Workshop Bayesian Corpus Studies

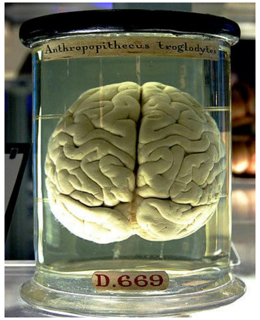
Christoph Finkensiep

Würzburg, Feb 2024

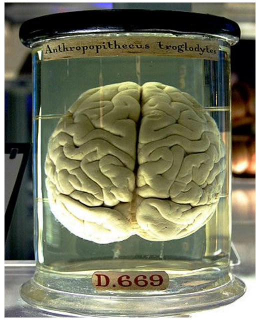
Session 1: Generative Modeling

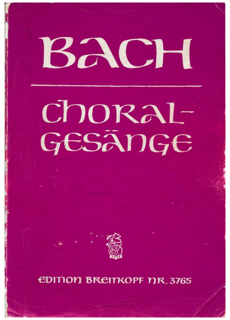
Motivation

Corpus Studies

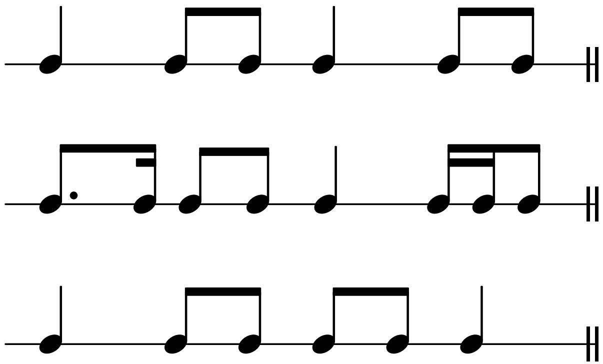


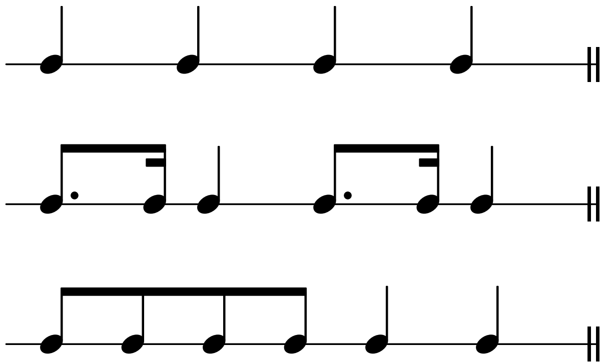
Corpus Studies





A Small Corpus Study





In which group are the patterns more regular?

Corpus Studies

The classical approach:

1. find / create a corpus
2. “operationalize” the quantity of interest
3. measure, do statistics

$$\begin{tikzcd}
\text{conclusions} \\
\text{observations} \arrow[u, "\textit{operationalize}"']
\end{tikzcd}$$

Corpus Studies

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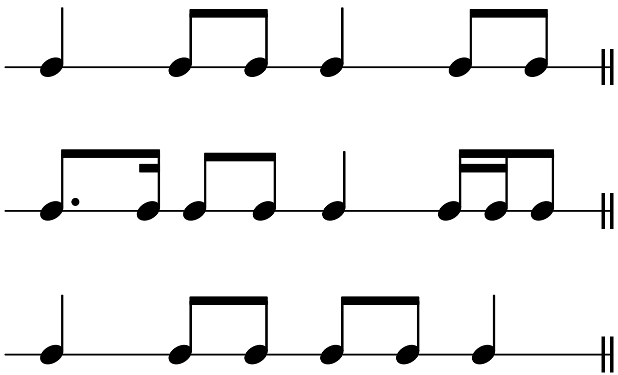
conclusions

$\uparrow$ *operationalize*

observations



Corpus Studies

The classical approach:

1. find / create a corpus
2. $\boxed{\text{“operationalize” the quantity of interest}}$???
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conclusions

operationalize

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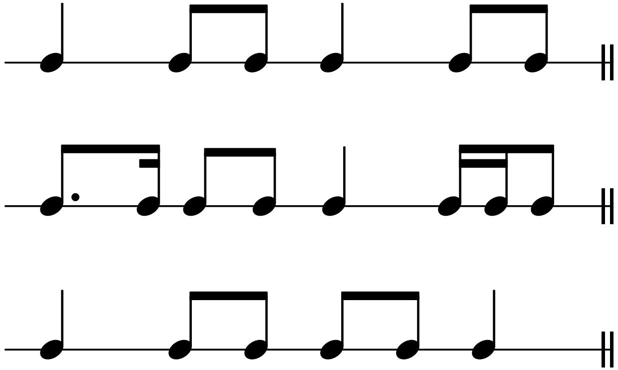
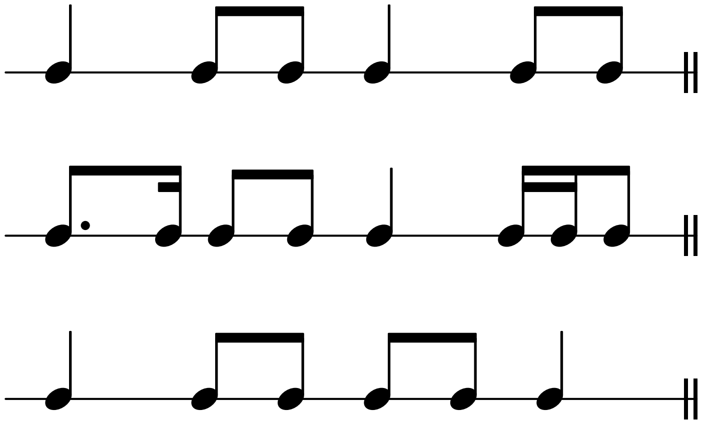


Problems with the Standard Approach

$$\mathrm{reg}(x) = \frac{1}{\#\ \text{unique beats in } x}$$



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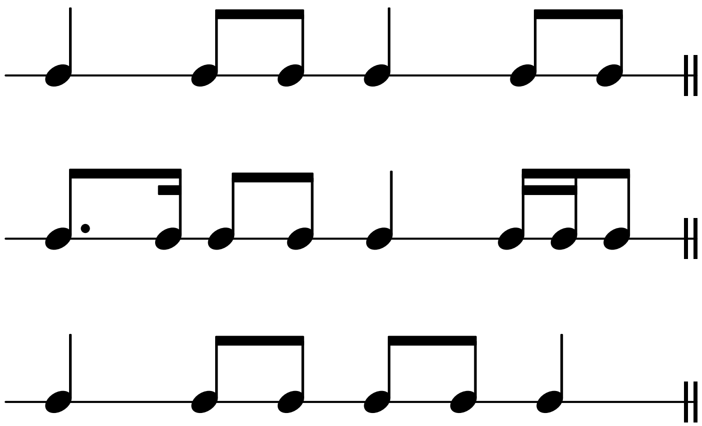

Measure:

- adequate?
- arbitrary, ad-hoc?
- overarching theory?

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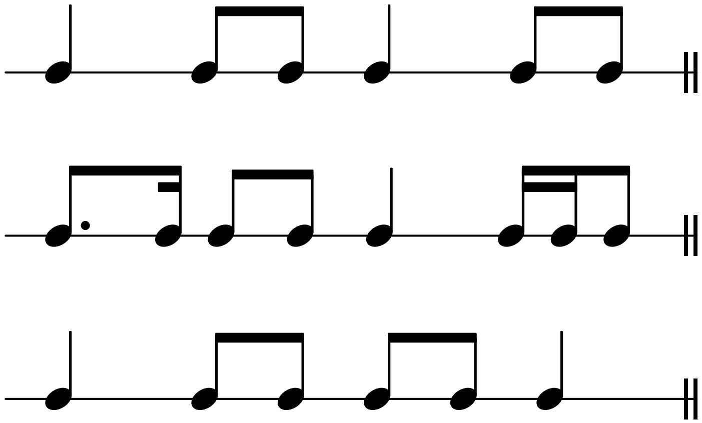

Measure:

- adequate?
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- overarching theory?

Stats:

- which statistic?
  - arithmetic mean: $\frac{1}{N}\sum_i x_i$
  - geometric mean: $\sqrt[N]{\prod_i x_i}$
- which test?

Alternative: Work with Models

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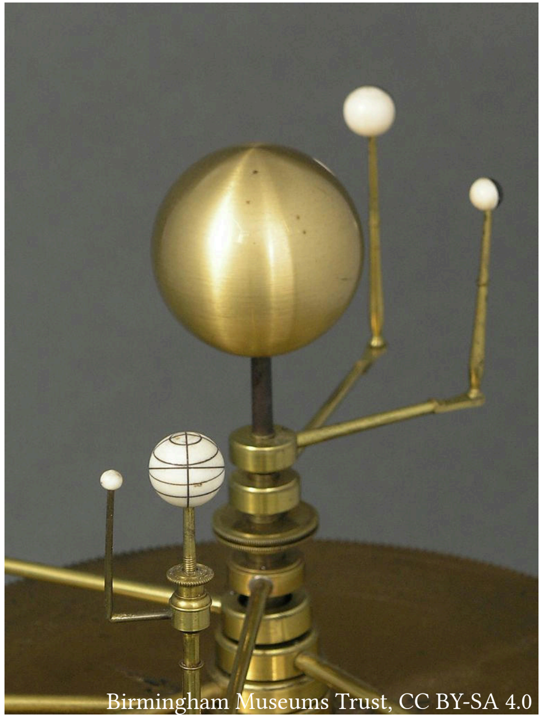
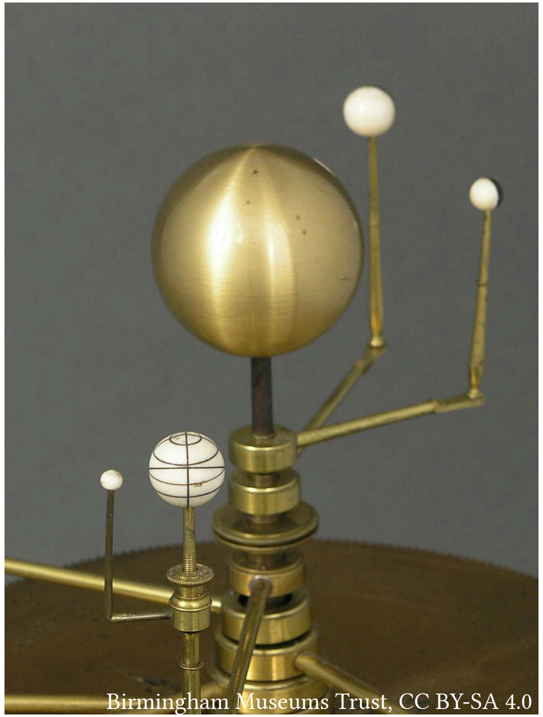
Models


Birmingham Museums Trust, CC BY-SA 4.0

Alternative: Work with Models

Models

- describe a segment of the world (simplified)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Alternative: Work with Models

Models

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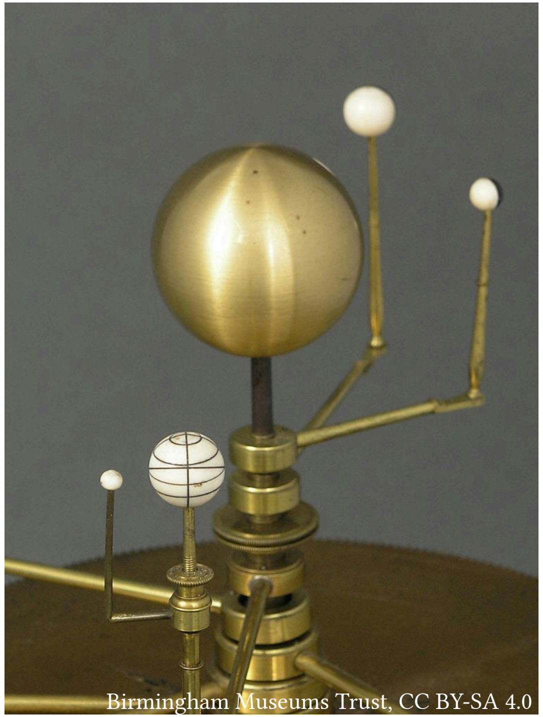
- relevant **entities**, **properties** and **relations**



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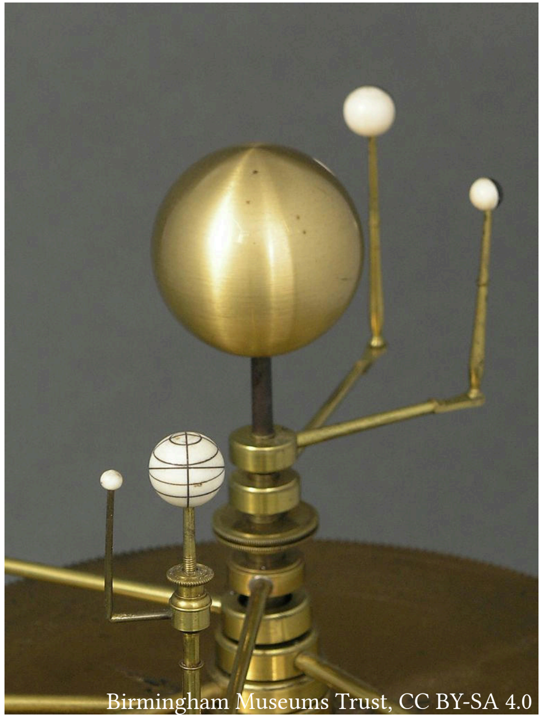

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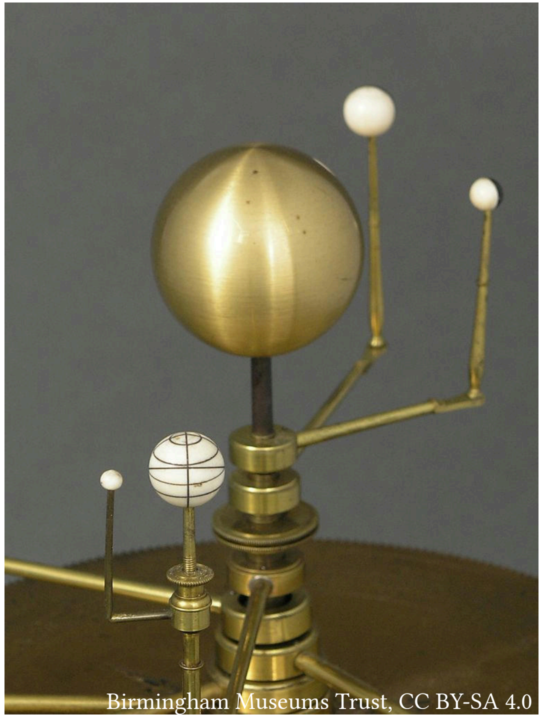

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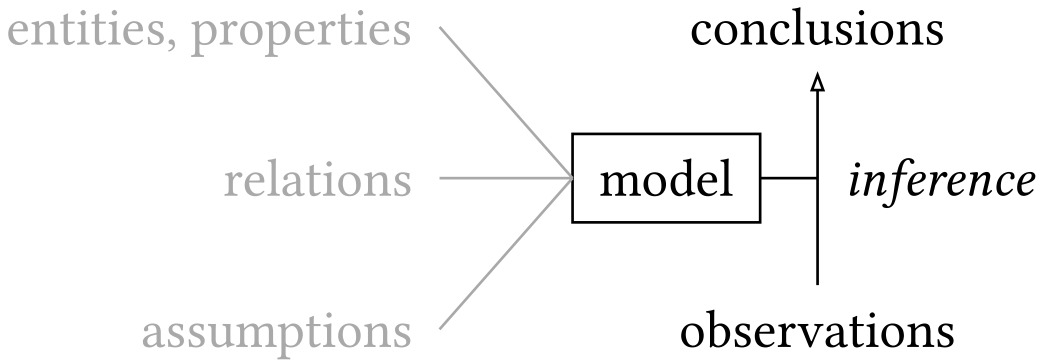
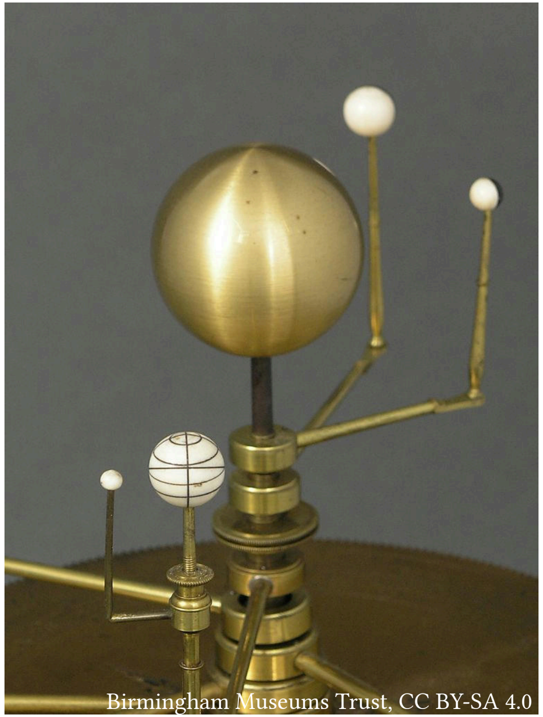
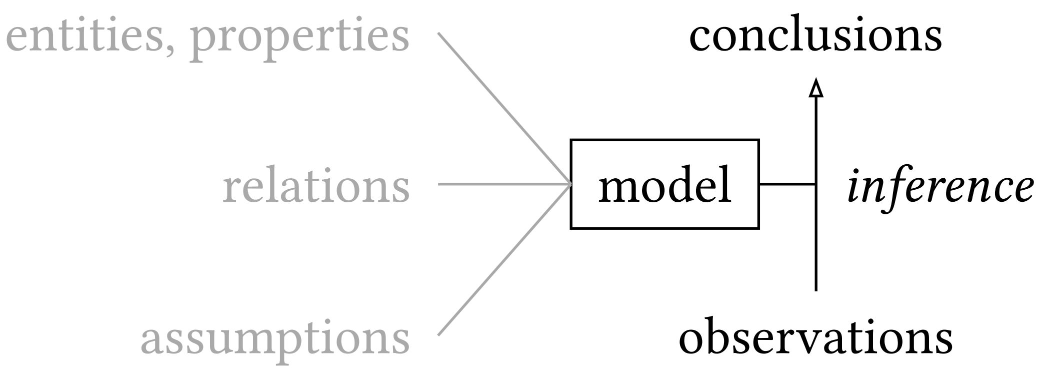
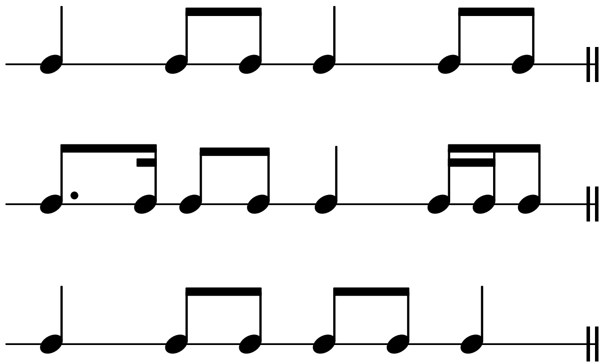




A Model of Rhythmic Regularity





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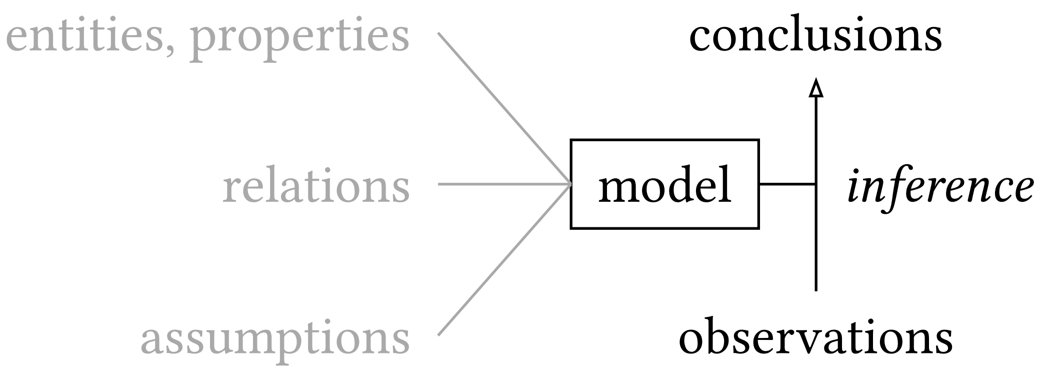
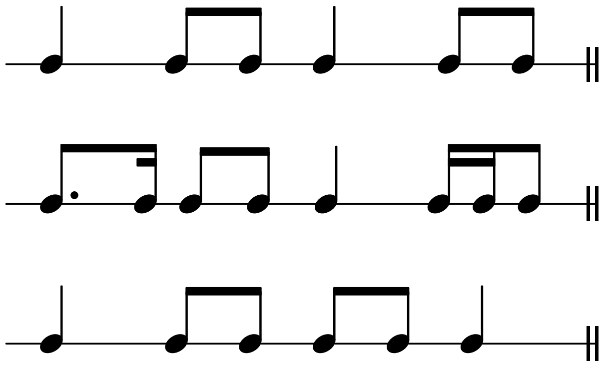
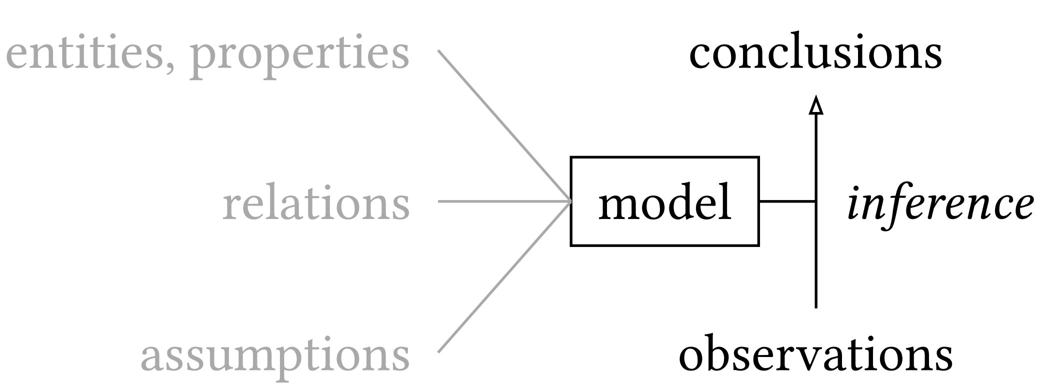
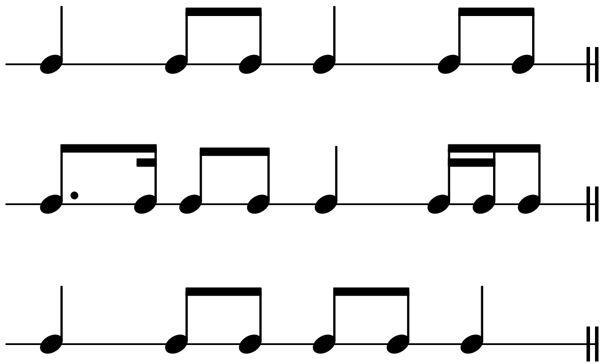




Entities and properties:

A Model of Rhythmic Regularity





Entities and properties:

- pattern
- note
- group of patterns
  - regularity

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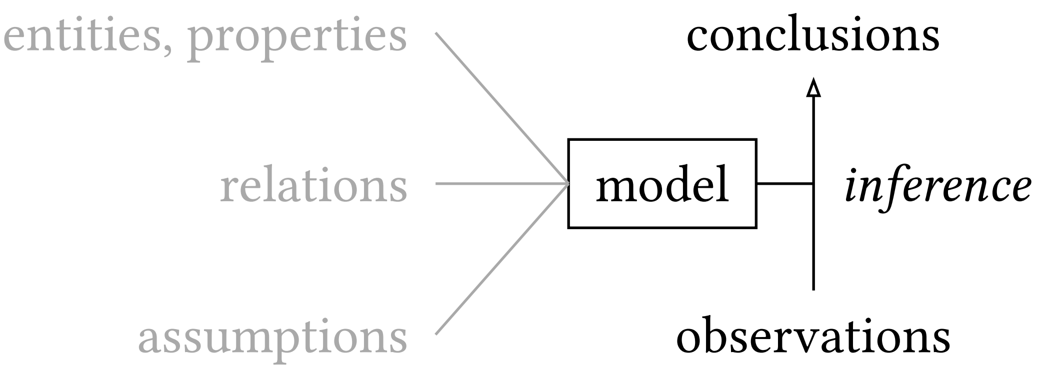

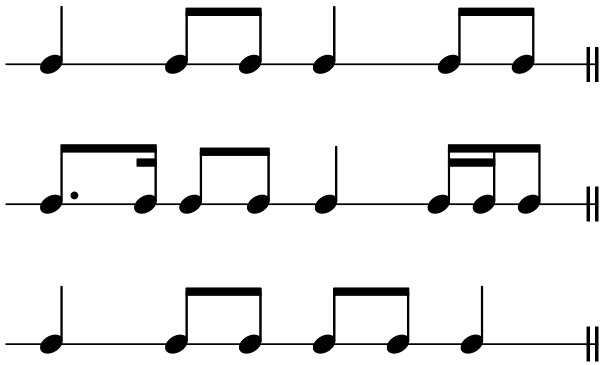

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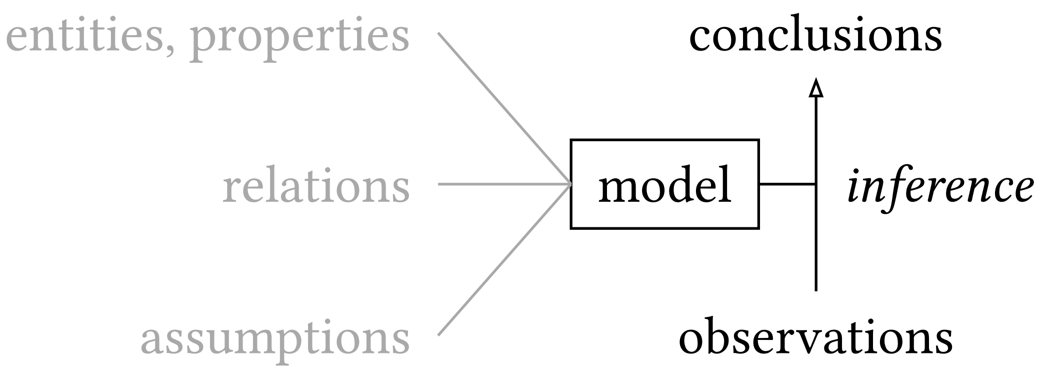
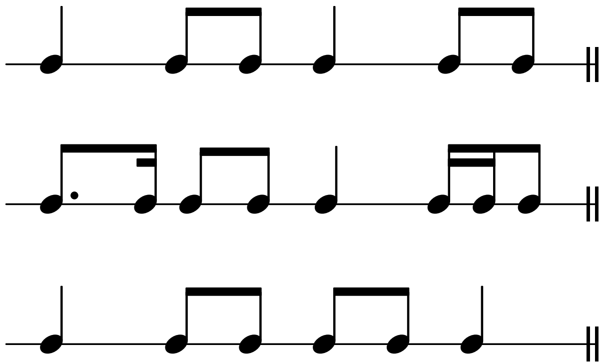
Relations:

A Model of Rhythmic Regularity





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- patterns consist of notes
- groups have patterns
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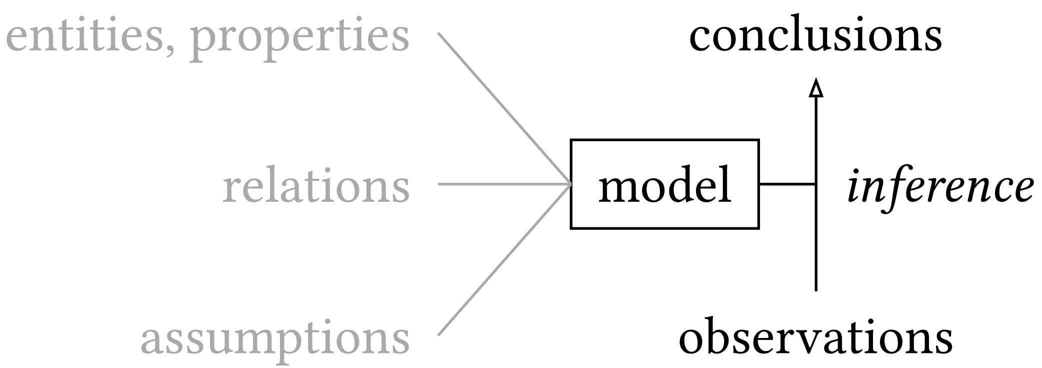

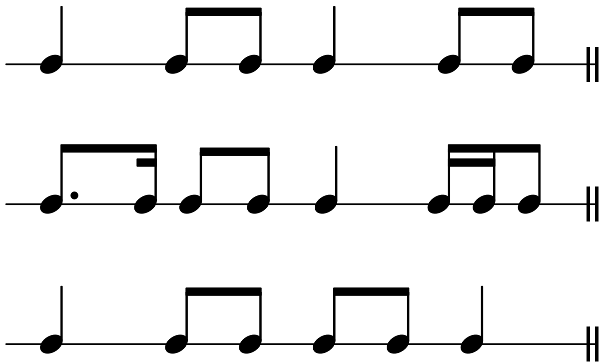

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Inference:

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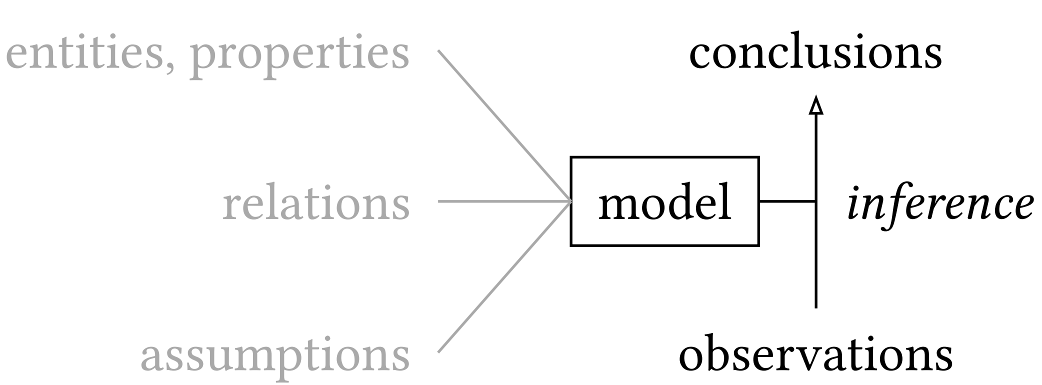

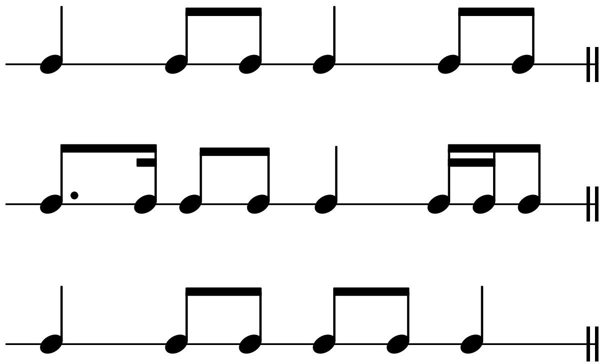

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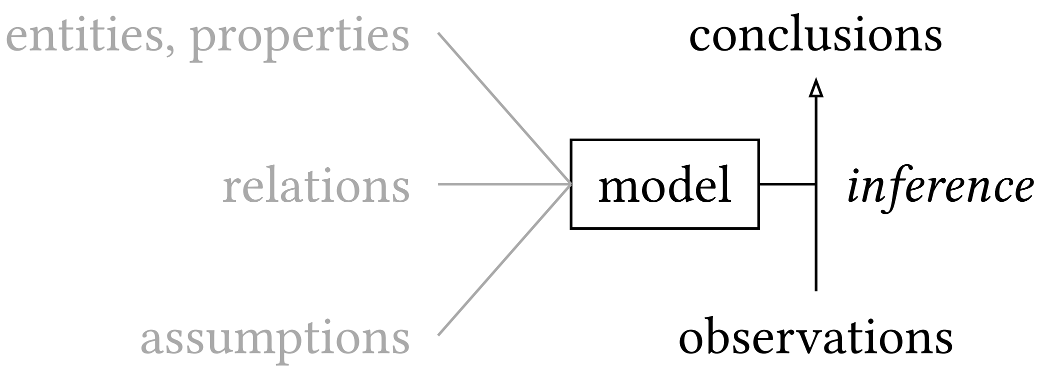

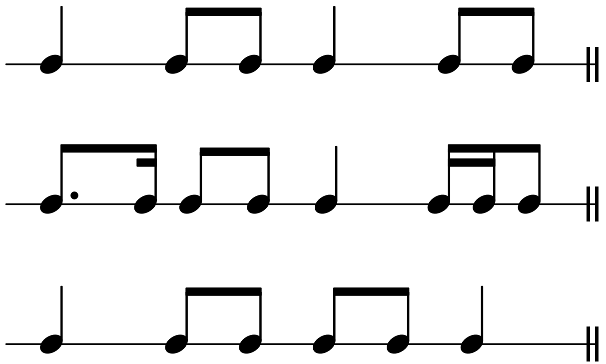

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Simulation:

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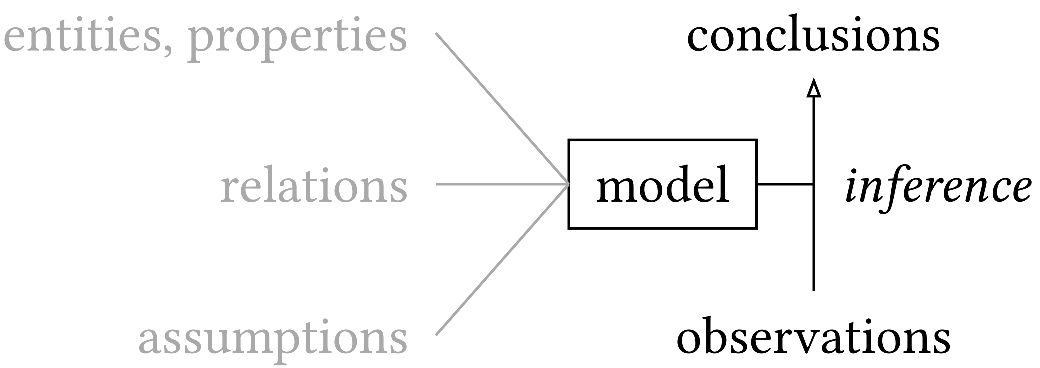

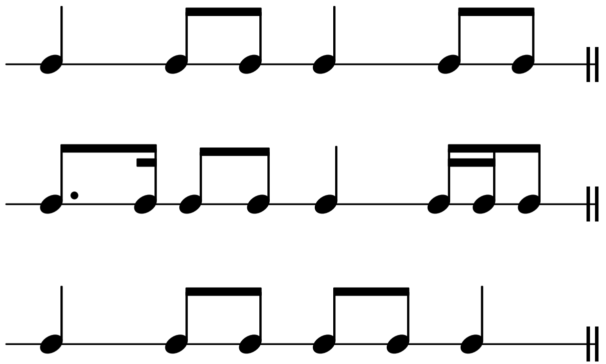

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Inference:

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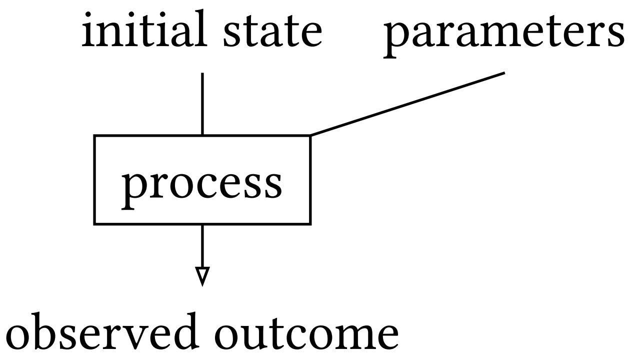
Simulation:

- generate new patterns

Generative Models

The Process Behind the Data



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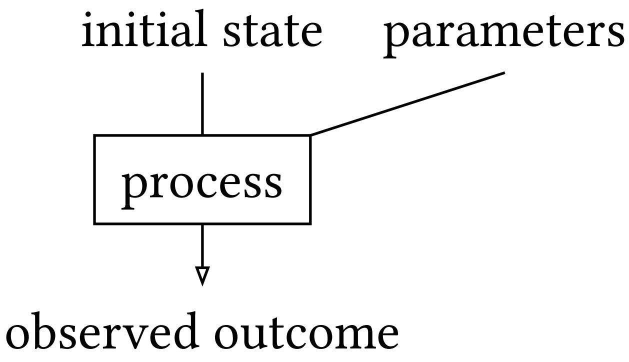

Simulation:

- run the process, change parameters

Inference:

- find plausible parameters

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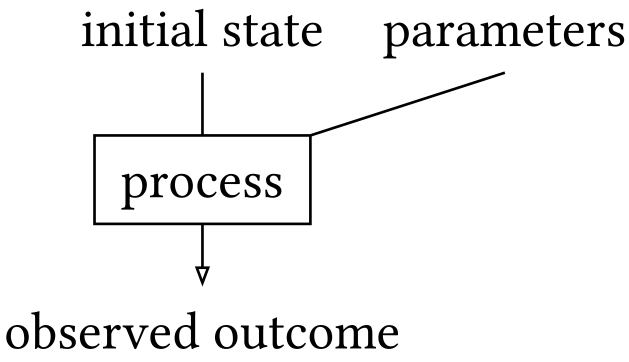

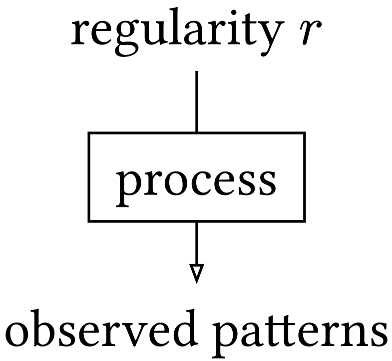

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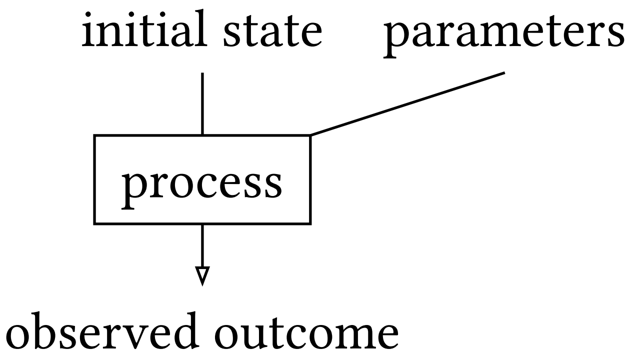

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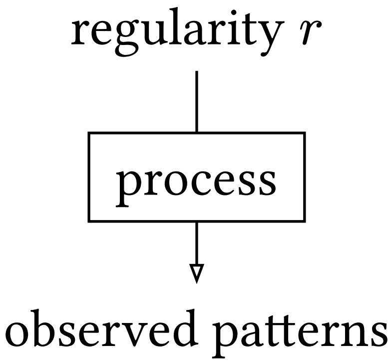
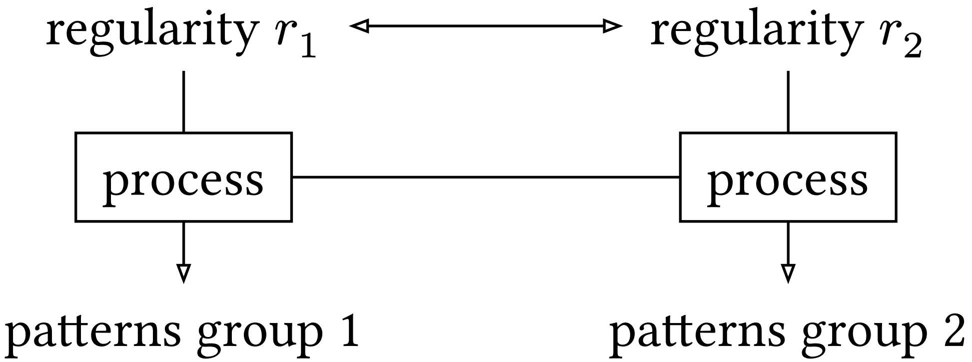
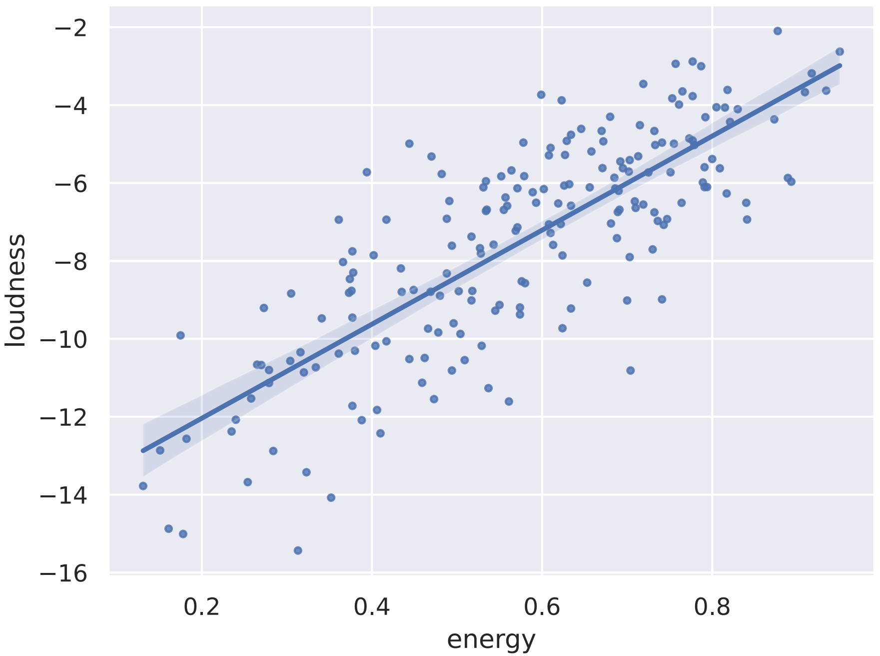




A Simple Example

Linear Regression: x is linearly related to y

$$y \approx a \cdot x + b$$



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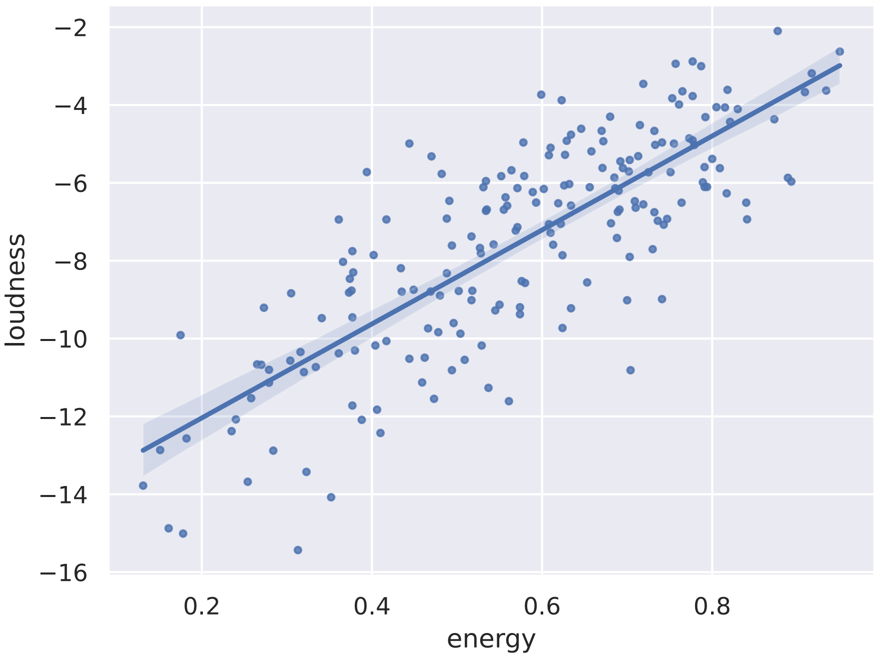

Observations: points (x and y)

Parameters: slope a, intercept b, variance σ^2

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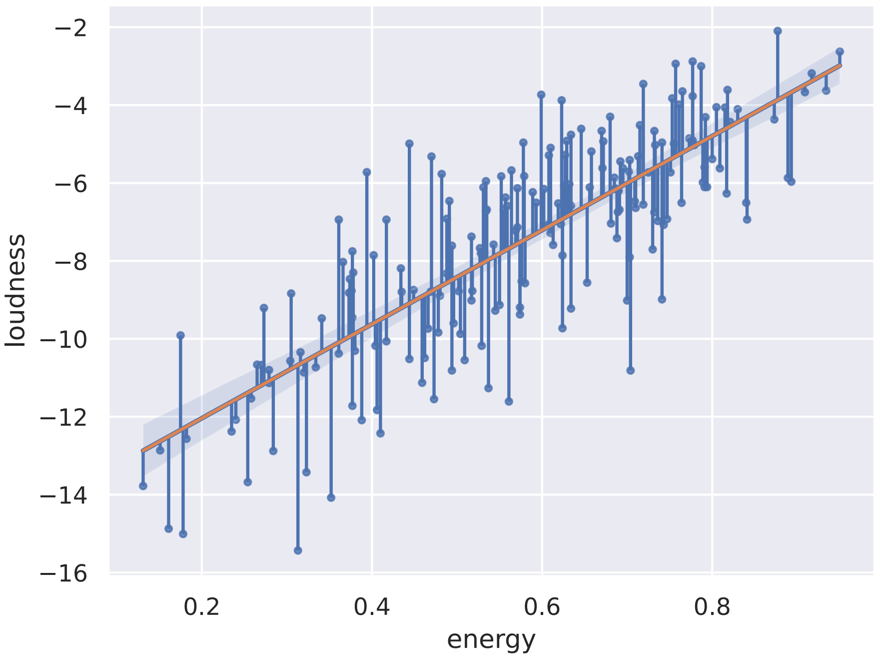

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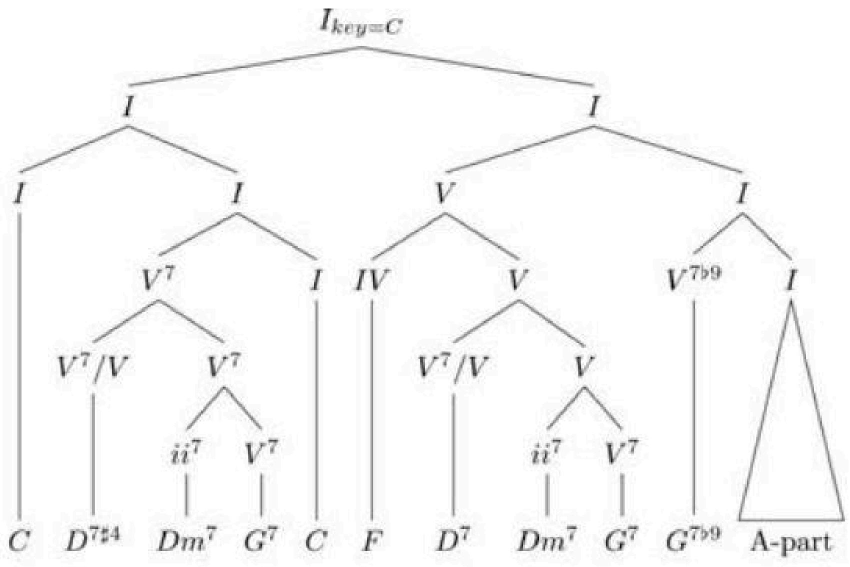
Generative Process:

- choose parameters a, b, σ
- for each point i:
  - choose x_i
  - compute $f(x_i) = a \cdot x_i + b$
  - pick $y_i \sim \mathcal{N}(f(x_i), \sigma)$

A More Involved Example



$$I \longrightarrow I\ I$$
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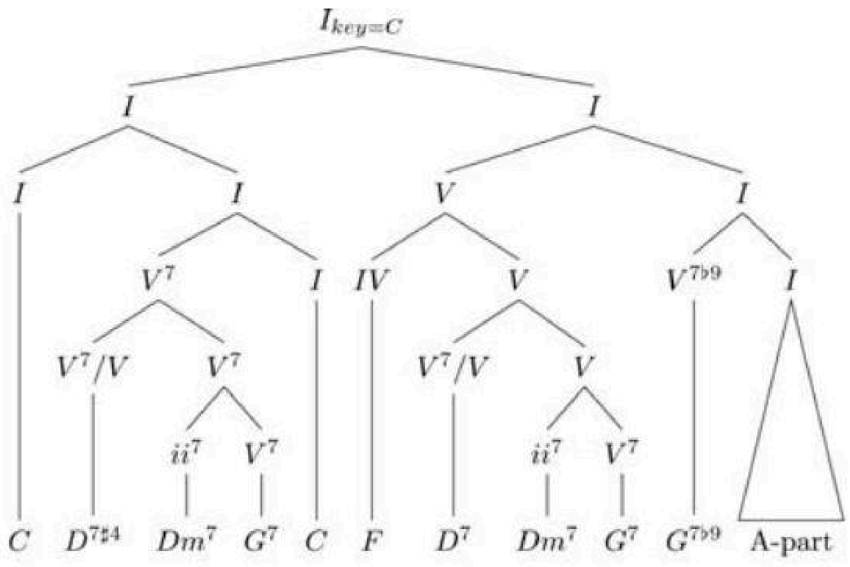

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Observations: chord sequences (pieces)

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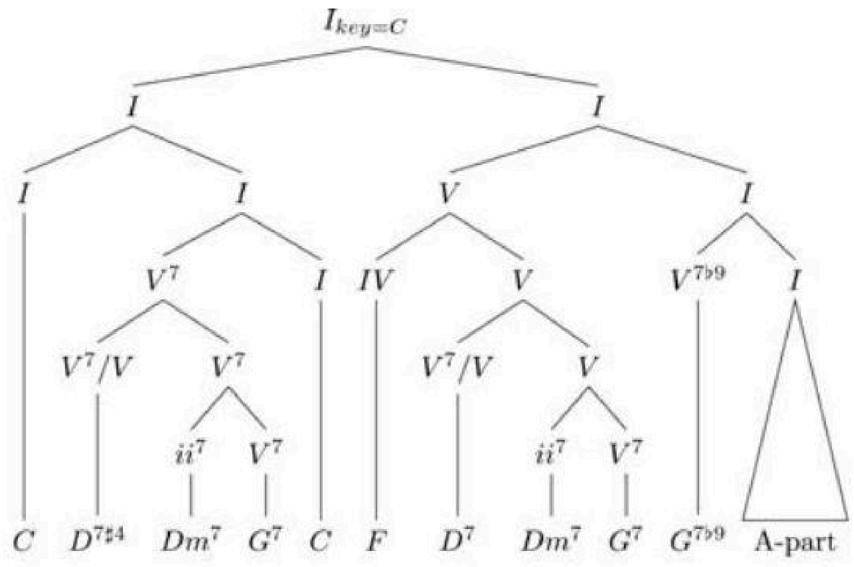

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Observations: chord sequences (pieces)

Parameters: Rules and probabilities

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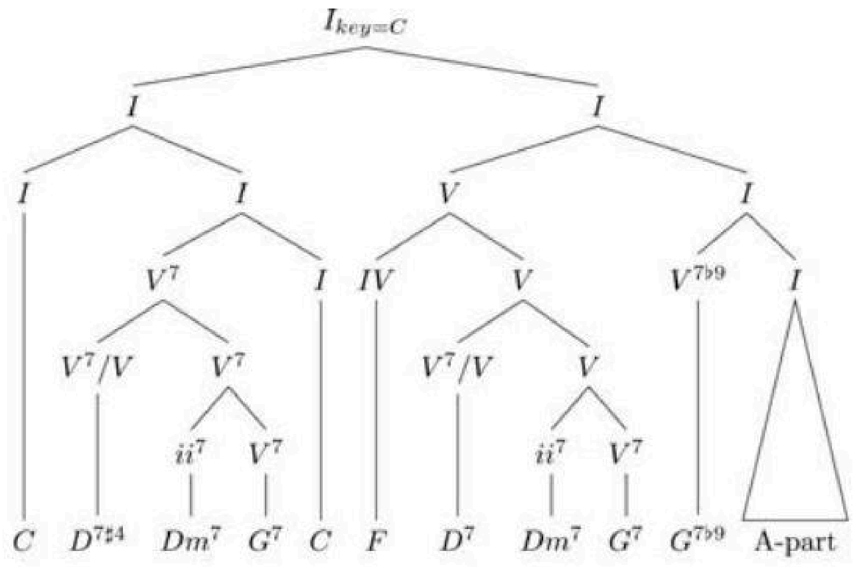

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Observations: chord sequences (pieces)

Parameters: Rules and probabilities

Process:

- choose grammar rules R
- choose rule probabilities p_R
- for each piece i:
  - sample a derivation d_i
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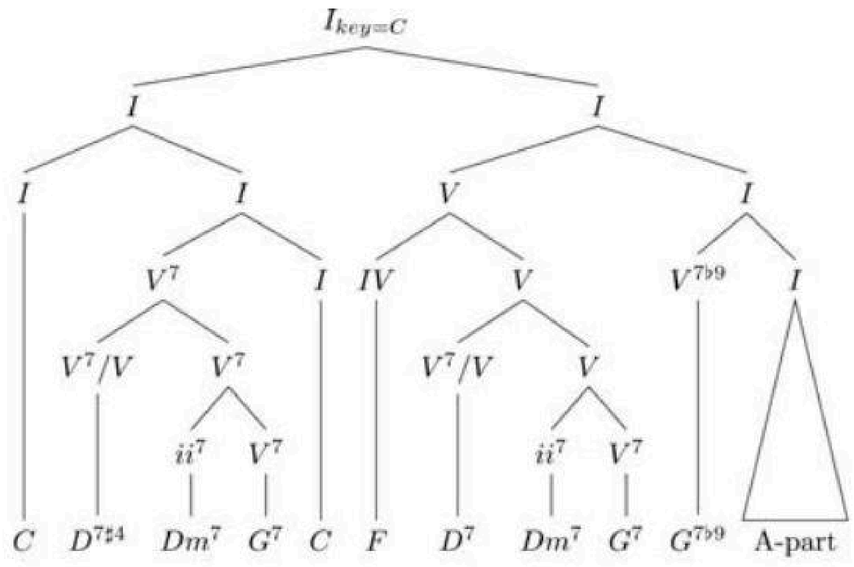

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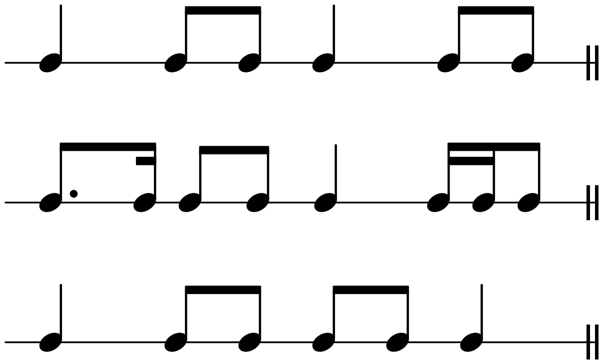
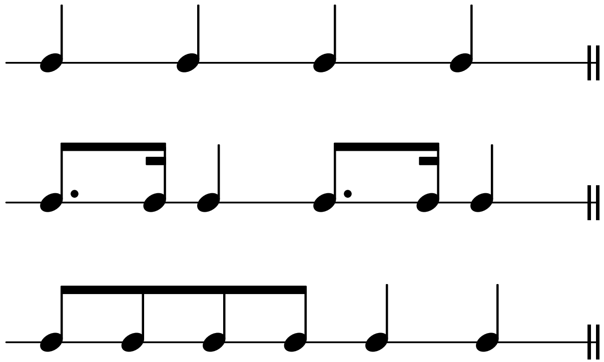
Inference:

- most plausible derivation d_i for each piece
- most plausible rules and probabilities

A *Generative* Model of Rhythmic Regularity

Generate this!





`abab` `aaaa`

`cbad` `caca`

`abba` `bbaa`

Simplifying assumptions: always 4/4, we don't look inside "beats".

Possible Solutions

Generate corpus:

- for each group g:
  - choose regularity r_g:
  - for each pattern i:
    - sample pattern p_{gi} using r_g

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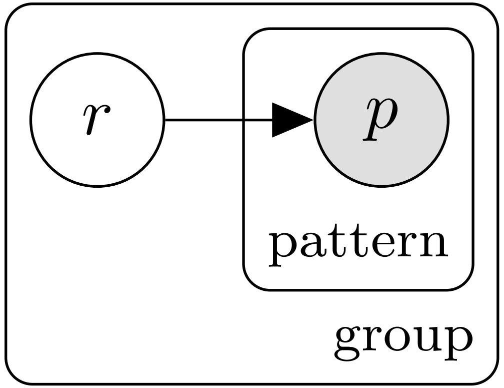

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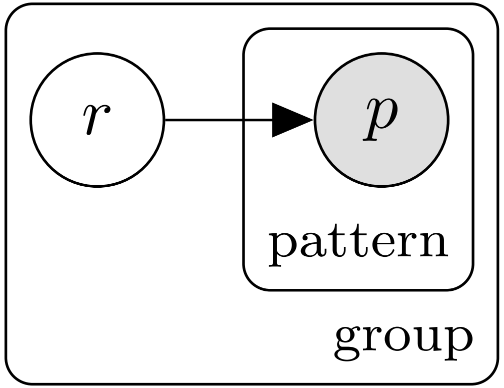
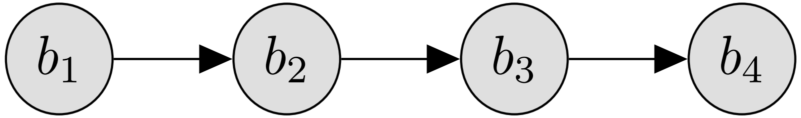


Generate pattern (based on predecessor):

- choose b_1 randomly
- for each following beat i:
  - flip coin (r_g):
    - heads: $b_i = b_{i-1}$
    - tails: choose new beat for b_i



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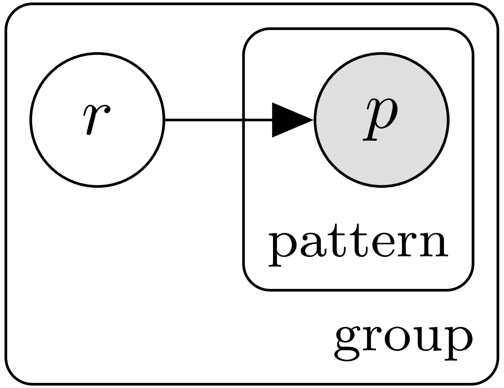
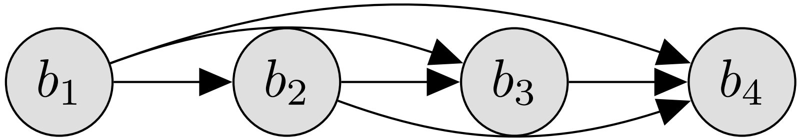


Generate pattern (based on position):

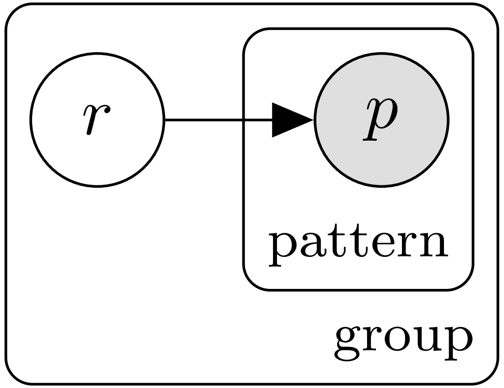
- choose b_1 randomly
- flip coin (r_g):
  - heads: $b_2 = b_1$
  - tails: new beat for b_2
- flip coin (r_g):
  - heads: repeat first half
  - tails:
    - repeat or new b_3 ...
    - repeat or new b_4 ...



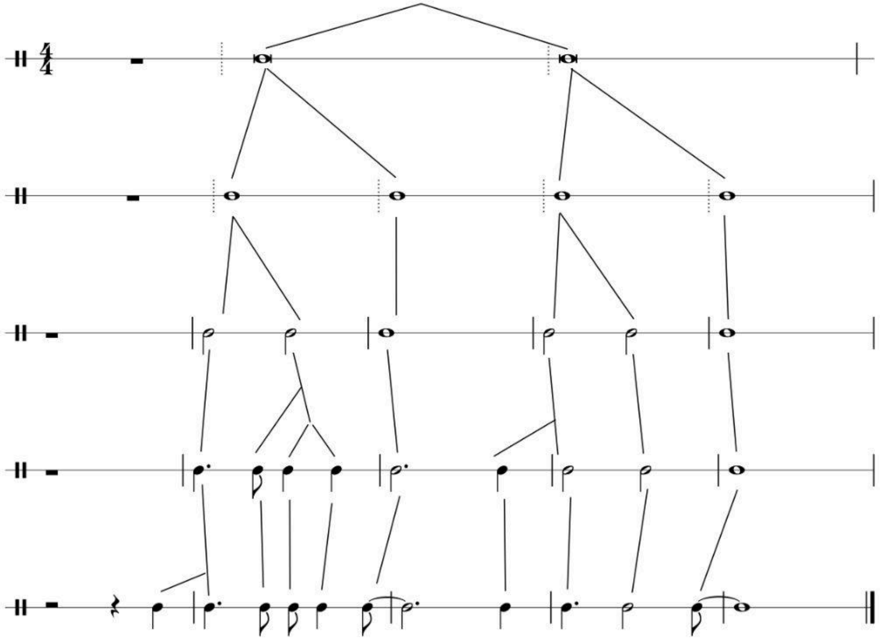
Possible Solutions

Generate corpus:

- for each group g:
  - choose regularity r_g:
  - for each pattern i:
    - sample pattern p_{gi} using r_g



A more detailed model:

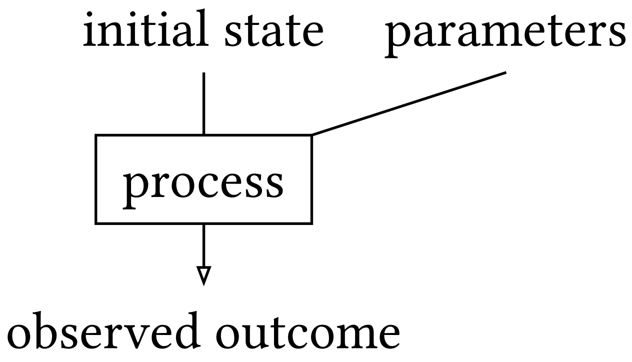


generate full rhythm, flip coins to repeat

Advantages of Generative Models

Very Explicit:

- write down the entire generative process
  - this is how you think/pretend it works
  - links entities (observations, parameters)



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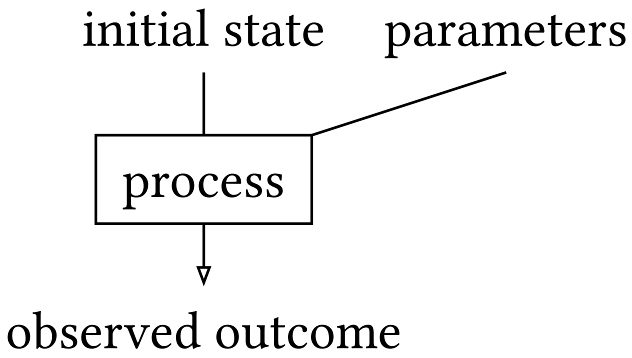

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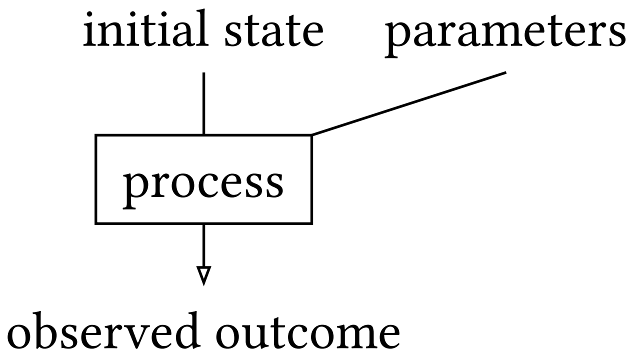

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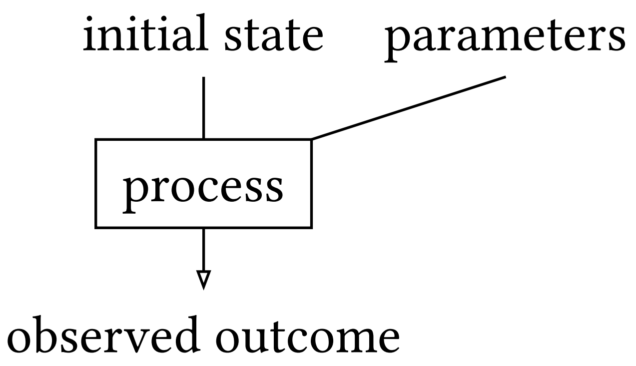

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- run the generative process
- manipulate process / parameters

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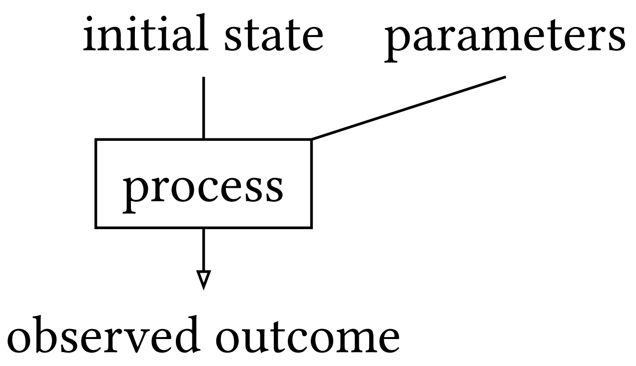

Simulation:

- run the generative process
- manipulate process / parameters

Inference: ???

Inference

Quantifying Uncertainty

"Random variable" X: uncertain quantity or property

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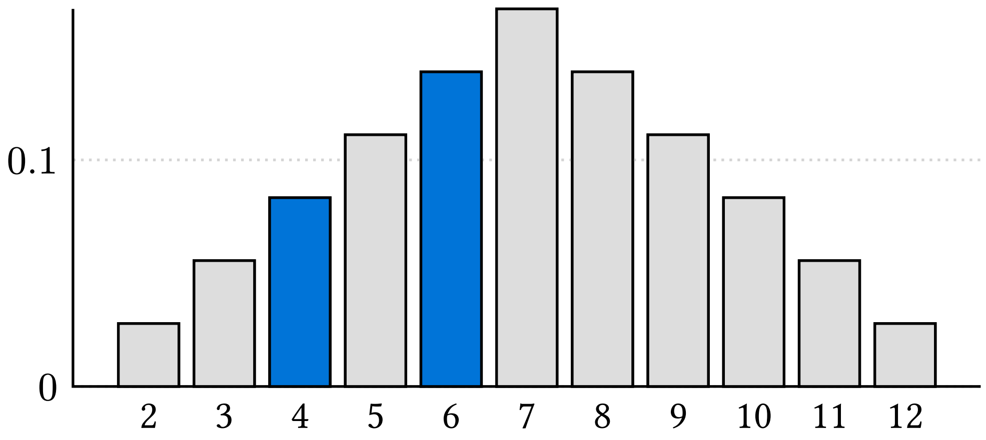

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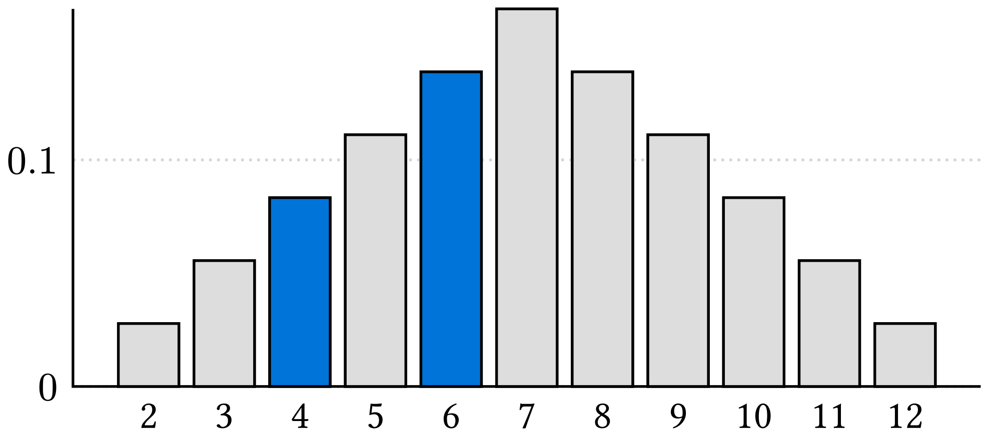

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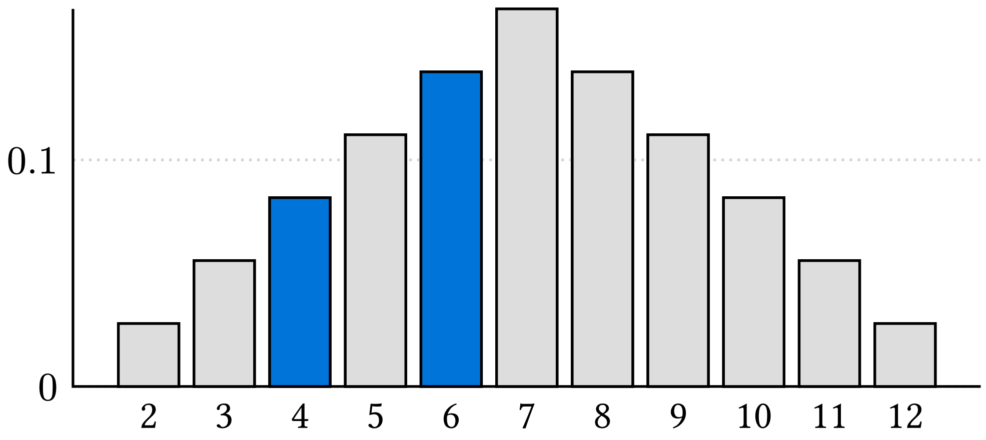

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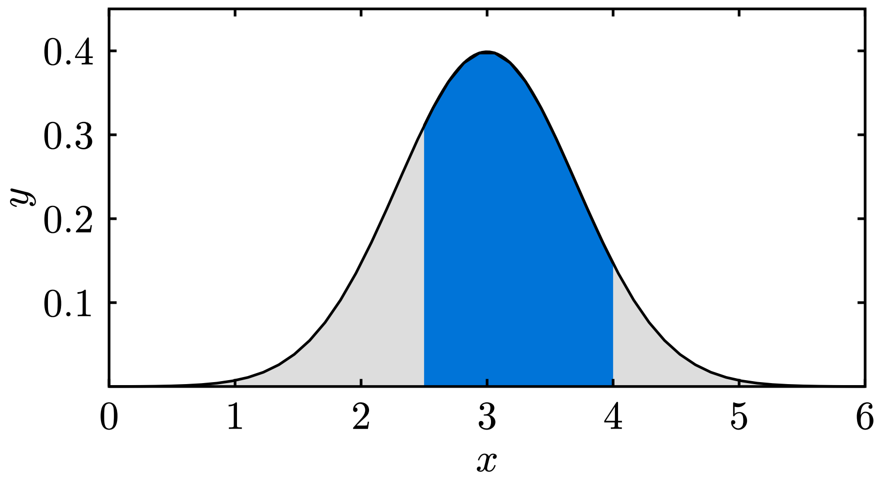

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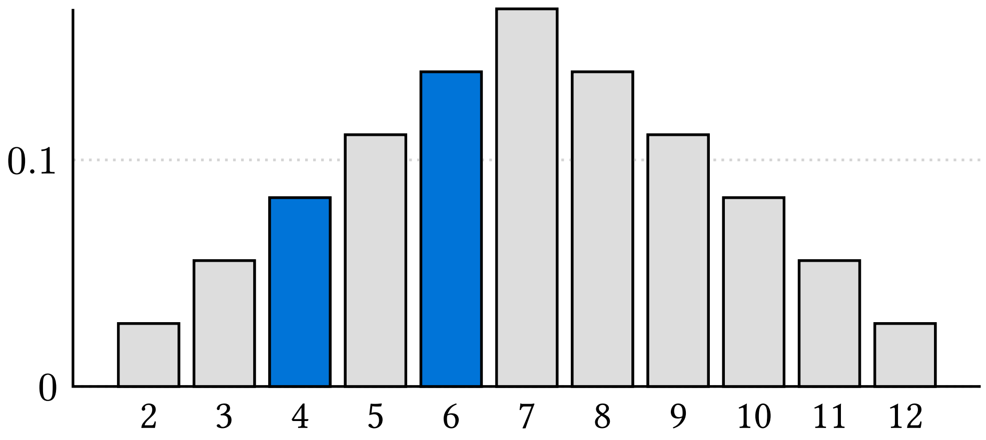

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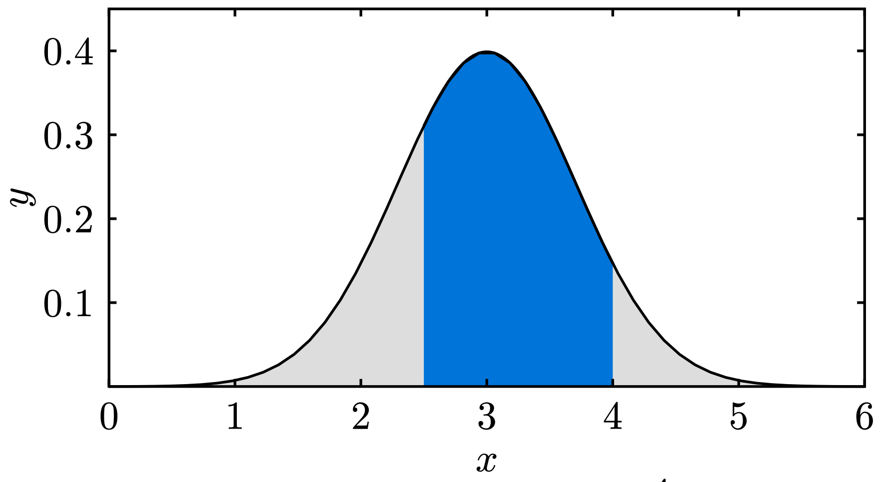

$$P(2.5 \leq X \leq 4) = \int_{2.5}^{4} p(x)\,\mathrm{d}x$$

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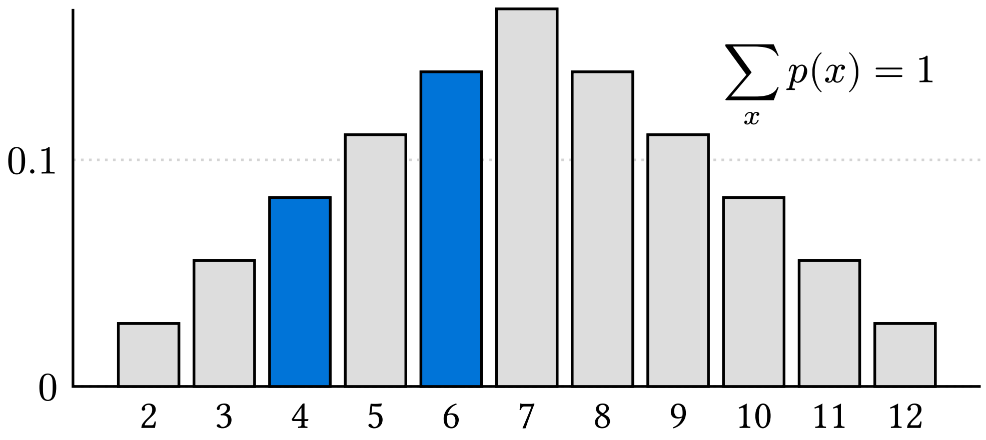

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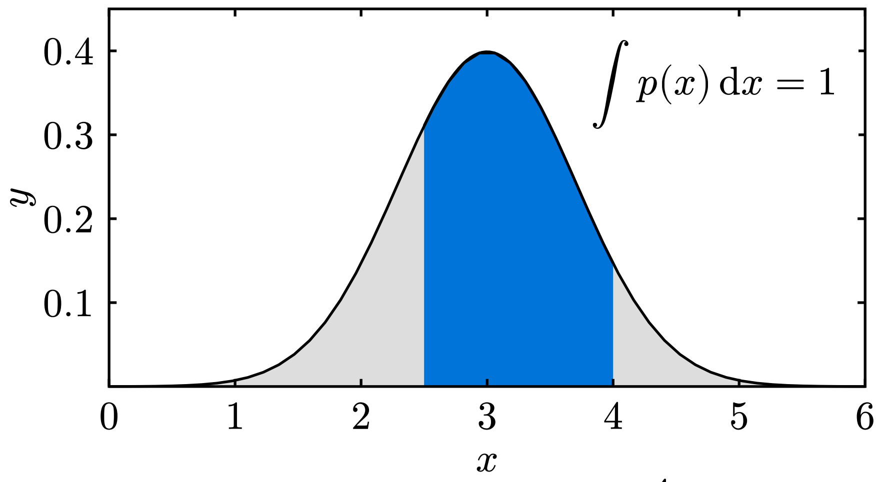

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Distributions over Several Variables

Several variables: **joint** distribution

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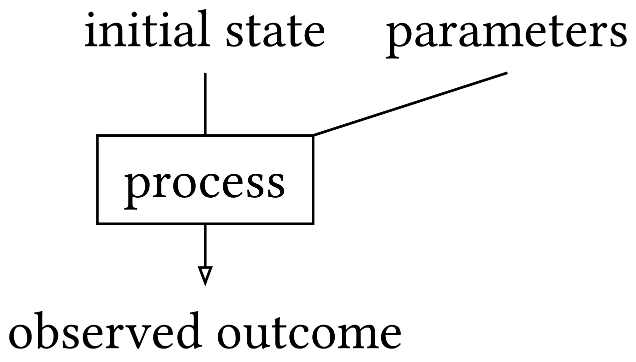
"observation": **conditional** distribution

$$\begin{aligned} p(x \mid y, z) &\quad x \text{ given } y \text{ and } z \\ p(x, z \mid y) &\quad x \text{ and } z \text{ given } y \\ p(y \mid x) &\quad y \text{ given } x \text{ (ignoring } z) \end{aligned}$$

Inference



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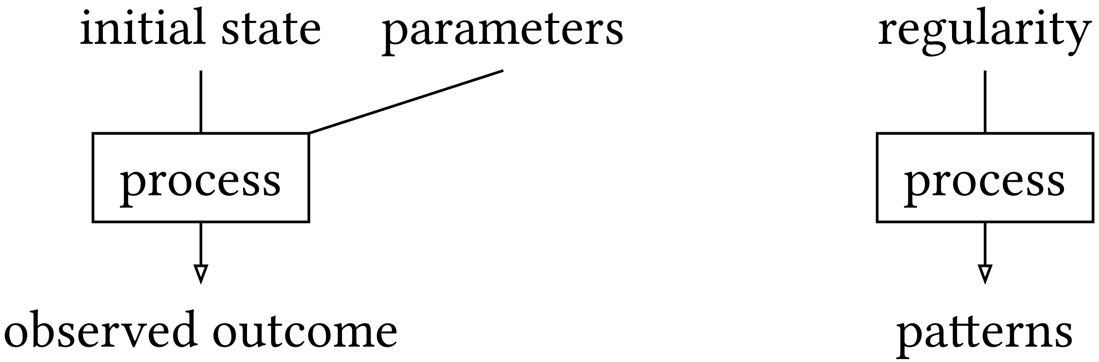
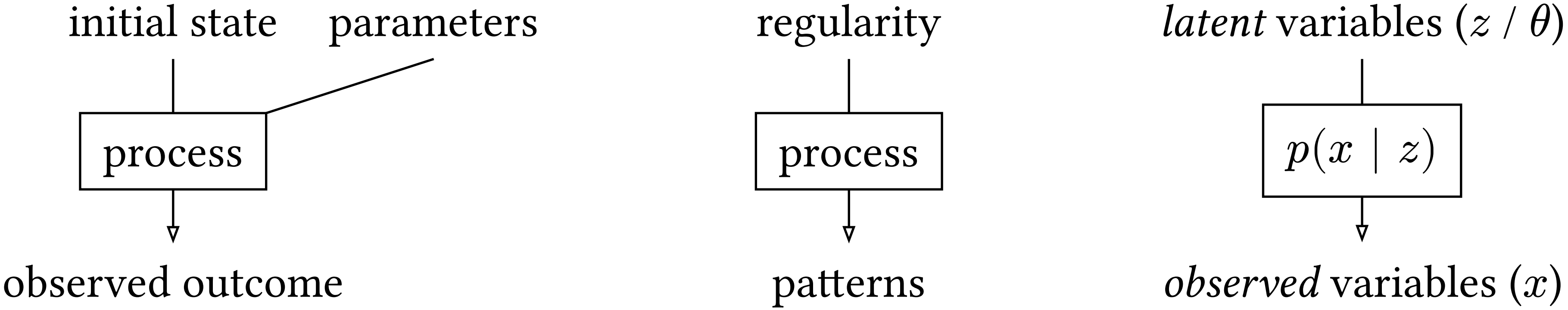
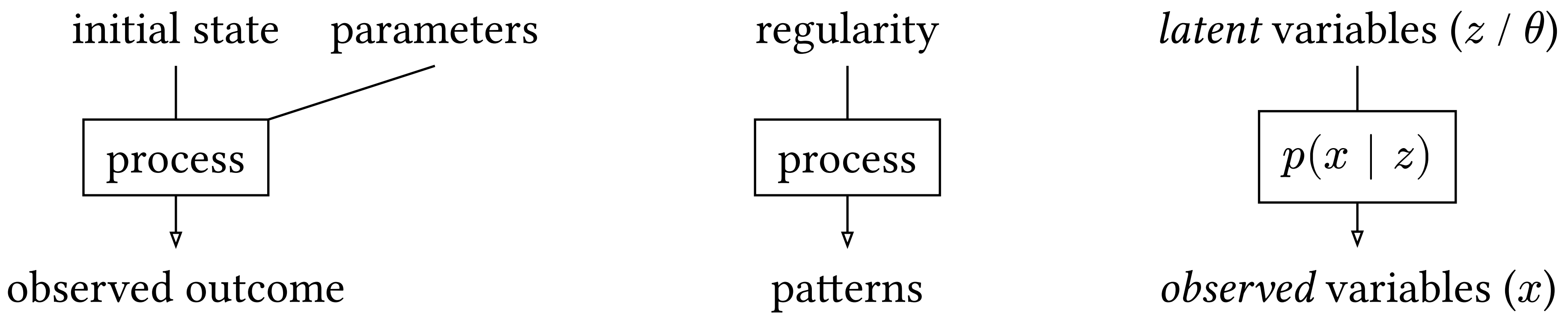

initial state
parameters
process
observed outcome
regularity
process
patterns

Inference


initial state
parameters
process
observed outcome
regularity
process
patterns
latent variables (z / θ)
$p(x \mid z)$
observed variables (x)

Inference



The Inference Button™:

$$\underbrace{p(x, z)}_{\substack{\text{model} \\ p(z) \cdot p(x|z)}} \longrightarrow \underbrace{p(z \mid x)}_{\substack{\text{posterior} \\ \text{distribution}}}$$

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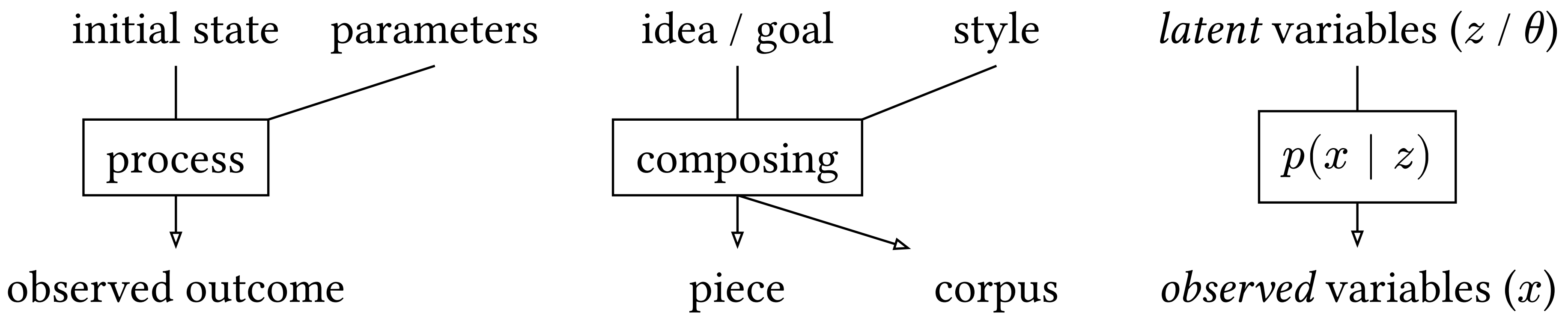

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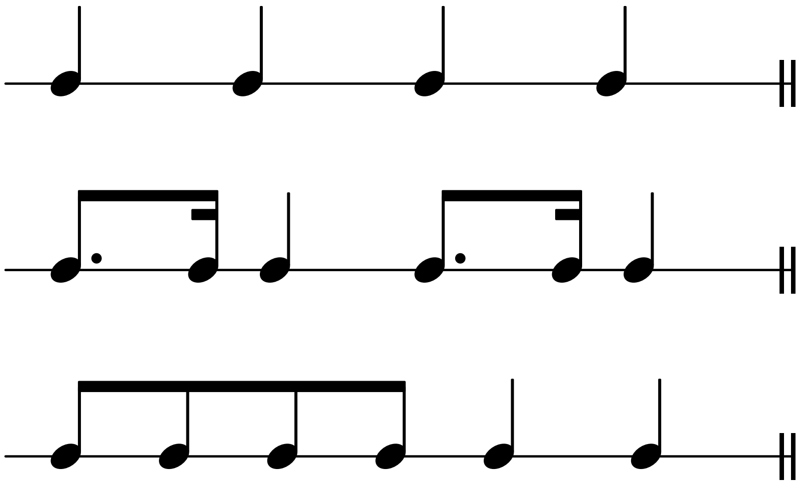
Back to our Corpus Study

Variables:

- regularity $r = ?$
- corpus C



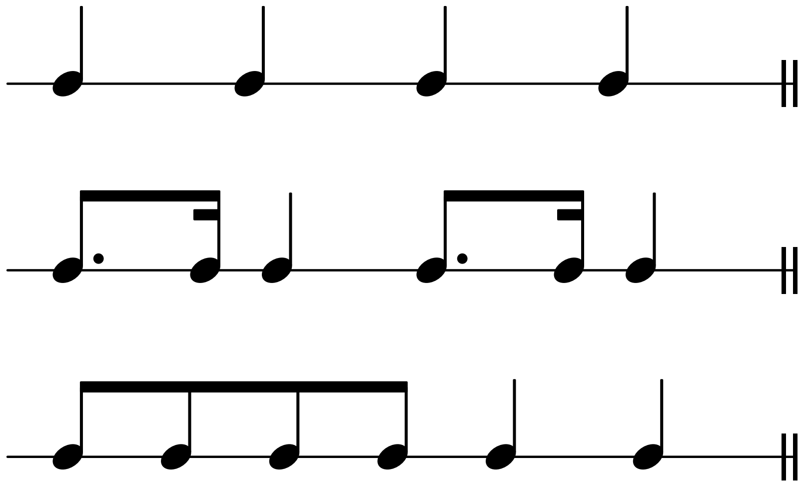
Back to our Corpus Study

Variables:

- regularity $r = ?$
- corpus C

Model:

- choose r
- for each pattern:
  - beat 1 random
  - for beat 2-4:
    - flip coin (r):
      - heads: repeat
      - tails: don't repeat



Back to our Corpus Study

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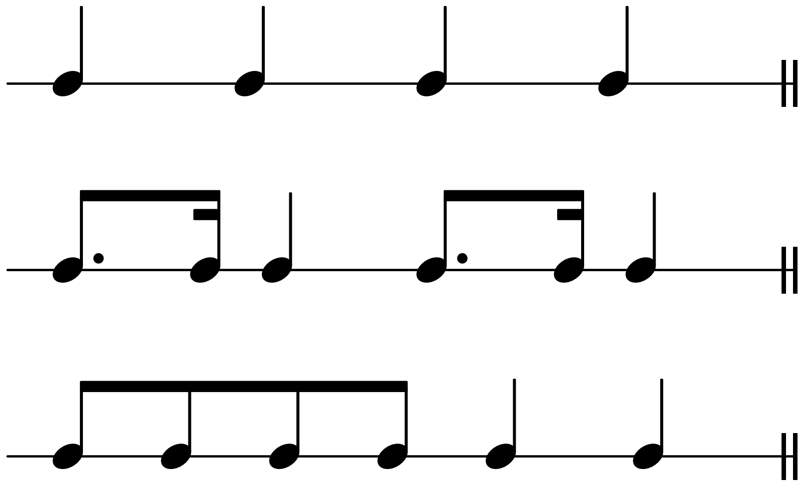

Back to our Corpus Study

Variables:

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Model:

- choose $r \sim$ unknown distribution
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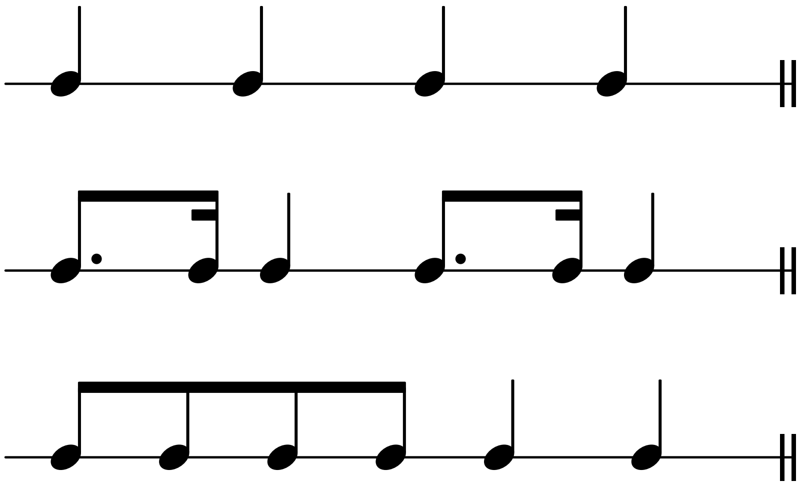

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Variables:

- regularity $r = ?$
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Model:

- choose $r \sim$ unknown distribution
- for each pattern:
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  - for beat 2-4:
    - flip coin (r): $\sim \text{Bernoulli}(r)$
      - heads: repeat
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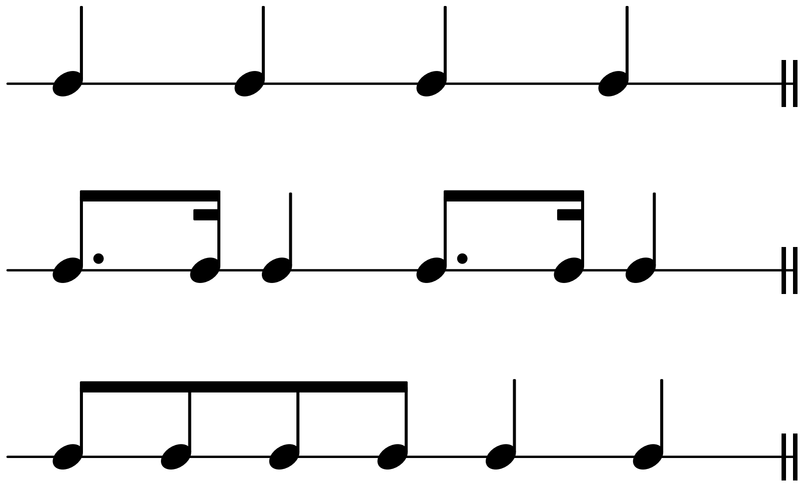

Looking at the Likelihood

The probability of the data depends on r:

$$p(C \mid r) = p([*\mathtt{rrr}, *\mathtt{nnn}, *\mathtt{rnr}] \mid r) = r^5 \cdot (1 - r)^4$$

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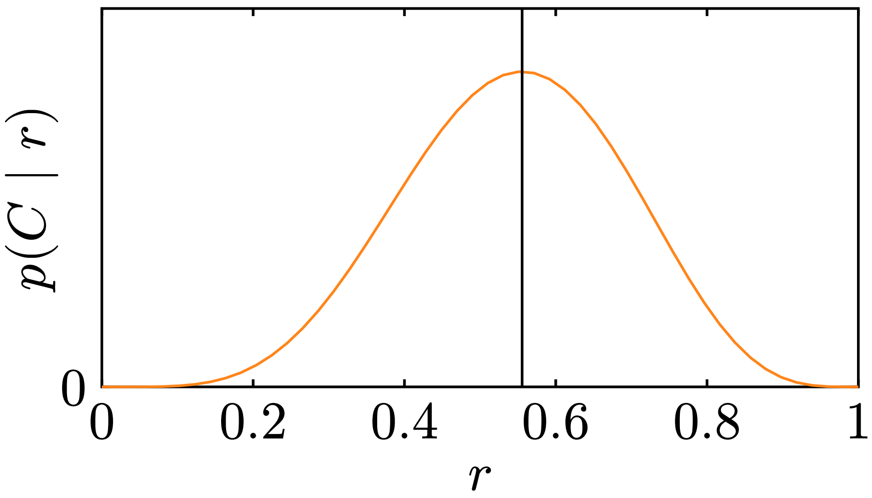

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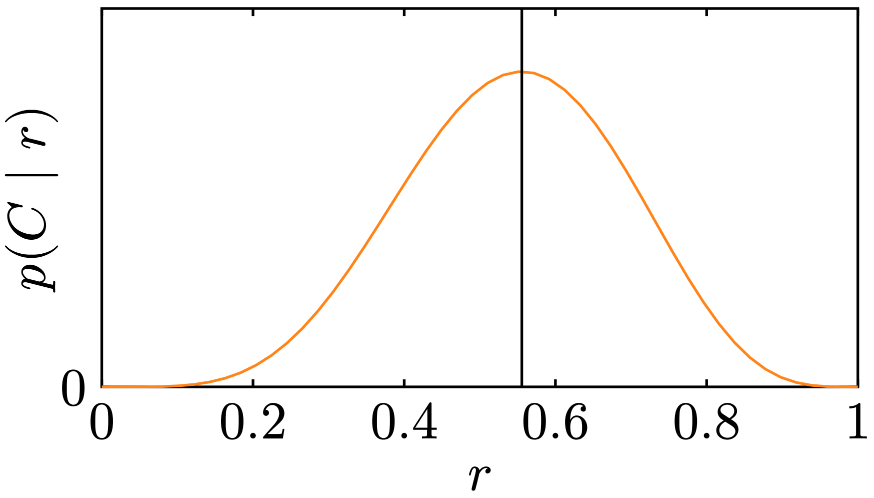

$$\arg\max_r p(C \mid r) = \frac{5}{9}$$

Make it Bayesian

$$\underbrace{p(r, C)}_{p(r)\cdot p(C|r)} \longrightarrow p(r \mid C)$$

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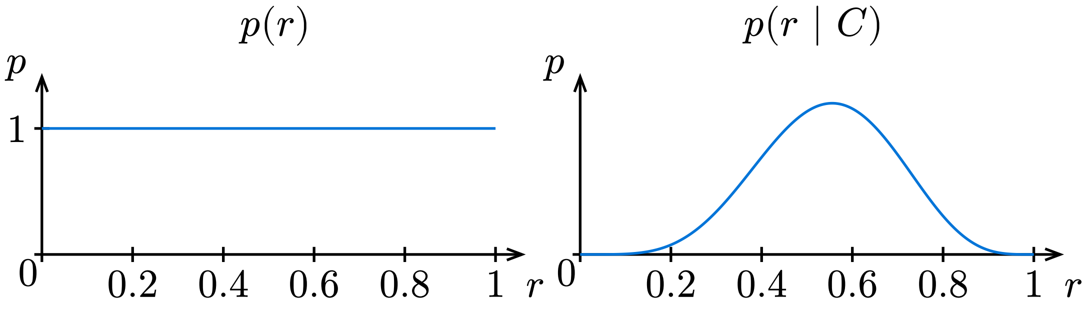
$$\underbrace{p(r, C)}_{p(r) \cdot p(C | r)} \longrightarrow p(r \mid C)$$

Model:

- choose $r \sim \text{Uniform}(0, 1)$
- for each pattern i:
  - choose $\vec{c}_i \sim 3 \times \text{Bernoulli}(r)$

Make it Bayesian



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Make it Bayesian

$$\underbrace{p(r, C)}_{p(r) \cdot p(C|r)} \longrightarrow p(r \mid C)$$

Model:

- choose $r \sim \text{Beta}(0.5, 0.5)$
- for each pattern i:
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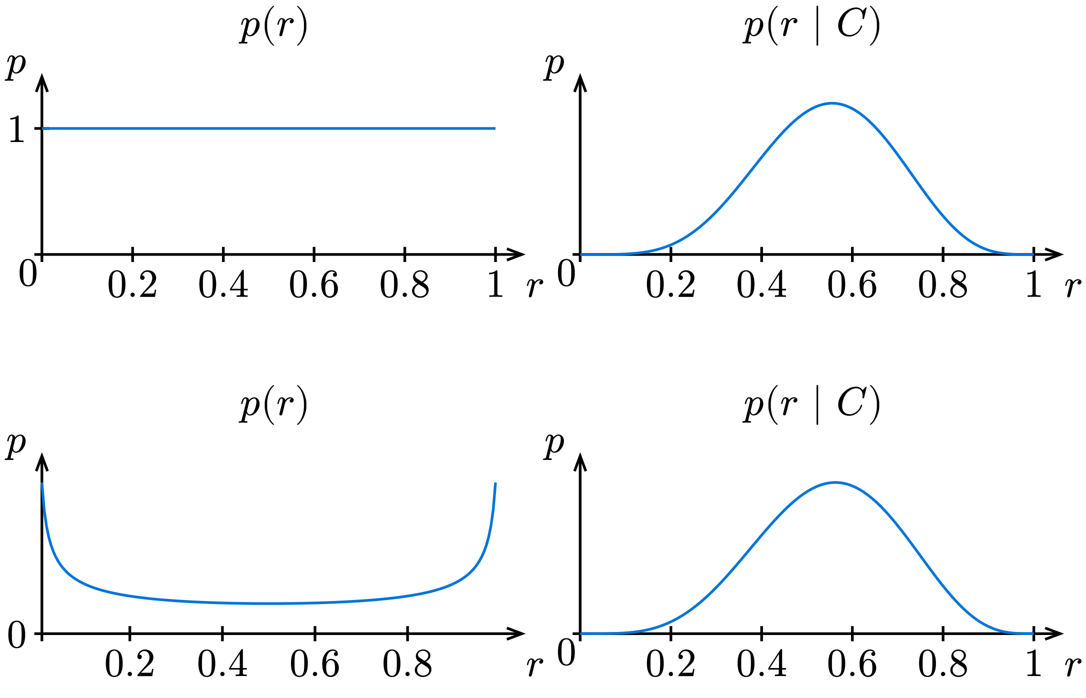

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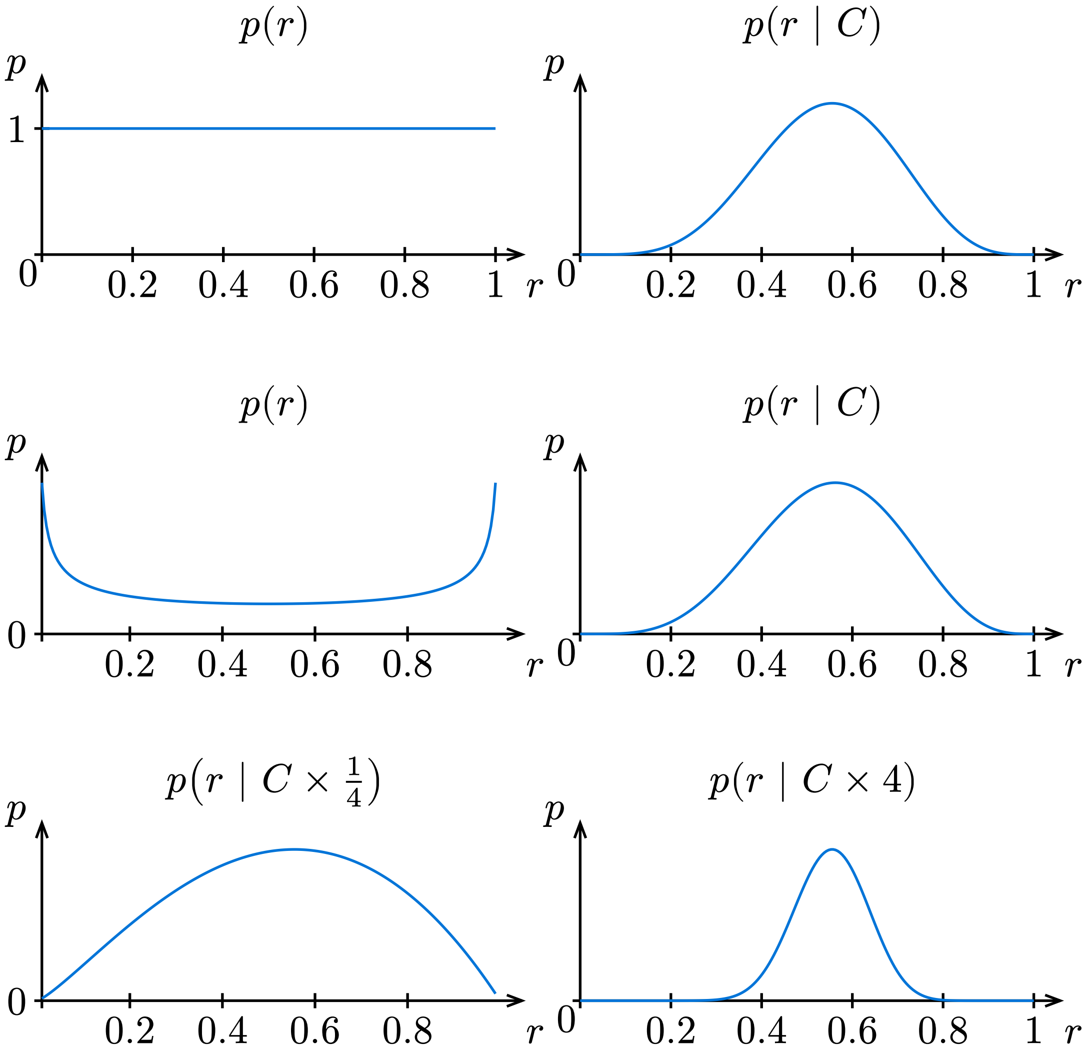

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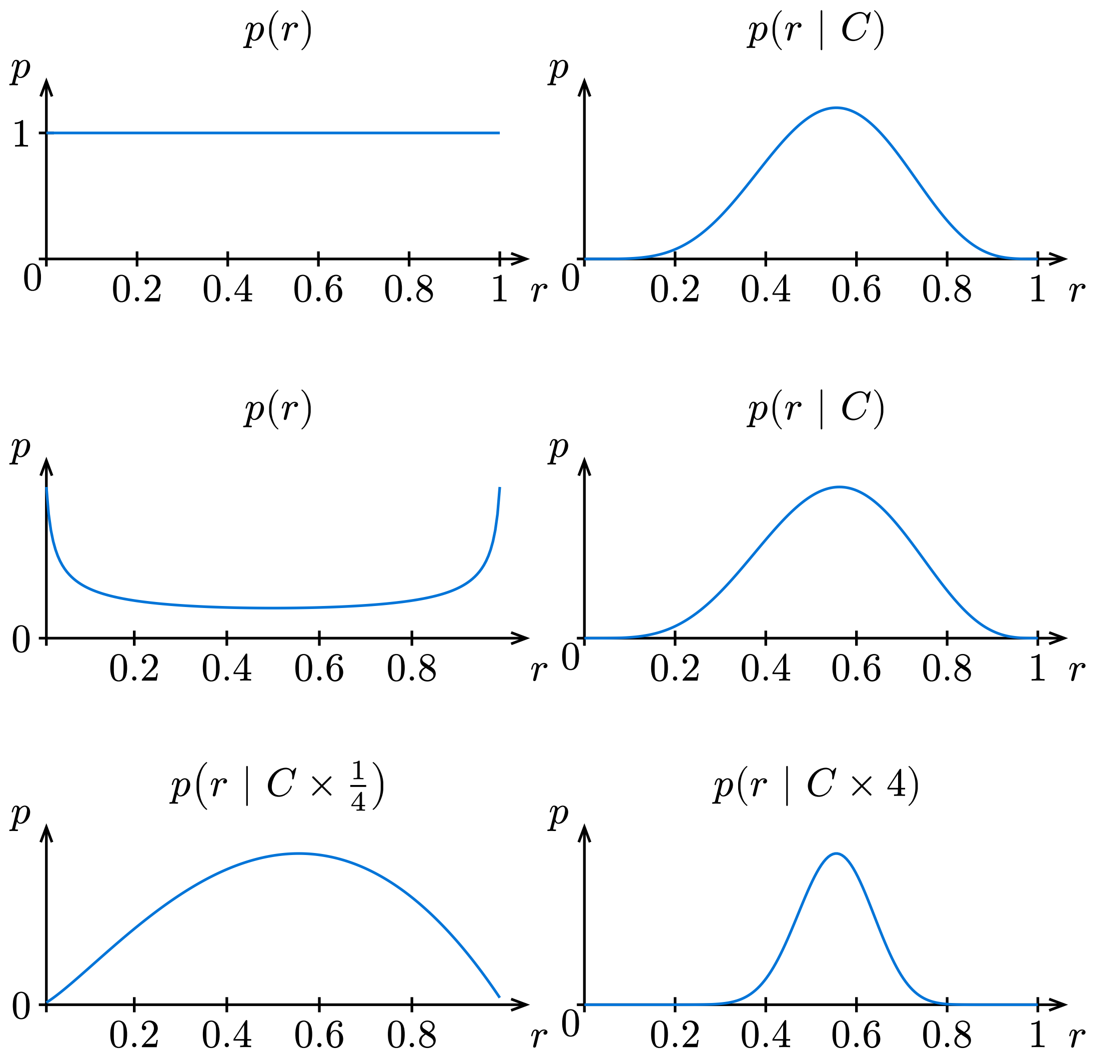
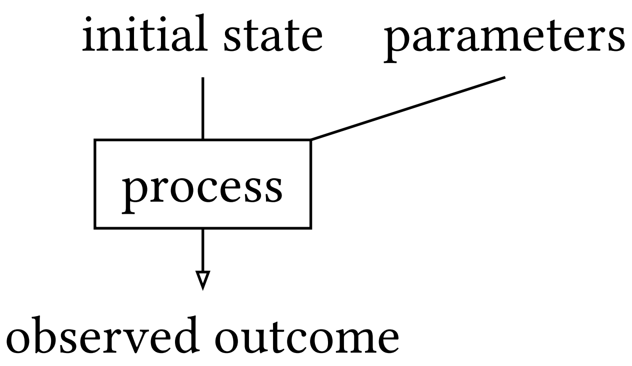
Problem:

How do we compute this?



Summary: The Three Ingredients



$$+ \qquad p(x, z) \qquad \longrightarrow \qquad p(z \mid x)$$

generative process (series of decisions)

decision probabilities

inference: conditioning

Practical Exercises

https://github.com/Amsterdam-Music-Lab/gmth23-bayes-workshop