Probabilistic Programming

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Project 1

Topic Modeling in PyMC

Topic Modeling

- Probabilistic models for uncovering the underlying semantic structure of a document collection based on a hierarchical Bayesian analysis of the original texts
- Topic models are algorithms for discovering the main themes that pervade a large and otherwise unstructured collection of documents. Topic models can organize the collection according to the discovered themes.

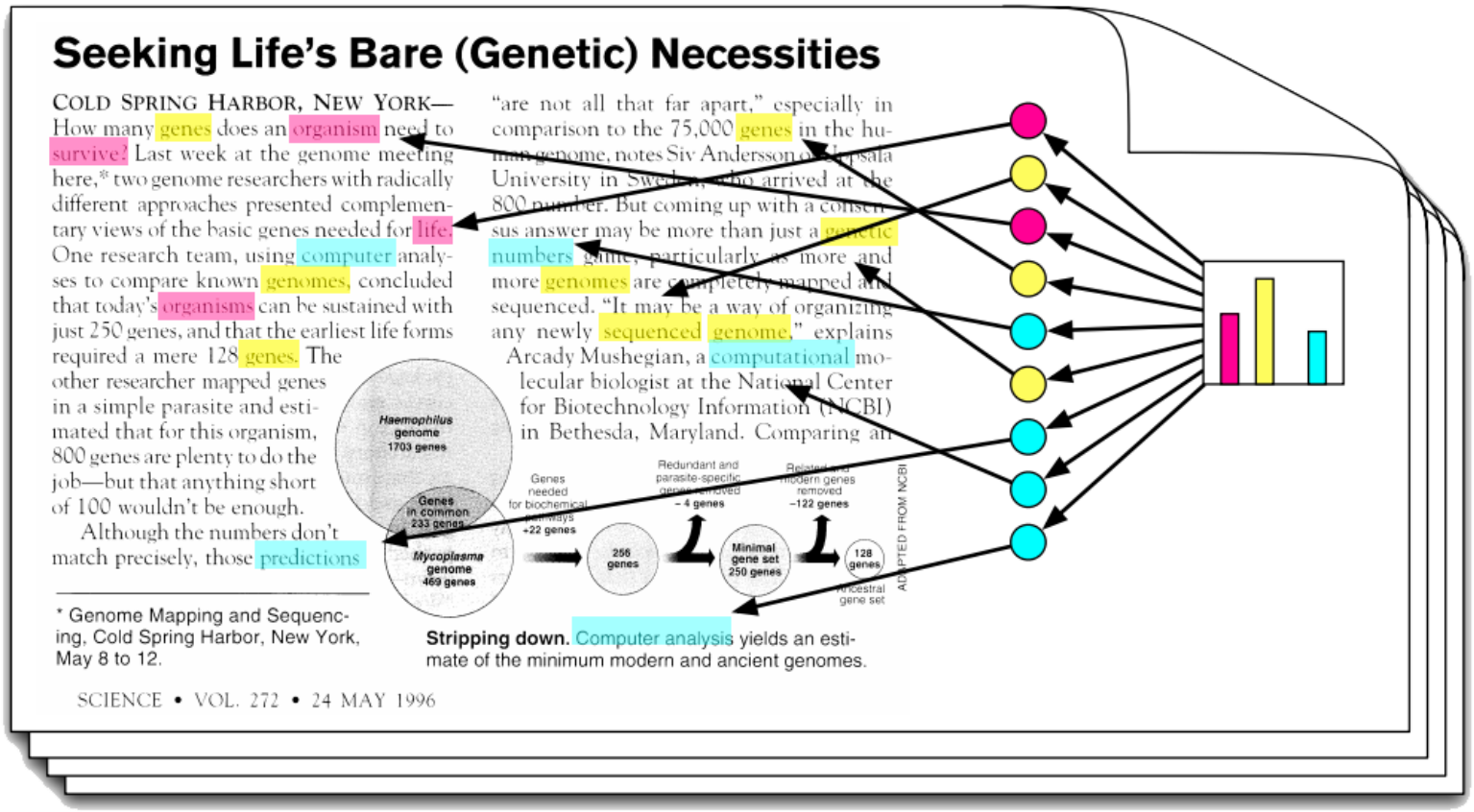
Topics *Documents* *Topic proportions & assignments*

gene	0.04
dna	0.02
genetic	0.01
...	

life	0.02
evolve	0.01
organism	0.01
...	

brain	0.04
neuron	0.02
nerve	0.01
...	

data	0.02
number	0.02
computer	0.01
...	



Seeking Life's Bare (Genetic) Necessities

COLD SPRING HARBOR, NEW YORK—How many genes does an organism need to survive? Last week at the genome meeting here,* two genome researchers with radically different approaches presented complementary views of the basic genes needed for life. One research team, using computer analyses to compare known genomes, concluded that today's organisms can be sustained with just 250 genes, and that the earliest life forms required a mere 128 genes. The other researcher mapped genes in a simple parasite and estimated that for this organism, 800 genes are plenty to do the job—but that anything short of 100 wouldn't be enough.

Although the numbers don't match precisely, those predictions "are not all that far apart," especially in comparison to the 75,000 genes in the human genome, notes Siv Andersson of Uppsala University in Sweden, who arrived at the 800 number. But coming up with a consensus answer may be more than just a genetic numbers game, particularly as more and more genomes are completely mapped and sequenced. "It may be a way of organizing any newly sequenced genome," explains Arcady Mushegian, a computational molecular biologist at the National Center for Biotechnology Information (NCBI) in Bethesda, Maryland. Comparing all

* Genome Mapping and Sequencing, Cold Spring Harbor, New York, May 8 to 12.

Stripping down. Computer analysis yields an estimate of the minimum modern and ancient genomes.

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Latent Dirichlet Allocation (LDA)

- In the LDA model, each document is viewed as a mixture of topics that are present in the corpus. The model proposes that each word in the document is attributable to one of the document's topics
- The idea behind LDA is to model documents as arising from multiple topics, where a topic is defined to be a distribution over a fixed vocabulary of terms
- Given a dataset of documents, LDA backtracks and tries to figure out what topics would create those documents in the first place

http://www.cs.columbia.edu/~blei/papers/Blei2012.pdf

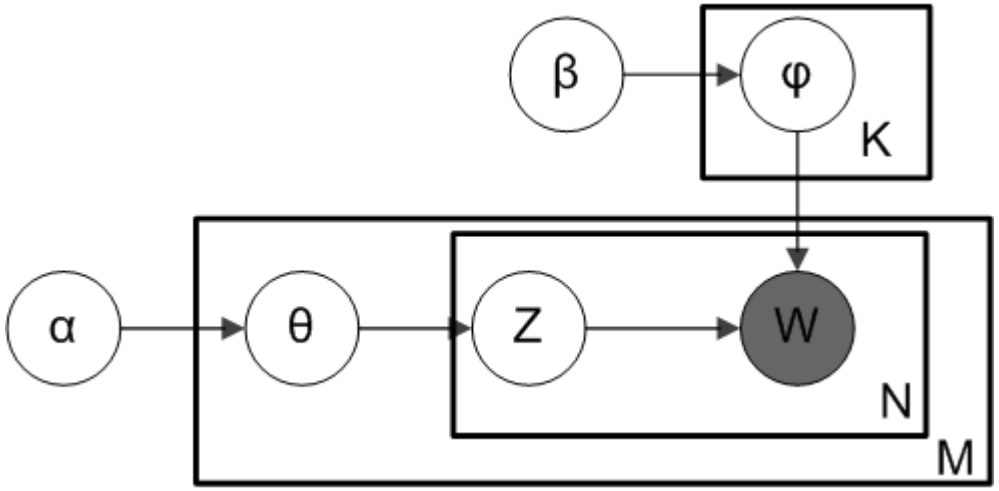
http://www.cs.columbia.edu/~blei/papers/BleiLafferty2009.pdf (sections 1 and 2)

https://en.wikipedia.org/wiki/Latent_Dirichlet_allocation

LDA: Generative Process

- Suppose, M is the number of documents in our collection. N_m is the number of words in the m-th document and V the size of the vocabulary. K is the number of predefined-topics. In this case, the number of topics is not automatically inferred. Instead, we will manually set the value of K based on our intuition.
- Documents are represented as random mixtures over latent topics, where each topic is characterized by a distribution over all the words

LDA: Generative Process



For each topic k, we draw its word distribution, which is denoted as φ_k. As prior for the per-topic word distribution we will use a V-dimensional symmetric Dirichlet distribution:

$$\varphi_k \sim \mathrm{Dir}(\beta), 1 \leq k \leq K$$

For each document m, we draw a topic distribution, which is denoted as θ_m. As prior for the per-document topic distribution we will use a K-dimensional symmetric Dirichlet distribution:

$$\theta_m \sim \mathrm{Dir}(\alpha), 1 \leq m \leq M$$

The hyperparameters α and β are assumed to be fixed typically sparse $\alpha, \beta \leq 1$ (set them to 1)
For each word n in document d (for each of the word positions), we draw a topic for that word according to a Multinomial (Categorical) distribution:

$$z_{m,n} \sim \mathrm{Multinomial}(\theta_m), 1 \leq m \leq M, 1 \leq n \leq N_m$$

Draw the physical word itself from the word distribution associated with its selected topic. Each word is denoted as $w_{m,n}$:

$$w_{m,n} \sim \mathrm{Multinomial}\left(\varphi_{z_{m,n}}\right), 1 \leq m \leq M, 1 \leq n \leq N_m$$

The Task 1

Document 1: I had a peanuts butter sandwich for breakfast.
Document 2: I like to eat almonds, peanuts and walnuts.
Document 3: My neighbor got a little dog yesterday.
Document 4: Cats and dogs are mortal enemies.
Document 5: You mustn't feed peanuts to your dog.

- Start with a corpus (document collection)
- Build the observed variable:

$$w_{m,n}\ 1 \le m \le M, 1 \le n \le N_m$$

- Infer the hidden topic structure:

$$\theta_m\ 1 \le m \le M$$

$$\varphi_k\ 1 \le k \le K$$

```
[[0, 1, 2, 3, 4, 5, 6, 7]
 [0, 8, 9, 10, 11, 3, 12, 13]
 [14, 15, 16, 2, 17, 18, 19]
 [20, 12, 21, 22, 23, 24]
 [25, 26, 27, 3, 9, 28, 18]]

 [[ 0.19633795  0.80366205]]
 [[ 0.18370696  0.81629304]]
 [[ 0.2242626  0.7757374]]
 [[ 0.72682194  0.27317806]]
 [[ 0.19720619  0.80279381]]

[[ 0.0153611   0.04258534  0.02096933  0.03742481  0.01003486  0.00337692
   0.03687246  0.01759574  0.0308261   0.01290443  0.01975973  0.01656558
   0.08359959  0.00988798  0.00815009  0.02573078  0.00934478  0.02543332
   0.0267678   0.01700781  0.07986162  0.15302288  0.09572016  0.06718658
   0.03541293  0.01084848  0.00399011  0.02285365  0.06090502]]
[[ 0.0475376   0.03045149  0.03110808  0.07166036  0.02985312  0.03845919
   0.03310906  0.03910945  0.05759958  0.0438841   0.0596612   0.02342716
   0.03582313  0.02156961  0.03489024  0.01348096  0.00848053  0.04888085
   0.03665115  0.02932311  0.02411743  0.01016715  0.02278155  0.04463838
   0.00023452  0.05109875  0.03079135  0.04895514  0.03225578]]
```

Sanity Check

```
docs = [["aaa", "bbb", "aaa"],
        ["bbb", "aaa", "bbb"],
        ["aaa", "bbb", "bbb", "aaa"],
        ["uuu", "vvv"],
        ["uuu", "vvv", "vvv"],
        ["uuu", "vvv", "vvv", "uuu"]]
```

```
[[0, 1, 0] [1, 0, 1] [0, 1, 1, 0] [2, 3] [2, 3, 3] [2, 3, 3, 2]]
```

```
[[ 0.56363964  0.43636036]]
[[ 0.3713786  0.6286214]]
[[ 0.96678627  0.03321373]]
[[ 0.04743652  0.95256348]]
[[ 0.26409289  0.73590711]]
[[ 0.24063087  0.75936913]]
```

- Start with a corpus (document collection)
- Build the observed variable:

$$w_{m,n}\ 1 \le m \le M, 1 \le n \le N_m$$

- Infer the hidden topic structure:

$$\theta_m\ 1 \le m \le M$$

$$\varphi_k\ 1 \le k \le K$$

- Trace also:

$$z_{m,n} 1 \le m \le M, 1 \le n \le N_m$$

```
[[ 0.49718579  0.33963714  0.08627236  0.07690471]]
[[ 0.02396443  0.10446209  0.5212554   0.35031808]]
```

```
[0 0 0]
[0 0 0]
[0 0 0 0]
[1 1]
[1 1 1]
[1 1 1 1]
```

The Task 2

- Can the topic model be used to define a topic-based similarity measure between documents?
- What about a new document? How can topics be assigned to it?