

OPERATION RESEARCH

MC305 Lab

ANEESH PANCHAL

2K20/MC/021



Department of Applied
Mathematics,
Delhi Technological University

Submitted To –

Prof. Anjana Gupta
and Ms. Anjali

INDEX

S. No.	Experiment	Date	Sign & Remark
01.	Solving Linear Programming Problem using Simplex Method in Tora	08/Aug/2022	
02.	Solving Linear Programming Problem using Two Phase Method in Tora	22/Aug/2022	
03.	Solving Linear Programming Problem using Big M Method in Tora	29/Aug/2022	
04.	Solving Linear Programming Problem using Dual Simplex Method in Tora	05/Sept/2022	
05.	Solving Linear Programming Problem and applying Sensitivity Analysis using Tora	12/Sept/2022	
06.	Solving Integer Programming Problem (IPP) using Branch and Bound Method in Tora	19/Sept/2022	
07.	To solve Transportation Problem in Tora. Finding Initial Basic Feasible Solution (IBFS) using, 1. N-W Corner Method (NWC) 2. Least Call Method (LCM) 3. Vogel's Approx. Method (VAM)	26/Sept/2022	
08.	To solve Assignment Problem in Tora.	10/Oct/2022	
09.	To implement Critical Path Method (CPM) using Tora.	17/Oct/2022	
10.	To implement Project Evaluation & Review Technique (PERT) using Tora.	31/Oct/2022	

TORA Software

Link to download **TORA software (.rar)** files:

<https://www.mediafire.com/file/myz0i84sd13osp2/Tora+System.rar/file>

Link to download **WinRAR** (.rar file extractor):

<https://www.win-rar.com/start.html?&L=0>

Password for TORA software (.rar) files:

123

Procedure to download and start using TORA:

Download the TORA software and use the password 123 to extract the files using WinRAR. Install the setup after extraction is complete, then look for the TORA software shortcut. First and foremost, TORA will support 1024x768 resolutions. Therefore, adjust the screen's resolutions accordingly. Then launch the software and you're ready to solve the problem.

History:

In **2002**, TORA is created by **Hamdy A. Taha**. (University Professor Emeritus of Industrial Engineering with the University of Arkansas, where he taught and conducted research in operations research and simulation.)

About:

Temporary-Ordered Routing Algorithm (TORA) – An Operations Research Software TORA is an algorithm i.e. a mathematical set of instructions or programs (mathematical-software). Simply one can say that TORA is a **menu-driven optimization system**. It is an optimization system in the area of operations research which is very easy to use. Further, TORA is menu-driven and Windows-based which makes it very user friendly.

The software can be executed in **automated** or **tutorial mode**. The automated mode reports the final solution of the problem, usually in the standard format followed in commercial packages, while the tutorial mode keeps on giving step-wise information about the methodology and solution.

The software provides a number of **tutorial features**:

1. TORA allows both user-guided and automated use of the software
2. In User-guide option, steps of the algorithms are reproduced exactly as presented in the book
3. All the details needed to use an algorithm are given directly on the screen, thus precluding the need for a user's manual.

TORA software deals with the following algorithms:

1. Solution of simultaneous linear equations
2. Linear programming
3. Transportation model
4. Integer programming
5. Network models
6. Project analysis by CPM/PERT
7. Poisson queuing models
8. Zero-sum games

Limitation of TORA Software:

We can't use the TORA software in solving Linear Programming, Problem (LPP) when the matrix associated to the **initial basis is not an identity matrix**.

Experiment 1

Aim:

Solving Linear Programming Problem using Simplex Method in Tora

Problem 1:

$$\text{Max } z = 2x_1 + 4x_2$$

$$\text{s.t. } x_1 + 4x_2 \leq 5$$

$$x_1 + x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

Input:

	x1	x2	Enter <, >, or =	R.H.S.
Var. Name				
Maximize	2.00	4.00		
Constr 1	1.00	4.00	<=	5.00
Constr 2	1.00	1.00	<=	4.00
Lower Bound	0.00	0.00		
Upper Bound	infinity	infinity		
Unrestr'd (y/n)?	n	n		

Output:

Iteration 1					
Basic	x1	x2	sx3	sx4	Solution
z (max)	-2.00	-4.00	0.00	0.00	0.00
sx3	1.00	4.00	1.00	0.00	5.00
sx4	1.00	1.00	0.00	1.00	4.00
Lower Bound	0.00	0.00			
Upper Bound	infinity	infinity			
Unrestr'd (y/n)?	n	n			
Iteration 2					
Basic	x1	x2	sx3	sx4	Solution
z (max)	-1.00	0.00	1.00	0.00	5.00
x2	0.25	1.00	0.25	0.00	1.25
sx4	0.75	0.00	-0.25	1.00	2.75
Lower Bound	0.00	0.00			
Upper Bound	infinity	infinity			
Unrestr'd (y/n)?	n	n			
Iteration 3					
Basic	x1	x2	sx3	sx4	Solution
z (max)	0.00	0.00	0.67	1.33	8.67
x2	0.00	1.00	0.33	-0.33	0.33
x1	1.00	0.00	-0.33	1.33	3.67
Lower Bound	0.00	0.00			
Upper Bound	infinity	infinity			
Unrestr'd (y/n)?	n	n			

Conclusion:

Using Simplex Method, we get the solution of LPP as $x_1 = 3.67$ and $x_2 = 0.33$

Problem 2:

$$\text{Max } z = 3x_1 + 2x_2$$

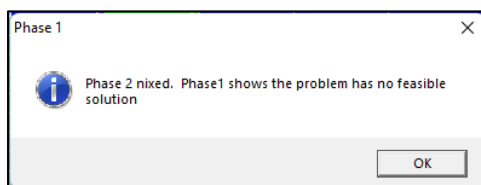
$$\text{s.t. } 2x_1 + x_2 \leq 2$$

$$3x_1 + 4x_2 \geq 12$$

$$x_1, x_2 \geq 0$$

Input:

	x1	x2	Enter <, >, or =	R.H.S.
Var. Name				
Maximize	3.00	2.00		
Constr 1	2.00	1.00	<=	2.00
Constr 2	3.00	4.00	>=	12.00
Lower Bound	0.00	0.00		
Upper Bound	infinity	infinity		
Unrestr'd (y/n)?	n	n		

Output:

Phase 1 (Iter 1)						
Basic	x1	x2	Sx3	sx4	Rx5	Solution
z (min)	3.00	4.00	-1.00	0.00	0.00	12.00
sx4	2.00	1.00	0.00	1.00	0.00	2.00
Rx5	3.00	4.00	-1.00	0.00	1.00	12.00
Lower Bound	0.00	0.00				
Upper Bound	infinity	infinity				
Unrestr'd (y/n)?	n	n				
Phase 1 (Iter 2)						
Basic	x1	x2	Sx3	sx4	Rx5	Solution
z (min)	-5.00	0.00	-1.00	-4.00	0.00	4.00
x2	2.00	1.00	0.00	1.00	0.00	2.00
Rx5	-5.00	0.00	-1.00	-4.00	1.00	4.00
Lower Bound	0.00	0.00				
Upper Bound	infinity	infinity				
Unrestr'd (y/n)?	n	n				

Conclusion:

We can't use simplex method here because we have \geq constraint. Hence, we use Two Phase method for this problem.

Using Two Phase Method, we get no feasible solution for the given LPP.

Problem 3:

$$\text{Max } z = 3x_1 + 2x_2$$

$$\text{s.t. } -x_1 + 2x_2 \leq 4$$

$$x_1 - x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Input:

	x1	x2	Enter <, >, or =	R.H.S.
Var. Name				
Maximize	3.00	2.00		
Constr 1	-1.00	2.00	<=	4.00
Constr 2	1.00	-1.00	<=	3.00
Lower Bound	0.00	0.00		
Upper Bound	infinity	infinity		
Unrestr'd (y/n)?	n	n		

Output:

Iteration 1					
Basic	x1	x2	sx3	sx4	Solution
z (max)	-3.00	-2.00	0.00	0.00	0.00
sx3	-1.00	2.00	1.00	0.00	4.00
sx4	1.00	-1.00	0.00	1.00	3.00
Lower Bound	0.00	0.00			
Upper Bound	infinity	infinity			
Unrestr'd (y/n)?	n	n			
Iteration 2					
Basic	x1	x2	sx3	sx4	Solution
z (max)	0.00	-5.00	0.00	3.00	9.00
sx3	0.00	1.00	1.00	1.00	7.00
x1	1.00	-1.00	0.00	1.00	3.00
Lower Bound	0.00	0.00			
Upper Bound	infinity	infinity			
Unrestr'd (y/n)?	n	n			
Iteration 3					
Basic	x1	x2	sx3	sx4	Solution
z (max)	0.00	0.00	5.00	8.00	44.00
x2	0.00	1.00	1.00	1.00	7.00
x1	1.00	0.00	1.00	2.00	10.00
Lower Bound	0.00	0.00			
Upper Bound	infinity	infinity			
Unrestr'd (y/n)?	n	n			

Conclusion:

Using Simplex Method, we get the solution of LPP as $x_1 = 10$ and $x_2 = 7$

Experiment 2

Aim:

Solving Linear Programming Problem using Two Phase Method in Tora

Problem:

$$\begin{aligned} \text{Min } z &= 2x - 3y + 6z \\ \text{s.t. } 3x - 4y - 6z &\leq 2 \\ 2x + y + 2z &\geq 11 \\ x + 3y - 2z &\leq 5 \\ x, y, z &\geq 0 \end{aligned}$$

First of all, standardize the given problem,

$$\begin{aligned} \text{Min } z &= 2x - 3y + 6z \\ \text{s.t. } 3x - 4y - 6z + s_1 &= 2 \\ 2x + y + 2z - s_2 &= 11 \\ x + 3y - 2z + s_3 &= 5 \\ x, y, z, s_1, s_2, s_3 &\geq 0 \end{aligned}$$

Let us take Initial Basic Feasible Solution for the given problem,

$$x = y = z = 0$$

Putting these values we get,

$$s_1 = 2, s_2 = -11 \text{ and } s_3 = 5$$

Which contradict the basic solution conditions that $s_1, s_2, s_3 \geq 0$

So, we can't use simplex method here. Hence, we use Two Phase method for this problem.

Input:

	x1	x2	x3	Enter <, >, or =	R.H.S.
Var. Name					
Minimize	2.00	-3.00	6.00		
Constr 1	3.00	-4.00	-6.00	<=	2.00
Constr 2	2.00	1.00	2.00	>=	11.00
Constr 3	1.00	3.00	-2.00	<=	5.00
Lower Bound	0.00	0.00	0.00		
Upper Bound	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n		

Output:

Phase 1 (Iter 1)								
Basic	x1	x2	x3	Sx4	sx5	Rx6	sx7	Solution
z (min)	2.00	1.00	2.00	-1.00	0.00	0.00	0.00	11.00
sx5	3.00	-4.00	-6.00	0.00	1.00	0.00	0.00	2.00
Rx6	2.00	1.00	2.00	-1.00	0.00	1.00	0.00	11.00
sx7	1.00	3.00	-2.00	0.00	0.00	0.00	1.00	5.00
Lower Bound	0.00	0.00	0.00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 1 (Iter 2)								
Basic	x1	x2	x3	Sx4	sx5	Rx6	sx7	Solution
z (min)	0.00	3.67	6.00	-1.00	-0.67	0.00	0.00	9.67
x1	1.00	-1.33	-2.00	0.00	0.33	0.00	0.00	0.67
Rx6	0.00	3.67	6.00	-1.00	-0.67	1.00	0.00	9.67
sx7	0.00	4.33	0.00	0.00	-0.33	0.00	1.00	4.33
Lower Bound	0.00	0.00	0.00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 1 (Iter 3)								
Basic	x1	x2	x3	Sx4	sx5	Rx6	sx7	Solution
z (min)	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00
x1	1.00	-0.11	0.00	-0.33	0.11	0.33	0.00	3.89
x3	0.00	0.61	1.00	-0.17	-0.11	0.17	0.00	1.61
sx7	0.00	4.33	0.00	0.00	-0.33	0.00	1.00	4.33
Lower Bound	0.00	0.00	0.00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 2 (Iter 4)								
Basic	x1	x2	x3	Sx4	sx5	Rx6	sx7	Solution
z (min)	0.00	6.44	0.00	-1.67	-0.44	blocked	0.00	17.44
x1	1.00	-0.11	0.00	-0.33	0.11	0.33	0.00	3.89
x3	0.00	0.61	1.00	-0.17	-0.11	0.17	0.00	1.61
sx7	0.00	4.33	0.00	0.00	-0.33	0.00	1.00	4.33
Lower Bound	0.00	0.00	0.00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 2 (Iter 5)								
Basic	x1	x2	x3	Sx4	sx5	Rx6	sx7	Solution
z (min)	0.00	0.00	0.00	-1.67	0.05	blocked	-1.49	11.00
x1	1.00	0.00	0.00	-0.33	0.10	0.33	0.00	4.00
x3	0.00	0.00	1.00	-0.17	-0.06	0.17	-0.14	1.00
x2	0.00	1.00	0.00	0.00	-0.00	0.00	0.23	1.00
Lower Bound	0.00	0.00	0.00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 2 (Iter 6)								
Basic	x1	x2	x3	Sx4	sx5	Rx6	sx7	Solution
z (min)	-0.50	0.00	0.00	-1.50	0.00	blocked	-1.50	9.00
sx5	9.75	0.00	0.00	-3.25	1.00	3.25	0.25	39.00
x3	0.63	0.00	1.00	-0.38	0.00	0.38	-0.13	3.50
x2	0.75	1.00	0.00	-0.25	0.00	0.25	0.25	4.00
Lower Bound	0.00	0.00	0.00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					

Conclusion:

Using 2 Phase Method, we get the solution of LPP as $x_2 = 4$ and $x_3 = 3.5$

Experiment 3

Aim:

Solving Linear Programming Problem using Big M Method in Tora

Problem:

$$\text{Max } z = x_1 - x_2 + 3x_3$$

$$\text{s.t. } x_1 + x_2 \leq 20$$

$$x_1 + x_3 = 5$$

$$x_2 + x_3 \geq 10$$

$$x_1, x_2, x_3 \geq 0$$

First of all, standardize the given problem,

$$\text{Max } z = x_1 - x_2 + 3x_3$$

$$\text{s.t. } x_1 + x_2 + s_1 = 20$$

$$x_1 + x_3 = 5$$

$$x_2 + x_3 - s_2 = 10$$

$$x_1, x_2, x_3, s_1, s_2 \geq 0$$

Let us take Initial Basic Feasible Solution for the given problem,

$$x_1 = x_2 = x_3 = 0$$

Putting these values we get,

$$s_1 = 20 \text{ and } s_2 = -10$$

Which contradict the basic solution conditions that $s_1, s_2 \geq 0$

So, we can't use simplex method here. Hence, we use Big M method for this problem.

Input:

	x1	x2	x3	Enter <, >, or =	R.H.S.
Var. Name					
Maximize	1.00	-1.00	3.00		
Constr 1	1.00	1.00	0.00	<=	20.00
Constr 2	1.00	0.00	1.00	=	5.00
Constr 3	0.00	1.00	1.00	>=	10.00
Lower Bound	0.00	0.00	0.00		
Upper Bound	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n		

Output:

Iteration 1								
Basic	x1	x2	x3	Sx4	sx5	Rx6	Rx7	Solution
z (max)	-101.00	-99.00	-203.00	100.00	0.00	0.00	0.00	-1500.00
sx5	1.00	1.00	0.00	0.00	1.00	0.00	0.00	20.00
Rx6	1.00	0.00	1.00	0.00	0.00	1.00	0.00	5.00
Rx7	0.00	1.00	1.00	-1.00	0.00	0.00	1.00	10.00
Lower Bound	0.00	0.00	0.00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Iteration 2								
Basic	x1	x2	x3	Sx4	sx5	Rx6	Rx7	Solution
z (max)	102.00	-99.00	0.00	100.00	0.00	203.00	0.00	-485.00
sx5	1.00	1.00	0.00	0.00	1.00	0.00	0.00	20.00
x3	1.00	0.00	1.00	0.00	0.00	1.00	0.00	5.00
Rx7	-1.00	1.00	0.00	-1.00	0.00	1.00	1.00	5.00
Lower Bound	0.00	0.00	0.00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Iteration 3								
Basic	x1	x2	x3	Sx4	sx5	Rx6	Rx7	Solution
z (max)	3.00	0.00	0.00	1.00	0.00	104.00	99.00	10.00
sx5	2.00	0.00	0.00	1.00	1.00	1.00	-1.00	15.00
x3	1.00	0.00	1.00	0.00	0.00	1.00	0.00	5.00
x2	-1.00	1.00	0.00	-1.00	0.00	-1.00	1.00	5.00
Lower Bound	0.00	0.00	0.00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					

Conclusion:

Using Big M Method, we get the solution of LPP as $x_2 = 5$ and $x_3 = 5$

Experiment 4

Aim:

Solving Linear Programming Problem using Dual Simplex Method in Tora

Problem:

$$\begin{aligned} \text{Min } z &= 2x_1 + 2x_2 + 4x_3 \\ \text{s.t. } 2x_1 + 3x_2 + 5x_3 &\geq 2 \\ 3x_1 + x_2 + 7x_3 &\leq 3 \\ x_1 + 4x_2 + 6x_3 &\leq 5 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Input:

	x1	x2	x3	Enter <, >, or =	R.H.S.
Var. Name					
Minimize	2.00	2.00	4.00		
Constr 1	2.00	3.00	5.00	>=	2.00
Constr 2	3.00	1.00	7.00	<=	3.00
Constr 3	1.00	4.00	6.00	<=	5.00
Lower Bound	0.00	0.00	0.00		
Upper Bound	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n		

Output:

Iteration 1							
Basic	x1	x2	x3	Sx4	Sx5	Sx6	Solution
z (min)	-2.00	-2.00	-4.00	0.00	0.00	0.00	0.00
Sx4	-2.00	-3.00	-5.00	1.00	0.00	0.00	-2.00
Sx5	3.00	1.00	7.00	0.00	1.00	0.00	3.00
Sx6	1.00	4.00	6.00	0.00	0.00	1.00	5.00
Lower Bound	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n				
Iteration 2							
Basic	x1	x2	x3	Sx4	Sx5	Sx6	Solution
z (min)	-0.67	0.00	-0.67	-0.67	0.00	0.00	1.33
x2	0.67	1.00	1.67	-0.33	0.00	0.00	0.67
Sx5	2.33	0.00	5.33	0.33	1.00	0.00	2.33
Sx6	-1.67	0.00	-0.67	1.33	0.00	1.00	2.33
Lower Bound	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n				

Conclusion:

Using Dual Simplex Method, we get the solution of LPP as $x_2 = 0.67$

Experiment 5

Aim:

Solving Linear Programming Problem and applying Sensitivity Analysis using Tora

Problem:

$$\text{Max } z = 50x_1 + 60x_2 + 55x_3$$

$$\text{s.t. } x_1 + x_2 + x_3 \leq 100$$

$$2x_1 + 3x_2 + 2x_3 \leq 300$$

$$2x_1 + 3x_2 + 2x_3 \geq 250$$

$$x_1 + 2x_3 \geq 60$$

$$x_1, x_2, x_3 \geq 0$$

Input:

	x1	x2	x3	Enter <, >, or =	R.H.S.
Var. Name					
Maximize	50.00	60.00	55.00		
Constr 1	1.00	1.00	1.00	<=	100.00
Constr 2	2.00	3.00	2.00	<=	300.00
Constr 3	2.00	3.00	2.00	>=	250.00
Constr 4	1.00	0.00	2.00	>=	60.00
Lower Bound	0.00	0.00	0.00		
Upper Bound	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n		

Output:

Variable	Value	Obj Coeff	Obj Val Contrib	
x1:	0.00	50.00	0.00	
x2:	70.00	60.00	4200.00	
x3:	30.00	55.00	1650.00	
Constraint	RHS	Slack-/Surplus+		
1 (<=)	100.00	0.00		
2 (<=)	300.00	30.00-		
3 (>=)	250.00	20.00+		
4 (>=)	60.00	0.00		
*** Sensitivity Analysis***				
Variable	Current Obj Coeff	Min Obj Coeff	Max Obj Coeff	Reduced Cost
x1:	50.00	-infinity	57.50	7.50
x2:	60.00	55.00	infinity	0.00
x3:	55.00	40.00	60.00	0.00
Constraint	Current RHS	Min RHS	Max RHS	Dual Price
1 (<=)	100.00	93.33	110.00	60.00
2 (<=)	300.00	270.00	infinity	0.00
3 (>=)	250.00	-infinity	270.00	0.00
4 (>=)	60.00	0.00	100.00	-2.50

Q1. State the optimal Solution.

Soln.

Optimal Solution is $x_2 = 70$ and $x_3 = 30$

Q2. What is the optimal Objective Function value ?

Soln.

Optimal Objective Function Value is $z_{max} = 5850$

Q3. What would happen to the Objective Function if,

- RHS of constraint 1 increases by 5
- RHS of constraint 4 changes to 44

Soln.

- Min obj. coeff. for constraint 1 is 93.33 and Max obj. coeff. for constraint 1 is 110
But the Objective Value will increase by 300 as dual price is 60 for constraint 1
New Objective Function Value = $5850 + 300 = 6150$

Variable	Value	Obj Coeff	Obj Val Contrib	
x1:	0.00	50.00	0.00	
x2:	75.00	60.00	4500.00	
x3:	30.00	55.00	1650.00	
Constraint	RHS	Slack-/Surplus+		
1 (<=)	105.00	0.00		
2 (<=)	300.00	15.00-		
3 (>=)	250.00	35.00+		
4 (>=)	60.00	0.00		
Sensitivity Analysis				
Variable	Current Obj Coeff	Min Obj Coeff	Max Obj Coeff	Reduced Cost
x1:	50.00	-infinity	57.50	7.50
x2:	60.00	55.00	infinity	0.00
x3:	55.00	40.00	60.00	0.00
Constraint	Current RHS	Min RHS	Max RHS	Dual Price
1 (<=)	105.00	93.33	110.00	60.00
2 (<=)	300.00	285.00	infinity	0.00
3 (>=)	250.00	-infinity	285.00	0.00
4 (>=)	60.00	30.00	130.00	-2.50

- Min obj. coeff. for constraint 4 is 0 and Max obj. coeff. for constraint 4 is 0
But the Objective Value will increase by 40 as dual price is -2.5 for constraint 4
New Objective Function Value = $5850 + 40 = 5890$

Variable	Value	Obj Coeff	Obj Val Contrib	
x1:	0.00	50.00	0.00	
x2:	78.00	60.00	4680.00	
x3:	22.00	55.00	1210.00	
Constraint	RHS	Slack-/Surplus+		
1 (<=)	100.00	0.00		
2 (<=)	300.00	22.00-		
3 (>=)	250.00	28.00+		
4 (>=)	44.00	0.00		
Sensitivity Analysis				
Variable	Current Obj Coeff	Min Obj Coeff	Max Obj Coeff	Reduced Cost
x1:	50.00	-infinity	57.50	7.50
x2:	60.00	55.00	infinity	0.00
x3:	55.00	40.00	60.00	0.00
Constraint	Current RHS	Min RHS	Max RHS	Dual Price
1 (<=)	100.00	90.67	107.33	60.00
2 (<=)	300.00	278.00	infinity	0.00
3 (>=)	250.00	-infinity	278.00	0.00
4 (>=)	44.00	0.00	100.00	-2.50

Conclusion:

Constraint 1 will not affect the optimal solution if the objective coefficient will lie in $[93.33, 110]$ with dual price of 60.
 Constraint 4 will not affect the optimal solution if the objective coefficient will lie in $[0, 100]$ with dual price of -2.5.
 Constraint 2 will not affect the optimal solution and objective function value if the objective coefficient will lie in $[270, \infty)$.
 Constraint 3 will not affect the optimal solution and objective function value if the objective coefficient will lie in $(-\infty, 270]$.

Experiment 6

Aim:

Solving Integer Programming Problem (IPP) using Branch and Bound Method in Tora.

Problem:

$$\text{Max } z = 7x_1 + 9x_2$$

$$\text{s.t. } -x_1 + 3x_2 \leq 6$$

$$7x_1 + x_2 \leq 35$$

$$x_1, x_2 \geq 0 \text{ and integer}$$

Input:

	x1	x2	Enter <, >, or =	R.H.S.
Var. Name				
Maximize	7.00	9.00		
Constr 1	-1.00	3.00	<=	6.00
Constr 2	7.00	1.00	<=	35.00
Lower Bound	0.00	0.00		
Upper Bound	infinity	infinity		
Unrestr'd (y/n)?	n	n		
Integer (y/n)?	y	y		

Output:

Title: IPP
(Current) Best Objective Value (Max) =55
Found at Iteration 3
Optimality verified at Iteration 5

FEASIBLE SOLUTIONS (in improved order)

Subproblem	ObjVal, z	x1	x2
3	55	4	3

B&B Search completed

Conclusion:

Using Branch and Bound Method,

we get the solution of Integer Programming Problem (IPP) as $x_1 = 4$ and $x_2 = 3$

Experiment 7

Aim:

To solve Transportation Problem in Tora.

Finding Initial Basic Feasible Solution (IBFS) using,

1. North-West Corner Cell Method (NWC)
2. Least Call Cell Method (LCM)
3. Vogel's Approximation Method (VAM)

Problem:

	d_1	d_2	d_3	d_4	Supply
O_1	6	14	21	23	12
O_2	26	12	18	16	17
O_3	14	20	21	14	20
O_4	9	11	18	16	11
Demand	14	18	20	8	

Input:

		D1	D2	D3	D4	Supply
	S/D Name					
S1	O1	6.00	14.00	21.00	23.00	12
S2	O2	26.00	12.00	18.00	16.00	17
S3	O3	14.00	20.00	21.00	14.00	20
S4	O4	9.00	11.00	18.00	16.00	11
Demand		14	18	20	8	

Output:

1. North-West Corner Cell Method (NWC)

Initialize u or v

u1=0

Next Iteration

All Iterations

Write to Printer

Iter 1	ObjVal =	903.00	D1	D2	D3	D4	Supply
	Name						
			v1=6.00	v2=-8.00	v3=-7.00	v4=-9.00	
S1	O1	u1=0.00	6.00	14.00	21.00	23.00	12
			12				
			0.00	-22.00	-28.00	-32.00	
S2	O2	u2=20.00	26.00	12.00	18.00	16.00	17
			2	15			
			0.00	0.00	-5.00	-5.00	
S3	O3	u3=28.00	14.00	20.00	21.00	14.00	20
				3	17		
			20.00	0.00	0.00	5.00	
S4	O4	u4=25.00	9.00	11.00	18.00	16.00	11
					3	8	
			22.00	6.00	0.00	0.00	
	Demand		14	18	20	8	

2. Least Call Cell Method (LCM)

<div>Initialize u or v</div> <div>u1=0</div> <div>Next Iteration All Iterations Write to Printer</div>							
Iter 1	ObjVal =	805.00	D1	D2	D3	D4	Supply
	Name						
			v1=6.00	v2=8.00	v3=14.00	v4=7.00	
S1	01	u1=0.00	6.00	14.00	21.00	23.00	12
			12				
			0.00	-6.00	-7.00	-16.00	
S2	02	u2=4.00	26.00	12.00	18.00	16.00	17
				9	8		
			-16.00	0.00	0.00	-5.00	
S3	03	u3=7.00	14.00	20.00	21.00	14.00	20
					12	8	
			-1.00	-5.00	0.00	0.00	
S4	04	u4=3.00	9.00	11.00	18.00	16.00	11
			2	9			
			0.00	0.00	-1.00	-6.00	
	Demand		14	18	20	8	

3. Vogel's Approximation Method (VAM)

<div>Initialize u or v</div> <div>u1=0</div> <div>Next Iteration All Iterations Write to Printer</div>							
Iter 1	ObjVal =	805.00	D1	D2	D3	D4	Supply
	Name						
			v1=6.00	v2=8.00	v3=14.00	v4=7.00	
S1	01	u1=0.00	6.00	14.00	21.00	23.00	12
			12				
			0.00	-6.00	-7.00	-16.00	
S2	02	u2=4.00	26.00	12.00	18.00	16.00	17
				9	8		
			-16.00	0.00	0.00	-5.00	
S3	03	u3=7.00	14.00	20.00	21.00	14.00	20
					12	8	
			-1.00	-5.00	0.00	0.00	
S4	04	u4=3.00	9.00	11.00	18.00	16.00	11
			2	9			
			0.00	0.00	-1.00	-6.00	
	Demand		14	18	20	8	

Conclusion:

The Initial Basic Feasible solution for the given Transportation problem is given in the Tableau above.
The Optimum Objective Value of given Transportation Problem using,

1. North-West Corner Cell Method (NWC) is **903**
2. Least Call Cell Method (LCM) is **805**
3. Vogel's Approximation Method (VAM) is **805**

Experiment 8

Aim:

To solve Assignment Problem in Tora.

Problem:

Workers	Job A	Job B	Job C	Job D
1	10	25	15	20
2	15	30	12	15
3	35	20	5	25
4	17	25	24	20

Input:

		D1	D2	D3	D4	Supply
	S/D Name	A	B	C	D	
S1	1	10.00	25.00	15.00	20.00	1
S2	2	15.00	30.00	12.00	15.00	1
S3	3	35.00	20.00	5.00	25.00	1
S4	4	17.00	25.00	24.00	20.00	1
Demand		1	1	1	1	

Output:

Iter 1	ObjVal =	55.00	D1	D2	D3	D4	Supply
	Name		A	B	C	D	
			v1=10.00	v2=25.00	v3=7.00	v4=10.00	
S1	1	u1=0.00	10.00	25.00	15.00	20.00	1
			0.00	0.00	-8.00	-10.00	
S2	2	u2=5.00	15.00	30.00	12.00	15.00	1
			0.00	0.00	0.00	0.00	
S3	3	u3=-2.00	35.00	20.00	5.00	25.00	1
			-27.00	3.00	0.00	-17.00	
S4	4	u4=0.00	17.00	25.00	24.00	20.00	1
			-7.00	0.00	-17.00	-10.00	
Demand			1	1	1	1	

Iter 2	ObjVal =	55.00	D1	D2	D3	D4	Supply
	Name		A	B	C	D	
			v1=10.00	v2=22.00	v3=7.00	v4=10.00	
S1	1	u1=0.00	10.00	25.00	15.00	20.00	1
			0.00	-3.00	-8.00	-10.00	
S2	2	u2=5.00	15.00	30.00	12.00	15.00	1
			0.00	-3.00	0.00	0.00	
S3	3	u3=-2.00	35.00	20.00	5.00	25.00	1
			-27.00	0.00	0.00	-17.00	
S4	4	u4=3.00	17.00	25.00	24.00	20.00	1
			-4.00	0.00	-14.00	-7.00	
Demand			1	1	1	1	

Conclusion:

The Optimum Objective Value of given Assignment problem is **55**.

Assignment of Jobs: Job A to 1, Job B to 4, Job C to 3 and Job D to 2.

Experiment 9

Aim:

To implement Critical Path Method (CPM) using Tora.

Problem:

Consider the following table summarizing the details of a project,

Activity	Precedence	Duration
A	-	3
B	A	5
C	A	4
D	B	6
E	B, C	7
F	D, E	4

Input:

Row	From Node	To Node	Activity Symbol	Duration
1	1	2	A	3.00
2	2	3	B	5.00
3	2	4	C	4.00
4	3	5	D	6.00
5	3	6	E	7.00
6	4	6	E	7.00
7	5	7	F	4.00
8	6	7	F	4.00

Output:

Forward Pass			Backward Pass		
Step	Node	Earliest Time	Step	Node	Latest Time
1	1	0.00	8	7	19.00
2	2	3.00	9	5	15.00
3	3	8.00	10	6	15.00
4	4	7.00	11	3	8.00
5	5	14.00	12	4	8.00
6	6	15.00	13	2	3.00
7	7	19.00	14	1	0.00
Forward pass completed			Backward pass completed		
Activity	Duration	Earliest Start	Latest Completion	Total Float	Free Float
A	3.00	0.00	3.00	0.00	0.00
B	5.00	3.00	8.00	0.00	0.00
C	4.00	3.00	8.00	1.00	0.00
D	6.00	8.00	15.00	1.00	0.00
E	7.00	8.00	15.00	0.00	0.00
E	7.00	7.00	15.00	1.00	1.00
F	4.00	14.00	19.00	1.00	1.00
F	4.00	15.00	19.00	0.00	0.00
Critical activities highlighted in red					

Conclusion:

The Critical activities for project completion are **Activity A, B, E & F**

The minimum duration for the completion of the project is **19 units**.

Experiment 10

Aim:

To implement Project Evaluation & Review Technique (PERT) using Tora.

Problem:

The following table shows the jobs of a network along with their time estimation,

Activity	Activity Symbol	Optimistic Time (a)	Most Likely Time (m)	Pessimistic Time (b)
1-2	A	3	5	7
1-3	B	4	6	8
2-3	C	1	3	5
2-4	D	5	8	11
3-5	E	1	2	3
3-6	F	9	11	13
4-5	Dummy	0	0	0
4-6	G	1	1	1
5-6	H	10	12	14

Input:

Row	From Node	To Node	Activity Symbol	a	m	b
1	1	2	A	3.00	5.00	7.00
2	1	3	B	4.00	6.00	8.00
3	2	3	C	1.00	3.00	5.00
4	2	4	D	5.00	8.00	11.00
5	3	5	E	1.00	2.00	3.00
6	3	6	F	9.00	11.00	13.00
7	4	5	Dummy	0.00	0.00	0.00
8	4	6	G	1.00	1.00	1.00
9	5	6	H	10.00	12.00	14.00

Output:

Node	Longest Path Based on Mean Durations	Mean Duration	Std. Deviation
2	1- 2	5.00	0.67
3	1- 2- 3	8.00	0.94
4	1- 2- 4	13.00	1.20
5	1- 2- 4- 5	13.00	1.20
6	1- 2- 4- 5- 6	25.00	1.37

Conclusion:

The Critical activities for project completion are **Activity A, D, Dummy & H**

The Expected duration for the completion of the project is **25 units**.