.5, 0.0\}$, and report the same statistics as in 1c. Explain the reason for the changes in the results.