The De Havilland DHC-2 ‘Beaver’ aircraft

- **We have 12 equations need to be arranged for integration :**
- **Equations 7-12 are OK.**

Euler rates

$$\boxed{\begin{bmatrix}\dot{\Phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}} = \begin{bmatrix}1 & S_\Phi \tan\theta & C_\Phi \tan\theta\\ 0 & C_\Phi & -S_\Phi\\ 0 & S_\Phi \sec\theta & C_\Phi \sec\theta\end{bmatrix}\boxed{\begin{bmatrix}p\\ q\\ r\end{bmatrix}}$$

angular velocities in the body frame

$$\boxed{\begin{bmatrix}\dfrac{dx}{dt}\\ \dfrac{dy}{dt}\\ \dfrac{dz}{dt}\end{bmatrix}} = [C]\boxed{\begin{bmatrix}u\\ v\\ w\end{bmatrix}}$$

Absolute Velocity Components along the fixed frame

Velocity Components along the body axes

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- **Equation 1-3 need small arrangement:**

$$F_x - mg\sin(\theta) = m(\dot{u} + qw - rv)$$

$$F_y + mg\cos(\theta)\sin(\phi) = m(\dot{v} + ru - pw)$$

$$F_z + mg\cos(\theta)\cos(\phi) = m(\dot{w} + pv - qu)$$

- **What about equations 4-6 ?**

Moments

$$\boxed{L} = I_x\dot{p}_i - I_{xz}\dot{r} + qr(I_z - I_y) - I_{xz}pq$$

$$\boxed{M} = I_y\dot{q} + qr(I_x - I_z) + I_{xz}(p^2 - r^2)$$

$$\boxed{N} = I_{xz}\dot{p}_i + I_z\dot{r} + qp(I_y - I_z) - I_{xz}rq$$

12 equations, 12 unknowns/variables: x, y, z; ψ, ϕ, θ; u, v, w; p, q, r

Manufacturer	De Havilland Aircraft of Canada Ltd.
Serial no.	1244
Type of aircraft	Single engine, high-wing, seven seat, all-metal aircraft
Wing span b	14.63 m
Wing area S	23.23 m^2
Mean aerodynamic chord $\overline{c}$	1.5875 m
Wing sweep	$0°$
Wing dihedral	$1°$
Wing profile	NACA 64 A 416
Fuselage length	9.22 m
Max. take-off weight	22800 N
Empty weight	14970 N
Engine	Pratt and Whitney Wasp Jr. R-985
Max. power	450 Hp at $n = 2300\ RPM$, $p_z = 26'' Hg$
Propeller	Hamilton Standard, two-bladed metal regulator propeller
Diameter of the propeller	2.59 m
Total contents of fuel tanks	521 l
Contents fuselage front tank	131 l
Contents fuselage center tank	131 l
Contents fuselage rear tank	95 l
Contents wing tanks	2 x 82 l
Most forward admissible c.g. position	17.36% $\overline{c}$ at 16989 N; 29.92% $\overline{c}$ at 22800 N
Most backward admissible c.g. position	40.24% $\overline{c}$

$x_{c.g.}$	$=$	0.5996	$[m]$	in F_M
$y_{c.g.}$	$=$	0.0	$[m]$	in F_M
$z_{c.g.}$	$=$	-0.8851	$[m]$	in F_M
I_x	$=$	5368.39	$[kg\,m^2]$	in F_R
I_y	$=$	6928.93	$[kg\,m^2]$	in F_R
I_z	$=$	11158.75	$[kg\,m^2]$	in F_R
J_{xy}	$=$	0.0	$[kg\,m^2]$	in F_R
J_{xz}	$=$	117.64	$[kg\,m^2]$	in F_R
J_{yz}	$=$	0.0	$[kg\,m^2]$	in F_R
m	$=$	2288.231	$[kg]$	
h	$=$	1828.8	$[m]$	$(= 6000\ [ft$
ρ	$=$	1.024	$[kg\,m^{-3}]$	

$$X_a = C_{X_a} \cdot \frac{1}{2}\rho V^2 S$$
$$Y_a = C_{Y_a} \cdot \frac{1}{2}\rho V^2 S$$
$$Z_a = C_{Z_a} \cdot \frac{1}{2}\rho V^2 S$$

and:

$$L_a = C_{l_a} \cdot \frac{1}{2}\rho V^2 S b$$
$$M_a = C_{m_a} \cdot \frac{1}{2}\rho V^2 S \bar{c}$$
$$N_a = C_{n_a} \cdot \frac{1}{2}\rho V^2 S b$$

Aerodynamic force and moment coefficients measured in the body-fixed reference frame:

$$
\begin{aligned}
C_{X_a} &= C_{X_0} + C_{X_\alpha}\alpha + C_{X_{\alpha^2}}\alpha^2 + C_{X_{\alpha^3}}\alpha^3 + C_{X_q}\frac{q\bar{c}}{V} + C_{X_{\delta_r}}\delta_r + C_{X_{\delta_f}}\delta_f + C_{X_{\alpha\delta_f}}\alpha\delta_f \\
C^*_{Y_a} &= C_{Y_0} + C_{Y_\beta}\beta + C_{Y_p}\frac{pb}{2V} + C_{Y_r}\frac{rb}{2V} + C_{Y_{\delta_a}}\delta_a + C_{Y_{\delta_r}}\delta_r + C_{Y_{\delta_r\alpha}}\delta_r\alpha \\
C_{Z_a} &= C_{Z_0} + C_{Z_\alpha}\alpha + C_{Z_{\alpha^3}}\alpha^3 + C_{Z_q}\frac{q\bar{c}}{V} + C_{Z_{\delta_e}}\delta_e + C_{Z_{\delta_e\beta^2}}\delta_e\beta^2 + C_{Z_{\delta_f}}\delta_f + C_{Z_{\alpha\delta_f}}\alpha\delta_f \\
C_{l_a} &= C_{l_0} + C_{l_\beta}\beta + C_{l_p}\frac{pb}{2V} + C_{l_r}\frac{rb}{2V} + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r + C_{l_{\delta_a\alpha}}\delta_a\alpha \\
C_{m_a} &= C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\alpha^2}}\alpha^2 + C_{m_q}\frac{q\bar{c}}{V} + C_{m_{\delta_e}}\delta_e + C_{m_{\beta^2}}\beta^2 + C_{m_r}\frac{rb}{2V} + C_{m_{\delta_f}}\delta_f \\
C_{n_a} &= C_{n_0} + C_{n_\beta}\beta + C_{n_p}\frac{pb}{2V} + C_{n_r}\frac{rb}{2V} + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r + C_{n_q}\frac{q\bar{c}}{V} + C_{n_{\beta^3}}\beta^3
\end{aligned}
$$

C_X		C_Y		C_Z	
parameter	value	parameter	value	parameter	value
0	-0.03554	0	-0.002226	0	-0.05504
α	0.002920	β	-0.7678	α	-5.578
α^2	5.459	$\frac{pb}{2V}$	-0.1240	α^3	3.442
α^3	-5.162	$\frac{rb}{2V}$	0.3666	$\frac{q\bar{c}}{V}$	-2.988
$\frac{q\bar{c}}{V}$	-0.6748	δ_a	-0.02956	δ_e	-0.3980
δ_r	0.03412	δ_r	0.1158	$\delta_e\beta^2$	-15.93
δ_f	-0.09447	$\delta_r\alpha$	0.5238	δ_f	-1.377
$\alpha\delta_f$	1.106	$\frac{\dot{\beta}b}{2V}$	-0.1600	$\alpha\delta_f$	-1.261

C_l		C_m		C_n	
parameter	value	parameter	value	parameter	value
0	0.0005910	0	0.09448	0	-0.003117
β	-0.06180	α	-0.6028	β	0.006719
$\frac{pb}{2V}$	-0.5045	α^2	-2.140	$\frac{pb}{2V}$	-0.1585
$\frac{rb}{2V}$	0.1695	$\frac{q\bar{c}}{V}$	-15.56	$\frac{rb}{2V}$	-0.1112
δ_a	-0.09917	δ_e	-1.921	δ_a	-0.003872
δ_r	0.006934	β^2	0.6921	δ_r	-0.08265
$\delta_a\alpha$	-0.08269	$\frac{rb}{2V}$	-0.3118	$\frac{q\bar{c}}{V}$	0.1595
		δ_f	0.4072	β^3	0.1373

C_X		C_Y		C_Z	
parameter	value	parameter	value	parameter	value
dpt	0.1161	–	–	dpt	-0.1563
$\alpha \cdot dpt^2$	0.1453				

C_l		C_m		C_n	
parameter	value	parameter	value	parameter	value
$\alpha^2 \cdot dpt$	-0.01406	dpt	-0.07895	dpt^3	-0.003026

$$dpt \equiv \frac{\Delta p_t}{\frac{1}{2}\rho V^2} = C_1 + C_2 \frac{P}{\frac{1}{2}\rho V^3} \quad \text{with:} \quad \left\{ \begin{array}{lcl} C_1 & = & 0.08696 \\ C_2 & = & 191.18 \end{array} \right.$$

- Engine power P, $[N m\, s^{-1}]$:

$$P = 0.7355 \left\{ -326.5 + \left(0.00412(p_z + 7.4)(n + 2010) + (408.0 - 0.0965n) \left(1.0 - \frac{\rho}{\rho_0} \right) \right) \right\}$$

Equations

- Dimensional forces, $[N]$:

$$\begin{aligned} X_a &= C_{X_a}\ q_{dyn}\, S \\ Y_a &= C_{Y_a}\ q_{dyn}\, S \\ Z_a &= C_{Z_a}\ q_{dyn}\, S \end{aligned}$$

- Dimensional moments, $[Nm]$:

$$\begin{aligned} L_a &= C_{l_a}\ q_{dyn}\, S\, b \\ M_a &= C_{m_a}\ q_{dyn}\, S\, \overline{c} \\ N_a &= C_{n_a}\ q_{dyn}\, S\, b \end{aligned}$$

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Steady wings-level flight.
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Give desired airspeed [m/s],                35

Give (initial) altitude [m],                2000*0.3048

Give heading [deg], default = 0:

Give flap angle [deg], default = 0:

Give engine speed [RPM], default = 1800:

Give manifold pressure pz ["Hg], default = 20:
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X= $V \quad \alpha \quad \beta \quad p \quad q \quad r \quad \psi \quad \theta \quad \varphi \quad x_e \quad y_e \quad H$

U= $\delta_e \quad \delta_a \quad \delta_r \quad \delta_f \quad n \quad p_z$

$$u = V \cos\alpha \cos\beta$$

$$v = V \sin\beta$$

$$w = V \sin\alpha \cos\beta$$

```
State vector (trimmed):

x =  3.5000e+001
     2.1131e-001
    -2.0667e-002
               0
               0
               0
               0
     1.9190e-001
               0
               0
               0
               0

Input vector (trimmed):

u = -9.3083e-002
     9.6242e-003
    -4.9506e-002
               0
     1.8000e+003
     2.0000e+001
```

1.7 ISA Atmospheric Model

For the atmospheric data an approximation of the International Standard Atmosphere (ISA) is used [Mulder et al., 2000].

$$
\begin{aligned}
T &= T_0 - 0.0065h \\
\rho &= \rho_0 e^{-\frac{g}{287.05T}h} \\
a &= \sqrt{1.4 \times 287.05T}
\end{aligned}
$$

where $T_0 = 288.15$ is the temperature at sea level and $\rho_0 = 1.225$ is the air density at sea level. This atmospheric model is only valid in the troposphere ($h < 11000$ m). Given the aircraft's altitude (h in meters) it returns the current temperature (T in Kelvin), the current air density (ρ in kg/m^3) and the speed of sound (a in m/s).