# Computational Modeling and Comparison of Single and Differential Learning Rates,Inverse Temperatures and Salience of probability of Aversaive Stimuli in a Probabilistic Reversed Reinforced Learning Task among High and Low Anxious Populations

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***Abstract*—The study of anxiety and its effects on human behavior has been a topic of interest in psychology and neuroscience for many years. Anxiety is a complex and multifaceted experience that can have negative impacts on various aspects of a person's life, including their emotional well-being, physical health, and overall functioning. The relationship between anxiety and decision-making has been a topic of particular interest, as anxiety can lead to decreased cognitive control and altered decision-making processes. The study of decision-making processes and their relationship to anxiety has been facilitated by the development of computational models, which allow researchers to simulate and quantify the effects of anxiety on decision-making. One such model is the reinforcement learning model, which considers the relationship between the outcomes of a decision and the value assigned to each choice. This model has been used to study anxiety and its effects on decision-making by manipulating the parameters of the model to reflect different levels of anxiety. In this paper, we aim to examine the relationship between anxiety and decision-making using the reinforcement learning models. Our study focuses on a specific type of decision-making task, where participants are presented with two options and assigned options between them based on their perceived likelihood of receiving an aversive outcome. We will simulate data from three different reinforcement learning models and fit these models to real data from participants to determine the best-fitting model. Additionally, we evaluate the reliability and identifiability of the models and their parameters using recovery methods, while also determining the affect of different parameters and different experimental settings affects on models. The findings of this study will contribute to our understanding of the relationship between anxiety and decision-making and the potential implications for the development of anxiety treatments.**

## I. Introduction

Anxiety is a common mental health disorder that affects millions of people worldwide. It is characterized by excessive and persistent fear or worry about future events, often accompanied by physical symptoms such as sweating, heart palpitations, and shaking. Trait anxiety, a stable tendency to experience anxiety across different situations, has been shown to be related to differences in cognitive processing, particularly in decision-making and learning. The ability to learn from previous experiences and adjust behavior accordingly is crucial in avoiding aversive outcomes and maintaining well-being. However, the precise mechanisms underlying these differences in learning and decision-making among individuals with high trait anxiety remain unclear.

### A. Background

Intuitively, anxiety is related to worry and being pessimistic about negative events occurring in the future. We are interested in understanding whether participants with anxiety differ in how they learn to predict negative events, in the context of a reinforcement learning task. We will focus on modelling behavioural impairments during a probabilistic reversal learning task inspired by an experiment conducted by Andrea D'Olimpio as part of his Masters by Research in the Seri'es' Lab. In this experiment, participants had to perform an avoidance learning task composed by two parts: a Pavlovian phase and an instrumental phase. The participants were recruited from the general population as those who self-identified as either anxious or calm. All participants were asked, prior to the beginning of the task, to complete the State-Trait Anxiety Inventory form, a commonly used questionnaire to assess anxiety levels. This questionnaire consists of two different forms which measure, respectively, state and trait anxiety. The values of each form range from 20 (not anxious) to 80 (very anxious). During the task, participants were asked to choose between two stimuli so that, at each trial, one stimulus would lead to a loud unpleasant noise (i.e., an aversive outcome) whereas the other would lead to a neutral outcome (i.e., no sound). During the Pavlovian phase, the computer would choose the stimulus at each trial and subjects would only observe. In the instrumental phase, participants were asked to make their own choices to maximise their changes of avoiding the aversive outcome.

### B. Postulate Statement

We hypothesize that trait anxiety is associated with alterations in avoidance learning and decision-making processes. Specifically, we predict that individuals with high trait anxiety will exhibit a reduced learning rate and altered decision-making processes, as reflected by changes in the inverse temperature parameter, in an avoidance learning task. We aim to test this hypothesis using a computational modeling approach, which will allow us to estimate the parameters underlying learning and decision-making processes and compare them between individuals with high and low trait anxiety. Our findings may shed light on the cognitive mechanisms underlying anxiety and inform the development of new treatment approaches for this common mental health disorder.

## II. Literature Review

### A. Decision Making Under Uncertainty

Reinforcement learning models have been widely used to study decision-making under uncertainty. These models propose that individuals make decisions based on the prediction of the outcomes of their choices and the associated learning rates. The learning rate determines the speed at which an individual updates their predictions, and the decision-making parameters determine the probability of choosing a certain option based on the predicted outcomes. Several studies have investigated the relationship between anxiety and reinforcement learning. For example, Bach et al. (2014) found that individuals with higher levels of trait anxiety showed a reduced learning rate in a probabilistic classification task, indicating that they had difficulty learning the association between cues and outcomes. Additionally, they found that these individuals had a lower degree of neural adaptation to prediction errors in the anterior insula, a region implicated in learning and decision-making. These findings suggest that anxiety may affect learning by altering the neural mechanisms underlying decision-making.

### *B. On the use of Reinforcement Models in Psychology*

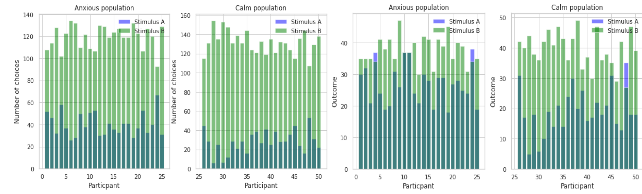
The use of reinforcement learning models in psychology has a long history. Several studies have used these models to investigate the relationship between anxiety and decision-making. Guitart-Masip et al. (2011) found that individuals with higher levels of trait anxiety showed increased sensitivity to negative outcomes, but reduced sensitivity to positive outcomes. This suggests that anxious individuals may be more likely to learn from negative outcomes, but less likely to learn from positive outcomes. Furthermore, the authors found that higher trait anxiety was associated with a reduced learning rate in the task, which could contribute to the reduced sensitivity to positive outcomes. Olimpio et al. (2017) aimed to address the gap in the literature by investigating the relationship between trait anxiety and avoidance learning using a reinforcement learning model. They found that anxious individuals had a lower learning rate, suggesting that they had difficulty learning the association between choices and outcomes. Additionally, they found that anxious individuals had a lower inverse temperature parameter, indicating that they were less likely to choose the option with the higher probability of a positive outcome. These results suggest that trait anxiety may be associated with changes in both learning and decision-making parameters.

## III. Data and Study Area

The data used in this analysis was simulated and provided by D'Olimpio for the purpose of investigating the relationship between trait anxiety and avoidance learning. The simulated data consists of the choices and outcomes of 50 participants during an instrumental phase of a task in which they had to infer the probabilities of two stimuli leading to aversive noise and avoid the stimuli accordingly. The probabilities of each stimulus leading to the aversive noise changed every 40 trials. The first 25 participants were recruited from an anxious population, and the last 25 from a calm population. The study area is in the field of cognitive neuroscience, specifically in the study of decision-making and learning in individuals with trait anxiety.

## IV. Explanatory Data Analysis

The results of the analysis of the STAI-Y2 scores show that the mean score is 42.72 with a standard deviation of 14.73 and a median score of 40.00. Out of the 50 participants, 29 were considered healthy based on a cutoff score of 43. The indices of the healthy participants were [1, 5, 17, 18, 26-50]. Regarding the number of times each participant chose Stimulus A, the average was found to be $21.54\%$ with a range of 0.03 to 0.42. Furthermore, the number of times each participant chose Stimulus B, the average was found to be $78.43\%$ with a range of 0.58 to 0.97 The expected number of aversive sounds experienced by randomly selected participants was $38.46\%$. The Expected number of aversive sounds experienced by random selected participants, $38.46\%$.To determine whether the participants performed well, it would be necessary to compare their behavior to the expected number of aversive sounds experienced by random participants e.g.'$38.46\%$'. If the actual participants chose option A less frequently than the expected number e.g.'$21.54\% < 38.46\%$', it would suggest that they were avoiding the aversive sounds and performing the task well. On the other hand, if they chose option A more frequently e.g.'$21.54\%! > 38.46\%$', it would suggest that they did not perform the task well.




(a) Stimuli Choice (b) Aversaive Sound

Fig. 1

The choice of stimulus A may be indicative of a higher level of anxiety in the anxious participants, as stimulus A is associated with a higher number of aversive sounds. When analyzing the outcome of the participants' choices in terms of aversive sounds experienced, we can observe a clear difference between the anxious and calm participants. For the anxious participants, choosing either option A or option B leads to a similar probability of experiencing an aversive sound. On the other hand, for the calm participants, choosing option A leads to a lower probability of experiencing an aversive sound compared to choosing option B. This difference in behavior may be explained by the buffer time between making a choice and experiencing the outcome, which may allow the calm participants to reconsider their decision and choose a safer option.

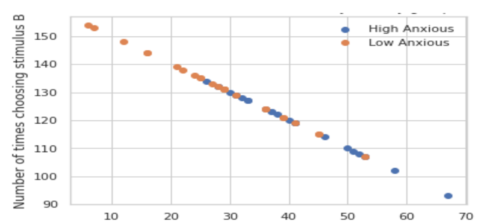



Fig. 2

Number of times choosing stimulus A vs. B, shown in Fig. 2 above can provide insight into the behavior of the participants in the study. It appears that calm participants tend to choose stimulus B more frequently, while anxious participants tend to choose stimulus A. This behavior can be understood in terms of the data as calm participants only come from the healthy population. It is possible that the healthy participants have a greater sense of control over their choices, leading them to choose stimulus B more frequently. On the other hand, anxious participants may have a greater sense of uncertainty or impulsivity, leading them to choose stimulus A more frequently. These findings suggest that the degree of anxiety may play a role in shaping the decision-making behavior of participants in a probabilistic reversed reinforcement learning task.

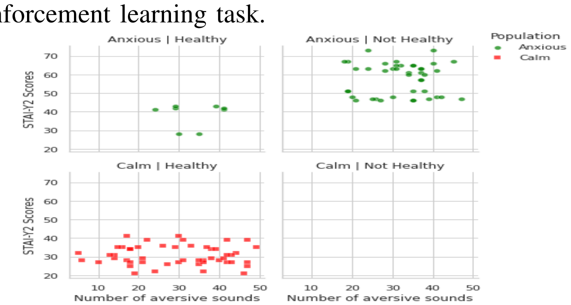



Fig. 3

Anxiety Score for Different Participants Characteristics shown in Fig. 3 above, provides insight into the relationship between anxiety levels (as measured by the STAI-Y2 scores) and the likelihood of being classified as healthy or unhealthy. The results show a clear distinction between the high and low anxious populations, with healthy high anxious participants having a mean score of 37, and unhealthy high anxious participants having a mean score of 68. On the other hand, calm and healthy participants had a mean score of 30, with no calm and unhealthy participants. These findings align with previous studies that have shown a strong correlation between anxiety levels and mental health status.

## V. Proposed Method

### A. *Technical Background*

Computational psychiatry is an interdisciplinary field that uses computational methods and models to study mental disorders. Reinforcement learning, a type of machine learning that involves decision-making based on rewards and punishments, is a key approach used in computational psychiatry to study decision-making behavior in individuals with mental disorders. This approach has been applied to study various mental disorders, including depression, anxiety, and addiction. Reinforcement learning models have provided valuable insights into the underlying mechanisms of mental disorders and have potential for developing new treatments.. A study by Hsu et al. (2015) found that individuals with social anxiety disorder exhibit an increased sensitivity to punishment and decreased sensitivity to reward compared to healthy controls. Similarly, Whelan et al. (2017) found that individuals with depression exhibit decreased sensitivity to reward and increased sensitivity to punishment. In conclusion, reinforcement learning models have been widely used to study decision-making in individuals with mental disorders and have the potential to provide valuable insights into the underlying mechanisms of mental disorders and aid in the development of new treatments.

### B. *Modeling Scheme*

The present study used three reinforcement learning models to investigate the relationship between trait anxiety and decision-making in an avoidance task. Reinforcement learning is a computational approach to studying how an agent can learn from feedback in an environment. In this study, the models used to capture the behavior of participants in the avoidance task were variants of the Rescorla Wagner model (Rescorla & Wagner, 1972), which is a commonly used reinforcement learning model in psychology.

*1) **Simple Rescorla-Wagner Model:*** , The first model used in the study is a simple Rescorla-Wagner model that updates the value of the chosen stimulus based on the observed outcome using a learning rate parameter, $\alpha$. The probability of choosing stimulus A as opposed to stimulus B on trial t is modeled using a softmax function with an inverse temperature parameter, $\beta$. The model is described by the following equations,

$$V(t+1) = V(t) + \alpha(o(t) - V(t)) \tag{1}$$

$$p(action|V(t),\beta) = \frac{exp(-\beta V(t))}{exp(-\beta V(t)) + exp(-\beta V(t))} \tag{2}$$

*2) **Rescorla-Wagner Model with Salience:*** , The second model is a variant of the Rescorla-Wagner model that includes an additional parameter, A, which scales the value of the chosen stimulus on each trial. This parameter represents the salience or importance of the stimulus to the participant. The model is described by the following equations,

$$V(t+1) = AV(t) + \alpha(o(t) - V(t)) \tag{3}$$

$$p(action|V(t),\beta) = \frac{exp(-\beta V(t))}{exp(-\beta V(t)) + exp(-\beta V(t))} \tag{4}$$

*3) **Rescorla-Wagner Model with Differential Learning Rates:*** , The third model is a variant of the Rescorla-Wagner model that includes two separate learnig rate parameters, $\alpha^+$ and $\alpha^-$, to capture differential learning rates for positive (no sound) and negative (sound) outcomes. The model is described by the following equations,

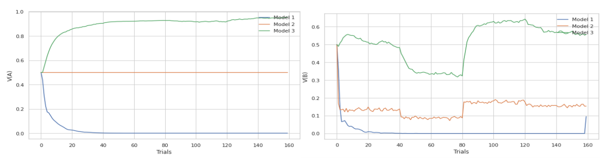
$$V(t+1) = V(t) + ((1 - o(t))\alpha + +o(t)\alpha-))(o(t) - V(t)) \tag{5}$$

$$p(action|V(t),\beta) = \frac{exp(-\beta V(t))}{exp(-\beta V(t)) + exp(-\beta V(t))} \tag{6}$$

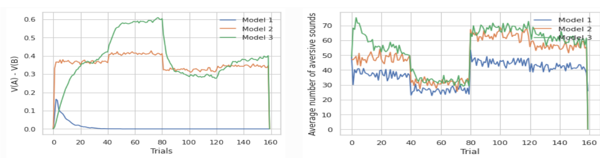
## VI. Experimentation of Generative Models

### A. *Experimental Simulations and Settings*

We will be using the generative model to generate data for our study. We have developed a function that takes in the known parameters as inputs and generates data based on equations (1)-(6) . This phase of the experiment consists of 160 trials, during which the probability of each stimulus leading to an aversive sound changed every 40 trials. The probabilities changed according to the values of 70/30, 80/20, 60/40, and 65/35, meaning that in the first 40 trials, stimulus A leads to the aversive sound 70% of the time and stimulus B 30 of the time. In the following 40 trials, stimulus A leads to the aversive sound 80% of the time and stimulus B 20% of the time, and so on. Participants were not informed of either the probabilities or their changes and had to infer them from the observed outcomes (loud sound or no sound) in order to avoid the aversive outcome as much as possible. The baseline parameters used are the following, for Model 1, $\alpha$ = 0.4, $\beta$ = 7. For Model 2, $\alpha$ = 0.4, $\beta$ = 5,$A$=0.5. For Model 3, $\alpha^{+}$ = 0.25, $\alpha^{-}$=0.5, $\beta$ = 6. Where $V_o$ = 0.5, Number of trials 160 and Number of Simulations 1000 is fixed for all simulations.




(a) Stimuli A Utility (b) Stimuli B Utility




(c) Difference in Utility (d) Aversaive Outcome

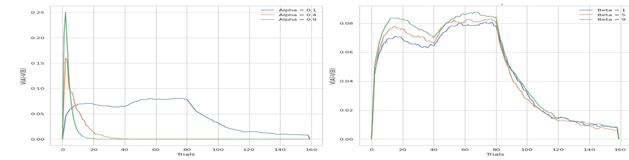
Fig. 4: Mechanism of Generative Model Simulation

These values reflect the subjective value or expected utility of each stimulus for the participant, and are updated based on the outcomes of previous trials. In the original research, VA and VB are modeled as latent variables that are updated in real time as the participant makes decisions and experiences outcomes. Each Model exhibit different mechanism in modeling the behaviour of a set of participants, using the baseline parameters to simulate the data. In Model 1, V(A) decreases over trials and reaches zero, while V(B) shows the same behavior, starting with a decrease and reaching 0.1 in the last trial. In Model 2, V(A) stays constant at 0.5, while V(B) drops initially and then stabilizes, showing a jump at trial 80. In Model 3, V(A) increases over trials and reaches one, while V(B) shows an initial drop followed by stabilization. The difference between V(A) and V(B) in all models increases and decreases before stabilizing at different levels. On average, Model 3 leads to the highest number of aversive sounds compared to Models 1 and 2.

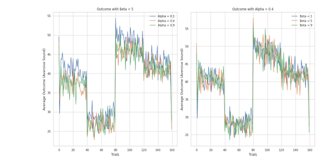
### *B. Sensitivity Analysis of Parameter and Model fitting*

To evaluate the impact of different values of parameters on each model. By varying individual parameters while keeping the rest constant, we aim to understand how changes in these parameters affect the outcome of the model and the behavior of participants.

*1) **Simple Rescorla-Wagner Model**:* ,



(a) Stimuli Utilities

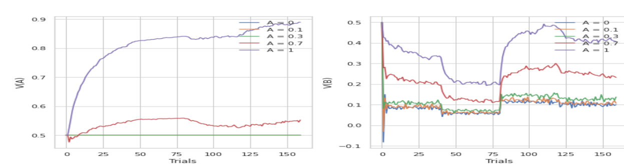


(b) Aversaive Outcome

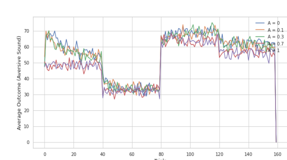
Fig. 5: Simple Generative Model Settings

In the first modeling scheme, the behavior of V(A) and V(B) is analyzed in terms of parameters setting. The model behavior of V(A) - V(B) is observed to change as the values of alpha and beta are altered. When beta is held constant, as alpha increases from 0.1 to 0.4 to 0.9, the trend of V(A) - V(B) becomes more pronounced with a narrower range of values. Similarly, when alpha is held constant and beta is increased from 1 to 5 to 9, the trend of V(A) - V(B) forms two hills and then decreases as beta increases, with the height of the hills increasing. The number of aversive sounds remains relatively stable across each set of 40 trials, with lower values of alpha and beta resulting in a higher number of aversive sounds.

*2) **Rescorla-Wagner Model with Salience**:* ,



(a) Stimuli Utilities

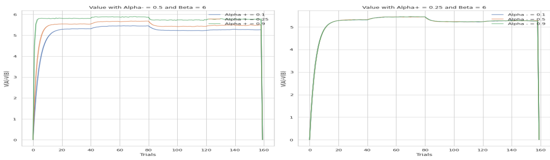


(b) Aversaive Outcome

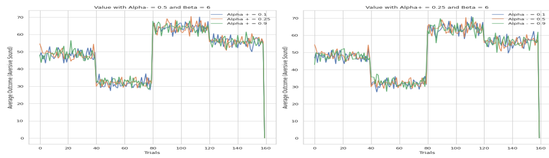
Fig. 6: Salience Generative Model Settings

In the second modeling scheme, the behavior of V(A) changes depending on the value of A, which represents the salience of the probability of the aversive stimuli. When alpha and beta are held constant and A is increased from 0 to 1, the V(A) value steadily increases by 0.1 for all values beyond 0.5. On the other hand, V(B) remains constant at around 0.1 for values of A between 0 and 0.5 and converges to V0/A for values of A beyond 0.5. This indicates that 'A' has a significant impact on the behavior of the participants in this model. Additionally, the mean count of aversive sounds across trials is higher for higher values of A, suggesting that a higher salience of the probability of the aversive stimuli leads to a higher number of aversive sounds experienced by the participants.

*3) **Rescorla-Wagner Model with Differential Learning Rates**:* ,



(a) Difference in Stimuli Utilities



(b) Aversaive Outcome

Fig. 7: Differential Learning Rates Generative Model Settings

In the third modeling scheme, the difference between V(A) and V(B) is analyzed when $\alpha^-$ is held constant and Beta is also held constant while changing $\alpha^+$ between 0.1, 0.25, and 0.9. The trend shows that the height of the utility increases with increasing $\alpha^+$. However, when $\alpha^-$ is changed, the difference between V(A) and V(B) remains the same for all values of $\alpha^-$ (0.1, 0.5, 0.9). The average number of aversive outcomes is the same for both sets of parameters. This suggests that the parameters $\alpha^-$ and $\alpha^+$ have a limited effect on the modeling of the participants' behavior.

## VII. Analysis on Participant's Data

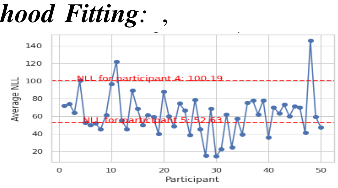
### A. Likelihood Optimization

*1) **Optimization Function:*** , To model the behavior of each participant, the parameter values that best capture their behavior must be found. This is done by defining the likelihood of the parameters using a function that takes in the data of an individual, including their choices and outcomes, and a vector of parameters such as learning rate and inverse temperature.

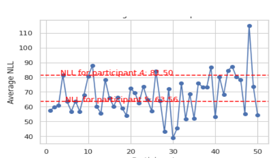
$$NLL = -\sum_{c \in chioces} logp(c|V, \theta) \tag{7}$$

The function returns the negative log likelihood (NLL) of the parameters, which is a measure of how well the parameters fit the data. The NLL is calculated using the parameter vector, which contains both alpha and beta, to evaluate the fit of the model to the data.

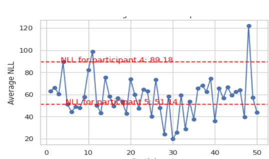
*2) **Likelihood Fitting:*** ,



(a) Simple Model



(b) Salience Model



(c) Differential Model

Fig. 8: Likelihood Fitting for Participant's

The results showed that the NLL scores for the simple model ranged from 20-150, with a mean of 60. Participants 4 and 5 had NLL scores of 100.19 and 52.63, respectively. For the second model, the NLL scores ranged from 40-115, with a mean of 75. Participants 4 and 5 had NLL scores of 81.50 and 63.56, respectively. For the third model, the NLL scores ranged from 20-120, with a mean of 50. Participants 4 and 5 had NLL scores of 89.18 and 51.14, respectively.

### B. Parameter and Model Fitting

The process of model fitting involves optimizing the parameters of the reinforcement learning model to best fit the behavior data of participants. The process starts with defining the negative log likelihood (NLL) of the parameters, which reflects the goodness of fit of the model to the participant's behavior. Next, the parameters of the model are optimized for each participant by minimizing the NLL. This is done using a optimization algorithm, such as Nelder-Mead, that finds the parameters that result in the lowest NLL. The process is repeated for all participants, resulting in a set of optimized parameters for each participant.

*1) **Simple Rescorla-Wagner Model:*** ,

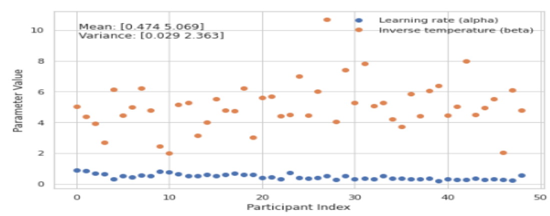



Fig. 9: Fitted Parameters

For the first modeling schema, the fitted parameters, which are the learning rate and inverse temperature, are plotted for each participant. The distribution of parameters are $\alpha \sim (\mu = 0.474, \sigma^2 = 0.029)$ and $\beta \sim (\mu = 5.069, \sigma^2 = 2.363)$.Its also evident that $\mu^{\alpha}_{calm} = 0.59, \mu^{\beta}_{calm} = 5.57$ and $\mu^{\alpha}_{anx} = 0.36, \mu^{\alpha}_{anx} = 0.4.65$

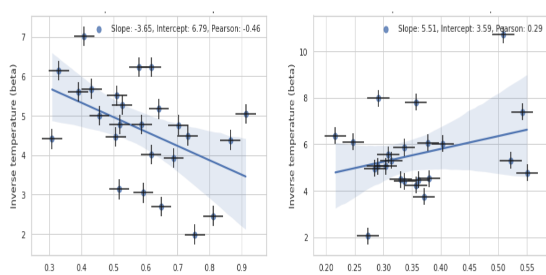



Fig. 10: Correlation Between Fitted Parameters

The relationship between the learning rate and the inverse temperature has been analyzed for both the calm and anxious population. The correlation for the calm population is moderate and negative (-0.46), meaning as the learning rate increases, the inverse temperature decreases, and participants with a higher learning rate tend to have a lower sensitivity to the outcomes of the task. On the other hand, the correlation for the anxious population is weak and positive (0.29), meaning as the learning rate increases, the inverse temperature also increases, and participants with a higher learning rate tend to have a higher sensitivity to the outcomes of the task.

*2) **Rescorla-Wagner Model with Salience:*** ,

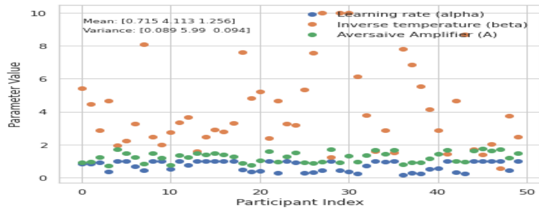



Fig. 11: Fitted Parameters

For the second modeling schema, the fitted parameters, which are the learning rate , inverse temperature and salience, are plotted for each participant. The distribution of parameters are $\alpha \sim (\mu = 0.715, \sigma^2 = 0.089)$, $\beta \sim (\mu = 4.113, \sigma^2 = 5.99)$ and $A \sim (\mu = 1.250, \sigma^2 = 0.094)$.Its also evident that $\mu^{\alpha}_{calm} = 0.63, \mu^{\beta}_{calm} = 4.56, \mu^{A}_{calm} = 1.29$ and $\mu^{\alpha}_{anx} = 0.83, \mu^{\alpha}_{anx} = 3.65, \mu^{A}_{anx} = 1.22$.

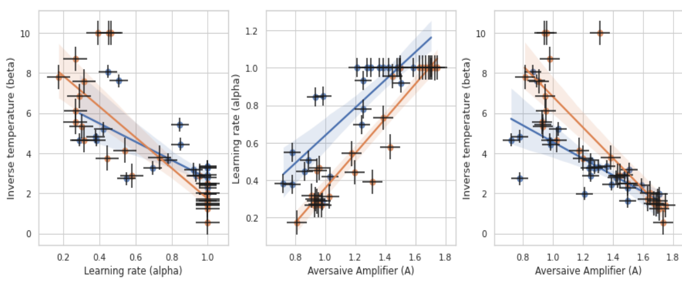



Fig. 12: Correlation Between Fitted Parameters

The results show that the correlation between Beta and Alpha was found to be strong and negative (-0.81 and -0.71) in both populations, meaning that as the learning rate increases, the inverse temperature decreases. This suggests that participants with a higher learning rate tend to be less sensitive to the outcomes of the task. The correlation between Alpha and A was strong and positive (0.83 and 0.95), meaning that as the learning rate increases, the probability of observing an aversive sound also increases. This suggests that participants with a higher learning rate tend to experience more aversive outcomes in the task. The correlation between Beta and A were strong and negative (-0.72 and -0.83), meaning that as the inverse temperature increases, the probability of observing an aversive outcome decreases. This suggests that participants with a higher inverse temperature tend to observe less aversive sound in the task.

*3) **Rescorla-Wagner Model with Differential Learning:*** ,

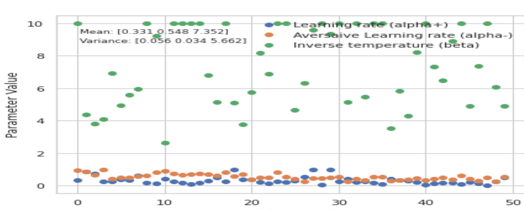



Fig. 13: Fitted Parameters

For the third modeling schema, the fitted parameters, which are the differential learning rates and inverse temperature, are plotted for each participant. The distribution of parameters are $\alpha^{+} \sim (\mu = 0.331, \sigma^2 = 0.056)$,$\alpha^{-} \sim (\mu = 0.548, \sigma^2 = 0.034)$ and $\beta \sim (\mu = 7.352, \sigma^2 = 5.662)$.Its also evident that $\mu^{\alpha^+}_{calm} = 0.36, \mu^{\alpha^-}_{calm} = 0.67, \mu^{\beta}_{calm} = 6.88$ and $\mu^{\alpha^+}_{anx} = 0.26, \mu^{\alpha^-}_{anx} = 0.38, \mu^{\beta}_{anx} = 7.17$.

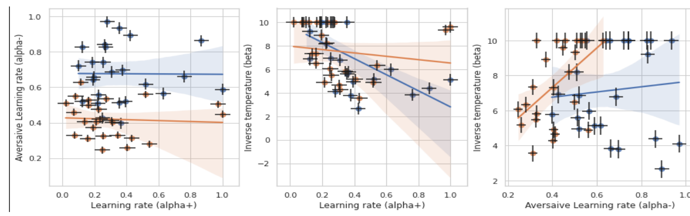



Fig. 14: Correlation Between Fitted Parameters

The results show that the correlation between learning rate and inverse temperature was negligible for both populations (-0.05 and -0.01). However, the relationship between Beta and $\alpha^+$ was moderate, weak and negative for both populations (-0.56 and -0.17), meaning that as $\alpha^+$ increases, $\beta$ decreases. Participants with a higher inverse temperature tend to have a lower sensitivity to non-aversive outcomes, while those with a lower inverse temperature tend to have a higher sensitivity. On the other hand, the relationship between Beta and $\alpha^-$ was moderate, strong, and positive for both populations (0.32 and 0.78), meaning that as $\alpha^-$ increases, $\beta$ increases. Participants with a higher inverse temperature tend to experience higher sensitivity to aversive outcomes while those with a lower inverse temperature tend to experience lower sensitivity.

### C. *Populations Comparison*

To determine whether both population groups have different parameters we measure the significance of difference in mean values of parameters. Assuming a significance level of 5% to test hypothesis.

| | Parameter | |
|---|---|---|
| | Alpha | Beta |
| T-Statistic | 6.219 | -1.997 |
| P-Value | 1.25e-7 | 0.0515 |
| Degrees of Freedom | 48 | 48 |

TABLE I: Simple Model Population Parameter T-test

Table 1 , shows the significance for both populations parameters in Simple Rescorla-Wagner Model. We can determine from the p-value of the tests that Alpha is significantly different for both populations where beta is not very significantly different for both populations.

| | Parameter | | |
|---|---|---|---|
| | Alpha | Beta | A |
| T-Statistic | 2.055 | -1.229 | -0.874 |
| P-Value | 0.045 | 0.199 | 0.386 |
| Degrees of Freedom | 48 | 48 | 48 |

TABLE II: Salience Model Population Parameter T-test

Table 2, shows the significance for both populations parameters in Rescorla-Wagner Model with Salience. We can determine from the p-value of the tests that Alpha is different for both populations where beta and A are not very significantly different for both populations.

| | Parameter | | |
|---|---|---|---|
| | Alpha + | Alpha- | Beta |
| T-Statistic | 0.929 | 6.776 | -0.532 |
| P-Value | 0.357 | 1.62e-8 | 0.597 |
| Degrees of Freedom | 48 | 48 | 48 |

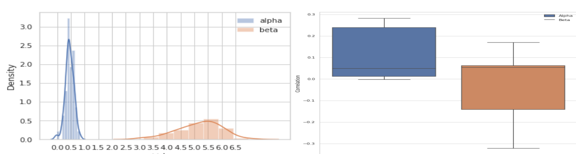
TABLE III: Differential Rate Model Population Parameter T-test

Table 3, shows the significance for both populations parameters in Rescorla-Wagner Model with Differential Learning Rates. We can determine from the p-value of the tests that $\alpha^{-}$ is significantly different for both populations where the rest of the parameters are not very significantly different for both populations.

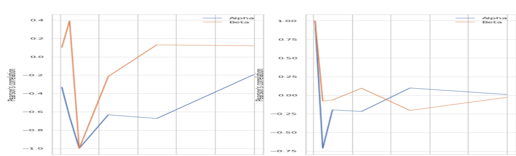
## VIII. Reliability and Identifiability of Modeling Scheme on Participant's Data

### A. Parameter Recovery

Parameter estimates were tested by sampling 50 sets parameters from a multivariate normal distribution for each modeling scheme. The mean and variance were set based on the findings of previous section with covariance set to zero, note that if the variance $>> 1$ we set it to 0.05. The sampled values were used to simulate data and fit new parameters, which were then compared to the original parameters using Pearson's correlation. This process was repeated 5 times, with the Pearson's correlation reported each time to assess the consistency of the results.



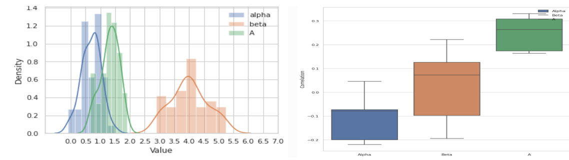
(a) Distribution of Sampled Parameters (b) Correlation Between Sampled and Fitted Parameters



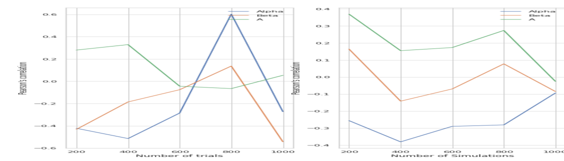
(c) Affect of Trials and Simulations on Correlation between Parameters

Fig. 15: Simple Rescorla-Wagner Model Diagnostics

Simple Rescorla-Wagner Model parameter recovery is shown in Fig 15, which includes three plots: the distribution of sampled parameters, the box plot of correlations between sampled and fitted parameters, and the effect of simulations and number of trials on the correlation between sampled and fitted parameters. The normal distribution of the sampled parameters, as shown in Fig. 15a, indicates that the results are unbiased and that the parameters have been sampled from each interval across a multivariate distribution. Fig 15b shows the following values for the correlations between sampled and fitted parameters for alpha and beta: [0.28, 0.01, 0.05, -0.00, 0.24] and [0.06, 0.05, -0.32, 0.17, -0.14], respectively. As the number of trials and simulations increased, the correlation between the fitted and sampled parameters eventually reached zero.



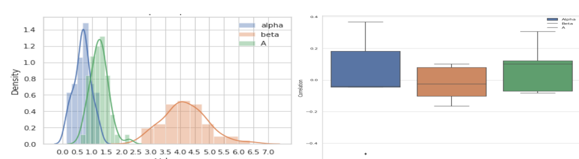
(a) Distribution of Sampled Parameters (b) Correlation Between Sampled and Fitted Parameters



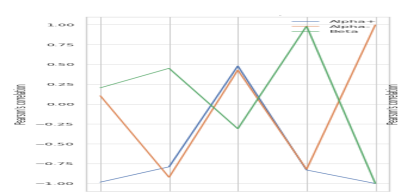
(c) Affect of Trials and Simulations on Correlation between Parameters

Fig. 16: Rescorla-Wagner Model with Salience Diagnostics

The parameter recovery of Simple Rescorla-Wagner Model with salience is analyzed, showing the distribution of sampled parameters, correlation between sampled and fitted parameters, and the effect of simulations and number of trials on this correlation. The normal distribution of the sampled parameters suggests that the conclusions are not biased. The correlation between sampled and fitted parameters of alpha, beta, and A is shown in a box plot. The mean correlation between the sampled and fitted alpha is -0.08 with a range of values from -0.2 to -0.07 over 5 simulations. The mean correlation between the sampled and fitted beta is 0.07 with a range of values from -0.1 to 0.13. The mean correlation between the sampled and fitted A is 0.27 with a range from 0.17-0.31. As the number of trials increase correlations between parameters have different behaviour when experimenting with about 620 trials the sampleed and fitted parameters are closest but diverges as we increase number if trials, but as number of Simulations increase the correlation between parameters approaches zero.



(a) Distribution of Sampled Parameters (b) Correlation Between Sampled and Fitted Parameters

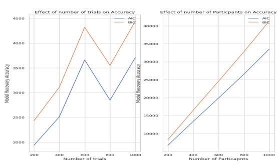


(c) Affect of Trials on Correlation between Parameters

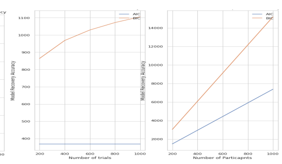
Fig. 17: Rescorla-Wagner Model with Differntial Rates Diagnostics

The parameter recovery of Rescorla-Wagner Model with Differential Rates is analyzed, showing the distribution of sampled parameters, correlation between sampled and fitted parameters, and the effect of simulations and number of trials on this correlation. The normal distribution of the sampled parameters suggests that the conclusions are not biased. The correlation between sampled and fitted parameters of $\alpha^{+}$, $\alpha^{-}$, and $\beta$ is shown in a box plot. The mean correlation between the sampled and fitted $\alpha^{+}$is -0.02 with a range of values from -0.04 to 0.18 . The mean correlation between the sampled and fitted $\alpha^{-}$ is 0.01 with a range of values from -0.01 to 0.1 The mean correlation between the sampled and fitted $\beta$ is 0.14 with a range from -0.05-0.15. As the number of trials increase correlations behave differently when experimenting with about 600 trials the sampled and fitted parameters are closest but diverges as we increase number if trials, but as number of Simulations increase the correlation between parameters approaches zero.

### *B. Model Recovery*



(a) Simple Model



(b) Salience Model

(c) Differential Rate Model

Fig. 18: Affects of Trials and Participants on Model Recovery

The effect of the number of trials and number of participants on model recovery was determined by plotting the AIC and BIC scores versus the number of trials and simulations for each model. The number of trials and participants were varied from 200 to 1000. For the simple model, as seen in Fig 18a, the AIC and BIC scores increase as the number of trials increases, then decrease and then increase again, but as the number of participants increases, the scores continuously increase. For the Salience model, seen in Fig 18b, as the number of trials increases, the AIC score remains the same while the BIC score increases parabolic-ally. As the number of participants increases, both the AIC and BIC scores increase linearly. The Differential rate model in Fig 18c follows a similar pattern to the Salience model.

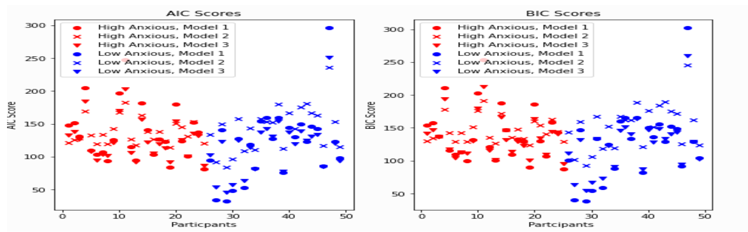
## IX. Results

### *A. Models Comparison*

The comparison between the two models (simple Rescorla-Wagner model and the Rescorla-Wagner model with salience) is made using AIC and BIC scores. The negative log likelihood values are compared between the models for each participant. The AIC and BIC scores are calculated for each model and summed up for each participant. The results are reported and commented upon, with the "best" model being chosen based on the AIC and BIC scores. The calculation of AIC and BIC is based on the negative log likelihood, the number of parameters, and the number of observations in the experiment. The formulas used for the calculations are provided for reference, where p is the number of estimated aprameters and n is the number of trials in an expirmental task.

$$AIC = NLL + 2p \tag{8}$$

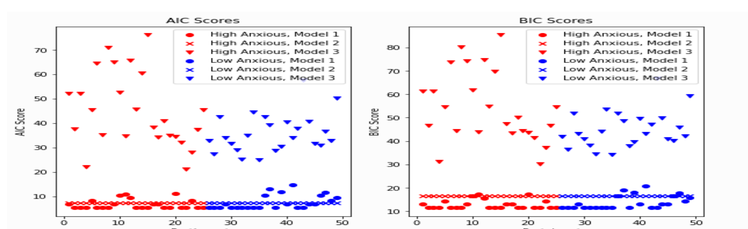
$$BIC = NLL + plog(n) \tag{9}$$




(a) Simple Model

Fig. 19: Model Scores of Participants Data

For the participant data, AIC and BIC scores were calculated for three different models (Model 1, Model 2, and Model 3) for both the calm and anxious populations. The sum of AIC scores for Model 1,2 and 3 are 6369.16, 7043.37, and 6020.70 respectively. For BIC scores, Model 1,2, and 3 are 6676.68, 7504.64, and 6481.97 respectively. When considering only the high anxious participants, the sum of AIC scores for Model 1,2, and 3 are 3523.80, 3597.31, and 3278.35 respectively. BIC scores for Model 1,2, and 3 are 3683.71, 3837.17, and 3518.22 respectively. For the low anxious participants, the sum of AIC scores for Model 1,2, and 3 are 2845.37, 3446.06, and 2742.34. Furthermore, sum of BIC scores for Model 1,2, and 3 are 2992.98, 3667.47, and 2963.76. Based on the AIC and BIC scores, Model 3 is the best fit for the participant data as it has the lowest AIC and BIC scores among all the models.



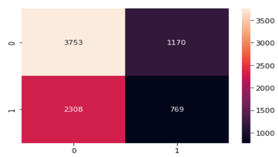

(a) Simple Model

Fig. 20: Model Scores of Simulated Data

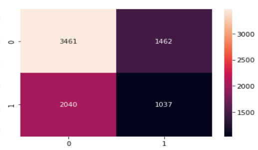
For the simulated data, AIC and BIC scores were calculated for three different models (Model 1, Model 2, and Model 3) for both the calm and anxious populations. The sum of AIC scores for Model 1,2, and 3 are 358.05, 369.31, and 2009.82 respectively. For BIC scores, Model 1,2, and 3 are 665.57, 830.59, and 2471.09 respectively. When considering only the high anxious participants, the sum of AIC scores for Model 1,2, and 3 are 166.15, 184.66, and 1125.96 respectively. BIC scores for Model 1,2, and 3 are 319.91, 415.30, and 1356.59 respectively. For the low anxious participants, the sum of AIC scores for Model 1,2, and 3 are 181.35, 177.27, and 860.10. Furthermore, sum of BIC scores for Model 1,2, and 3 are 328.96, 398.68, and 1081.51. Based on the AIC and BIC scores, Model 1 is the best fit for the simulated data as it has the lowest AIC and BIC scores among all the models.

### B. Predictability of Modeling

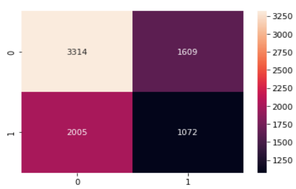
To determine the most accurate model for predicting the generated data, the results of the confusion matrix, precision accuracy, and F1 scores were compared. The confusion matrix was used to calculate the number of true positive, true negative, false positive, and false negative predictions made by each model.By comparing the results of these three metrics, a clear understanding of the predictability of each model was obtained and the best-performing model was selected as the one that generated the data.



(a) Simple Model



(b) Salience Model



(c) Differential Rate Model

Fig. 21: Affects of Trials and Participants on Model Recovery

The results of the confusion matrix were used to evaluate the model predictibility and compare the performance of Model 1, Model 2, and Model 3. The results of Model 1's confusion matrix showed that 3778 samples were correctly classified as the true class and 1170 were incorrectly classified as the false class. On the other hand, 2308 samples were incorrectly classified as the true class and 769 were correctly classified as the false class. The results of Model 2 showed that 3461 samples were correctly classified as the true class and 1462 were incorrectly classified as the false class. Meanwhile, 2040 samples were incorrectly classified as the true class and 1037 were correctly classified as the false class. The results of Model 3's confusion matrix showed that 3314 samples were correctly classified as the true class and 1069 were incorrectly classified as the false class. 2005 samples were incorrectly classified as the true class and 1072 were correctly classified as the false class.Based on the calculated results of precision, accuracy, and F1 scores, Model 1 has the highest precision score of 0.7635 , followed by Model 3 with 0.7561 and Model 2 with 0.7030. In terms of accuracy, Model 3 has the highest score of 0.5879, followed by Model 1 with 0.5666 and Model 2 with 0.5622. Finally, Model 1 also has the highest F1 score of 0.6847, followed by Model 3 with 0.6832 and Model 2 with 0.6640.

## X. Summary of Findings and recommendations

Findings:

- Model 3, Rescorla-Wagner Model with Differential Rates, showed the lowest AIC and BIC scores among all three models, suggesting that it is the best fit for the participant data.
- Model 1, Simple Rescorla-Wagner Model, had the highest precision, accuracy, and F1 scores among the models, making it the best-performing model for predicting the generated data.
- Model 1 had the highest precision score of 0.7635, followed by Model 3 with 0.7561 and Model 2 with 0.7030.
- Model 3 had the highest accuracy score of 0.5879, followed by Model 1 with 0.5666 and Model 2 with 0.5622.
- Model 1 also had the highest F1 score of 0.6847, followed by Model 3 with 0.6832 and Model 2 with 0.6640.

Recommendations:

- Based on the results, it is recommended to use Model 3 for further research on anxiety treatment for calm and anxious populations.
- Further research could focus on improving the predictability of the Model 3 for anxiety treatment and finding ways to increase the precision and accuracy scores.

Measures for anxiety treatments:

- Based on the results of the study, it is suggested to use Model 3 for further research on anxiety treatments.
- The study provides a foundation for exploring the effectiveness of different treatments for calm and anxious populations, which can help in developing targeted interventions.
- Further research can focus on improving the predictability of Model 3 and finding ways to increase the precision and accuracy scores, which will help in developing more effective treatments for anxiety.
- It is also recommended to consider additional factors, such as the individual's unique characteristics and the environment, when developing treatments for anxiety, as these factors may play a significant role in the effectiveness of the treatments.

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