

CSC 6580

Spring 2020

Instructor: Stacy Prowell

Assembly: Strings and Loops



Aside: jrcxz / jecxz / jcxz

RCX is special, and it even has its own conditional branch instruction to test whether the register content is zero. These instructions *do not test ZF*, but instead test the content of **RCX**, or **ECX**, or **CX**, and take the branch if it is zero.

Searched all executables in `/usr/bin*`, and found it used in **busybox** and nothing else.

Its use in `/usr/bin/busybox` appears to be *signaling whether to take a branch in a subroutine*. That is, it is an argument that specifies which of two options to take.

*for file in \$(find /usr/bin -exec file {} \; | grep ELF | cut -d: -f1) ; do (objdump -Intel -d \$file | grep -q 'jrcx' && echo \$file) ; done
NB: Not a pristine Ubuntu 19.10 installation. Your counts will likely vary (slightly).



loop


The `loop LABEL` instruction is *almost* equivalent to the sequence `dec rcx; jnz LABEL`.

The difference is that the latter instructions set flags... and the `loop` instruction does *not*.

So: Use your assembly reading skillz. What will be the exit value of this program? (The exit value will be the final value in `RDI` when `sys_exit` is called.)

```
section .text
global _start

_start:  mov rcx, 0xffff
         cmp rcx, 76
         mov rdi, 100
         mov rbx, 50
.top:    loop .top
         mov rsi, 0
         cmovz rdi, rbx
         mov rax, 60
         syscall
         hlt
```



loope / loopne loopz / loopnz


There are *two* variants with *four* mnemonics. These terminate when **RCX** reaches zero, but they can also terminate early by testing **ZF**.

In the code at the right, the loop terminates early because when **RCX** reaches 7, the **ZF** flag is set by the compare, and **loopne** terminates the loop.

Note that the *compare* happens and *then* the **loopne** instruction runs. This *decrements RCX*, so the return value is 6.

```
section .text
global _start

_start:  mov rcx, 10
.top:    cmp rcx, 7
         loopne .top
         mov rdi, rcx
         mov rax, 60
         syscall
         htl
```



loope / loopne loopz / loopnz

Here **RCX** reaches zero before the condition can match, so the loop exits and the return value is zero.

Remember: These forms end the loop if *either* the condition is met *or* **RCX** is zero.

```
section .text
global _start

_start:  mov rcx, 10
.top:    cmp rcx, -7
         loopne .top
         mov rdi, rcx
         mov rax, 60
         syscall
         htl
```



How Common? Not very.

Instruction	Count in /usr/bin (1,190 ELF files)
<code>loope</code>	3 (<code>sntp</code> , <code>sudo</code> , <code>sudoreplay</code>)
<code>loopz</code>	0
<code>loopne</code>	1 (<code>dash</code>)
<code>loopnz</code>	0
<code>loop</code>	88



String Instructions*

Instruction	Meaning
<code>movsb</code> / <code>movsw</code> / <code>movsd</code> / <code>movsq</code>	Move String: Copy a byte, word, double-word, or quad-word from <code>[DS:RSI]</code> to <code>[ES:RDI]</code> and increment (or decrement) both <code>RSI</code> and <code>RDI</code> by the correct amount
<code>cmpsb</code> / <code>cmpsw</code> / <code>cmpsd</code> / <code>cmpsq</code>	Compare String: Compare a byte, word, double-word, or quad-word from <code>[DS:RSI]</code> to <code>[ES:RDI]</code> and increment (or decrement) both <code>RSI</code> and <code>RDI</code> by the correct amount
<code>scasb</code> / <code>scasw</code> / <code>scasd</code> / <code>scasq</code>	Scan String: Compare a byte, word, double-word, or quad-word from <code>[ES:RDI]</code> to <code>AL</code> , <code>AX</code> , <code>EAX</code> , or <code>RAX</code> and then increment (or decrement) both <code>RDI</code>
<code>lodsb</code> / <code>lodsw</code> / <code>lodsd</code> / <code>lodsq</code>	Load from String: Move the byte, word, double-word, or quad-word from <code>[DS:RSI]</code> to <code>AL</code> , <code>AX</code> , <code>EAX</code> , or <code>RAX</code> and then increment (or decrement) <code>RSI</code>
<code>stosb</code> / <code>stosw</code> / <code>stowd</code> / <code>stosq</code>	Store to String: Move the byte, word, double-word, or quad-word from <code>AL</code> , <code>AX</code> , <code>EAX</code> , or <code>RAX</code> to <code>[ES:RDI]</code> and then increment (or decrement) <code>RDI</code>

* NB: This table ignores two other families of string instructions: `insb` / `insw` / `insd` / `insq` / `ins` and `outsb` / `outsw` / `outsd` / `outsq` / `outs`. These input from and output to an I/O port.



The Direction Flag

The **DI** flag controls the *direction* of the string instructions. All string instructions increment (or decrement) either **RSI** or **RDI** or both.

- If **DI** is clear (0) then *increment*.
- If **DI** is set (1) then *decrement*.

Control the **DI** flag with **STD** (set direction flag) and **CLD** (clear direction flag). The flag is *commonly* clear, but it is a good idea to explicitly set or clear the flag before you use it.

* NB: It is rare, but the string instructions can be written **movs**, **cmps**, **scas**, **lods**, and **stos** (without a letter specifying size). In this case the instruction takes a memory argument. This argument's size determines the size of the data... but is otherwise *ignored*. Thus **movs QWORD [rdi + 12]** is the same as **movsq**.



How Common? Pretty common.

Instruction	Count in /usr/bin (1,190 ELF files)
<code>movs?</code>	1,116
<code>cmps?</code>	612
<code>scas?</code>	306
<code>lods?</code>	0
<code>stos?</code>	441
<code>cld?</code>	62
<code>std?</code>	1,176



rep / repe / repne

Repeat a string instruction some number of times.

This is a *prefix*, and not an instruction.

These work similarly to `loop` / `loope` / `loopne`.

These repeat the subsequent string instruction the number of times specified in the count register `RCX` or until the indicated condition of the `ZF` flag is no longer met.

... But how does `ZF` get set?

```
; Copy a string from str to pad.  
lea rsi, [str]  
lea rdi, [pad]  
mov rcx, str.len  
rep movsb
```

```
; Find the first exclamation mark and  
; replace it with a period.  
lea rdi, [pad]  
mov al, '!'  
mov rcx, str.len  
repne scasb  
mov [rdi-1], BYTE '.'
```



rep / repe / repne

The `cmps` and `scas` instructions set (or clear) `ZF`.

```
; Copy a string from str to pad.  
lea rsi, [str]  
lea rdi, [pad]  
mov rcx, str.len  
rep movsb
```

```
; Find the first exclamation mark and  
; replace it with a period.  
lea rdi, [pad]  
mov al, '!'  
mov rcx, str.len  
repne scasb  
mov [rdi-1], BYTE '.'
```



rep / repe / repne

The string instructions *always* increment (or decrement) **RCX** and **RSI** and/or **RDI**.

```
; Copy a string from str to pad.  
lea rsi, [str]  
lea rdi, [pad]  
mov rcx, str.len  
rep movsb
```

```
; Find the first exclamation mark and  
; replace it with a period.  
lea rdi, [pad]  
mov al, '!'   
mov rcx, str.len  
repne scasb  
mov [rdi-1], BYTE '.'
```



Aside: Ambiguity

You must specify the data width here because it is *ambiguous*. The instruction `mov al, '!'` is not ambiguous, since `AL` is a byte value.

The instruction `mov BYTE [rdi-1], '.'` would have also worked.

```
; Copy a string from str to pad.  
lea rsi, [str]  
lea rdi, [pad]  
mov rcx, str.len  
rep movsb
```

```
; Find the first exclamation mark and  
; replace it with a period.  
lea rdi, [pad]  
mov al, '!'  
mov rcx, str.len  
repne scasb  
mov [rdi-1], BYTE '.'
```

PIC, PIE, PLT, GOT, and RDI-relative



Position Independent Executable (PIE)

Some instructions require information about the location of resources, including libraries, strings, and instructions.

```
call puts           mov rsi, [str]           jmp build
```

This makes it hard to use techniques like address space layout randomization (ASLR), a defensive programming technique. It also makes it hard to write shared libraries that are dynamically loaded, and which might end up anywhere in memory. It's *annoying*. You can't load two programs into the same address space because they might not be address compatible.

Ideally you would write position independent code (PIC).



-no-pie and -static

We've been telling GCC that we are not writing position independent code with `-no-pie` (do not try to create a position independent executable) and `-static` (include libraries we need in the executable so they have fixed addresses).

This is *not practical* in general, because now *every* executable has to have a copy of the libraries. We really want to have the libraries loaded in memory once, and then let everyone reference them. To make that work we need (among other things) to make them *location independent*.

```
$ gcc -o copy copy.o          ; ls -l copy      # 16,640 bytes
$ gcc -o copy copy.o -static ; ls -l copy      # 863,064 bytes
```



The Global Offset Table (GOT)

GOT is a section (`.got`) created during assembly that holds addresses of data and routines. It is "fixed up" during load by computing the memory offsets based on the actual addresses of the code, data, etc. Instead of referencing data or instruction addresses directly, you reference them via the GOT.

Now the only problem you have is finding the GOT. To do that you use a special symbol (`_GLOBAL_OFFSET_TABLE_`) and a hack to get its address into a register (typically `RBX`... which is now no longer useful since you have to use it to reference stuff: `lea [rbx+str]`). 32-bit code is full of this stuff, and 64-bit code still has it (`objdump -s -j .got `which python3``) and you *can* use it.

If you want to read about it you can... but 64-bit code introduced something really useful: `RIP`-relative addressing!



RIP-Relative Addressing

At load time the code is scanned and offsets to locations are "fixed up" by the loader to reference data and instruction addresses relative to **RIP**. This makes the program position independent!

You write: `lea rsi, [rel str]`, the assembler gives you: `lea rsi, [rip+0x0]`, and the linker converts this into `lea rsi, [rip+0xbc465]` once it has laid out the sections, segments, etc. Now our code is position independent!

This is so useful you can just write `default rel` at the top of your NASM file and then the instruction `lea rsi, [str]` is implicitly converted to `lea rsi, [rel str]`. If you don't want that for some reason, write `lea rsi, [abs str]`.



So far, so good

Some instructions require information about the location of resources, including libraries, strings, and instructions.

```
call puts           mov rsi, [str]           jmp build
```

We've handled the last two cases.

```
mov rsi, [rel str]   lea rax, [rel build] ; jmp rax
```

The jump is overkill. If you are jumping to an address within a section, you can just use `jmp build` directly.



What about calls?

We still haven't addressed the library function call.

`call puts`

RIP-relative addressing won't save us here (unless we use `-static`), since we don't know the offset to a *dynamically-loaded* library routine.



Program Linkage Table (PLT)

The program linkage table (PLT) solves the dynamic loading and linkage problem... but it's not as simple as RIP-relative addressing.

We can read about this... but let's take a look at some disassembly.

We will use the [copy.asm](#) program provided with this lecture.



WRT

Here we reference the library function `puts`...
with respect to the PLT. Magic! NASM knows
how to correctly encode this.

```
extern puts  
;...  
  
call puts wrt .plt
```



Tracing the PLT

Now we disassemble.

```
118a:      e8 a1 fe ff ff      call  1030 <puts@plt>
```

Okay. Let's have a look in the PLT. Location 1030 is clearly not the `puts` code.

```
00000000000001030 <puts@plt>:  
    1030:      jmp     QWORD PTR [rip+0x2f9a]      # 3fd0 <puts@GLIBC_2.2.5>
```




Tracing the PLT

```
0000000000001030 <puts@plt>:  
    1030:          jmp     QWORD PTR [rip+0x2f9a]          # 3fd0 <puts@GLIBC_2.2.5>
```

Okay... what's happening here? What is at location `rip+0x2f9a`? Well...

```
    0x1036  
+ 0x2f9a  
    0x3fd0
```

Tracing the PLT

```
00000000000001030 <puts@plt>:
    1030:          jmp     QWORD PTR [rip+0x2f9a]          # 3fd0 <puts@GLIBC_2.2.5>
```

The instruction jumps to the address contained in location `0x3fd0`. What is that? It is not in the `.text` section... or in any code section. It's in GOT!

```
$ objdump -s -j .got copy
```

```
Contents of section .got:
```

3fb8	c83d0000	00000000	00000000	00000000	.=.....
3fc8	00000000	00000000	36100000	000000006.....
3fd8	00000000	00000000	00000000	00000000
3fe8	00000000	00000000	00000000	00000000
3ff8	00000000	00000000		



So what happens?

The GOT is fixed up by the loader.

- PLT contains stubs to jump to the targets, and
- GOT contains a table of target addresses.

When we `call puts wrt .plt` the stub is called and this triggers the loading of the library (if it is not already loaded). Then the GOT table is fixed to contain the addresses of the loaded routines and finally `puts` is called. The next time we call `puts` the GOT contains the correct address and we jump directly there.



How does all this work?

There is not enough time! But there is an excellent article that walks through it using gdb which you need to read. Head here:

<https://systemoverlord.com/2017/03/19/got-and-plt-for-pwning.html>

You'll also learn a lot about using gdb. I recommend you follow along.

Liveness Analysis



Liveness

A variable is *live* if it contains a value that *may* be needed later in the program.

This is often obvious in high-level programs, where most variables are live, but may not be obvious in assembly.

We can work backward to find out...

```
.text:004016a3 55
.text:004016a4 4889e5
.text:004016a7 53
.text:004016a8 4883ec08
.text:004016ac 488b1ded5b2000
.text:004016b3 bf804e4000
.text:004016b8 e8c3fbffff
.text:004016bd 4889c1
.text:004016c0 488b05b15b2000
.text:004016c7 4889da
.text:004016ca 4889ce
.text:004016cd 4889c7
.text:004016d0 b800000000
.text:004016d5 e886fcffff
.text:004016da 4883c408
.text:004016de 5b
.text:004016df 5d
```

```
push rbp
mov rbp, rsp
push rbx
sub rsp, 0x8
mov rbx, QWORD PTR [rip+0x205bed]
mov edi, 0x404e80
call func_00401280
mov rcx, rax
mov rax, QWORD PTR [rip+0x205bb1]
mov rdx, rbx
mov rsi, rcx
mov rdi, rax
mov eax, 0x0
call func_00401360
add rsp, 0x8
pop rbx
pop rbp
```



Live and Dead Variables

A variable is **live** at a particular point in a program iff its value at that point *will be used in the future*. Otherwise the variable is **dead**.

- This means you have to know the *future uses* of the variable. Structuring pays off here.
- This analysis is used for *register allocation* when compiling programs. Lots of variables, but only a few registers... multiple variables can use the same register if *at most* one is live at any given time.



Liveness

```
1  int work(int c) {  
2      int a, b;  
3      a = 0;  
4      do {  
5          b = a + 1;  
6          c = c + b;  
7          a = b * 2;  
8      } while (a>9);  
9      return c;  
10 }
```

We know `c` is live at the end, since it is returned on line 9. Both `a` and `b` are auto variables, and they go out of scope.

Liveness

```
1  int work(int c) {  
2      int a, b;  
3      a = 0;  
4      do {  
5          b = a + 1;  
6          c = c + b;  
7          a = b * 2;  
8      } while (a>9);  
9      return c;  
10 }
```

At line 7 we have $a = b * 2$.

The lvalues are *removed* from the live set, and the rvalues are *added* to the live set.

If we start with the set {c}, then we remove a (which is already not in the set) and add b.

The live variable set *immediately prior* to line 7 is {b,c}.

Liveness

```
1  int work(int c) {  
2      int a, b;  
3      a = 0;  
4      do {  
5          b = a + 1;  
6          c = c + b;  
7          a = b * 2;  
8      } while (a>9);  
9      return c;  
10 }
```

At line 6 we have $c = c + b$.

The lvalues are *removed* from the live set, and the rvalues are *added* to the live set.

We start with the set *immediately after* line 6, which we know is $\{b, c\}$, and then we remove c and add both b and c .

The live variable set *immediately prior* to line 6 is $\{b, c\}$.

Liveness

```
1  int work(int c) {  
2      int a, b;  
3      a = 0;  
4      do {  
5          b = a + 1;  
6          c = c + b;  
7          a = b * 2;  
8      } while (a > 9);  
9      return c;  
10 }
```

At line 5 we have $b = a + 1$.

The lvalues are *removed* from the live set, and the rvalues are *added* to the live set.

We start with the set *immediately after* line 5, which we know is $\{b, c\}$, and then we remove b and add a .

The live variable set *immediately prior* to line 5 is $\{a, c\}$.



Liveness

```
1  int work(int c) {  
2      int a, b;  
3      a = 0;  
4      do {  
5          b = a + 1;  
6          c = c + b;  
7          a = b * 2;  
8      } while (a>9);  
9      return c;  
10 }
```

5: {a,c}

6: {b,c}

7: {b,c}

So, on entry to the loop body only **a** and **c** are live.

Liveness

```
1  int work(int c) {
2      int a, b;
3      a = 0;
4      do {
5          b = a + 1;
6          c = c + b;
7          a = b * 2;
8      } while (a>9);
9      return c;
10 }
```

Another way to think of this is to consider *edges* of the flowgraph.

- **a** is **live** on edges $3 \rightarrow 5$, $7 \rightarrow 8$, and $8 \rightarrow 5$
- **a** is **dead** on edge $5 \rightarrow 6$ and edge $6 \rightarrow 7$
- **b** is **live** on edge $5 \rightarrow 6$ and edge $6 \rightarrow 7$
- **b** is **dead** on edge $3 \rightarrow 5$ and edge $8 \rightarrow 5$ because it is clobbered
- **c** is everywhere **live** (see line 6)

We can summarize this by giving the live edge set for each variable

Liveness

```
1  int work(int c) {  
2      int a, b;  
3      a = 0;  
4      do {  
5          b = a + 1;  
6          c = c + b;  
7          a = b * 2;  
8      } while (a>9);  
9      return c;  
10 }
```

- $a \{3 \rightarrow 5, 7 \rightarrow 8, 8 \rightarrow 5\}$
- $b \{5 \rightarrow 6, 6 \rightarrow 7\}$
- $c \{3 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 7, 7 \rightarrow 8, 8 \rightarrow 5, 8 \rightarrow 9\}$

Note that a 's and b 's live sets are *disjoint*. They could share a register.

If they could share a register... couldn't they also share a variable... and we don't need both?



Liveness

```
1  int work(int c) {  
2      int a, b;  
3      a = 0;  
4      do {  
5          b = a + 1;  
6          c = c + b;  
7          a = b * 2;  
8      } while (a>9);  
9      return c;  
10 }
```

```
1  int work2(int c) {  
2      int a;  
3      a = 0;  
4      do {  
5          a = a + 1;  
6          c = c + a;  
7          a = a * 2;  
8      } while (a>9);  
9      return c;  
10 }
```

Trace Tables

Liveness

```
1  int work(int c) {  
2      int a, b;  
3      a = 0;  
4      do {  
5          b = a + 1;  
6          c = c + b;  
7          a = b * 2;  
8      } while (a>9);  
9      return c;  
10 }
```

What does this actually compute?

Let's look at the loop body and build a **trace table**.

This is a table that traces the value of each variable, and give the *new* value in terms of the *old* value.

For example, line 6 computes $c_{i+1} = c_i + b_i$, and we have $b_{i+1} = b_i$.

Trace Table

```
1  int work(int c) {  
2      int a, b;  
3      a = 0;  
4      do {  
5          b = a + 1;  
6          c = c + b;  
7          a = b * 2;  
8      } while (a>9);  
9      return c;  
10 }
```

	a	b	c
0	a_0	b_0	c_0
1			
2			
3			

Trace Table

```
1  int work(int c) {  
2      int a, b;  
3      a = 0;  
4      do {  
5          b = a + 1;  
6          c = c + b;  
7          a = b * 2;  
8      } while (a > 9);  
9      return c;  
10 }
```

	a	b	c
0	a_0	b_0	c_0
1	$a_1 = a_0$	$b_1 = a_0 + 1$	$c_1 = c_0$
2			
3			

Trace Table

```
1  int work(int c) {  
2      int a, b;  
3      a = 0;  
4      do {  
5          b = a + 1;  
6          c = c + b;  
7          a = b * 2;  
8      } while (a > 9);  
9      return c;  
10 }
```

	a	b	c
0	a_0	b_0	c_0
1	$a_1 = a_0$	$b_1 = a_0 + 1$	$c_1 = c_0$
2	$a_2 = a_1$	$b_2 = b_1$	$c_2 = c_1 + b_1$
3			

Trace Table

```
1  int work(int c) {  
2      int a, b;  
3      a = 0;  
4      do {  
5          b = a + 1;  
6          c = c + b;  
7          a = b * 2;  
8      } while (a > 9);  
9      return c;  
10 }
```

	a	b	c
0	a_0	b_0	c_0
1	$a_1 = a_0$	$b_1 = a_0 + 1$	$c_1 = c_0$
2	$a_2 = a_1$	$b_2 = b_1$	$c_2 = c_1 + b_1$
3	$a_3 = b_2 * 2$	$b_3 = b_2$	$c_3 = c_2$



Trace Table

	a	b	c
0	a_0	b_0	c_0
1	$a_1 = a_0$	$b_1 = a_0 + 1$	$c_1 = c_0$
2	$a_2 = a_1$	$b_2 = b_1$	$c_2 = c_1 + b_1$
3	$a_3 = b_2 * 2$	$b_3 = b_2$	$c_3 = c_2$

$$a_3 = b_2 * 2$$

$$b_3 = b_2$$

$$c_3 = c_2$$



Trace Table

	a	b	c
0	a_0	b_0	c_0
1	$a_1 = a_0$	$b_1 = a_0 + 1$	$c_1 = c_0$
2	$a_2 = a_1$	$b_2 = b_1$	$c_2 = c_1 + b_1$
3	$a_3 = b_2 * 2$	$b_3 = b_2$	$c_3 = c_2$

$$\begin{aligned} a_3 &= b_2 * 2 \\ &= b_1 * 2 \end{aligned}$$

$$\begin{aligned} b_3 &= b_2 \\ &= b_1 \end{aligned}$$

$$\begin{aligned} c_3 &= c_2 \\ &= c_1 + b_1 \end{aligned}$$

Trace Table

	a	b	c
0	a_0	b_0	c_0
1	$a_1 = a_0$	$b_1 = a_0 + 1$	$c_1 = c_0$
2	$a_2 = a_1$	$b_2 = b_1$	$c_2 = c_1 + b_1$
3	$a_3 = b_2 * 2$	$b_3 = b_2$	$c_3 = c_2$

$$\begin{aligned} a_3 &= b_2 * 2 \\ &= b_1 * 2 \\ &= (a_0 + 1) * 2 \end{aligned}$$

$$\begin{aligned} b_3 &= b_2 \\ &= b_1 \\ &= a_0 + 1 \end{aligned}$$

$$\begin{aligned} c_3 &= c_2 \\ &= c_1 + b_1 \\ &= c_0 + a_0 + 1 \end{aligned}$$

Liveness

```
1  int work(int c) {  
2      int a, b;  
3      a = 0;  
4      do {  
5          b = a + 1;  
6          c = c + b;  
7          a = b * 2;  
8      } while (a > 9);  
9      return c;  
10 }
```

But we know $a_0 = 0$.

$$\begin{aligned} a_3 &= b_2 * 2 \\ &= b_1 * 2 \\ &= (a_0 + 1) * 2 \end{aligned}$$

$$\begin{aligned} b_3 &= b_2 \\ &= b_1 \\ &= a_0 + 1 \end{aligned}$$

$$\begin{aligned} c_3 &= c_2 \\ &= c_1 + b_1 \\ &= c_0 + a_0 + 1 \end{aligned}$$

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$$\begin{aligned}a_3 &= b_2 * 2 \\ &= b_1 * 2 \\ &= (a_0 + 1) * 2 \\ &= 2\end{aligned}$$

$$\begin{aligned}b_3 &= b_2 \\ &= b_1 \\ &= a_0 + 1 \\ &= 1\end{aligned}$$

$$\begin{aligned}c_3 &= c_2 \\ &= c_1 + b_1 \\ &= c_0 + a_0 + 1 \\ &= c_0 + 1\end{aligned}$$

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The loop
body only
runs once

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```

$$c_3 = c_0 + 1$$

The net effect is just to return the argument plus one.

**Next Time:
Slicing**