Formalization of SPARK Subset in Coq

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Outline

- Motivation
- Formalization Work
- Demo
- Future Work

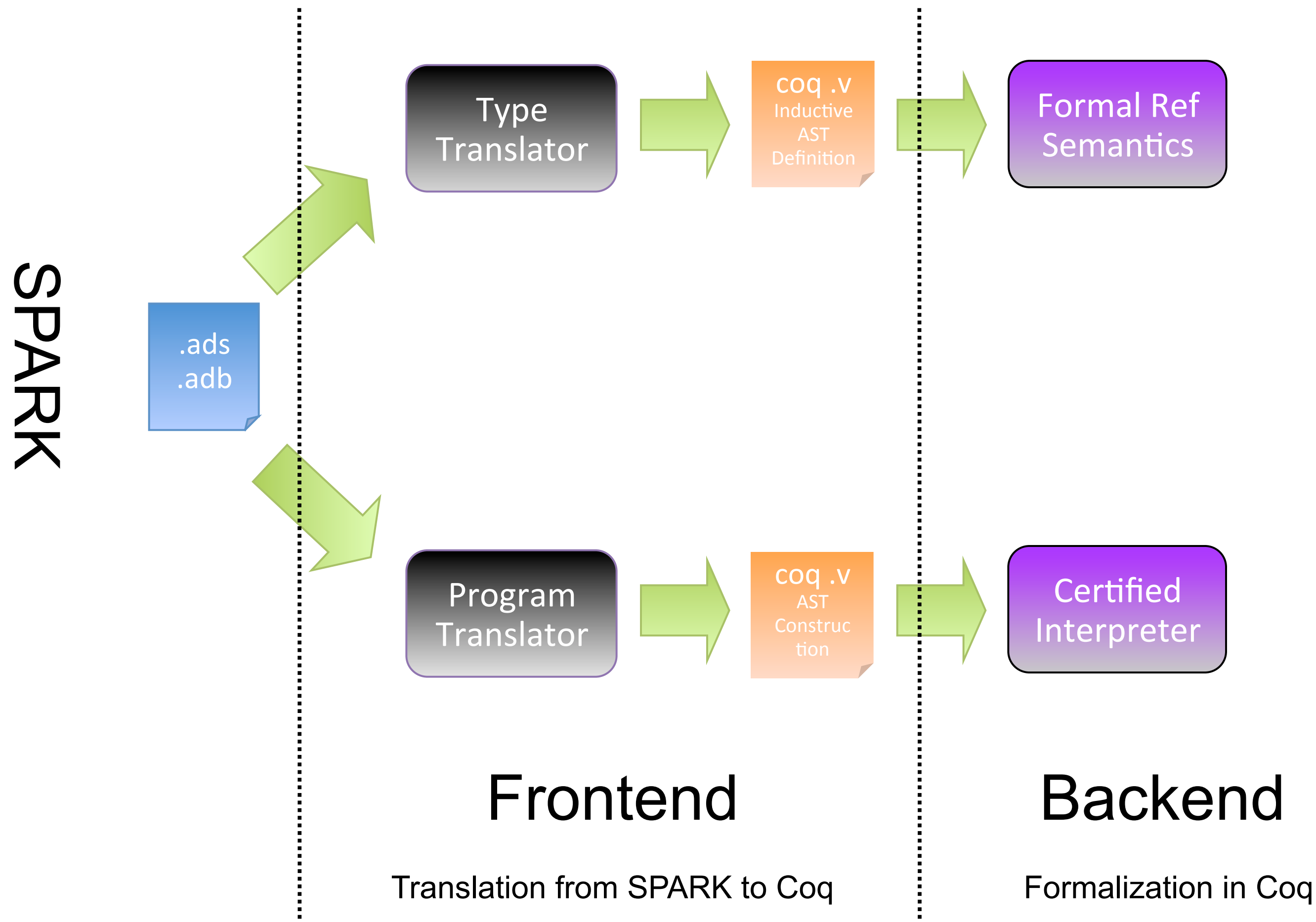
Our Work

- Formalize dynamic semantics of SPARK subset in Coq
  - Perform necessary run time checks
  - Prove correctness for well-formed programs
  - Build a tool chain from SPARK to Coq

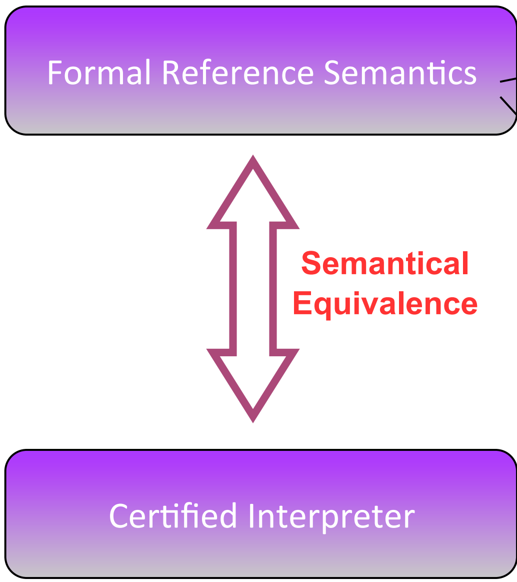
Motivation

- Define Formal Semantics for SPARK
  - Basis for SPARK certification technology
- Strengthen GNATprove Toolchain
  - Justify "*all necessary RTC are in inserted*"
- Provide SPARK Infrastructure for
  - Machine-verified proofs of static analysis
  - Certified SPARK frontend for CompCert

SPARK 2014 To Coq Tool Chain



SPARK 2014 To Coq Tool Chain



✓ **Verify Run Time Checks (RTC)**

It's proved that for any SPARK program, if all necessary RTC are inserted, then it should behave as prescribed by the reference semantics

✓ **Verify SPARK Program Correctness**

It's proved that for any well-formed terminating SPARK program, it will terminate in a valid state

Certified Interpreter

- For formal method practicer
  - validate formalized SPARK semantics experimentally by testing
- For users
  - familiarize oneself with the SPARK 2014 semantics, and
  - help to fix the program if the program exhibits undefined behavior

SPARK Subset Language

```
expr  ::=  c
         | x
         | expr bop expr
         | uop expr

stmt  ::=  x := expr
         | if expr then stmt end if
         | while expr loop stmt end loop
         | stmt; stmt
```

SPARK Subset

```
Inductive expr: Type :=
    | Literal            : constant → expr
    | Identifier         : id → expr
    | Binary_Operation   : binary_operator → expr → expr → expr
    | Unary_Operation    : unary_operator → expr → expr

Inductive stmt: Type :=
    | Assignment : id → expr → stmt
    | If         : expr → stmt → stmt
    | While_Loop : expr → stmt → stmt
    | Sequence   : stmt → stmt → stmt
```

Inductive Definition in Coq

Example

```
If (N <= 1) then
   Result := false;
end if;
```

SPARK Code

```
If (Binary_Operation Less_Than_Or_Equal (Identifier N) (Literal (Integer_Literal 1)))
 (Assignment Result (Literal (Boolean_Literal false)))
```

SPARK AST in Coq

Program States

```
Inductive state: Type :=
    | Normal  : store → state
    | Run_Time_Error : state
    | Unterminated        : state
    | Abnormal             : state
```

Classification of Errors (Ada RM):

- Errors that are required to be detected prior to run time
- Errors that are required to be detected at run time
- Bounded errors
- Erroneous execution

In the future, we would refine the abnormal state into these more precise categories.

Run Time Check Semantics

Checking Rules Mark *What* and *Where* to Check

- ***Do_Division_Check***
  - This flag is set on a division operator (/, mod, rem) to indicate that a zero divide check is required;
- ***Do_Overflow_Check***
  - This flag is set on an operator where an overflow is required on the operation;

Run Time Check Semantics

Run Time Check Flags

```
Inductive run_time_checks: Type :=
    | Do_Division_Check  : run_time_checks
    | Do_Overflow_Check : run_time_checks
```

Semantics for Run Time Checks

```
Inductive do_check: binary_operator → value → value → bool → Prop :=
    | Do_Overflow_Check_On_Plus : forall v1 v2 b,
        (((v1 + v2) >= min_signed)  && ((v1 + v2) <= max_signed)) = b →
        do_check Plus (Int v1) (Int v2) b
    | Do_Division_And_Overflow_Check_on_Divide : forall v1 v2 b,
        ((v2 <> 0) && ((v1 / v2) >= min_signed)  && ((v1 / v2) <= max_signed)) = b →
        do_check Divide (Int v1) (Int v2) b
    …
```

Example

X / Y	Binary_Operation Divide (Identifier X) (Identifier Y)	(Y <> 0) and ((-2^31) <= (X / Y) <= (2^31 -1))
SPARK Expr	SPARK AST in Coq	Run Time Checking (32-bit singed integer)

Language Semantics

- Formal Reference Semantics

  ref_eval: state → procedure → state → Prop

- Certified Interpreter

  certified_eval (s: state) (p: procedure): state

- Semantical Equivalence

  ref_eval s f t ↔ certified_eval (s, f) = t

Certify GNATprove Frontend

Do-178-C Standard

- It allows formal verification to replace some forms of testing in the software certification process;

Do-333 Supplement (formal method supplement to Do-178-C)

- It recommends that when using formal methods *all assumptions related to each formal analysis be described and justified*;

Certify GNATprove Frontend

- Ideally, we want certified frontend, but …
- GNATprove relies on *uncertified GNAT Compiler* to insert the necessary run time check flags
- We want to *formalize and certify* these run time checks to make sure that GNATprove compiler inserts the appropriate run time checks at appropriate places

Check Flags Generator

Run Time Checking Rules

```
Inductive check_flags: expr → run_time_check_set → Prop :=
    | CF_Literal : forall c,
          check_flags (Literal c) nil
    | CF_Plus : forall e1 e2,
          check_flags (Binary_Operation Plus e1 e2)
                      (Do_Overflow_Check :: nil)
    | CF_Divide : forall e1 e2,
          check_flags (Binary_Operation Divide e1 e2)
                      (Do_Division_Check :: Do_Overflow_Check :: nil)
    …
```

Example

SPARK Expr	SPARK AST in Coq	Check Flags
2	Literal (Integer_Literal 2)	nil
X / Y	Binary_Operation Divide (Identifier X) (Identifier Y)	[Do_Division_Check, Do_Overflow_Check]

Semantics With Flagged Checks

- Formal Reference Semantics

  do complete checks

  ref_eval: state → procedure → state → Prop

- Semantics With Flagged Checks

  do selected checks

  ref_eval': *check_points* → state → procedure → state → Prop

- Semantical Correctness

  ref_eval' *checks* s f t → ref_eval s p t

  (where checks are checks generated by the checking rules)

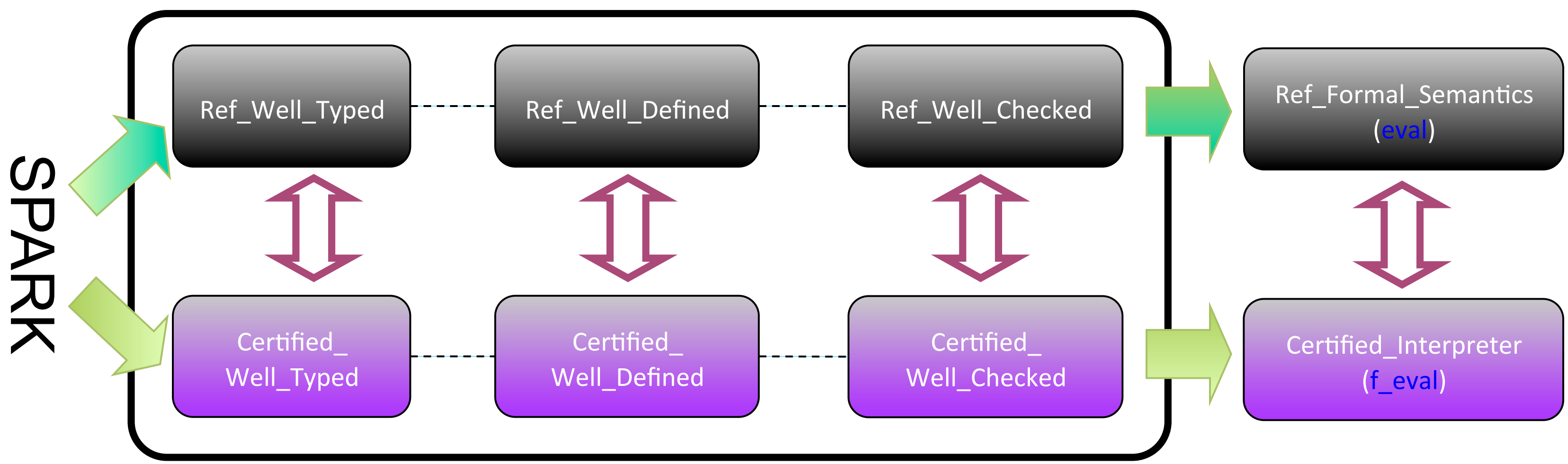
Static Analysis

- Well-Typed
  - programs are correct with respect to the typing rules
  - values with correct in/out mode
- Well-Defined
  - all used variables have been initialized
- Well-Checked
  - necessary checks are inserted in the AST tree

Program Correctness Proof

Machine-verified SPARK Program Correctness

Theorem Program_Correctness: forall f,
    Ref_Well_Typed f →
    Ref_Well_Defined f →
    Ref_Well_Checked f →
    (forall s,
        (exists t s', ref_eval s f t /\ (t = Normal s' \/ t = Run_Time_Error) \/
        (forall k, certified_eval s f = Unterminated)
)



Demo

- Show How the tool chain works
  - Translation from SPARK to Coq
- Run the certified interpreter
  - Its result should be the same as the reference semantics
  - It can capture necessary run time errors
- Static analysis
  - Run certified static analysis for checking program's well-formness
  - All well-formed terminating program will behave correctly

Example 1

```
procedure Test_for_Coq
is
   N: Integer := 25;
   Result: Boolean;
   I: Integer;
   X: Integer;
begin
   Result := true;
   if N <= 1 then
      Result := false;
   end if;
   I := 2;
   while I*I <= N loop
      X := N / I;
      if N = X * I then
         Result := false;
      end if;
      I := I + 1;
   end loop;
end Test_for_Coq;
```

SPARK

```
Definition f_prime :=
Procedure 3 (
    mkprocedure_body 4
     (** Procedure Name *)
     (** Test_for_Coq *) 1
     (** Specification *)
     (nil)
     (** Parameters *)
     (nil)
     (** Variable Declarations *)
     (
     mkobject_declaration 5 (** N *) 1 1 (Some (E_Literal 6 (Integer_Literal 25))) ::
     mkobject_declaration 7 (** Result *) 2 2 None ::
     mkobject_declaration 8 (** I *) 3 1 None ::
     mkobject_declaration 9 (** X *) 4 1 None :: nil)
     (** Procedure Body *) (
      S_Sequence 10 (
       S_Assignment 11 ((** Result *) 2) (E_Literal 12 (Boolean_Literal true)) ) (
       S_Sequence 13 (
        S_If 14 (E_Binary_Operation 15 Less_Than_Or_Equal (E_Identifier 16 (** N *) 1) (E_Literal 17 (Integer_Literal 1))) (
          S_Assignment 18 ((** Result *) 2) (E_Literal 19 (Boolean_Literal false))
         ) ) (
        S_Sequence 20 (
         S_Assignment 21 ((** I *) 3) (E_Literal 22 (Integer_Literal 2)) ) (
         S_While_Loop 23 (E_Binary_Operation 24 Less_Than_Or_Equal
             (E_Binary_Operation 25 Multiply (E_Identifier 26 (** I *) 3) (E_Identifier 27 (** I *) 3)) (E_Identifier 28 (** N *) 1)) (
           S_Sequence 29 (
            S_Assignment 30 ((** X *) 4) (E_Binary_Operation 31 Divide (E_Identifier 32 (** N *) 1) (E_Identifier 33 (** I *) 3)) ) (
            S_Sequence 34 (
             S_If 35 (E_Binary_Operation 36 Equal
                 (E_Identifier 37 (** N *) 1) (E_Binary_Operation 38 Multiply (E_Identifier 39 (** X *) 4) (E_Identifier 40 (** I *) 3)))
               S_Assignment 41 ((** Result *) 2) (E_Literal 42 (Boolean_Literal false))
              ) ) (
             S_Assignment 43 ((** I *) 3) (E_Binary_Operation 44 Plus (E_Identifier 45 (** I *) 3) (E_Literal 46 (Integer_Literal 1)))
          ) ) ) )
     )).
```

SPARK AST in Coq

1) Translation from SPARK to AST in Coq
2) Check program's well-formedness with certified static analyses (well-typed, well-defined, well-checked)
3) Run certified interpreter on the Coq AST

Example 2

```
procedure Test_for_Coq
is
   N: Integer := 25;
   Result: Boolean;
   I: Integer;
   X: Integer;
begin
   Result := true;
   if N <= 1 then
      Result := false;
   end if;
   I := 0; -- Error !
   while I*I <= N loop
      X := N / I;
      if N = X * I then
         Result := false;
      end if;
      I := I + 1;
   end loop;
end Test_for_Coq;
```

SPARK

```
Definition f_prime :=
Procedure 3 (
    mkprocedure_body 4
     (** Procedure Name *)
     (** Test_for_Coq *) 1
     (** Specification *)
     (nil)
     (** Parameters *)
     (nil)
     (** Variable Declarations *)
     (
     mkobject_declaration 5 (** N *) 1 1 (Some (E_Literal 6 (Integer_Literal 25))) ::
     mkobject_declaration 7 (** Result *) 2 2 None ::
     mkobject_declaration 8 (** I *) 3 1 None ::
     mkobject_declaration 9 (** X *) 4 1 None :: nil)
     (** Procedure Body *) (
      S_Sequence 10 (
       S_Assignment 11 ((** Result *) 2) (E_Literal 12 (Boolean_Literal true)) ) (
       S_Sequence 13 (
        S_If 14 (E_Binary_Operation 15 Less_Than_Or_Equal (E_Identifier 16 (** N *) 1) (E_Literal 17 (Integer_Literal 1))) (
          S_Assignment 18 ((** Result *) 2) (E_Literal 19 (Boolean_Literal false))
         ) ) (
        S_Sequence 20 (
         S_Assignment 21 ((** I *) 3) (E_Literal 22 (Integer_Literal 0)) ) (
         S_While_Loop 23 (E_Binary_Operation 24 Less_Than_Or_Equal
             (E_Binary_Operation 25 Multiply (E_Identifier 26 (** I *) 3) (E_Identifier 27 (** I *) 3)) (E_Identifier 28 (** N *) 1)) (
           S_Sequence 29 (
            S_Assignment 30 ((** X *) 4) (E_Binary_Operation 31 Divide (E_Identifier 32 (** N *) 1) (E_Identifier 33 (** I *) 3)) ) (
            S_Sequence 34 (
             S_If 35 (E_Binary_Operation 36 Equal
                 (E_Identifier 37 (** N *) 1) (E_Binary_Operation 38 Multiply (E_Identifier 39 (** X *) 4) (E_Identifier 40 (** I *) 3)))
               S_Assignment 41 ((** Result *) 2) (E_Literal 42 (Boolean_Literal false))
              ) ) (
             S_Assignment 43 ((** I *) 3) (E_Binary_Operation 44 Plus (E_Identifier 45 (** I *) 3) (E_Literal 46 (Integer_Literal 1)))
          ) ) ) )
     )).
```

SPARK AST in Coq

1) Translation from SPARK to AST in Coq
2) Check program's well-formedness with certified static analyses (well-typed, well-defined, well-checked)
3) Run certified interpreter, which captures the *division by zero* exception

Future Work

- Extend the language subset
  - Add function call
  - Add array, records, subtypes
  - … and so on
- Add run time checks optimizations and prove its correctness
- Certified CompCert frontend for SPARK

END !

Thanks