German University in Cairo - GUC
Faculty of Media Engineering and Technology - MET
Department of Digital Media Engineering and Technology
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Saturday, October 28th, 2023

DMET 502 Computer Graphics

Winter Semester 2023/2024

Midterm Exam (Version I) Model Answers

Barcode

Major: Pick one

- [ ] DMET
- [ ] CSEN

Instructions: **Read Carefully Before Proceeding.**

1- Non-programmable calculators are allowed.
2- This is a **closed book exam**.
3- Write your solutions in the space provided.
4- The exam consists of **(3) questions.**
5- This exam booklet contains **(7) pages** including this page. The last page is a formula sheet. **Keep it attached**.
6- Total time allowed for this exam is **(120) minutes.**
7- When you are told that time is up, stop working on the test.

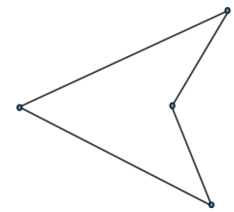
Good Luck!

Don't write anything below ;-)

Question	1	2		3	∑
		a	b		
Possible Marks	24	8	18	17	67
Final Marks					

Question 1 [2D Graphics]:

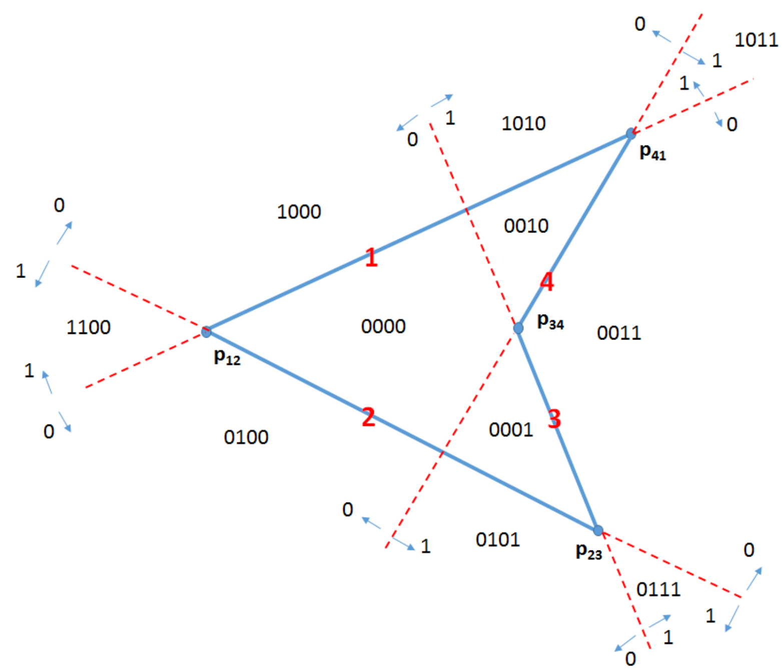
a) [9 marks] In Cohen-Sutherland Algorithm (listed in the Formula Sheet) for line clipping, if the clip polygon used is a **concave** 4-sided polygon like the one shown, determine the minimum length of each outcode (i.e., the number of binary digits), the number of outcodes and all the outcodes in this case.



Answers to Question 1 a):

In the original algorithm, the clip polygon was a rectangle; thus, the minimal space required corresponds to the number of borders (i.e., 4 bits). This is the same case here as it is a 4-sided polygon. [2 marks]

Notice that the number of regions = 4 (number of borders) * 2 (sides per border) + 3 (clip regions) = 11 regions. Thus, we have 11 outcodes. [1.5 marks]



Consider the figure and the outcodes. The borders of the polygon are numbered. According to the borders of the polygon, the sides of each border take either 0 or 1. For example, the left bit corresponds to border 1; any left bit above this border will be assigned 1 or 0 otherwise. (Note that the 1 and 0 can be swapped.) The same goes for the other borders.

[0.5 mark each outcode = 5.5 marks]

b) [8 marks] Propose an algorithm to generate these outcodes.

Answers to Question 1 b):

1. Determine the linear equations for each of the sides.
2. Determine the vertices by intersecting lines; $\mathbf{p}_{12}$, $\mathbf{p}_{23}$, $\mathbf{p}_{34}$, $\mathbf{p}_{41}$
3. Outcode = 0000
4. Apply vertex $\mathbf{p}_{23}$ to the border line 1. For each region on the same side of border 1 as $\mathbf{p}_{23}$. Outcode OR 1000
5. Apply vertex $\mathbf{p}_{41}$ to the border line 2. For each region on the same side of border 2 as $\mathbf{p}_{41}$. Outcode OR 0100
6. Apply vertex $\mathbf{p}_{12}$ to the border line 3. For each region on the same side of border 3 as $\mathbf{p}_{12}$. Outcode OR 0010
7. Apply vertex $\mathbf{p}_{12}$ to the border line 4. For each region on the same side of border 4 as $\mathbf{p}_{12}$. Outcode OR 0001
8. Return outcode

[1 mark each step = 8 marks]

c) [7 marks] Modify the Cohen-Sutherland Algorithm to work with the clip concave polygon.

Answers to Question 1 c):

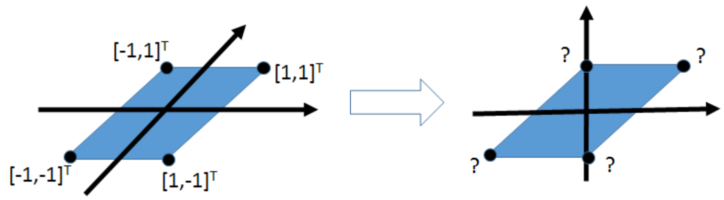
1. Determine outcode for each endpoint. [1 mark]
2. Dealing with the two outcodes of a border: [1 mark]
    a. Bitwise-OR the bits. If this results in 000, or 0010, or 0001, trivially accept. [1 mark]
    b. Otherwise, if both outcodes are equal, trivially reject. . [2 marks]
    c. Otherwise, segment the line. The outpoint is replaced by the intersection point. Go to Step 2. [1 mark]
3. If trivially accepted, draw the line. [1 mark]

Any other logical alternative can be considered

Question 2 [2D Transformations]:

a) [8 marks] Consider the coordinate system shown on the left where the angle between the axes is not 90°.

If the axes are sheared to get back to the Cartesian coordinate system, estimate an inhomogeneous matrix (and a homogeneous matrix) that can be used to obtain the new coordinates of the vertices. Determine these new coordinates.



Answers to Question 2 a):

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \text{[3 marks]}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{[1 mark]}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \text{[1 mark]}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \text{[1 mark]}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix} \text{[1 mark]}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{[1 mark]}$$

b) [18 marks] Derive the reflection matrix about a line having a slope of 0.5 and *y*-intercept of 3.

Answers to Question 2 b):

Either a homogeneous matrix or an inhomogeneous equation is acceptable.

Angle of inclination = $\tan^{-1}(0.5) = 26.565°$ [2 marks]
Steps:

1. Translate using $[0,-3]^T$ [2 marks]
2. Rotate through -26.565 ➔ R(-26.565) [2 marks]
3. Reflect about the x-axis [2 marks]
4. Rotate back R(26.565) [1 mark]
5. Translate back $[0,3]^T$ [1 mark]

$$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \text{[1 mark]}$$

$$R_2 = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta)1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(-26.565) & -\sin(-26.565) & 0 \\ \sin(-26.565) & \cos(-26.565) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 0 \\ -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{[1 mark]}$$

$$Ref_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{[1 mark]}$$

$$R_4 = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) & 0 \\ \sin(-\theta) & \cos(-\theta)1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(26.565) & -\sin(26.565) & 0 \\ \sin(26.565) & \cos(26.565) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 0 \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{[1 mark]}$$

$$T_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \text{[1 mark]}$$

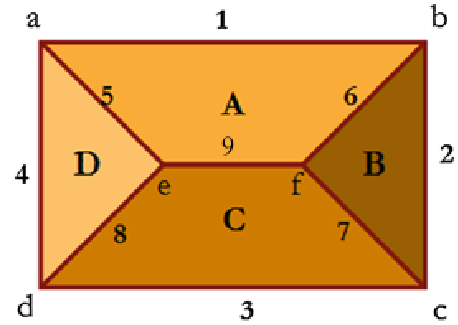
$$M = T_5 R_4 Ref_x R_2 T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 0 \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 0 \\ -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \text{[2 marks]}$$

$$M = \begin{bmatrix} 0.6 & 0.8 & -2.4 \\ 0.8 & -0.6 & 4.8 \\ 0 & 0 & 1 \end{bmatrix}$$

[1 mark]

Question 3 [3D Modeling]:

[17 marks] The roof of a house is shown where the vertices are indicated by lowercase letters (i.e., “a,” “b,” “c,” . . .), the faces by uppercase letters (i.e., “A,” “B,” “C,” . . .) and the edges by digits (i.e., “1,” “2,” “3,” . . .). The coordinates of the vertices are $[x_a, y_a, z_a]^T$, $[x_b, y_b, z_b]^T$, etc.



a) Write down **all** the entries of the required **tables** if it is represented as **wireframe**.

b) Write down the entries of the **edge table** of **Baumgart's Winged-Edge Data Structure**. Consider only edges 5, 6, 7, 8 and 9.

Answers to Question 3:

[3 marks + 4 marks + 10 marks]

Vertex	Coordinates
a	$[x_a, y_a, z_a]^T$
b	$[x_b, y_b, z_b]^T$
c	$[x_c, y_c, z_c]^T$
d	$[x_d, y_d, z_d]^T$
e	$[x_e, y_e, z_e]^T$
f	$[x_f, y_f, z_f]^T$

Edge	Start vertex	End vertex
1	a	b
2	b	c
3	c	d
4	d	a
5	a	e
6	f	b
7	c	f
8	e	d
9	f	e

If wireframe tables are not written, consider the edges and vertices columns of the Winged-edge table (if correct) as 0.556 of the wireframe edge table. ➔ vertices table = 0 and edge table = 2.5

Edges	Vertices		Faces		Left traverse		Right traverse	
	Start	End	Left	Right	Pred.	Succ.	Pred.	Succ.
5	a	e	A	D	9	1	4	8
6	b	f	B	A	7	2	1	9
7	c	f	C	B	9	3	2	6
8	d	e	D	C	5	4	3	9
9	e	f	A	C	6	5	8	7

Formula Sheet

Input: x_0, y_0, x_1, y_1

$steep = |y_1 - y_0| > |x_1 - x_0|$
if $(steep = TRUE)$ **then**
  swap (x_0, y_0)
  swap (x_1, y_1)
end if

if $(x_0 > x_1)$ **then**
  swap (x_0, x_1)
  swap (y_0, y_1)
end if

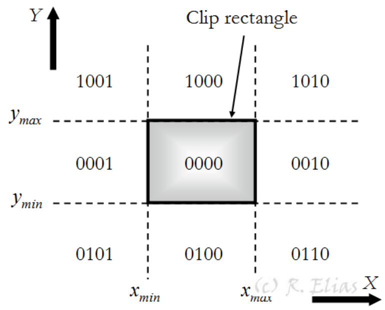
if $(y_0 > y_1)$ **then**
  $\delta y = -1$
else
  $\delta y = 1$
end if

$\Delta x = x_1 - x_0$
$\Delta y = |y_1 - y_0|$
$y = y_0$
$error = 0$

for $(x = x_0$ to $x_1)$ **do**
  if $(steep = TRUE)$ **then**
    Plot $[y, x]^T$
  else
    Plot $[x, y]^T$
  end if
  $error = error + \Delta y$
  if $(2 \times error \geq \Delta x)$ **then**
    $y = y + \delta y$
    $error = error - \Delta x$
  end if
end for

end

1. Determine outcode for each endpoint.
2. Dealing with the two outcodes:
   a. Bitwise-OR the bits. If this results in 0000, trivially accept.
   b. Otherwise, bitwise-AND the bits. If this results in a value other than 0000, trivially reject.
   c. Otherwise, segment the line. The outpoint is replaced by the intersection point. Go to Step 2.
3. If trivially accepted, draw the line.



$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathbf{p}}_2} = \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathbf{p}}_1} + \underbrace{\begin{bmatrix} t_x \\ t_y \end{bmatrix}}_{\mathbf{t}}.$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}}_{\mathbf{p}_2} = \underbrace{\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}}_{\mathtt{T}([t_x,t_y]^T)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}}_{\mathbf{p}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathbf{p}}_2} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\dot{\mathtt{R}}(\theta)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathbf{p}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathbf{p}}_2} = \underbrace{\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}}_{\dot{\mathtt{S}}(s_x,s_y)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathbf{p}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathbf{p}}_2} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\dot{\mathtt{Ref}}_x} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathbf{p}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathbf{p}}_2} = \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}}_{\dot{\mathtt{Ref}}_y} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathbf{p}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathbf{p}}_2} = \underbrace{\begin{bmatrix} 1 & sh_x \\ 0 & 1 \end{bmatrix}}_{\dot{\mathtt{Sh}}_x(sh_x)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathbf{p}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathbf{p}}_2} = \underbrace{\begin{bmatrix} 1 & 0 \\ sh_y & 1 \end{bmatrix}}_{\dot{\mathtt{Sh}}_y(sh_y)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}_{\dot{\mathbf{p}}_1}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -t_x \\ -t_y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \frac{1}{s_x} & 0 \\ 0 & \frac{1}{s_y} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

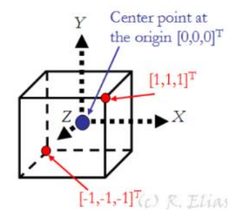
Vertex #	x	y	z

Edge #	Start vertex	End vertex

Edge	Vertices		Faces		Left traverse		Right traverse	
Name	Start	End	Left	Right	Pred	Succ	Pred	Succ

Vertex	edge

Face	edge



translate(scale(Block, < 1, 1.5, 1.5 >), < 1, 2, 3 >)

Face	Vertices
A	$[x_1, y_1, z_1]^T$, $[x_2, y_2, z_2]^T$, $[x_3, y_3, z_3]^T$
B	$[x_2, y_2, z_2]^T$, $[x_4, y_4, z_4]^T$, $[x_3, y_3, z_3]^T$
$\vdots$	$\vdots$

Vertex	Coordinates
1	$[x_1, y_1, z_1]^T$
2	$[x_2, y_2, z_2]^T$
$\vdots$	$\vdots$

Face	Vertices
A	1, 2, 3
B	2, 4, 3
$\vdots$	$\vdots$