# **CSC 6580 Spring 2020**

Instructor: Stacy Prowell

## Midterm

## Changes

Adjusted scores from best ten answers to best eight answers.

Average change of +10 points

New high: 100

New average: 84

New grades are in iLearn

# Steensgaard's Algorithm

## Points-to Analysis in Almost Linear Time

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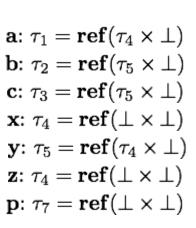
We present a

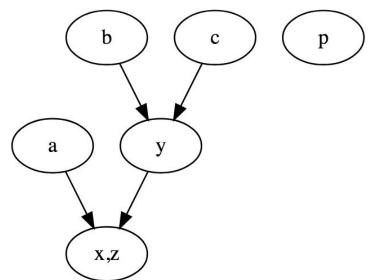
- flow insensitive,
- interprocedural
- points-to analysis algorithm

that has a desirable

- **linear space** and
- almost linear time complexity and is also very fast in practice.

Figure out the *type* of thing pointed to by each variable (that is a pointer).





```
\alpha ::= \tau \times \lambda
\tau ::= (\bot) | \mathbf{ref}(\alpha)
\lambda ::= \bot | \mathbf{lam}(\alpha_1 \dots \alpha_n)(\alpha_{n+1} \dots \alpha_{n+m})
```

The "bottom" of the type lattice; think of it as not specified, or not determined

Basically anything not known to be a location or a pointer to a location

```
\alpha ::= \tau \times \lambda 

[\tau ::= \bot | \mathbf{ref}(\alpha)] 

\lambda ::= \bot | \mathbf{lam}(\alpha_1 \dots \alpha_n)(\alpha_{n+1} \dots \alpha_{n+m})
```

The type of some thing in memory

```
\alpha ::= \tau \times \lambda
\tau ::= \bot \mid \mathbf{ref}(\alpha)
[\lambda ::= \bot \mid \mathbf{lam}(\alpha_1 \dots \alpha_n)(\alpha_{n+1} \dots \alpha_{n+m})]
```

The type of of a function

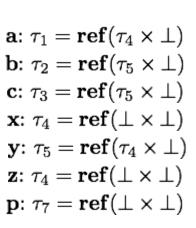
```
\begin{array}{ll}
\left(\alpha & ::= & \tau \times \lambda\right) \\
\tau & ::= & \bot \mid \mathbf{ref}(\alpha) \\
\lambda & ::= & \bot \mid \mathbf{lam}(\alpha_1 \dots \alpha_n)(\alpha_{n+1} \dots \alpha_{n+m})
\end{array}
```

Either one or the other (but probably not both)

```
\alpha ::= \tau \times \lambda
\tau ::= \perp | \mathbf{ref}(\alpha)
\lambda ::= \perp | \mathbf{lam}(\alpha_1 \dots \alpha_n)(\alpha_{n+1} \dots \alpha_{n+m})
```

Note the recursion

Figure out the *type* of thing pointed to by each variable (that is a pointer).



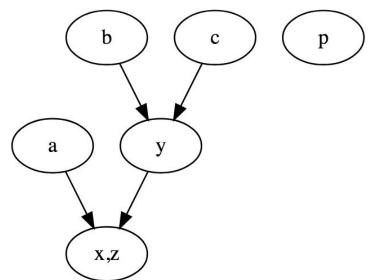
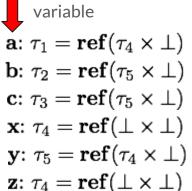


Figure out the *type* of thing pointed to by each variable (that is a pointer).



 $\mathbf{p}$ :  $\tau_7 = \mathbf{ref}(\bot \times \bot)$ 

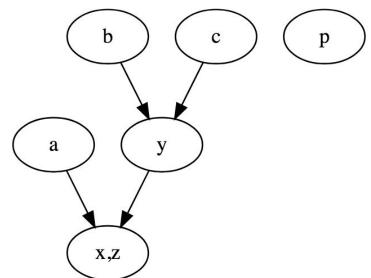
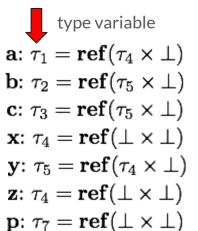


Figure out the *type* of thing pointed to by each variable (that is a pointer).



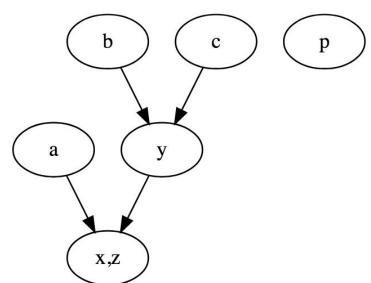


Figure out the *type* of thing pointed to by each variable (that is a pointer).

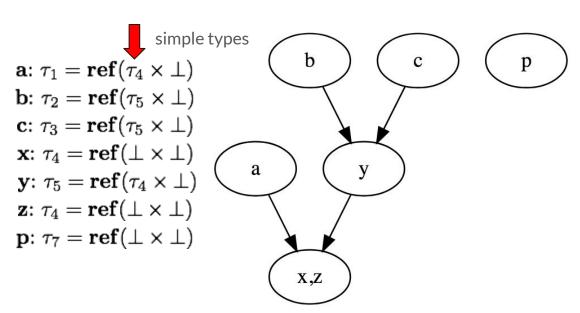


Figure out the *type* of thing pointed to by each variable (that is a pointer).

function pointer types



$$\mathbf{a}$$
:  $\tau_1 = \mathbf{ref}(\tau_4 \times \bot)$ 

**b**: 
$$\tau_2 = \mathbf{ref}(\tau_5 \times \bot)$$

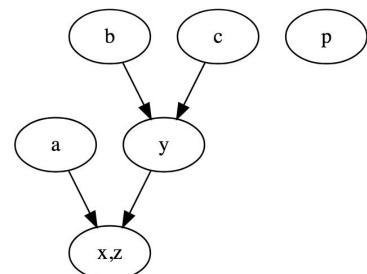
$$\mathbf{c}$$
:  $\tau_3 = \mathbf{ref}(\tau_5 \times \bot)$ 

$$\mathbf{x}$$
:  $\tau_4 = \mathbf{ref}(\bot \times \bot)$ 

$$\mathbf{y}$$
:  $\tau_5 = \mathbf{ref}(\tau_4 \times \bot)$ 

$$\mathbf{z}$$
:  $\tau_4 = \mathbf{ref}(\bot \times \bot)$ 

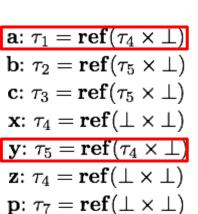
$$\mathbf{p}$$
:  $\tau_7 = \mathbf{ref}(\bot \times \bot)$ 

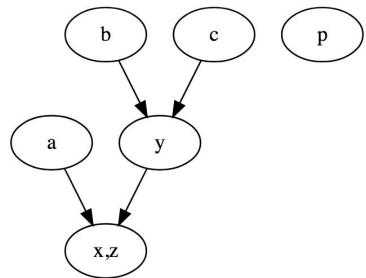


The algorithm works by:

- assuming every variable has a unique type
- merging type variables as it walks through the program

(See prior class notes.)





We need to be specific about when we can merge. We introduce a rule.

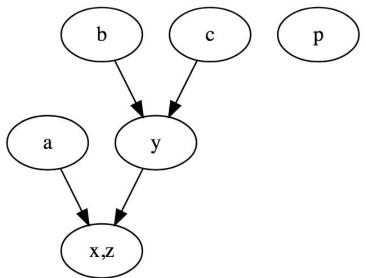
$$A \vdash \mathbf{x} : \mathbf{ref}(\alpha_1)$$

$$A \vdash \mathbf{y} : \mathbf{ref}(\alpha_2)$$

$$\alpha_2 \leq \alpha_1$$

$$A \vdash welltyped(\mathbf{x} = \mathbf{y})$$

a: 
$$\tau_1 = \mathbf{ref}(\tau_4 \times \bot)$$
  
b:  $\tau_2 = \mathbf{ref}(\tau_5 \times \bot)$   
c:  $\tau_3 = \mathbf{ref}(\tau_5 \times \bot)$   
x:  $\tau_4 = \mathbf{ref}(\bot \times \bot)$   
y:  $\tau_5 = \mathbf{ref}(\tau_4 \times \bot)$   
z:  $\tau_4 = \mathbf{ref}(\bot \times \bot)$   
p:  $\tau_7 = \mathbf{ref}(\bot \times \bot)$ 



$$A \vdash \mathbf{x} : \mathbf{ref}(\alpha_1)$$

$$A \vdash \mathbf{y} : \mathbf{ref}(\alpha_2)$$

$$\alpha_2 \leq \alpha_1$$

$$A \vdash welltyped(\mathbf{x} = \mathbf{y})$$

$$\begin{cases}
A \vdash \mathbf{x} : \mathbf{ref}(\alpha_1) \\
A \vdash \mathbf{y} : \mathbf{ref}(\alpha_2) \\
\alpha_2 \leq \alpha_1
\end{cases}$$

$$A \vdash welltyped(\mathbf{x} = \mathbf{y})$$

If the things above the line are true...

$$A \vdash \mathbf{x} : \mathbf{ref}(\alpha_1)$$

$$A \vdash \mathbf{y} : \mathbf{ref}(\alpha_2)$$

$$\alpha_2 \leq \alpha_1$$

$$A \vdash welltyped(\mathbf{x} = \mathbf{y})$$

...then we can conclude that the thing below the line is true.

 $A \vdash \mathbf{x} : \mathbf{ref}(\alpha_1)$   $A \vdash \mathbf{y} : \mathbf{ref}(\alpha_2)$   $\alpha_2 \leq \alpha_1$   $A \vdash welltyped(\mathbf{x} = \mathbf{y})$ 

A program statement

$$A \vdash \mathbf{x} : \mathbf{ref}(\alpha_1)$$

$$A \vdash \mathbf{y} : \mathbf{ref}(\alpha_2)$$

$$\alpha_2 \leq \alpha_1$$

$$A \vdash welltyped(\mathbf{x} = \mathbf{y})$$

We want to conclude that our rule produces well-typed programs

$$A \vdash \mathbf{x} : \mathbf{ref}(\alpha_1)$$

$$A \vdash \mathbf{y} : \mathbf{ref}(\alpha_2)$$

$$\alpha_2 \leq \alpha_1$$

$$A \vdash welltyped(\mathbf{x} = \mathbf{y})$$

If our typing rule tells us (entails) that

- if x is a reference to some type α<sub>1</sub>
- and y is a reference to some type α<sub>2</sub>
- and if  $\alpha_2$  "fits in  $\bar{\alpha}_1$  then our typing rule correctly types  $\mathbf{x} = \mathbf{y}$ .

We defined a partial order among types as follows.

$$t_1 \leq t_2 \Leftrightarrow (t_1 = \bot) \vee (t_1 = t_2)$$

$$(t_1 \times t_2) \trianglelefteq (t_3 \times t_4) \Leftrightarrow (t_1 \trianglelefteq t_3) \land (t_2 \trianglelefteq t_4).$$

$$t_1 \unlhd t_2 \Leftrightarrow (t_1 = \bot) \lor (t_1 = t_2)$$

Defines a partial order among types.

Type  $t_1$  is "less than"  $t_2$  iff  $t_1$  is bottom or the types are equal.

$$(t_1 \times t_2) \trianglelefteq (t_3 \times t_4) \Leftrightarrow (t_1 \trianglelefteq t_3) \land (t_2 \trianglelefteq t_4).$$

$$t_1 \leq t_2 \Leftrightarrow (t_1 = \bot) \lor (t_1 = t_2)$$

Defines a partial order among types.

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$$(t_1 \times t_2) \trianglelefteq (t_3 \times t_4) \Leftrightarrow (t_1 \trianglelefteq t_3) \land (t_2 \trianglelefteq t_4).$$

We extend the partial order to pairs by comparing the components. This allows us to compare more complex data types, like structures.

Consider what this means if we can conclude that the left hand side is true, from our typing rule.

$$[t_1 \unlhd t_2] \Leftrightarrow (t_1 = \bot) \lor (t_1 = t_2)$$

If we can conclude that the lhs is true from our typing rule, then the rhs must also be true.

$$[t_1 \unlhd t_2] \Leftrightarrow (t_1 = \bot) \lor (t_1 = t_2)$$

So either:

- $t_1$  is bottom or

If the types are the same, we should merge them!

$$t_1 \leq t_2 \Leftrightarrow (t_1 = \bot) \lor (t_1 = t_2)$$
$$(t_1 \times t_2) \leq (t_3 \times t_4) \Leftrightarrow (t_1 \leq t_3) \land (t_2 \leq t_4).$$

So if the lhs is  $\bot$ , we don't need to merge, but if the lhs is *not*  $\bot$ , then we do.

We call this a *conditional join*, and represent it by **cjoin**. This is different from the *always join* represented by **join**.

$$t_1 \leq t_2 \Leftrightarrow (t_1 = \bot) \lor (t_1 = t_2)$$
$$(t_1 \times t_2) \leq (t_3 \times t_4) \Leftrightarrow (t_1 \leq t_3) \land (t_2 \leq t_4).$$

So if the lhs is  $\bot$ , we don't need to merge, but if the lhs is *not*  $\bot$ , then we do.

But it is possible that we later change the type of a variable (join), and it is no longer  $\bot$ . (We did this several times in the prior example.) Then the above relation might no longer hold for the prior lines.

We don't want to go back (backtracking is expensive and messes up our time complexity), so instead:

- for every  $\bot$ , keep a list of type variables to join with if we should ever change  $\bot$  to something else (let's call this set "pending") and...
- if we do change  $\perp$  to something else, merge all the type variables then.

#### **Joins**

```
\begin{aligned} & \textbf{cjoin}(e_1, e_2): \\ & \text{if } \textbf{type}(e_2) = \bot \text{ then} \\ & \textbf{pending}(e_2) \leftarrow \{e_1\} \cup \textbf{pending}(e_2) \\ & \text{else} \\ & \textbf{join}(e_1, e_2) \end{aligned}
```

This defines the conditional join of two types, exactly as we defined it earlier... but it is a bit hard to read. Either we add to the pending set, or we just join.

- for every ⊥, keep a list of type variables to join with if we should ever change ⊥ to something else (let's call this set "pending") and...
- if we do change ⊥ to something else, merge all the type variables then.

#### **Joins**

```
\begin{aligned} \textbf{cjoin}(e_1, e_2): \\ & \text{if } \textbf{type}(e_2) = \bot \text{ then} \\ & \textbf{pending}(e_2) \leftarrow \{e_1\} \cup \textbf{pending}(e_2) \\ & \text{else} \\ & \textbf{join}(e_1, e_2) \end{aligned}
```

Perform the join, watching for cases where we can merge the pending sets and then join all those variables.

This is where we pick up a bit more time complexity.

```
\mathbf{join}(e_1, e_2):
    let t_1 = type(e_1)
        t_2 = \mathbf{type}(e_2)
        e = \operatorname{ecr-union}(e_1, e_2) in
       if t_1 = \bot then
          type(e) \leftarrow t_2
         if t_2 = \bot then
             pending(e) \leftarrow pending(e_1) \cup
                                   pending(e_2)
          else
             for x \in \mathbf{pending}(e_1) do \mathbf{join}(e, x)
      else
          \mathbf{type}(e) \leftarrow t_1
         if t_2 = \bot then
             for x \in \mathbf{pending}(e_2) do \mathbf{join}(e, x)
          else
             \mathbf{unify}(t_1,t_2)
```

#### **Joins**

```
\begin{aligned} & \textbf{cjoin}(e_1, e_2): \\ & \text{if } \textbf{type}(e_2) = \bot \text{ then} \\ & \textbf{pending}(e_2) \leftarrow \{e_1\} \cup \textbf{pending}(e_2) \\ & \text{else} \\ & \textbf{join}(e_1, e_2) \end{aligned}
```

```
unify(ref(\tau_1 \times \lambda_1), ref(\tau_2 \times \lambda_2)):

if \tau_1 \neq \tau_2 then join(\tau_1, \tau_2)

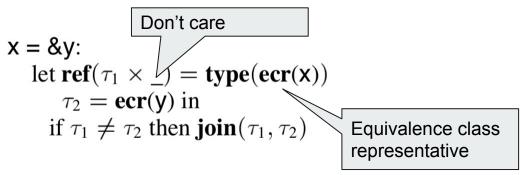
if \lambda_1 \neq \lambda_2 then join(\lambda_1, \lambda_2)
```

Now unify the types by recursively joining their components.

```
\mathbf{join}(e_1, e_2):
   let t_1 = \mathbf{type}(e_1)
        t_2 = \mathbf{type}(e_2)
        e = \operatorname{ecr-union}(e_1, e_2) in
      if t_1 = \bot then
         type(e) \leftarrow t_2
         if t_2 = \bot then
            pending(e) \leftarrow pending(e_1) \cup
                                   pending(e_2)
         else
            for x \in \mathbf{pending}(e_1) do \mathbf{join}(e, x)
      else
         type(e) \leftarrow t_1
         if t_2 = \bot then
            for x \in \mathbf{pending}(e_2) do \mathbf{join}(e, x)
         else
            \mathbf{unify}(t_1,t_2)
```

### Rules

The paper builds rules for all the statements. The one we used in our prior example was this.



```
Consider: \mathbf{y} = \mathbf{\&x}; \mathbf{x}: \tau_4 = \mathbf{ref}(\bot \times \bot)

\mathbf{y}: \tau_5 = \mathbf{ref}(\tau_6 \times \bot)

Substitute into the rule:

\mathbf{x} = \mathbf{\&y}: \mathbf{y} = \mathbf{\&x}:

\det \mathbf{ref}(\tau_1 \times \bot) = \mathbf{type}(\mathbf{ecr}(\mathbf{x}))
\tau_2 = \mathbf{ecr}(\mathbf{y}) \text{ in}
\text{if } \tau_1 \neq \tau_2 \text{ then } \mathbf{join}(\tau_1, \tau_2)
\text{if } \tau_1 \neq \tau_2 \text{ then } \mathbf{join}(\tau_1, \tau_2)
```

```
 \begin{aligned} \mathbf{x} &: \tau_4 = \mathbf{ref}(\bot \times \bot) \\ \mathbf{y} &: \tau_5 = \mathbf{ref}(\tau_6 \times \bot) \end{aligned} \qquad \begin{aligned} \mathbf{ecr}(\mathbf{x}) &= \tau_4 \\ \mathbf{ecr}(\mathbf{y}) &= \tau_5 \\ \mathbf{type}(\mathbf{ecr}(\mathbf{y})) &= \mathbf{type}(\tau_5) = \mathbf{ref}(\tau_6 \times \bot) \end{aligned} \\ \mathbf{y} &= \&\mathbf{x} : \\ \det \mathbf{ref}(\tau_1 \times \_) &= \mathbf{type}(\mathbf{ecr}(\mathbf{y})) \\ \tau_2 &= \mathbf{ecr}(\mathbf{x}) \text{ in} \\ \text{if } \tau_1 \neq \tau_2 \text{ then } \mathbf{join}(\tau_1, \tau_2) \end{aligned}
```

```
\mathbf{x}: \ \tau_4 = \mathbf{ref}(\bot \times \bot) \\ \mathbf{y}: \ \tau_5 = \mathbf{ref}(\tau_6 \times \bot) \\ \mathbf{ecr}(\mathbf{y}) = \tau_5 \\ \mathbf{type}(\mathbf{ecr}(\mathbf{y})) = \mathbf{type}(\tau_5) = \mathbf{ref}(\tau_6 \times \bot) \\ \mathbf{y} = \&\mathbf{x}: \\ \mathbf{p} = &\mathbf{x}: \\ \mathbf{y} = &\mathbf{y} = &\mathbf{x}: \\ \mathbf{y} = &\mathbf{y} = &\mathbf{y} = \mathbf{y} \\ \mathbf{y} = &\mathbf{y} = &\mathbf{y} = \mathbf{y} \\ \mathbf{y} = &\mathbf{y} = &\mathbf{y} = \mathbf{y} \\ \mathbf{y} = &\mathbf{y} = &\mathbf{y} = &\mathbf{y} \\ \mathbf{y} = &\mathbf{y} \\ \mathbf{y} = &\mathbf{y} = &\mathbf{y} \\ \mathbf{y} = &\mathbf{y} \\ \mathbf{y} = &\mathbf{y} = &\mathbf{y} \\ \mathbf{y} = &\mathbf{y}
```

 $\tau_2 = \tau_4$  in

if  $\tau_1 \neq \tau_2$  then **join** $(\tau_1, \tau_2)$ 

$$\begin{aligned} \mathbf{x} &: \tau_4 = \mathbf{ref}(\bot \times \bot) \\ \mathbf{y} &: \tau_5 = \mathbf{ref}(\tau_6 \times \bot) \end{aligned} & \mathbf{ecr}(\mathbf{x}) = \tau_4 \\ \mathbf{ecr}(\mathbf{y}) &= \tau_5 \\ \mathbf{type}(\mathbf{ecr}(\mathbf{y})) = \mathbf{type}(\tau_5) = \mathbf{ref}(\tau_6 \times \bot) \end{aligned}$$
 
$$\mathbf{y} = \&\mathbf{x} :$$
 
$$|\mathbf{et} \ \mathbf{ref}(\tau_1 \times \bot) = \mathbf{ref}(\tau_6 \times \bot)$$
 
$$\mathbf{y} = \&\mathbf{x} :$$

if  $\tau_6 \neq \tau_4$  then  $\mathbf{join}(\tau_6, \tau_4)$ 

The other rules are similar.

The most complex rules are for functions.

This is the rule for function invocation.

```
x_1 \dots x_m = p(y_1 \dots y_n):
    let ref(x \times \lambda) = type(ecr(p)) in
       if type(\lambda) = \perp then
           settype(\lambda, lam(\alpha_1 \dots \alpha_n)(\alpha_{n+1} \dots \alpha_{n+m}))
           where
                \alpha_i = \tau_i \times \lambda_i
                 [\tau_i, \lambda_i] = \mathbf{MakeECR}(2)
       let lam(\alpha_1 \dots \alpha_n)(\alpha_{n+1} \dots \alpha_{n+m}) = type(\lambda) in
          for i \in [1 \dots n] do
             let \tau_1 \times \lambda_1 = \alpha_i
                   ref(\tau_2 \times \lambda_2) = type(ecr(y_i)) in
                 if \tau_1 \neq \tau_2 then cjoin(\tau_1, \tau_2)
                 if \lambda_1 \neq \lambda_2 then cjoin(\lambda_1, \lambda_2)
          for i \in [1 \dots m] do
             let \tau_1 \times \lambda_1 = \alpha_{n+i}
                   \operatorname{ref}(\tau_2 \times \lambda_2) = \operatorname{type}(\operatorname{ecr}(\mathbf{x}_i)) in
                 if \tau_1 \neq \tau_2 then cjoin(\tau_2, \tau_1)
                 if \lambda_1 \neq \lambda_2 then cjoin(\lambda_2, \lambda_1)
```

# How has it held up?

# It still performs very well

# Which Pointer Analysis Should I Use?

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# It has multiple implementations

"Alias Analysis in LLVM," Sheng-Hsiu Lin, M.S. 2015

#### Abstract

Alias analysis is a study of the relations between pointers. It has important applications in code optimization and security. This research introduces the fundamental concepts of alias analysis. It explains different approaches of alias analysis with examples. It provides a survey of some very important pointer analysis algorithms. LLVM interface is introduced along with the alias analyses that are currently available on it. This research implementes a Steensgaard's pointer analysis on LLVM. The philosophy of this implementation is explained in detail. Evaluations of rule based basic alias analysis, Andersen's pointer analysis, Steensgaard's pointer analysis and data structure analysis are provided with experimental results on their precision, time and memory usage.

Homework Due: Tuesday, April 14

# **Structuring 1**

Starting with the provided solution to the basic block homework, or with your own solution, apply the constructive proof of the structure theorem. We will do this in pieces. For next time:

- 1. Find the entry point and see if it appears the C runtime is in use. If so, figure out where main is and add it to the addresses to extract.
- 2. Create a Python class for a Node. There should be three types of Node, possibly by subclass.
  - a. A function node that holds a basic block that has a single next address or no next address ("unknown")
  - b. A predicate node that holds a basic block that has two next addresses, one for true and one for false
  - c. A label assignment node that holds an address to assign to the label
- 3. Package all basic blocks into Node instances. Don't worry about the label assignments yet.
- 4. Don't worry about output yet. Turn your program in by Tuesday.

Please name your program structure.py.

# **Next Time: More!**