Workshop Bayesian Corpus Studies

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Session 2: Bayesian Inference

Benachrichtigung zu Ihrer Paketzustellung **Nr. 34632900-371?**

DPD

Ihr Paket konnte nicht zugestellt werden, da bei der Zustellung keine Person zur Unterschrift anwesend war.

Wir möchten Ihnen mitteilen, dass wir eine Adressbestätigung benötigen, um den Paketversand erneut zu bestätigen.

HIER ÜBERPRÜFEN

[Abbestellen..27]


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PAKETVERFOLGUNGSNUMMER
58412233520000
VERFOLGEN
dpd
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Operations on Distributions

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from joint to **conditional** distribution (“chain rule”):

$$p(x, z \mid y) = \frac{p(x, y, z)}{p(y)} \qquad p(y \mid x, z) = \frac{p(x, y, z)}{p(x, z)}$$

The Rule of Bayes (Bayes' Theorem)

x: observed variables

z: latent variables

$$p(z \mid x) = \frac{p(x, z)}{p(x)} = \frac{p(x \mid z) \cdot p(z)}{p(x)}$$

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posterior

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posterior prior

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evidence

Finding Spam

- 10% of my email is spam
- 90% of all spam emails contain the word "urgent"
- 20% of all non-spam emails contain the word "urgent"

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$\bar{\boldsymbol{u}}$	1%	72%

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$$p(s|u) = \frac{p(u,s)}{p(u)} = \frac{p(u,s)}{p(u,s) + p(u,\bar{s})} = \frac{0.09}{0.09 + 0.18} = \frac{0.09}{0.27} = \frac{1}{3}$$

Finding the Posterior Distribution

x: observed variables

z: latent variables

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Finding the Posterior Distribution

x: observed variables

z: latent variables

D: data

$$p(z \mid x = D) = \frac{p(x = D, z)}{p(x = D)} = \frac{p(x = D \mid z) \cdot p(z)}{p(x = D)}$$

Finding the Posterior Distribution

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join likelihood prior

posterior

$$p(z \mid x = D) = \frac{p(x = D, z)}{p(x = D)} = \frac{p(x = D \mid z) \cdot p(z)}{p(x = D)}$$

evidence

constant

distribution

function

Finding the Posterior Distribution

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z: latent variables

D: data

$$\underbrace{p(z \mid x = D)}_{\substack{\text{posterior} \\ Z \to \mathbb{P}}} = \frac{\overbrace{p(x = D, z)}^{\substack{Z \to \mathbb{R} \\ \text{joint}}}}{\underbrace{p(x = D)}_{\substack{\text{evidence} \\ \mathbb{P}}}} = \frac{\overbrace{p(x = D \mid z)}^{\substack{Z \to \mathbb{R} \\ \text{likelihood}}} \cdot \overbrace{p(z)}^{\substack{Z \to \mathbb{P} \\ \text{prior}}}}{\underbrace{p(x = D)}_{\text{evidence}}}$$

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The Problem

Computing $p(x = D)$ is hard (often impossible)!

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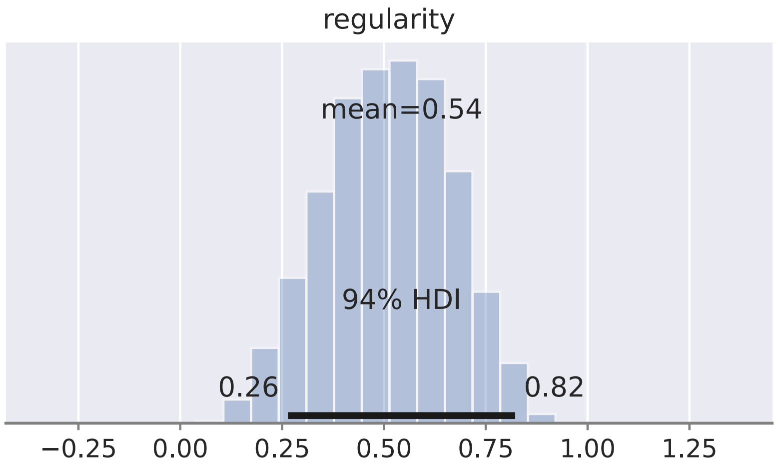
Solutions:

sample from $p(z \mid x)$



approximate $p(z \mid x)$