

# Competitive Programming Cheat Sheet

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# 1 Arrays, Strings & Sorting

## 1.1 Longest Common Prefix

# Usage: Find the longest common prefix among multiple strings  
 # Useful for: string processing, dictionary/trie problems, prefix analysis

```
def areEqual(strs, index):
    for i in range(1, len(strs)): # O(n)
        if strs[i][index] != strs[0][index]:
            return False
    return True
```

```
def longestCommonPrefix(strs: list[str]) -> str:
    # O(n * L) / O(L)
    minLength = len(strs[0])
    for i in range(1, len(strs)): # O(n)
        minLength = min(minLength, len(strs[i]))
    # minLength = min([len(word) for word in strs]) # O(n * L)
    longestCommonPrefix = ''
    for i in range(minLength): # O(L)
        if areEqual(strs, i): # O(n)
            longestCommonPrefix += strs[0][i] # O(1)
        else:
            break
    return longestCommonPrefix # O(L)
```

```
print(longestCommonPrefix(['flower',
                           'flow',
                           'flood',
                           'flair'])))
```

```
'''
Input: ['flower',
        'flow',
        'flood',
        'flair']
Output: 'fl'
'''
```

## 1.2 Group Anagrams

```
def groupAnagrams(strings: list[str]) -> list[list[str]]:
    # O(n log n)
    strings.sort(key=lambda word: ''.join(sorted(word)))
    currGroup = [strings[0]]
    groups = []
    for i in range(1, len(strings)):
        if sorted(strings[i]) == sorted(strings[i - 1]):
            currGroup.append(strings[i])
        else:
            groups.append(currGroup)
            currGroup = [strings[i]]
    groups.append(currGroup)
    return groups
```

```
print(groupAnagrams(['eat', 'tea', 'tan', 'ate', 'nat', 'bat']))
# Input: ['eat', 'tea', 'tan', 'ate', 'nat', 'bat']
# Output: [['bat'], ['nat', 'tan'], ['ate', 'eat', 'tea']]
# Usage: Group strings that are anagrams of each other
# Useful for: string manipulation, hashing, dictionary/trie problems
```

## 1.3 Count Binary Substrings

```
def BinarySubstrings(s: str) -> int:
    # O(n) / O(1)
    sol = 0
    len1 = 0
    len2 = 1
    for i in range(1, len(s)):
        if s[i] == s[i - 1]:
            len2 += 1
        else:
            sol += min(len1, len2)
            len1 = len2
            len2 = 1
    sol += min(len1, len2)
```

```
return sol
```

```
print(BinarySubstrings('00110011'))
```

```
# Input: '00110011'
# Output: 6
# Input: '10101'
# Output: 4
# Usage: Count binary substrings with equal consecutive 0s and 1s
# Useful for: string analysis, pattern counting, binary sequences
```

## 1.4 Rotate One To Right

```
def rotate(nums: list[int]) -> None:
    # O(n) / O(1)
    aux_val = nums[-1]
    for i in range(len(nums)-1, 0, -1):
        nums[i] = nums[i - 1]
    nums[0] = aux_val
    return nums
```

```
print(rotate([1, 2, 3, 4, 5]))
# Input: [1, 2, 3, 4, 5]
# Output: [5, 1, 2, 3, 4]
# Input: [4, -2, 13, 1]
# Output: [1, 4, -2, 13]
# Usage: Rotate elements of an array by one position to the right
# Useful for: array manipulation, cyclic shifts, in-place operations
```

## 1.5 Minimum Absolute Difference

```
def minimumAbsDifference(nums: list[int]) -> list[list[int]]:
    # O(n log n) / O(n^2)
    nums.sort()
    minDiff = nums[1] - nums[0]
    minDiffPairs = []
    for i in range(1, len(nums)):
```

```

curDiff = nums[i] - nums[i - 1]
if curDiff < minDiff:
    minDiff = curDiff
    minDiffPairs = [[nums[i - 1], nums[i]]]
elif curDiff == minDiff:
    minDiffPairs.append([nums[i - 1], nums[i]])
return minDiffPairs

print(minimumAbsDifference([4, 2, 1, 3]))
# Input: [4, 2, 1, 3]
# Output: [[1, 2], [2, 3], [3, 4]]
# Input: [3, 8, -10, 23, 19, -4, -14, 27]
# Output: [[-14, -10], [19, 23], [23, 27]]
# Usage: Find all pairs with minimum absolute difference
#         in a list
# Useful for: array sorting problems, consecutive pair
#             analysis

```

## 1.6 Best Time To Buy And Sell One Stock

```

def maxProfit(prices: list[int]) -> int:
    # O(n) / O(1)
    maxProfit = 0
    maxPrice = prices[-1]
    for buyDay in range(len(prices) - 2, -1, -1):
        currMaxProfit = maxPrice - prices[buyDay]
        maxProfit = max(maxProfit, currMaxProfit)
        maxPrice = max(maxPrice, prices[buyDay])
    return maxProfit

print(maxProfit([7, 1, 5, 3, 6, 4]))
# Input: [7, 1, 5, 3, 6, 4]
# Output: 5
# Input: [7, 6, 4, 3, 1]
# Output: 0
# Usage: Find maximum profit from a single buy/sell in
#         stock prices
# Useful for: array analysis, greedy problems, financial
#             algorithms

```

## 1.7 Increasing Triplet

```

def increasingTriplet(nums: list[int]) -> bool:
    # O(n) / O(n)
    suffixMax = [0] * len(nums)
    suffixMax[-1] = nums[-1]
    for i in range(len(nums) - 2, -1, -1):
        suffixMax[i] = max(suffixMax[i + 1], nums[i])

    prefixMin = nums[0]
    for j in range(1, len(nums) - 1):
        if prefixMin < nums[j] and suffixMax[j + 1] > nums[j]:
            return True
        prefixMin = min(prefixMin, nums[j])
    return False

print(increasingTriplet([2, 1, 5, 0, 4, 6]))
# Input: [5, 4, 3, 2, 1]
# Output: false
# Input: [2, 1, 5, 0, 4, 6]
# Output: true
# Usage: Check if an increasing triplet subsequence
#         exists in an array
# Useful for: subsequence problems, array analysis,
#             greedy patterns

```

## 1.8 Maximum Value And Number Of Occurrences

```

def maxValNumOfOccurrences(nums: list[int]) -> list[int]:
    # O(n) / O(1)
    maxVal = nums[0]
    counter = 0
    for num in nums:
        if num > maxVal:
            maxVal = num
            counter = 1
        elif num == maxVal:
            counter += 1
    return [maxVal, counter]

```

```

print(maxValNumOfOccurrences([2, 7, 11, 8, 11, 8, 3, 11]))
# Input: [2, 7, 11, 8, 11, 8, 3, 11]
# Output: [11, 3]
# Usage: Find the maximum value in an array and count its
#         occurrences
# Useful for: array analysis, frequency counting,
#             selection problems

```

## 1.9 Maximum Consecutive Ones

```

def findMaxConsecutiveOnes(nums: list[int]) -> int:
    # O(n) / O(1)
    counter = 0
    solution = 0
    for num in nums:
        if num == 1:
            counter += 1
        else:
            counter = 0
            solution = max(solution, counter)
    return counter

print(findMaxConsecutiveOnes([1, 1, 0, 1, 1, 1]))
# Input: [1, 1, 0, 1, 1, 1]
# Output: 3
# Input: [1, 0, 1, 1, 0, 1]
# Output: 2
# Usage: Find the maximum number of consecutive ones in a
#         binary array
# Useful for: array analysis, binary sequences, sliding
#             window problems

```

## 1.10 Majority Element

```

def majorityElement(nums: list[int]) -> int:
    # O(n log n) / O(1)
    counter = 1
    maxCounter = 1
    solution = nums[0]
    nums.sort()

```

```

for i in range(1, len(nums)):
    if nums[i] == nums[i - 1]:
        counter += 1
    else:
        counter = 1
    if counter > maxCounter:
        maxCounter = counter
        solution = nums[i - 1]
return solution

```

```

print(majorityElement([2, 2, 1, 1, 1, 3, 3, 3, 3]))
# Input: [3, 2, 3]
# Output: 3
# Input: [2, 2, 1, 1, 1, 2, 2]
# Output: 2
# Usage: Find the element that appears more than half of
#         the array
# Useful for: array analysis, frequency counting, voting
#             problems

```

## 1.11 Number Of Distinct Values

```

def numOfDistinctValues(nums: list[int]) -> int:
    # O(n log n) / O(1)
    sol = 1
    nums.sort()
    for i in range(1, len(nums)):
        if nums[i] != nums[i - 1]:
            sol += 1
    return sol

```

```

print(numOfDistinctValues([1, 5, -3, 1, -4, 2, -4, 7, 7]))
# Input: [1, 5, -3, 1, -4, 2, -4, 7, 7]
# Output: 6
# Usage: Count the number of distinct elements in an
#         array
# Useful for: array analysis, duplicates handling,
#             sorting-based problems

```

## 1.12 Single Number

```

def isSingleNumber(nums, index):
    if index > 0 and nums[index - 1] != nums[index]:
        return False
    if index < len(nums) - 1 and nums[index] != nums[
        index + 1]:
        return False
    return True

```

```

def singleNumber(nums: list[int]) -> int:
    # O(n log n)
    if len(nums) == 1:
        return nums[0]
    nums.sort()
    for i in range(0, len(nums)):
        if isSingleNumber(nums, i):
            return nums[i]

```

```

print(singleNumber([4, 1, 2, 1, 2]))
# Input: [2, 2, 1]
# Output: 1
# Input: [4, 1, 2, 1, 2]
# Output: 4
# Usage: Find the element that appears exactly once in an
#         array
# Useful for: array analysis, duplicates handling,
#             sorting-based problems

```

## 1.13 Find Duplicates

```

def isDuplicate(nums, index):
    if index > 0 and nums[index] == nums[index - 1]:
        return False
    if index == len(nums) - 1 or nums[index] != nums[
        index + 1]:
        return False
    return True

```

```

def findDuplicates(nums: list[int]) -> list[int]:

```

```

# O(n log n) / O(n)
nums.sort()
duplicates = []
for i in range(len(nums)):
    if isDuplicate(nums, i):
        duplicates.append(nums[i])
return duplicates

```

```

print(findDuplicates([1, 5, 1, 2, 3, 5, 4]))
# Input: [2, 3, 1, 1, 4, 3, 2, 1]
# Output: [2, 1, 3]
# Usage: Find all elements that appear more than once in
#         an array
# Useful for: array analysis, counting duplicates,
#             sorting-based problems

```

## 1.14 Find Second Largest - Solution 1

```

def secondLargest(nums: list[int]) -> int:
    # O(n log n) / O(1)
    nums.sort(reverse=True)
    for num in nums:
        if num != nums[0]:
            return num

```

```

print(secondLargest([2, 7, 11, 8, 11, 8, 3, 11]))
# Input: [2, 7, 11, 8, 11, 8, 3, 11]
# Output: 8
# Usage: Find the second largest element by sorting the
#         array
# Useful for: array analysis, selection problems

```

## 1.15 Find Second Largest - Solution 2

```

def secondLargest(nums: list[int]) -> int:
    # O(n) / O(1)
    largest = secondLargest = None
    for num in nums:
        if not largest or num > largest:
            secondLargest = largest
            largest = num

```

```

    elif num != largest and (not secondLargest or num
        > secondLargest):
        secondLargest = num
    return secondLargest

```

```

print(secondLargest([1000, 100, 100]))
# Input: [2, 7, 11, 8, 11, 8, 3, 11]
# Output: 8
# Input: [1000, 100, 100]
# Output: 100
# Usage: Find the second largest element in an array
# Useful for: array analysis, selection problems, in-
place operations

```

## 2 Nested Loops & Brute Force Algorithms

### 2.1 Index Of Substring

```

def isSubstring(haystack, needle, start): # O(m)
    for i in range(len(needle)):
        if needle[i] != haystack[start + i]:
            return False
    return True

def indexOf(haystack: str, needle: str) -> int:
    # O(n * m) / O(1)
    n = len(haystack)
    m = len(needle)
    for i in range(n - m + 1): # O(n - m)
        if isSubstring(haystack, needle, i): # O(m)
            return i
    return -1

```

```

print(indexOf('hello', 'll'))
# Input: "hello", "ll"
# Output: 2
# Input: "aaaaa", "bba"
# Output: -1

```

# Usage: Find the first occurrence of a substring in a string  
 # Useful for: string search, naive pattern matching problems

### 2.2 Longest Common Prefix Of Multiple Strings

```

def areEqual(strs, index):
    for i in range(1, len(strs)): # O(n)
        if strs[i][index] != strs[0][index]:
            return False
    return True

def longestCommonPrefix(strs: list[str]) -> str:
    # O(n * L) / O(L)
    minLength = len(strs[0])
    for i in range(1, len(strs)): # O(n)
        minLength = min(minLength, len(strs[i]))
    # minLength = min([len(word) for word in strs]) # O(n * L)
    longestCommonPrefix = ''
    for i in range(minLength): # O(L)
        if areEqual(strs, i): # O(n)
            longestCommonPrefix += strs[0][i] # O(1)
        else:
            break
    return longestCommonPrefix # O(L)

print(longestCommonPrefix(['flower',
                            'flow',
                            'flood',
                            'flair']))
'''

```

```

Input: ['flower',
        'flow',
        'flood',
        'flair']
Output: 'fl'
'''

```

# Usage: Find the longest common prefix among multiple strings  
 # Useful for: string processing, dictionary/trie problems

### 2.3 Repeated Substring Pattern

```

def isSolution(s, length): # O(n) / O(1)
    if len(s) % length:
        return False
    count = int(len(s) / length)
    for index in range(length): # O(length)
        for group in range(1, count): # O(count)
            if s[index] != s[index + group * length]:
                return False
    return True

```

```

def repeatedSubstringPattern(s: str) -> bool:
    # O(n^2) / O(1)
    for length in range(1, len(s)): # O(n)
        if isSolution(s, length): # O(n)
            return True
    return False

```

```

print(repeatedSubstringPattern('abcabcabcabc'))
# Input: 'abab'
# Output: true
# Input: 'aba'
# Output: false
# Input: 'abcabcabcabc'
# Output: true
# Usage: Check if a string is composed of repeated
substring(s)
# Useful for: string pattern matching, periodicity
detection

```

### 2.4 Count Triangles

```

def isTriangle(num1, num2, num3): # O(1)
    return num1 + num2 > num3 and num1 + num3 > num2 and
        num2 + num3 > num1

```

```
def countTriangles(nums: list[int]) -> int:
    # O(n^3) / O(1)
    solution = 0
    for i in range(len(nums)): # O(n)
        for j in range(i + 1, len(nums)): # O(n)
            for k in range(j+1, len(nums)): # O(n)
                if isTriangle(nums[i], nums[j], nums[k]):
                    solution += 1
    return solution

print(countTriangles([3, 5, 10, 7]))
# Input: [3, 5, 10, 7]
# Output: 2
# Explanation: (3, 5, 7), (5, 10, 7)
# Usage: Count number of triplets forming valid triangles
# Useful for: geometry problems, combinatorial enumeration
```

## 2.5 Max Sum Subarray - Solution 1

```
def maxSumSubArray(nums: list[int]) -> int:
    # O(n^3) / O(n)
    greatestSum = nums[0]
    for i in range(len(nums)): # O(n)
        for j in range(i, len(nums)): # O(n)
            currentSum = sum(nums[i: j + 1]) # O(n) / O(n)
            greatestSum = max(greatestSum, currentSum) # O(1)
    return greatestSum

print(maxSumSubArray([-2, -5, 6, -2, -3, 1, 5, -6]))
# Input: [-2, -5, 6, -2, -3, 1, 5, -6]
# Output: 7
# Explanation: sum([6, -2, -3, 1, 5]) = 7
# Usage: Brute-force maximum subarray sum
# Useful for: array subproblems, Kadanes algorithm comparison
```

## 2.6 Max Sum Subarray - Solution 2

```
def maxSumSubArray(nums: list[int]) -> int:
    # O(n^2) / O(1)
    greatestSum = nums[0]
    for i in range(len(nums)): # O(n)
        currentSum = 0
        for j in range(i, len(nums)): # O(n)
            currentSum += nums[j] # O(1)
            greatestSum = max(greatestSum, currentSum)
    return greatestSum

print(maxSumSubArray([-2, -5, 6, -2, -3, 1, 5, -6]))
# Input: [-2, -5, 6, -2, -3, 1, 5, -6]
# Output: 7
# Explanation: sum([6, -2, -3, 1, 5]) = 7
# Usage: Brute-force maximum subarray sum
# Useful for: array subproblems, Kadanes algorithm comparison
```

## 2.7 Sum Of Subarray Maximums - Solution 1

```
def maxValue(nums, left, right):
    maxValue = nums[left]
    for i in range(left + 1, right + 1):
        maxValue = max(maxValue, nums[i])
    return maxValue

def computeSum(nums: list[int]) -> int:
    # O(n^3) / O(n)
    totalSum = 0
    for i in range(len(nums)): # O(n)
        for j in range(i, len(nums)): # O(n)
            totalSum += max(nums[i: j + 1]) # O(n) / O(n)
            # totalSum += maxValue(nums, i, j) # O(n) / O(1)
    return totalSum
```

```
print(computeSum([2, 3, 4, 1]))
# Input: [2, 3, 4, 1]
```

```
# Output: 33
# Usage: Brute-force sum of maximums over all subarrays
# Useful for: subarray analysis, enumeration problems
```

## 2.8 Sum Of Subarray Maximums - Solution 2

```
def computeSum(nums: list[int]) -> int:
    # O(n^2) / O(1)
    totalSum = 0
    for i in range(len(nums)): # O(n)
        curMax = nums[i]
        for j in range(i, len(nums)): # O(n)
            curMax = max(curMax, nums[j])
            totalSum += curMax
    return totalSum
```

```
print(computeSum([2, 3, 4, 1]))
# Input: [2, 3, 4, 1]
# Output: 33
# Usage: Brute-force sum of maximums over all subarrays
# Useful for: subarray analysis, enumeration problems
```

# 3 Recursion

## 3.1 Recursive Array Sum

```
def sum(nums: list[int]) -> int:
    # O(n^2) / O(n^2)
    if not nums:
        return 0
    return nums[0] + sum(nums[1:]) # O(len) / O(len)

print(sum([1, 2, 3, 4, 5]))
# Input: [1, 2, 3, 4, 5]
# Output: 15
```

## 3.2 Recursive Reverse String

```
def reverse(s: str) -> str:
    # O(n^2) / O(n^2)
    if not s:
        return ""
    return s[-1] + reverse(s[:-1]) # O(len) / O(len)
```

```
print(reverse('abcde'))
# Input: 'abcde'
# Output: 'edcba'
```

### 3.3 Generate Pattern

```
def pattern(n: int) -> list[int]:
    # O(n^2) / O(n^2)
    if n == 0:
        return [] # O(1)
    halfPattern = pattern(n - 1) # O(len) / O(len)
    return halfPattern + [n] + halfPattern
```

```
print(pattern(4))
# Input: 3
# Output: [1, 2, 1, 3, 1, 2, 1]
# Input: 4
# Output: [1, 2, 1, 3, 1, 2, 1, 4, 1, 2, 1, 3, 1, 2, 1]
```

### 3.4 Recursive First Occurrence

```
def firstOccurrence(nums: list[int], value: int) -> int:
    # O(n^2) / O(n^2)
    if not nums:
        return -1
    index = firstOccurrence(nums[:-1], value) # O(len)
    if index != -1:
        return index
    if value == nums[-1]:
        return len(nums) - 1
    return -1
```

```
print(firstOccurrence([1, 3, 5, 7, 9], 11))
```

```
# Input: [2, 4, 8, 6, 8, 10], 8
# Output: 2
# Input: [1, 3, 5, 7, 9], 11
# Output: -1
```

### 3.5 Flatten Multidimensional Array

```
def flatten(item):
    # O(number of int items * maxDepth)
    # / O(number of int items * maxDepth)
    if type(item) is int: # base case
        return [item]
    flattened_array = []
    for inner_item in item:
        flattened_array += flatten(inner_item)
    return flattened_array
```

```
print(flatten([[[[1, 2], 3], [[5, [6]], 7], 8]])
# Input: [0, [1, [2]], [[3]]]
# Output: [0, 1, 2, 3]
```

## 4 Backtracking

## 5 Stacks

## 6 Two Pointers & Sliding Window

## 7 Partial Sums

## 8 Graphs

### 8.1 DFS Find If Path Exists In Graph

```
from collections import defaultdict

def validPath(n, edges, source, destination):
    # O(n + m) / O(n + m)
    def dfs(node):
```

```
visited.add(node) # Total: O(n)
for adj_node in graph[node]: # Total: O(m)
    if adj_node not in visited:
        dfs(adj_node)
```

```
graph = defaultdict(list[int]) # / O(m)
visited = set() # / O(n)
for edge in edges: # O(m)
    graph[edge[0]].append(edge[1])
    graph[edge[1]].append(edge[0])
```

```
dfs(source)
return destination in visited
```

```
print(validPath(6, [[0, 1], [0, 2], [2, 3], [3, 5],
                    [5, 4], [4, 3]], 0, 5))
```

```
'''
Input: n = 6
edges = [[0, 1], [0, 2], [2, 3], [3, 5], [5, 4], [4, 3]]
source = 0,
destination = 5
Output: True
'''
```

### 8.2 Word ladder - Solution 3 - Part 1

### 8.3 Word ladder - Solution 3 - Part 2

### 8.4 BFS Min Distance To Every Vertex

```
from collections import defaultdict, deque
```

```
def findMinDistances(n, edges, source):
    # O(n + m) / O(n + m)
    graph = defaultdict(list[int]) # / O(m)
    for edge in edges: # O(m)
        graph[edge[0]].append(edge[1])
        # graph[edge[1]].append(edge[0])
```



```

queue = deque([source])
minDist = [-1] * n # / O(n)
minDist[source] = 0

while queue: # Total: O(n + m)
    node = queue.popleft() # Total: O(n)
    for adj_node in graph[node]: # Total: O(m)
        if minDist[adj_node] == -1:
            minDist[adj_node] = minDist[node] + 1
            queue.append(adj_node)

return minDist

print(findMinDistances(8, [[0, 1], [0, 2], [0, 3],
                           [2, 1], [3, 4], [4, 2],
                           [4, 6], [4, 5], [5, 6],
                           [6, 7]], 0))
'''
Input: n = 8
edges = [[0, 1], [0, 2], [0, 3], [2, 1], [3, 4],
          [4, 2], [4, 6], [4, 5], [5, 6], [6, 7]]
source = 0,
Output: [0, 1, 1, 1, 2, 3, 3, 4]
'''

```

## 8.5 Shortest Path With Alternating Colors

```

from collections import defaultdict, deque

```

```

def getAnswer(dist1, dist2):
    if dist1 == -1:
        return dist2
    if dist2 == -1:
        return dist1
    return min(dist1, dist2)

```

```

def shortestAlternatingPaths(n, redEdges, blueEdges,
                             source):
    # O(n + m) / O(n + m)

```

```

graph = defaultdict(list)
for edge in redEdges:
    graph[edge[0]].append([edge[1], 0])
    # graph[edge[1]].append([edge[0], 0])
for edge in blueEdges:
    graph[edge[0]].append([edge[1], 1])
    # graph[edge[1]].append([edge[0], 1])

queue = deque([[source, 0], [source, 1]])
minDist = [[-1, -1] for _ in range(n)]
minDist[source][0] = minDist[source][1] = 0

```

```

while queue:
    [node, last_color] = queue.popleft()
    for [adj_node, edge_color] in graph[node]:
        if edge_color != last_color and minDist[adj_node][
            edge_color] == -1:
            minDist[adj_node][edge_color] = minDist[node][
                last_color] + 1
            queue.append([adj_node, edge_color])

```

```

answer = []
for node in range(n):
    answer.append(getAnswer(minDist[node][0], minDist[
        node][1]))

```

```

return answer

```

```

print(shortestAlternatingPaths(7, [[0, 1], [1, 3], [2,
3], [3, 5], [4, 5]],
                                [[0, 2], [2, 6], [2, 4], [3,
4]], 0))
'''

```

```

Input: n = 7
redEdges = [[0, 1], [1, 3], [2, 3], [3, 5], [4, 5]]
blueEdges = [[0, 2], [2, 6], [2, 4], [3, 4]]
source = 0,
Output: [0, 1, 1, 2, 3, 4, -1]
'''

```

## 8.6 Dijkstra's Algorithm

```

from collections import defaultdict
import heapq # MAX HEAP!!!

```

```

def findMinDistances(n, edges, source):
    # O((m + n) log m) / O(m)
    graph = defaultdict(list)
    for edge in edges:
        graph[edge[0]].append([edge[1], edge[2]])
        # graph[edge[1]].append([edge[0], edge[2]])

```

```

min_heap = []
heapq.heappush(min_heap, [0, source])
minDist = [-1] * n
minDist[source] = 0
while min_heap:
    [distance, node] = heapq.heappop(min_heap)
    distance *= -1
    if distance != minDist[node]:
        continue
    for [adj_node, weight] in graph[node]:
        currDist = minDist[node] + weight
        if minDist[adj_node] == -1 or minDist[adj_node]
            > currDist:
            minDist[adj_node] = currDist
            heapq.heappush(min_heap, [-currDist,
                adj_node])

```

```

return minDist

```

```

print(findMinDistances(7, [[0, 1, 6], [0, 2, 2], [2, 1,
3],
                           [1, 4, 2], [2, 3, 1], [3, 1, 1],
                           [3, 4, 2], [4, 5, 1], [4, 6, 3]],
                           0))
'''

```

```

Input: n = 7
edges = [[0, 1, 6], [0, 2, 2], [2, 1, 3], [1, 4, 2], [2,
3, 1],

```

```

        [3, 1, 1], [3, 4, 2], [4, 5, 1], [4, 6,
3]]

```

```

source = 0,

```

```
Output: [0, 4, 2, 3, 5, 6, 8]
,,,
```

## 8.7 Number Of Islands - Part 1

```
from collections import deque
```

```
def findMinDistances(n, m, grid):
    def discoverNewIsland():
        islands[i][j] = numOfIslands
        queue.append([i, j])

    while queue:
        node = queue.popleft()
        for x, y in [[node[0] + 1, node[1]],
                    [node[0], node[1] + 1],
                    [node[0] - 1, node[1]],
                    [node[0], node[1] - 1]]:
            if x < 0 or x >= n or y < 0 or y >= m or \
                grid[x][y] == 0 or islands[x][y] != -1:
                continue
            islands[x][y] = numOfIslands
            queue.append([x, y])

    queue = deque()
    islands = [[-1] * m for _ in range(n)]
    numOfIslands = 0

    for i in range(n):
        for j in range(m):
            if grid[i][j] == 1 and islands[i][j] == -1:
                numOfIslands += 1
                discoverNewIsland()

    print(grid)
    print(islands)
    return numOfIslands
```

```
print(findMinDistances(4, 5, [[1, 1, 0, 0, 0],
                              [1, 1, 0, 0, 0],
                              [0, 0, 1, 0, 0],
```

```
,,,
                                [0, 0, 0, 1, 1]]))
,,,
Input: n = 4, m = 5
grid = [[1, 1, 0, 0, 0],
        [1, 1, 0, 0, 0],
        [0, 0, 1, 0, 0],
        [0, 0, 0, 1, 1]]
Output: 3
,,,
```

## 8.8 Number Of Islands - Part 2

## 8.9 Word Ladder - Solution 1

## 8.10 Word Ladder - Solution 2 - Part 1

## 8.11 Word ladder - Solution 2 - Part 2

## 9 Hash Maps

## 10 Greedy

## 11 Linked Lists

## 12 Algorithmic Fundamentals

### 12.1 Time complexities

$O(1)$	constant time
$O(\log n)$	binary search, gcd, exponentiation
$O(n)$	linear scan, BFS, DFS
$O(n \log n)$	merge sort, quick sort, heap sort
$O(n^2)$	DP on substrings, Floyd-Warshall
$O(n^3)$	matrix multiplication (naive)

### 12.2 Math formulas

$$\gcd(a, b) = \begin{cases} b & a \bmod b = 0 \\ \gcd(b, a \bmod b) & \text{otherwise} \end{cases}$$

Fast exponentiation:

$$a^b \bmod m = \begin{cases} 1 & b = 0 \\ (a^{b/2})^2 \bmod m & b \text{ even} \\ a \cdot (a^{b-1} \bmod m) & b \text{ odd} \end{cases}$$

Sieve of Eratosthenes: find primes up to  $n$  in  $O(n \log \log n)$ .

### 12.3 Graph algorithms

Breadth First Search (BFS):  $O(V + E)$ .

Depth First Search (DFS):  $O(V + E)$ .

Dijkstra (using priority queue):  $O((V + E) \log V)$ .

Floyd-Warshall (all pairs shortest path):  $O(V^3)$ .

Kruskal (Minimum Spanning Tree with DSU):  $O(E \log E)$ .

## 12.4 Data structures

Segment Tree: range query + update in  $O(\log n)$ .

Fenwick Tree (BIT): prefix sums and updates in  $O(\log n)$ .

Disjoint Set Union (Union-Find): almost  $O(1)$  per operation with path compression.

## 12.5 String algorithms

KMP algorithm: prefix-function in  $O(n)$ .

Z-function: compute in  $O(n)$ .

Hashing: polynomial rolling hash,  $O(n)$  preprocessing.

## 12.6 Dynamic Programming patterns

1D DP:  $dp[i] = \min(dp[i-1] + cost)$ .

Knapsack:  $dp[i][w] = \max(dp[i-1][w], dp[i-1][w-w_i] + v_i)$ .

Longest Common Subsequence:  $dp[i][j]$  for prefixes.

## 12.7 Modular arithmetic

$$(a + b) \bmod m = ((a \bmod m) + (b \bmod m)) \bmod m$$

$$(a \cdot b) \bmod m = ((a \bmod m) \cdot (b \bmod m)) \bmod m$$

Modular inverse (if  $m$  prime):

$$a^{-1} \equiv a^{m-2} \pmod{m}$$

# 13 Essential Math Tables & Constants

## 13.1 Basic Geometry — Areas, Perimeters, Volumes

**Rectangle:**

$$\text{Area: } A = lw \quad \text{Perimeter: } P = 2(l + w)$$

**Square:**

$$\text{Area: } A = a^2 \quad \text{Perimeter: } P = 4a$$

**Triangle:**

$$\text{Area: } A = \frac{1}{2}bh \quad \text{Perimeter: } P = a + b + c$$

$$\text{Heron: } s = \frac{a+b+c}{2}, \quad A = \sqrt{s(s-a)(s-b)(s-c)}$$

**Equilateral triangle:**

$$\text{Area: } A = \frac{\sqrt{3}}{4}a^2$$

**Circle:**

$$\text{Area: } A = \pi r^2 \quad \text{Circumference: } C = 2\pi r \quad \text{Diameter: } d = 2r$$

**Ellipse:**

$$\text{Area: } A = \pi ab \text{ (semi-axes } a, b)$$

**Sphere:**

$$\text{Surface: } S = 4\pi r^2 \quad \text{Volume: } V = \frac{4}{3}\pi r^3$$

**Cylinder:**

$$\text{Volume: } V = \pi r^2 h \quad \text{Surface (incl. bases): } S = 2\pi r(h + r)$$

**Cone:**

$$\text{Volume: } V = \frac{1}{3}\pi r^2 h$$

**Regular polygon (n sides, side length a):**

$$\text{Area: } A = \frac{na^2}{4\tan(\pi/n)}$$

**Distance (2D):**

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

**Shoelace formula (polygon area):**

$$A = \frac{1}{2} \left| \sum_{i=0}^{n-1} (x_i y_{i+1} - x_{i+1} y_i) \right|, \quad (x_n, y_n) = (x_0, y_0)$$

## 13.2 Important Sequences & Small Tables

**Powers of 2:**

$$\begin{array}{lll} 2^0 = 1 & 2^5 = 32 & 2^{10} = 1024 \\ 2^1 = 2 & 2^6 = 64 & 2^{11} = 2048 \\ 2^2 = 4 & 2^7 = 128 & 2^{12} = 4096 \\ 2^3 = 8 & 2^8 = 256 & 2^{13} = 8192 \\ 2^4 = 16 & 2^9 = 512 & 2^{14} = 16384 \\ 2^{15} = 32768 & 2^{16} = 65536 & 2^{20} = 1048576 \end{array}$$

**Powers of 3:**

$$\begin{array}{lll} 3^0 = 1 & 3^5 = 243 & 3^{10} = 59049 \\ 3^1 = 3 & 3^6 = 729 & 3^{11} = 177147 \\ 3^2 = 9 & 3^7 = 2187 & 3^{12} = 531441 \\ 3^3 = 27 & 3^8 = 6561 & \\ 3^4 = 81 & 3^9 = 19683 & \end{array}$$

**Powers of 5:**

$$\begin{array}{lll} 5^0 = 1 & 5^5 = 3125 & 5^{10} = 9765625 \\ 5^1 = 5 & 5^6 = 15625 & \\ 5^2 = 25 & 5^7 = 78125 & \\ 5^3 = 125 & 5^8 = 390625 & \\ 5^4 = 625 & 5^9 = 1953125 & \end{array}$$

**Factorials:**

$$\begin{array}{lll} 0! = 1 & 5! = 120 & 10! = 3628800 \\ 1! = 1 & 6! = 720 & 11! = 39916800 \\ 2! = 2 & 7! = 5040 & 12! = 479001600 \\ 3! = 6 & 8! = 40320 & 13! = 6227020800 \\ 4! = 24 & 9! = 362880 & 14! = 87178291200 \\ & & 15! = 1307674368000 \end{array}$$

**Fibonacci numbers ( $F_0$  start):**

$$\begin{array}{lll} F_0 = 0 & F_3 = 2 & F_6 = 8 \\ F_1 = 1 & F_4 = 3 & F_7 = 13 \\ F_2 = 1 & F_5 = 5 & F_8 = 21 \\ & F_9 = 34 & F_{10} = 55 \end{array}$$

**Catalan numbers:**

$$\begin{array}{lll} C_0 = 1 & C_3 = 5 & C_6 = 132 \\ C_1 = 1 & C_4 = 14 & C_7 = 429 \\ C_2 = 2 & C_5 = 42 & \end{array}$$

**Small primes:**

2	19	47	79
3	23	53	83
5	29	59	89
7	31	61	97
11	37	67	
13	41	71	
17	43	73	

**Number of primes:**

30:	10
60:	17
100:	25
1000:	168
10000:	1229
100000:	9592
1000000:	78498
10000000:	664579

**Central Binomial Coefficients  $C(2n, n)$ :**

1:	2
2:	6
3:	20
4:	70
5:	252
6:	924
7:	3432
8:	12870
9:	48620
10:	184756
11:	705432
12:	2704156
13:	10400600
14:	40116600
15:	155117520

**Numbers with Most Divisors:**

$\leq 10^2$ :	60 with 12 divisors
$\leq 10^3$ :	840 with 32 divisors
$\leq 10^4$ :	7560 with 64 divisors
$\leq 10^5$ :	83160 with 128 divisors
$\leq 10^6$ :	720720 with 240 divisors
$\leq 10^7$ :	8648640 with 448 divisors
$\leq 10^8$ :	73513440 with 768 divisors
$\leq 10^9$ :	735134400 with 1344 divisors
$\leq 10^{10}$ :	6983776800 with 2304 divisors
$\leq 10^{11}$ :	97772875200 with 4032 divisors
$\leq 10^{12}$ :	963761198400 with 6720 divisors
$\leq 10^{13}$ :	9316358251200 with 10752 divisors
$\leq 10^{14}$ :	97821761637600 with 17280 divisors
$\leq 10^{15}$ :	866421317361600 with 26880 divisors
$\leq 10^{16}$ :	8086598962041600 with 41472 divisors
$\leq 10^{17}$ :	74801040398884800 with 64512 divisors
$\leq 10^{18}$ :	897612484786617600 with 103680 divisors

**13.3 Algebra & Series (basic sums)****Sum of first  $n$  integers:**

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

**Sum of squares:**

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

**Sum of cubes:**

$$\sum_{k=1}^n k^3 = \left( \frac{n(n+1)}{2} \right)^2$$

**Arithmetic progression:**

$$S_n = \frac{n}{2}(a_1 + a_n)$$

**Geometric progression:**

$$S_n = a \frac{1-r^n}{1-r} \quad (\text{if } r \neq 1)$$

**Finite geometric (special):**

$$1 + r + r^2 + \dots + r^{n-1} = \frac{1-r^n}{1-r}$$

**13.4 Basic Number Theory****GCD / LCM:**

$$\gcd(a, b) \text{ by Euclid (iterative)} \quad \text{lcm}(a, b) = \frac{|ab|}{\gcd(a, b)}$$

**Extended Euclid:** Solve  $ax + by = \gcd(a, b)$  for  $x, y$

**Modular exponentiation:**

Compute  $a^b \bmod m$  in  $O(\log b)$  by binary exponentiation

**Modular inverse:**

If  $\gcd(a, m) = 1$ , inverse exists. If  $m$  prime:  $a^{-1} \equiv a^{m-2} \pmod{m}$  (Fermat)

**Euler's totient:**

$$\varphi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$$

**Legendre formula (power of  $p$  in  $n!$ ):**

$$\nu_p(n!) = \sum_{i \geq 1} \left\lfloor \frac{n}{p^i} \right\rfloor$$

**Wilson's theorem:**

$$p \text{ prime} \iff (p-1)! \equiv -1 \pmod{p}$$

**13.5 Combinatorics (essentials)****Factorial/Permutations:**

$$P(n, k) = \frac{n!}{(n-k)!}$$

**Binomial coefficient:**

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Identities:  $\sum_{k=0}^n \binom{n}{k} = 2^n$  Pascal:  $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$

**Stars and bars:**

Number of solutions nonnegative:  $x_1 + \dots + x_k = n \Rightarrow \binom{n+k-1}{k-1}$

### Inclusion–Exclusion (2 sets):

$$|A \cup B| = |A| + |B| - |A \cap B|$$

## 13.6 Quick constants & reminders

**Golden ratio:**  $\varphi = \frac{1 + \sqrt{5}}{2} \approx 1.6180339887$

**Binet (Fibonacci closed):**  $F_n = \frac{\varphi^n - (1 - \varphi)^n}{\sqrt{5}}$

### Useful approximations:

Stirling:  $n! \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$

### Hardy–Ramanujan (partition approx):

$$p(n) \sim \frac{1}{4n\sqrt{3}} \exp\left(\pi\sqrt{\frac{2n}{3}}\right)$$

# 14 Graphs & Dynamic Programming

## 14.1 Graph Traversal

### BFS:

- Finds shortest path in unweighted graphs.
- Complexity  $O(n + m)$ .

### DFS:

- For connectivity, cycle detection, topological sort.
- Complexity  $O(n + m)$ .

## 14.2 Shortest Paths

### Dijkstra:

- Nonnegative edges.
- $O(m \log n)$  with heap.

### Bellman-Ford:

- Handles negative edges.
- $O(nm)$ . Detects negative cycles.

### Floyd-Warshall:

- All-pairs shortest paths.
- $O(n^3)$ .

### SPFA:

- Optimization of Bellman-Ford, average faster.

## 14.3 Minimum Spanning Tree (MST)

**Kruskal:** Sort edges, union-find.  $O(m \log m)$ .

**Prim:** Use PQ,  $O(m \log n)$ .

## 14.4 Flows & Matchings

### Max Flow:

- Edmonds-Karp  $O(nm^2)$ .
- Dinic  $O(n^2m)$ , often faster.
- Push-Relabel:  $O(n^3)$ .

### Min-Cost Max-Flow:

- Successive shortest path or cycle canceling.

### Bipartite Matching:

- Hopcroft–Karp:  $O(\sqrt{nm})$ .
- Hungarian Algorithm (Assignment):  $O(n^3)$ .

## 14.5 Connectivity & Components

### SCC (Kosaraju / Tarjan):

- $O(n + m)$ .

### Bridges & Articulation points:

- Low-link values with DFS.  $O(n + m)$ .

### 2-SAT:

- Build implication graph, SCC.  $O(n + m)$ .

## 14.6 Topological Sort

**Kahn's algorithm:** repeatedly remove nodes indegree=0.  $O(n + m)$ .

**DFS ordering:** reverse postorder.

## 14.7 Classic DP Problems

### Knapsack:

- 0/1:  $O(nW)$ .
- Unbounded:  $O(nW)$ .
- Optimizations: bitset, divide & conquer.

### LIS (Longest Increasing Subsequence):

- $O(n \log n)$  via patience sorting.

### Matrix Chain Multiplication:

- $dp[i][j] = \min_k (dp[i][k] + dp[k+1][j] + cost)$ .

### Edit Distance:

- $dp[i][j] = \min\{dp[i-1][j] + 1, dp[i][j-1] + 1, dp[i-1][j-1] + (a_i \neq b_j)\}$ .
- Complexity  $O(nm)$ .

### Subset Sum / Partition:

- Boolean DP:  $dp[i][s] = \text{true}$  if subset of first  $i$  sums to  $s$ .

### Bitmask DP (TSP):

- $dp[mask][i] = \text{shortest path covering set mask ending at } i$

$i$ .  
-  $O(n2^n)$ .

## 14.8 Tree DP

### Rerooting technique:

- Compute DP rooted at one node, then propagate to neighbors.

### Examples:

- Count paths, subtree sums, DP on independent sets, etc.

## 14.9 Important Complexity Table

Algorithm	Complexity
DFS/BFS	$O(n + m)$
Dijkstra (PQ)	$O(m \log n)$
Bellman-Ford	$O(nm)$
Floyd-Warshall	$O(n^3)$
MST (Kruskal/Prim)	$O(m \log n)$
Dinic	$O(n^2 m)$
Hopcroft-Karp	$O(\sqrt{n}m)$
Hungarian	$O(n^3)$
Knapsack (0/1)	$O(nW)$
LIS	$O(n \log n)$
TSP (DP)	$O(n2^n)$

## 15 Advanced combinatorics & Number Theory

### 15.1 Advanced Combinatorics

#### Binomial identities:

$$\sum_{k=0}^n \binom{n}{k} = 2^n, \quad \sum_{k=0}^n (-1)^k \binom{n}{k} = 0 \quad (n \geq 1)$$

#### Algebraic identities:

$$\binom{n}{k} = \binom{n}{n-k}, \quad \text{Pascal: } \binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

#### Vandermonde:

$$\sum_k \binom{r}{k} \binom{s}{n-k} = \binom{r+s}{n}$$

#### Multiset / combinations with repetition:

$$\binom{n+k-1}{k-1} \text{ ways to distribute } n \text{ identical items into } k \text{ bins}$$

#### Stirling numbers (2nd kind):

$S(n, k)$ : partitions of  $n$  labeled objects into  $k$  nonempty unlabeled subsets.

Recurrence:  $S(n, k) = kS(n-1, k) + S(n-1, k-1)$

#### Stirling numbers (1st kind, unsigned):

$c(n, k)$ : coefficients in falling factorials. Recurrence:  
 $c(n, k) = c(n-1, k-1) + (n-1)c(n-1, k)$

#### Bell numbers (partitions):

$$B_n = \sum_{k=0}^n S(n, k), \quad B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k$$

#### Inclusion-Exclusion (general):

$$\left| \bigcup_{i=1}^m A_i \right| = \sum_i |A_i| - \sum_{i < j} |A_i \cap A_j| + \sum_{i < j < k} |A_i \cap A_j \cap A_k| - \dots$$

#### Polya / Burnside (counting up to symmetry):

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |\text{Fix}(g)|$$

### 15.2 Multiplicative Functions and Transforms

**Multiplicative functions:**  $f$  is multiplicative if  $f(ab) = f(a)f(b)$  whenever  $\gcd(a, b) = 1$ .

Examples:  $1(n) = 1$ ,  $\text{id}(n) = n$ ,  $\varphi(n)$ ,  $\mu(n)$ ,  $d(n)$  (number of divisors).

#### Möbius function $\mu(n)$ :

$\mu(1) = 1$ . If  $n$  has a squared prime factor,  $\mu(n) = 0$ . Otherwise  $\mu(n) = (-1)^k$ , where  $k$  is the number of distinct primes dividing  $n$ .

#### Möbius inversion:

If  $g(n) = \sum_{d|n} f(d)$ , then  $f(n) = \sum_{d|n} \mu(d)g(n/d)$

#### Divisor count $d(n)$ and divisor sum $\sigma_k(n)$ :

If  $n = \prod p_i^{e_i}$ , then  $d(n) = \prod (e_i + 1)$ ,  $\sigma_k(n) = \prod \frac{p_i^{(e_i+1)k} - 1}{p_i^k - 1}$   
Especially  $\sigma_1(n) = \sigma(n)$  is the sum of divisors.

### 15.3 Euler Totient and Carmichael Function

#### Euler's totient $\varphi(n)$ :

$$\varphi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$$

If  $n = \prod p_i^{e_i}$ , then  $\varphi(n) = \prod p_i^{e_i-1}(p_i - 1)$

**Carmichael function  $\lambda(n)$ :** (smallest  $m$  such that  $a^m \equiv 1 \pmod{n}$  for all  $\gcd(a, n) = 1$ )

For prime powers:

$$\lambda(p^e) = \begin{cases} \varphi(2^e) = 2^{e-2} & \text{if } p = 2, e \geq 3, \\ \varphi(p^e) = p^{e-1}(p-1) & \text{if } p \text{ odd prime.} \end{cases}$$

For general  $n$ :  $\lambda(n) = \text{lcm}(\lambda(p_i^{e_i}))$

### 15.4 Theorems and Tools

#### Fermat's little theorem:

$a^p \equiv a \pmod{p}$ , if  $\gcd(a, p) = 1$  then  $a^{p-1} \equiv 1 \pmod{p}$

**Euler's theorem:**

$a^{\varphi(n)} \equiv 1 \pmod{n}$  if  $\gcd(a, n) = 1$

**Multiplicative order:**

$\text{ord}_n(a)$  is the smallest  $k$  with  $a^k \equiv 1 \pmod{n}$ ; always divides  $\lambda(n)$

**Primitive root:**

Exists for  $n = 2, 4, p^k, 2p^k$  with odd prime  $p$ .

**Chinese Remainder Theorem (CRT):**

If  $m_i$  are pairwise coprime, the system  $x \equiv a_i \pmod{m_i}$  has a unique solution  $\pmod{\prod m_i}$ .

**Lucas theorem (for  $\binom{n}{k} \pmod{p}$ ,  $p$  prime):**

Writing  $n, k$  in base  $p$ :  $n = \sum n_i p^i$ ,  $k = \sum k_i p^i$ , then

$$\binom{n}{k} \equiv \prod_i \binom{n_i}{k_i} \pmod{p}$$

**Lifting The Exponent (LTE):**

For evaluating  $\nu_p(x^n - y^n)$  there are formulas depending on  $p, x, y$  (handy in contests).

**15.5 Other Useful Numeric Facts****Prime number approximation:**

$$\pi(x) \sim \frac{x}{\ln x} \quad (\text{approximate number of primes } \leq x)$$

**Trial division and factorization:**

For factorization up to  $10^{12}$ , trial division up to  $\sqrt{n}$  or optimized sieve-based methods are fine; for larger numbers, Pollard Rho is recommended.

**Modular arithmetic (add/mul/div):**

Addition/subtraction/multiplication straightforward; division  $\Rightarrow$  multiply by modular inverse:  $a/b \equiv a \cdot b^{-1} \pmod{m}$  if inverse exists.

**16 String Algorithms, FFT-NTT, Factorization****16.1 String Algorithms****Prefix-function (KMP):**

Computes the length of the longest prefix which is also a suffix. Used in KMP for pattern matching.  
Complexity:  $O(n + m)$ .

**Z-function:**

$Z[i]$  = length of the longest prefix starting at  $s[i]$ .  
Complexity:  $O(n)$ .

**Manacher's algorithm (Palindrome radii):**

Computes all palindromic substrings in  $O(n)$ .

**Suffix Array + LCP:**

- Construct with radix sort in  $O(n \log n)$ , or SA-IS in  $O(n)$ .
- LCP with Kasai in  $O(n)$ .

Applications: search, longest repeated substring, count distinct substrings.

**Aho-Corasick Automaton:**

For multiple patterns (dictionary matching). Build:  $O(\sum |p|)$ , query  $O(|text| + occ)$ .

**Rolling Hash (Polynomial Hash):**

$H[i] = \sum_{k=0}^{i-1} s[k] \cdot p^k \pmod{M}$ .  
Applications: substring comparison, Rabin-Karp.

**16.2 FFT-NTT and Convolution****Discrete Fourier Transform (DFT):**

$$A_k = \sum_{j=0}^{n-1} a_j \omega^{jk}, \quad \omega = e^{2\pi i/n}.$$

**FFT:** Computes DFT in  $O(n \log n)$ .

Applications: polynomial multiplication, big integer multiplication.

**NTT (Number Theoretic Transform):**

Analog of FFT over a prime modulus  $p = k2^m + 1$ . Example:  $p = 998244353$ , primitive root=3.

**Convolution Complexity:**

Naive  $O(n^2)$ , Karatsuba  $O(n^{\log_2 3})$ , FFT/NTT  $O(n \log n)$ .

**16.3 Integer Factorization**

**Pollard Rho:** Probabilistic factorization algorithm for large integers. Average  $O(n^{1/4})$ .

Idea: function  $f(x) = (x^2 + c) \pmod{n}$ , find cycle with Floyd.

**Fermat factorization:** Good for numbers close to perfect squares:  $n = a^2 - b^2 = (a - b)(a + b)$ .

**Miller-Rabin primality test:**

For odd  $n$ , write  $n - 1 = 2^s d$ . Check  $a^d \equiv 1$  or  $a^{2^r d} \equiv -1$ . Otherwise composite.  
Probabilistic, with fixed witnesses can be deterministic up to  $2^{64}$ .

**16.4 Other Common Algorithms****Union-Find (DSU):**

Operations:  $\text{find}(x)$ ,  $\text{union}(x, y)$ . Optimizations: path compression, union by rank.  
Amortized complexity:  $O(\alpha(n))$  (inverse Ackermann, almost constant).

**Binary Lifting (LCA):**

Precompute  $up[v][k]$ :  $2^k$ -th ancestor.  
 $O(n \log n)$  build,  $O(\log n)$  query.

#### Segment Tree:

Supports sum, min, lazy propagation. Complexity  $O(\log n)$ .

#### Fenwick Tree (BIT):

Supports prefix sums in  $O(\log n)$ . Lighter memory than segment tree.

### 16.5 Pseudo-code Snippets

#### KMP prefix-function:

```
pi[0]=0
for i=1..n-1:
    j=pi[i-1]
    while j>0 and s[i]!=s[j]:
        j=pi[j-1]
    if s[i]==s[j]: j++
    pi[i]=j
```

#### Z-function:

```
l=0,r=0
for i=1..n-1:
    if i<=r: z[i]=min(r-i+1,z[i-1])
    while i+z[i]<n and s[z[i]]==s[i+z[i]]: z[i]++
    if i+z[i]-1>r: l=i; r=i+z[i]-1
```

#### Pollard Rho (sketch):

```
def f(x): return (x*x+c)%n
x,y,d=2,2,1
while d==1:
    x=f(x); y=f(f(y))
    d=gcd(abs(x-y),n)
if d!=n: return d
```

## 17 Computational Geometry & Miscellaneous

### 17.1 Vector Geometry

- Dot product:  $\vec{a} \cdot \vec{b} = |a||b| \cos \theta = a_x b_x + a_y b_y$
- Cross product (2D):  $\vec{a} \times \vec{b} = a_x b_y - a_y b_x$
- Orientation test:
  - $> 0$ : counter-clockwise
  - $< 0$ : clockwise
  - $= 0$ : collinear
- Distance from point  $P$  to line  $AB$ :  $d = \frac{|(B-A) \times (P-A)|}{|B-A|}$
- Distance from point  $P$  to segment  $AB$ : project  $P$  on  $AB$ , check if projection lies in segment, else take min distance to endpoints

### 17.2 Important Complexity Table

Algorithm / Operation	Complexity
Convex Hull (Graham/Andrew)	$O(n \log n)$
Convex Hull (Jarvis)	$O(nh)$
Closest Pair	$O(n \log n)$
Sweep Line	$O((n+k) \log n)$

### 17.3 Polygons

- Area (shoelace formula):  $A = \frac{1}{2} |\sum_{i=1}^n (x_i y_{i+1} - x_{i+1} y_i)|$
- Convex polygon check: all triples have same orientation
- Point in polygon (ray casting): count ray-edge intersections

### 17.4 Convex Hull

- Graham scan / Andrew monotone chain:  $O(n \log n)$
- Jarvis march (gift wrapping):  $O(nh)$ ,  $h$  = hull size

### 17.5 Closest Pair of Points

- Divide and conquer algorithm:  $O(n \log n)$

### 17.6 Line & Circle Intersections

- Lines  $p + ta$  and  $q + ub$ : solve  $p + ta = q + ub$  for  $(t, u)$
- Segment intersection: check orientation + bounding boxes
- Circle:  $(x - x_0)^2 + (y - y_0)^2 = r^2$
- Line-circle intersection: solve quadratic
- Circle-circle intersection: check distance between centers

### 17.7 Sweep Line

- Used for: segment intersection, closest pair, union of rectangles
- Complexity:  $O((n+k) \log n)$  with balanced BST,  $k$  = intersections

### 17.8 Voronoi & Delaunay

- Voronoi: partition plane by nearest site
- Delaunay: dual of Voronoi, maximizes minimum angle in triangles

### 17.9 Miscellaneous Useful Formulas

- Pick's theorem (lattice polygon):  $A = I + \frac{B}{2} - 1$
- Euler's formula (planar graphs):  $V - E + F = 2$
- Stirling's approximation:  $n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$



### 17.10 Coordinate Geometry Tricks

- Rotate  $(x, y)$  by  $\theta$ :  $(x', y') = (x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta)$
- Reflect point  $P$  over line  $AB$ : project  $P$  onto  $AB$ , then double it