STOCHASTIC PROCESSES

MC303 Lab

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S. No.	Experiment	Date	Sign & Remark
08.	Demonstrating Renewal Process. Write a program to find the expected waiting time until the nth renewal in case of a renewal process with renewal cycle length distributed 1. Normally with mean μ and standard deviation σ, ($\mu > 3\sigma$) 2. Exponentially with parameter λ	01/Nov/2022	
09.	Demonstrating Markov Chain. Write a program to find the n-step transition probability in case of a Markov Chain.	01/Nov/2022	

Experiment 1

Aim:
Demonstrating a Stochastic Process with Discrete Index Set

(a) with Discrete State Space

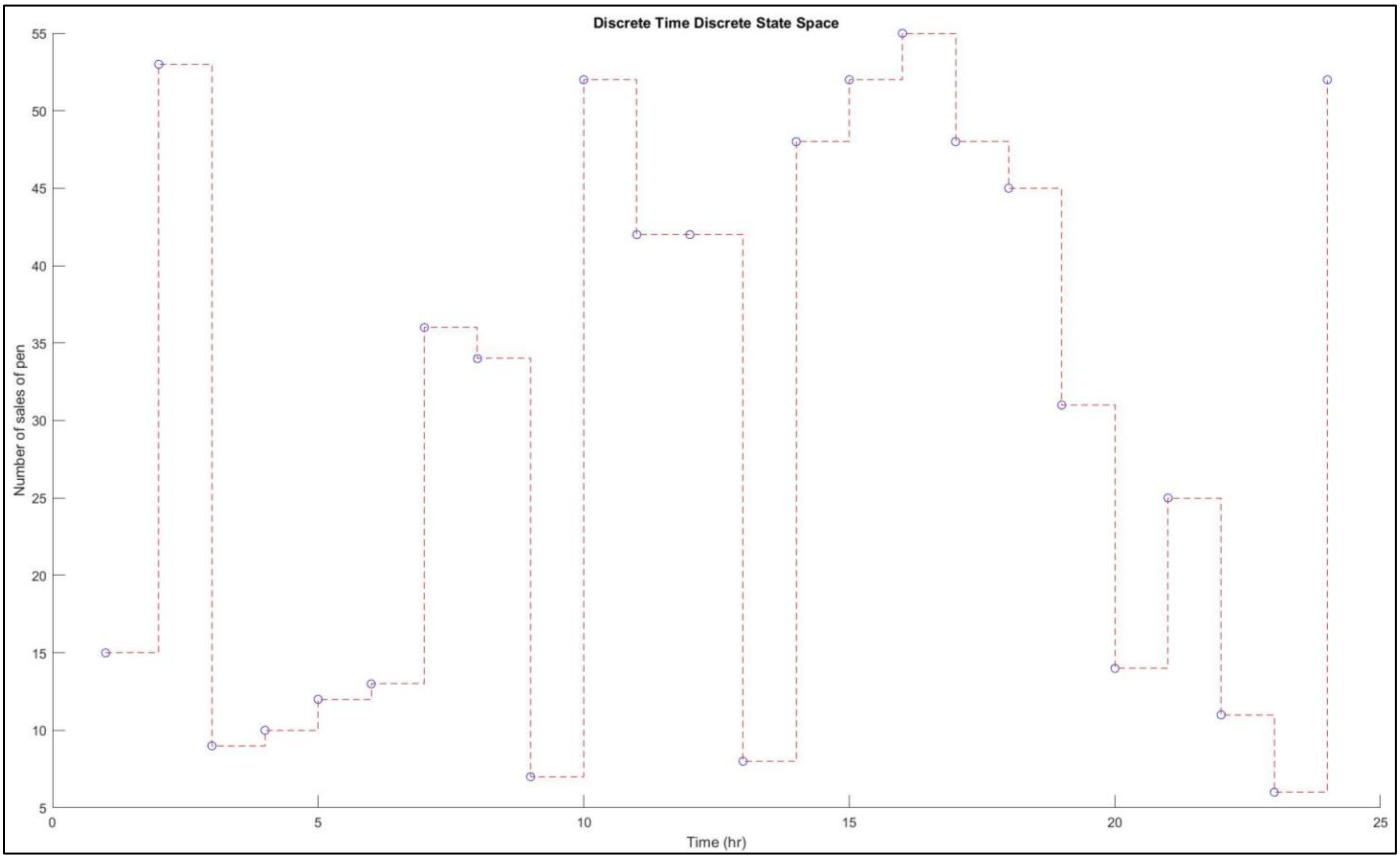
(b) with Continuous State Space

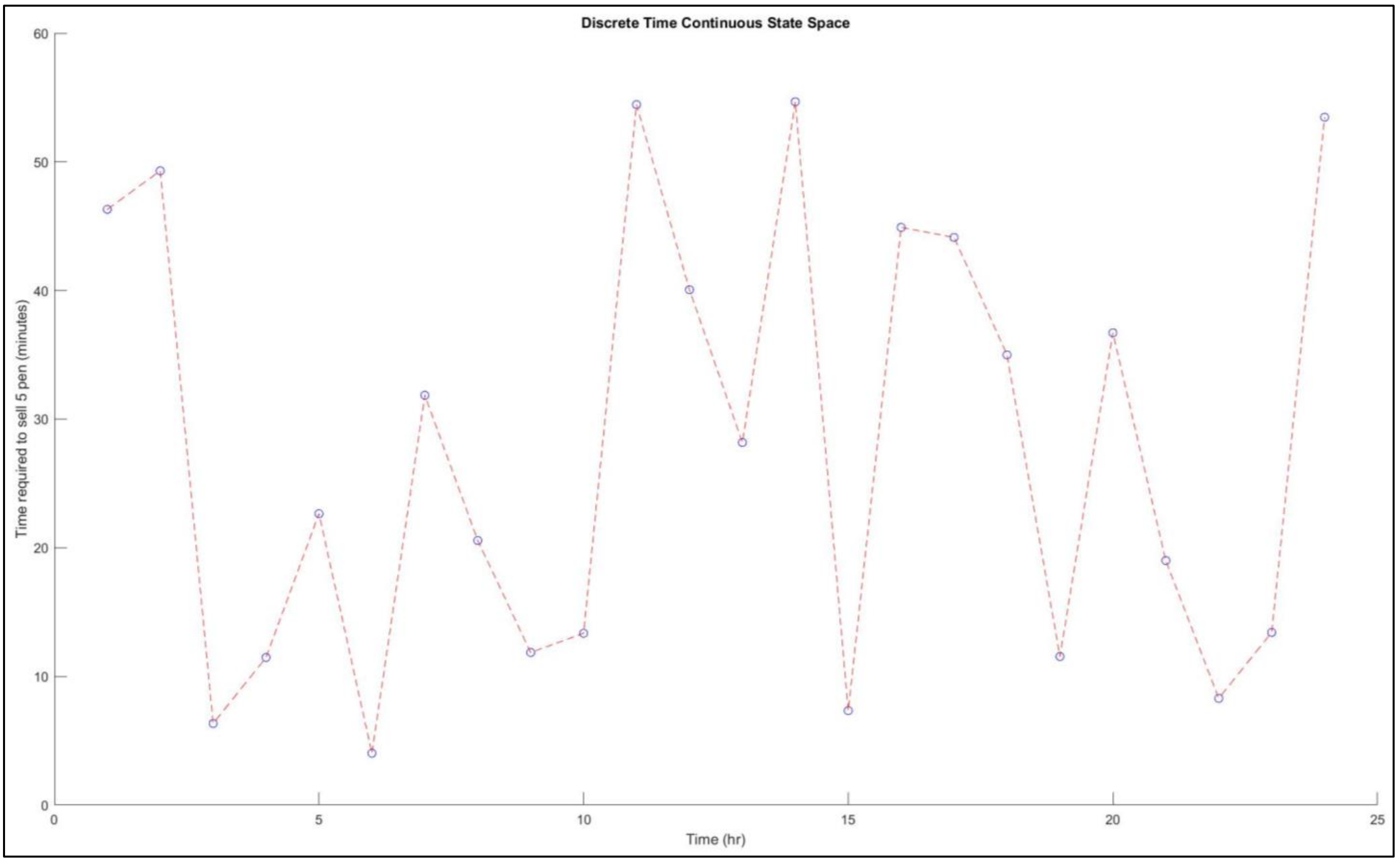
Theory:
A stochastic process is a family of random variables {X(t), t ∈ T} defined on a given probability space, indexed by parameter t, where t varies over an index set T. The values assumed by X(t) are called its states, and the set of all possible values forms the state space of the process. Stochastic processes are classified on the basis of the underlying index set T and state space S. If T = {0, 1, 2, …}, or T = {0, ±1, ±2,..}, the stochastic process is said to be discrete parameter process and is usually indicated by {Xn}.The state space is classified as discrete if it is finite or countable and process is classified as continuous if it consists of an interval, finite or infinite of the real line.

Examples:
1. *Discrete Time Discrete State Space –*
   Number of sales of pen from a shop in a day
2. *Discrete Time Continuous State Space –*
   Time required to sell 5 pens in each hour of day

Code:
```
%Number of hours in a day
x = [1:1:24]';

%Discrete Time Discrete State Space
%Lower Bound = 5, Upper Bound = 55
y = 5 + randi([0 50],24,1);
figure
scatter(x,y,'bo');
hold on
stairs(x,y,'r--');
xlabel('Time (hr)');
ylabel('Number of sales of pen');
title('Discrete Time Discrete State Space');

%Discrete Time Continuous State Space
%Lower Bound = 1, Upper Bound = 60
y = rand(24,1) + randi([1 59],24,1);
figure
scatter(x,y,'bo');
hold on
plot(x,y,'r--');
xlabel('Time (hr)');
ylabel('Time required to sell 5 pen (minutes)');
title('Discrete Time Continuous State Space');
```

Output:


Discrete Time Discrete State Space
Number of sales of pen
Time (hr)


Discrete Time Continuous State Space
Time required to sell 5 pen (minutes)
Time (hr)

Experiment 2

Aim:

Demonstrating a Stochastic Process with Continuous Index Set

(a) with Discrete State Space

(b) with Continuous State Space

Theory:

Stochastic processes are also often called random processes, random functions or simply processes. Depending on the choice of the index set T, we distinguish between the following types of stochastic processes:

If T consists of just one element (called, say, 1), then a stochastic process reduces to just one random variable X: Ω → R. So, the concept of a stochastic process includes the concept of a random variable as a special case. If T = {1, ..., n} is a finite set with n elements, then a stochastic process reduces to a collection of n random variables X1, ..., Xn defined on a common probability space. Such collection is called a random vector. So, the concept of a stochastic process includes the concept of a random vector as a special case. Stochastic processes with index set T = N, T = Z (or any other countable set) are called stochastic processes with discrete time. Stochastic processes with index set T = R, T = [a, b] (or other similar uncountable sets) are called stochastic processes with continuous time.

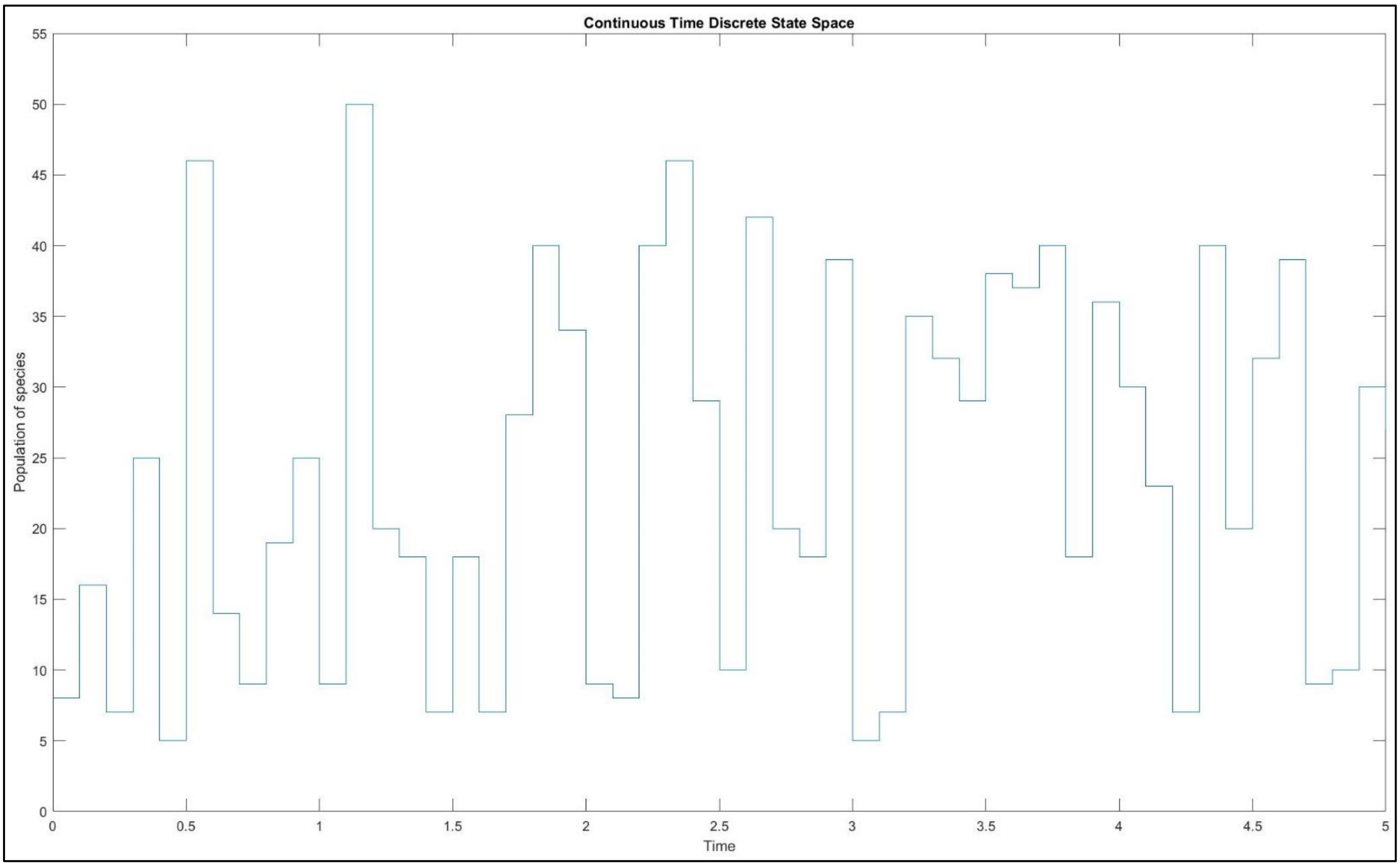
Examples:

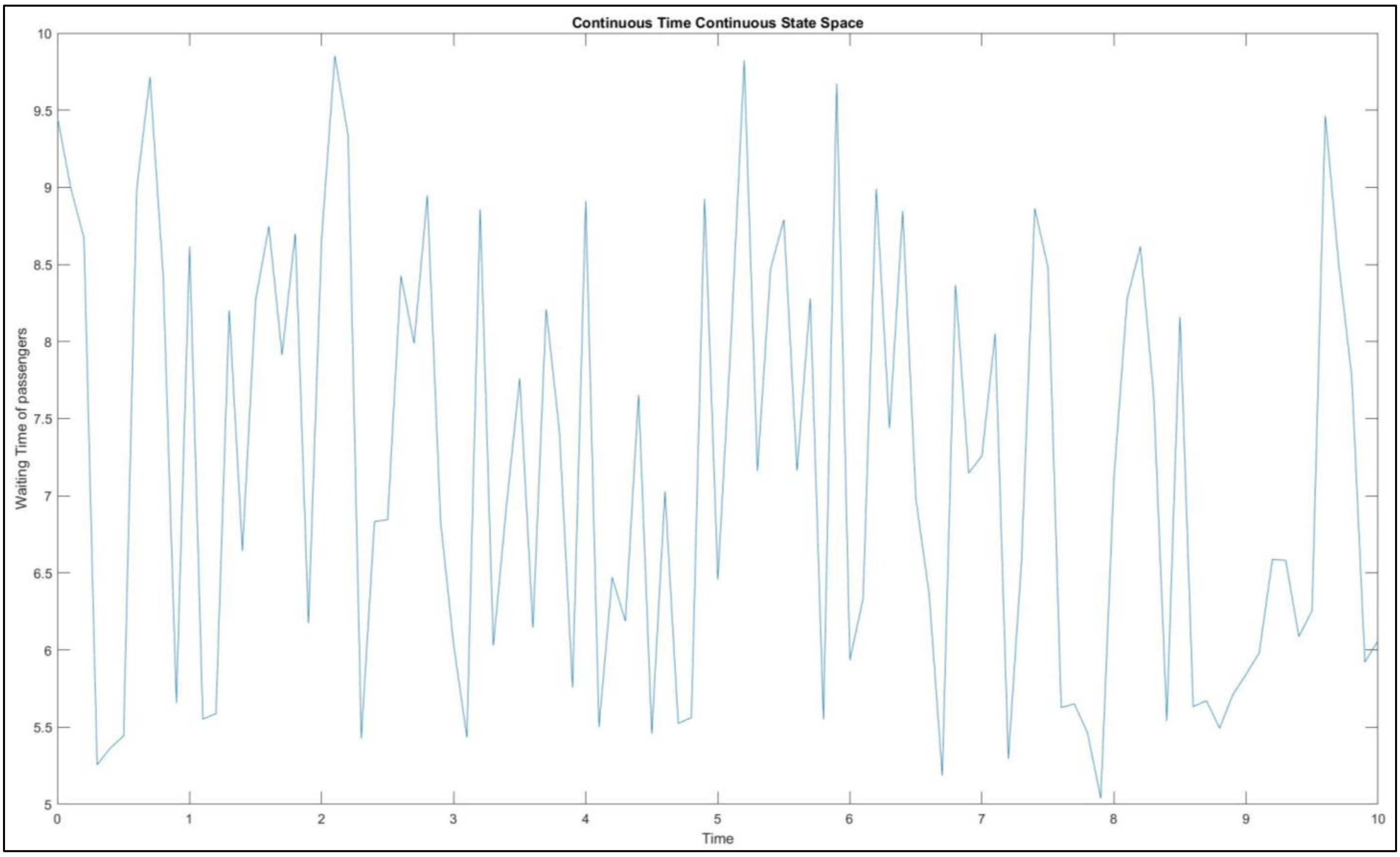
1. *Continuous Time Discrete State Space –*
   Population of a species at any specific point of time
2. *Continuous Time Continuous State Space –*
   Waiting time for the train of passengers arriving (rate 5) at a platform

Code:

```
%Continuous Time Discrete State Space
% Lower Bound = 5, Upper Bound = 55
x = [0:0.1:5];
y = randi([5 50],length(x),1);
figure
stairs(x,y);
ylim([0 55]);
title("Continuous Time Discrete State Space");
xlabel("Time");
ylabel("Population of species");

%Continuous Time Continuous State Space
% Lower Bound = 5, Upper Bound = 10
x = [0:0.1:10];
y = 5 + 5*rand(length(x),1);
figure
plot(x,y);
title("Continuous Time Continuous State Space");
xlabel("Time");
ylabel("Waiting Time of passengers");
```

Output:


Continuous Time Discrete State Space
Population of species
Time


Continuous Time Continuous State Space
Waiting Time of passengers
Time

Experiment 3

Aim:

Demonstrating Bernoulli process. Write a program to find the probability that in case a Bernoulli Process

(a) out of n trials k are successes

(b) kth success occurs at the nth trial

Theory:

A binomial distribution gives us the probabilities associated with independent, repeated Bernoulli trials. In a binomial distribution the probabilities of interest are those of receiving a certain number of successes, k, in n independent trials each having only two possible outcomes and the same probability, p of success.

Probability of getting k success out of n trials,

Prob_k = $\binom{n}{k} p^k (1-p)^{n-k}$

Probability of getting kth success at nth trial,

Probability of getting kth success at nth trial = Probability of getting (k-1) successes in (n-1) trials and getting success at nth trial.

Prob_kth = $\binom{n-1}{k-1} p^{k-1}(1-p)^{(n-1)-(k-1)}p = \binom{n-1}{k-1} p^k (1-p)^{n-k}$

Question:

Find the probability of getting 5 tails out of 10 trials, and probability of getting 5th tail at 10th trial.

Solution:

Probability of getting tail (success), p = 0.5

Number of trials, n = 10

Number of Successes, k = 5

Probability of getting 5 tails out of 10 trials = $\binom{10}{5} 0.5^5(1-0.5)^5 = \boldsymbol{0.246}$

Probability of getting 5th tail at 10th trial = $\binom{9}{4} 0.5^5(1-0.5)^5 = \boldsymbol{0.123}$

Code:

```
clc;
clear;

%Bernoulli Process inputs
n = input("Trials: ");
k = input("Successes: ");
p = input("Probability of Success: ");
q = 1-p;

%Out of n trials k are successes
m = n;
c = 1;
for i=1:k
    c = c*(m/i);
    m = m-1;
end

prob_k = vpa(c*(p^(k))*(q^(n-k)))
```

```
%kth success occur at nth trial
m = n-1;
c = 1;
for i=1:k-1
    c = c*(m/i);
    m = m-1;
end
prob_kth = vpa(c*(p^(k-1))*(q^(n-k))*p)
```

Output:

```
>> Exp3
Trials: 10
Successes: 5
Probability of Success: 0.5

prob_k =

0.24609375


prob_kth =

0.123046875

>>
```

Experiment 4

Aim:
Demonstrating Poisson process. Write a program to find the probability that in case of Poisson process with arrival rate λ, in a length of time t there are exactly k arrivals.

Theory:
The Poisson process is used in scenarios where we are counting the occurrences of certain events that appear to happen at a certain rate, but completely at random. Let λ>0 be fixed. The counting process {N(t), t∈ [0, ∞)} is called a Poisson process with rates λ if all the following conditions hold:

1) N (0) = 0
2) N(t) has independent increments;
3) the number of arrivals in any interval of length τ > 0 has Poisson($\lambda\tau$) distribution.

Question:
Assume that a circuit has an IC whose time to failure is an exponentially distributed random variable with expected lifetime of 3 months. If there are 10 spare IC's and time from failure to replacement is zero. What is the probability that the circuit can be kept operational for atleast 1 year ?

Solution:
Given values of number of trials, arrival rate and time is given as $n = 10, \lambda = \frac{1}{3}, t = 12$

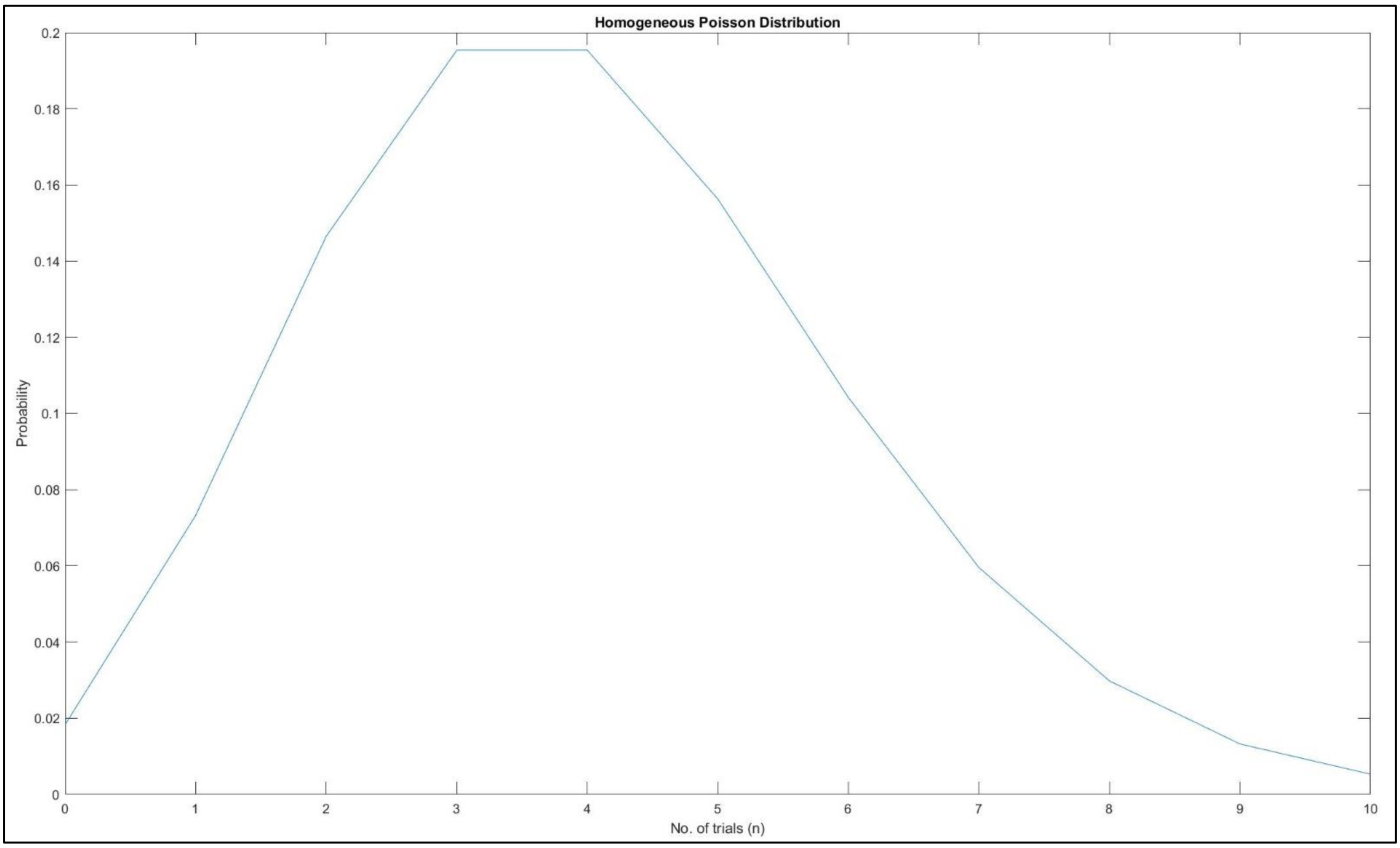
Probability circuit kept operational for atleast 1 year with 10 replacements, $P_{10} = e^{-\lambda t}\frac{(\lambda t)^{10}}{10!} = \mathbf{0.00529}$

Probability that circuit can be kept operational for atleast 1 year, $P_{n\leq 10} = \sum_{i=0}^{10} e^{-\lambda t}\frac{(\lambda t)^{i}}{i!} = \mathbf{0.99716}$

Code:
```
syms f(n)
lambda = input("Arrival Rate: ");
m = input("No. of Trials: ");
time = input("Time: ");

f(n) = (exp(-lambda*time)*((lambda*time)^n))/factorial(n);
probs(1) = vpa(f(0));
prob(1) = vpa(f(0));
for i=0:m-1
    probs(i+2) = probs(i+1) + vpa(f(i+1));
    prob(i+2) = vpa(f(i+1));
end

prob_k_Arrivals = prob(m+1)
prob_atmost_k_Arrivals = probs(m+1)
plot(0:m,prob);

title("Homogeneous Poisson Distribution");
xlabel("No. of trials (n)");
ylabel("Probability");
```

Output:

```
>> Exp4
Arrival Rate: 1/3
No. of Trials: 10
Time: 12

prob_k_Arrivals =

0.0052924766764201196350466880480869


prob_atmost_k_Arrivals =

0.99716023387948616847515029639603

>>
```



Experiment 5

Aim:

Demonstrating Poisson process (Non-Homogeneous). Write a program to find the probability that in case of Poisson process with arrival rate λ(t), in a length of time t there are exactly k arrivals.

Theory:

Let λ(t): [0, ∞) ↦ [0, ∞) be an integrable function. The counting process {N(t), t ∈ [0, ∞)} is called a nonhomogeneous Poisson process with rate λ(t) if all the following conditions hold.

1) N(0) = 0;
2) N(t) has independent increments;
3) for any t∈ [0, ∞), we have
   $P(N(t+\Delta) - N(t) = 0) = 1 - \lambda(t)\Delta + o(\Delta)$
   $P(N(t+\Delta) - N(t) = 1) = \lambda(t)\Delta + o(\Delta)$
   $P(N(t+\Delta) - N(t) \geq 2) = o(\Delta)$

More Specifically,

$N(t+s) - N(t) \sim Poisson(\int_t^{t+s} \lambda(\alpha)d\alpha)$

Question:

Find the probability of arrivals of (i) exact (ii) less than 10 cars in a filling station in 12 hours. Assume that arrivals of car follows Poisson distribution with arrival function λ(t) = 1/(1 + t) per hour.

Solution:

Given values of number of trials, arrival rate and time is given as $n = 10, \lambda(t) = \int_0^{12} \frac{1}{1+t} dt$

Probability of exact 10 cars arrival, $P_{10} = e^{-\lambda(t)} \frac{(\lambda(t))^{10}}{10!} = \mathbf{0.000261}$

Probability of atmost 10 cars arrival, $P_{n\leq 10} = \sum_{i=0}^{10} e^{-\lambda(t)} \frac{(\lambda(t))^i}{i!} = \mathbf{0.999923}$

Code:

```
%Non Homogeneous Poisson Process
syms t ip(t) g(n) lambda_func(t)
ip = input("Arrival Rate function: ");
m = input("No. of Trials: ");
time = input("Time: ");

lambda_func(t) = int(ip,t,0,t);
g(n) = subs((exp(-lambda_func)*((lambda_func)^n))/factorial(n),t,time);
probs(1) = vpa(g(0));
prob(1) = vpa(g(0));
for i=0:m-1
    probs(i+2) = probs(i+1) + vpa(g(i+1));
    prob(i+2) = vpa(g(i+1));
end

prob_k_Arrivals = prob(m+1)
prob_atmost_k_Arrivals = probs(m+1)
plot(0:m,prob);
```

```
title("Non Homogeneous Poisson Distribution");
xlabel("No. of trials (n)");
ylabel("Probability");
```

Output:

>> Exp5

Arrival Rate function: 1/(1+t)

No. of Trials: 10

Time: 12

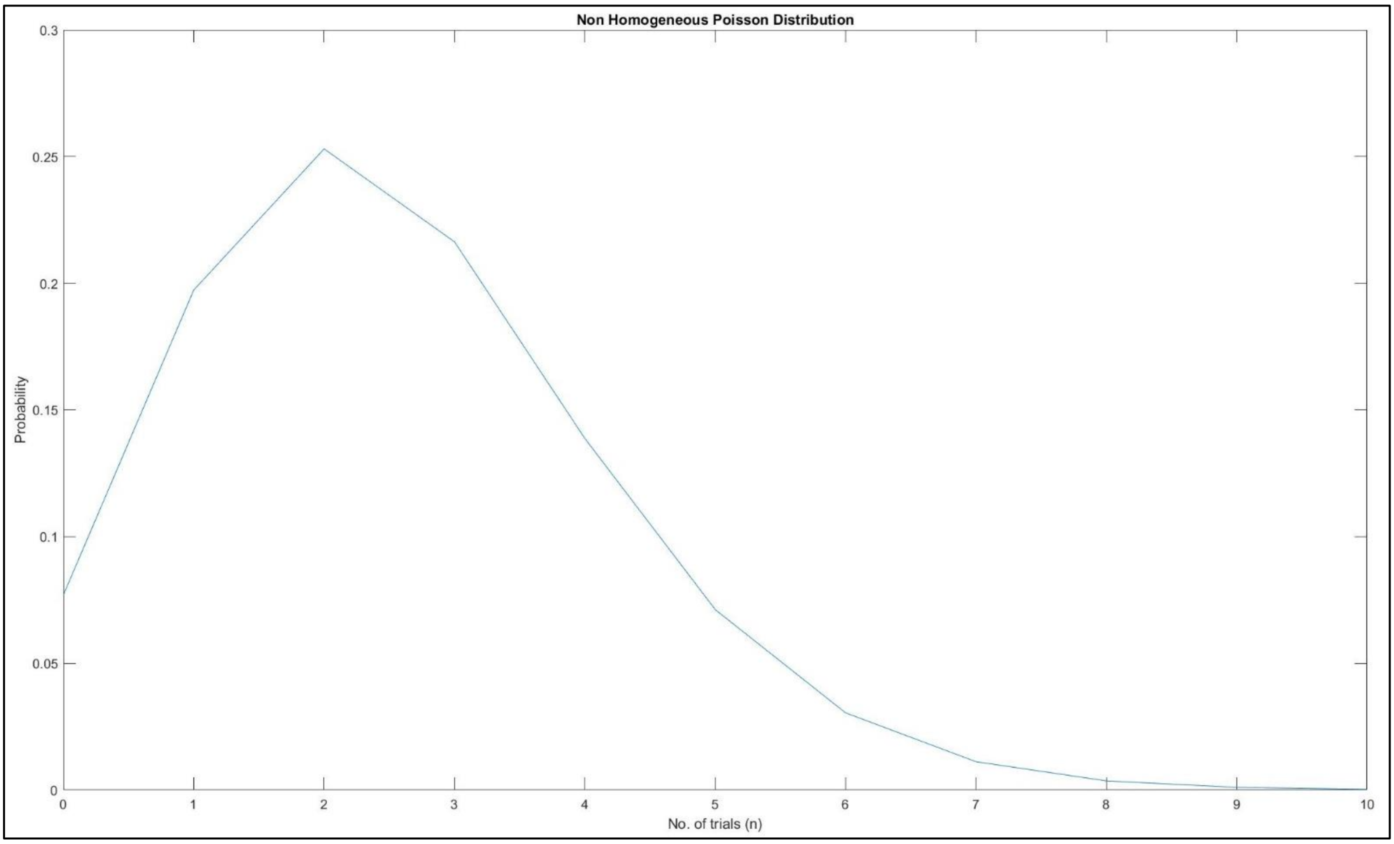
prob_k_Arrivals =

0.00026126519778019330469554958889 06

prob_atmost_k_Arrivals =

0.99992292169895653911625992713596

>>



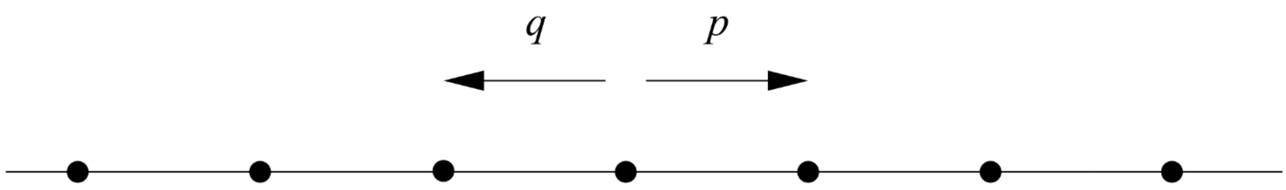
Experiment 6

Aim:
Demonstrating Simple Random Walk. Write a program to find the probability that in case of an unrestricted random walk, at the nth instant particle lies between two specified limits. Find the probability by taking suitable values of the parameters p, q, n, j and k.

Theory:
A random walk is a stochastic sequence $\{S_n\}$, with $S_O = 0$, defined by

$$S_n = \sum_{k=1}^{n} X_k$$

where $\{X_k\}$ are independent and identically distributed random variables $(i.i.d.)$. The random walk is simple if $X_k = 0\ or, \pm 1$, with $P(X_k = 1) = p$, $P(X_k = -1) = q$ and $P(X_k = 0) = 1 - p - q$.
Imagine a particle performing a random walk on the integer points of the real line, where it in each step moves to one of its neighboring points or, stay on its initial position.



Question:
Find probability that a drunken man (which can take only +1, 0, -1 m step at a time) will stay between 15 m left to the bar and 20 m right to the bar at his 100th step. Assume probability of taking +1 step be 0.4, probability of taking -1 step be 0.6 and he can't stay on his position at any step.

Solution:
Probability of taking +1 step, p = 0.4

Probability of taking -1 step, q = 0.6

Lower Limit, j = -15

Upper Limit, k = 20

No. of steps, n = 100

Probability = ***0.3415***

Code:
```
%Unrestricted Random Walk
p = input("Probability of +1 step: ");
q = input("Probability of -1 step: ");
j = input("Lower Limit: ");
k = input("Upper Limit: ");
n = input("No. of Instances/ Steps: ");
if p+q == 1
    c = 1;
else
    c = 0.5;
end
```

```
mean = p-q;
std_dev = sqrt(p+q-(p-q)^2);
up_lim = (k+c-n*mean)/(std_dev*sqrt(n));
low_lim = (j-c-n*mean)/(std_dev*sqrt(n));
probability = normcdf(up_lim) - normcdf(low_lim)
```

Output:

```
>> Exp6
Probability of +1 step: 0.4
Probability of -1 step: 0.6
Lower Limit: -15
Upper Limit: 20
No. of Instances/ Steps: 100

probability =

   0.3415

>>
```

Experiment 7

Aim:

Demonstrating Simple Random Walk. Write a program to find the probability that in case of

1. An unrestricted random walk the particle is at kth position at time n using CLT.
2. A random walk with 2 absorbing barriers, the probability of absorption at a specific barrier.
3. A random walk with 2 reflecting barriers, steady state probability distribution for the possible states.

Theory:

A random walk is a stochastic sequence S_n, with $S_O = 0$, defined by

$$S_n = \sum_{k=1}^{n} X_k$$

where X_k are independent and identically distributed random variables ($i.i.d.$). The random walk is simple if $X_k = 0\ or, \pm 1$, with $P(X_k = 1) = p$, $P(X_k = -1) = q$ and $P(X_k = 0) = 1 - p - q$.

Imagine a particle performing a random walk on the integer points of the real line, where it in each step moves to one of its neighboring points or, stay on its initial position.

1. Unrestricted Random Walk:
   When there is no boundary/ restrictions for the particle to stop at any point in (-∞,∞)
2. Random Walk with 2 Absorbing Barriers:
   Absorbing barriers are those barriers in which when the particle enters then it will stay at that position of the absorbing barrier only.
3. Random Walk with 2 Reflecting Barriers:
   Reflecting barriers are those barriers in which when the particle enters it can bounce back but can't go further in the direction of movement.

Examples:

1. ***Unrestricted Random Walk –***
   Drunk man walks with probability of taking step forward as 0.4 and taking step backward as 0.6 on an infinite road. Find the probability that drunk man will be at position +2 on 10^{th} step ?
   Solution: *0.1132*

2. ***Random Walk with 2 Absorbing Barriers –***
   Gambler's Ruin Problem
   2 players A & B play a game where if A wins a round he get 1 unit of wealth from B and vice versa with limited amount of capital for both. A player wins the game if opponent is ruined. Wealth of A is 20, wealth of B is 5, probability of A winning the round is 0.4 and probability of B winning the round is 0.5. Find the probabilities that,
   a) A wins the game or, B is ruined
   b) B wins the game

   Solution:
   a) Probability that B is ruined = ***0.9922***
   b) Probability that A is ruined = ***0.0078***

3. ***Random Walk with 2 Reflecting Barriers –***
   2 players A & B play a game where if A wins the round he get 1 unit of wealth from B until he reaches it max capacity after which he can only lose the wealth but can't win and vice versa with limited amount of capital for both. Wealth of A is 15, wealth of B is 0, probability of A winning the round is 0.5 and probability of B winning the round is 0.4.
   Plot the graph for the probability distribution for this case.

Code:

```
%-------------------------
%Unrestricted Random Walk
%-------------------------
fprintf("\n----------------------------\n");
fprintf("Unrestricted Random Walk");
fprintf("\n----------------------------\n\n");
p = input("Probability of +1 step: ");
q = input("Probability of -1 step: ");
k = input("Position: ");
n = input("No. of Instances/ Steps: ");
if p+q == 1
    c = 1;
else
    c = 0.5;
end
mean = p-q;
std_dev = sqrt(p+q-(p-q)^2);
up_lim = (k+c-n*mean)/(std_dev*sqrt(n));
low_lim = (k-c-n*mean)/(std_dev*sqrt(n));
probability = normcdf(up_lim) - normcdf(low_lim)

%---------------------
%2 Absorbing Barriers
%---------------------
fprintf("\n----------------------------\n");
fprintf("2 Absorbing Barriers");
fprintf("\n----------------------------\n\n");
p = input("Probability of +1 step: ");
q = input("Probability of -1 step: ");
a = input("Position of 1st(+ve) barrier: ");
b = input("Position of 2st(-ve) barrier: ");
if b <0
    b = -b;
end
if p == q
    prob_a = b/(a+b);
else
    prob_a = (p^a)*((p^b - q^b)/(p^(a+b) - q^(a+b)));
end
Probability_absorb_a = prob_a
Probability_absorb_b = 1 - prob_a

%----------------------
%2 Reflecting Barriers
%----------------------
```

```
fprintf("\n--------------------------\n");
fprintf("2 Reflecting Barriers");
fprintf("\n--------------------------\n\n");
p = input("Probability of +1 step: ");
q = input("Probability of -1 step: ");
a = input("Position of reflecting barrier: ");
if p==q
    pi(1) = 1/(a+1);
else
    pi(1) = (1-(p/q))/(1-(p/q)^(a+1));
end
for i=1:100
    k = (a*i)/100;
    pi(i+1) = pi(1)*(p/q)^k;
end
plot(0:a/100:a,pi);
title("2 Reflecting Barriers");
xlabel("Position of Particle");
ylabel("Probability Distribution");
```

Output:

>> Exp7

Unrestricted Random Walk

Probability of +1 step: 0.4

Probability of -1 step: 0.6

Position: 2

No. of Instances/ Steps: 10

probability =

0.1132

----

2 Absorbing Barriers

Probability of +1 step: 0.4

Probability of -1 step: 0.5

Position of 1st(+ve) barrier: 20

Position of 2st(-ve) barrier: 5

Probability_absorb_a =

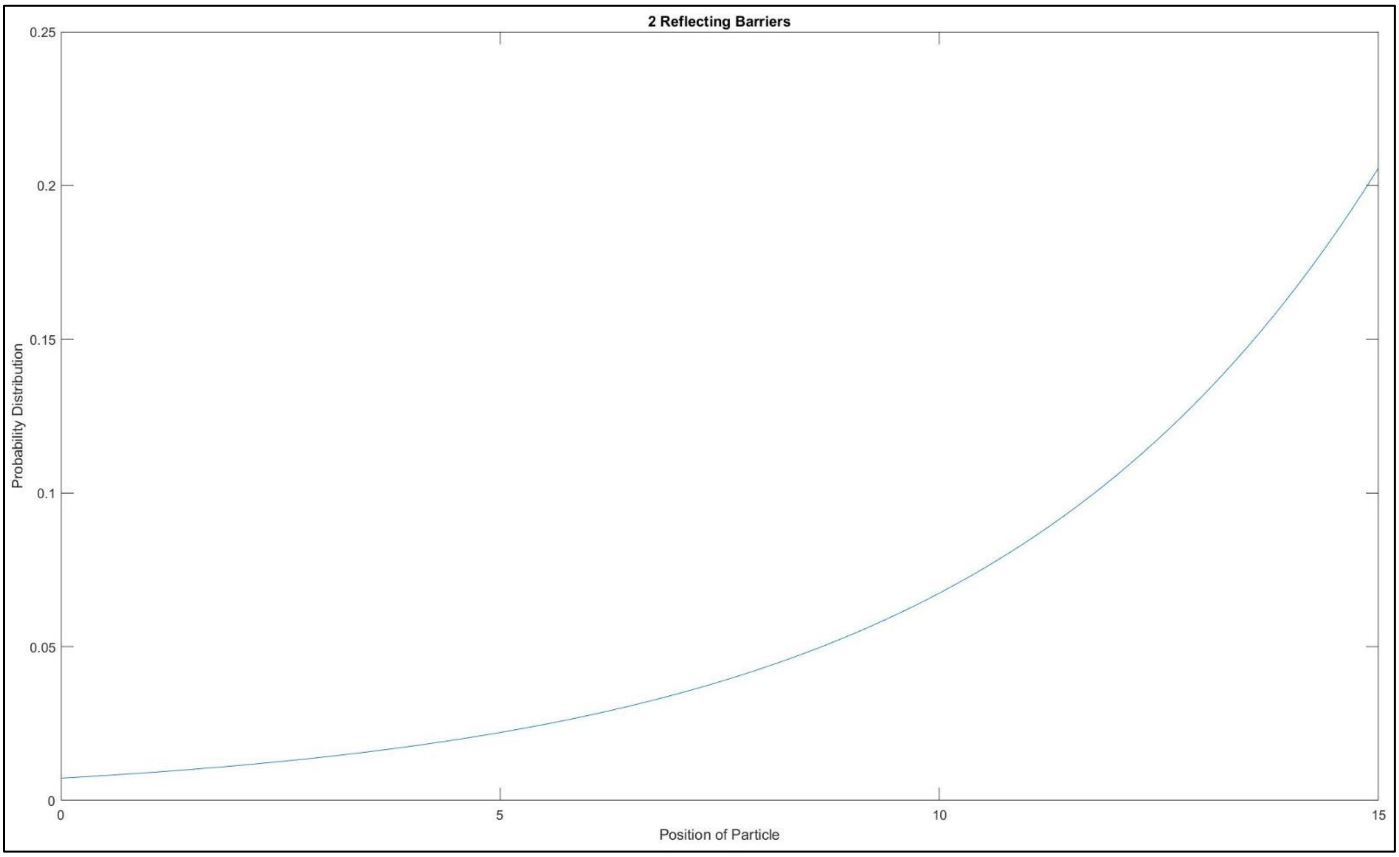
0.0078

Probability_absorb_b =

0.9922

----
2 Reflecting Barriers

Probability of +1 step: 0.5
Probability of -1 step: 0.4
Position of reflecting barrier: 15
>>



Experiment 8

Aim:
Demonstrating Renewal Process. Write a program to find the expected waiting time until the nth renewal in case of a renewal process with renewal cycle length distributed

1. Normally with mean μ and standard deviation σ, (μ > 3 σ)
2. Exponentially with parameter λ

Theory:
Renewal theory is the branch of probability theory that generalizes the Poisson process for arbitrary holding times. Instead of exponentially distributed holding times, a renewal process may have any independent and identically distributed (IID) holding times that have finite mean. Many quantities of interest in the study of renewal processes can be described by a special type of integral equation known as a renewal equation. Renewal equations almost always arise by conditioning on the time of the first arrival and by using the defining property of a renewal process—the fact that the process restarts at each arrival time, independently of the past.

Question:
Assuming that a circuit has an IC whose time to failure is distributed with expected time. If there are 10 spare IC's and time from failure to replacement is negligible. Find the expected waiting time until the 10th renewal in case of a renewal process with

a) Normally distributed with mean 15 minutes and standard deviation 3 minutes
b) Exponentially distributed with arrival rate (lambda) as 0.3 per minute

Solution:
Number of Renewals, n = 10

a) Mean, μ = 15 min.
   Standard Deviation, σ = 3 min.
   E[X] = nμ = 150 min.
b) Arrival Rate, λ = 0.3 per min.
   E[X] = n/λ = 33.333 min.

Code:
```
n = input("No. of Renewals: ");

%Normally distributed with mean & standard deviation
fprintf("\n---------------------------\n");
fprintf("Normal Distribution");
fprintf("\n---------------------------\n\n");
u = input("Mean: ");
std_dev = input("Standard Deviation: ");
Expected_wait_time = n*u

%Exponentially Distributed with lambda
fprintf("\n---------------------------\n");
fprintf("Exponentially Distribution");
fprintf("\n---------------------------\n\n");
```

```
lambda = input("Lambda: ");
Expected_wait_time = n/lambda
```

Output:

>> Exp8

No. of Renewals: 10

Normal Distribution

Mean: 15

Standard Deviation: 3

Expected_wait_time =

150

Exponentially Distribution

Lambda: 0.3

Expected_wait_time =

33.3333

>>

Experiment 9

Aim:
Demonstrating Markov Chain. Write a program to find the n-step transition probability in case of a Markov Chain.

Theory:
Consider a Markov chain X0, X1, X2..., with transition probability matrix P and set of states S. A state j is said to be accessible from i if for some $n \geq 0$ the probability of going from i to j in n steps is positive, that is, $\mathrm{p}_{\mathrm{n}_{\mathrm{ij}}} \geq 0$. We write $i \rightarrow j$ to represent this. If $i \rightarrow j$ and $j \rightarrow i$, we say that i and j communicate and denote it by $i \leftrightarrow j$. The state transition probability matrix of a Markov chain gives the probabilities of transitioning from one state to another in a single time unit.

The n-step transition probability for a Markov chain is,

$P_{ij}^{(n)} = Pr(X_{k+1} = j \mid X_k = i)$

We can find the n -step transition probability matrix through matrix multiplication.

Question:
Let there be just 2 steps available for a person (either +1 or -1). According to these 4 conditions,

Probability of staying at -1 step is 0.25

Probability of taking +1 step when previous step is -1 is 0.75

Probability of staying at +1 step is 0.5

Probability of taking -1 step when previous step is +1 is 0.5

Find the probability transition matrix and hence find the probabilities of 5th step (5 step transition matrix).

Solution:
Probability Transition Matrix, P = $\begin{pmatrix} 0.75 & 0.25 \\ 0.5 & 0.5 \end{pmatrix}$

5 step Probability Transition Matrix, $P^{(5)}$ = $P*P*P*P*P$

$P^{(5)}$ = $\begin{pmatrix} 0.75 & 0.25 \\ 0.5 & 0.5 \end{pmatrix} * \begin{pmatrix} 0.75 & 0.25 \\ 0.5 & 0.5 \end{pmatrix} * \begin{pmatrix} 0.75 & 0.25 \\ 0.5 & 0.5 \end{pmatrix} * \begin{pmatrix} 0.75 & 0.25 \\ 0.5 & 0.5 \end{pmatrix} * \begin{pmatrix} 0.75 & 0.25 \\ 0.5 & 0.5 \end{pmatrix}$

$P^{(5)} = \begin{pmatrix} 0.3994 & 0.6006 \\ 0.4004 & 0.5996 \end{pmatrix}$

Code:
```
Prob_Trans_Mat = input("Input Probability Transition Matrix: ")
k = size(Prob_Trans_Mat);
n = input("Input number of steps (n): ");
n_step_Transition_Matrix = eye(k(1));
for i=1:n
    n_step_Transition_Matrix = n_step_Transition_Matrix*Prob_Trans_Mat;
end
n_step_Transition_Matrix
```

Output:
>> Exp9

```
Input Probability Transition Matrix: [0.25,0.75;0.5,0.5]

Prob_Trans_Mat =

    0.2500    0.7500
    0.5000    0.5000

Input number of steps (n): 5

n_step_Transition_Matrix =

    0.3994    0.6006
    0.4004    0.5996

>>
```