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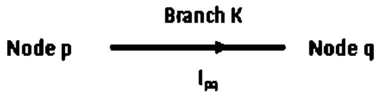
2K20/A6/56

Experiment 4

Aim:

To Verify Tellegen's Theorem.

Theory:



For any given time, the sum of power delivered to each branch of any electric network is zero. Thus for K^{th} branch, this theorem states that,

$$\sum_{k=1}^{n} V_k i_k = 0$$

n being the number of branches, V_k the drop in the branch and i_k the through current while V_p and V_q are voltages at p and q nodes. We have

$$V_k i_{pq} = \left(V_p - V_q\right) i_{pq} = V_k i_k$$
$$V_k i_{pq} = (V_q - V_p) i_{qp}$$
$$i_{pq} = -i_{qp}$$

Summing equations (i) and (ii)

$$2V_k i_k = \left(V_p - V_q\right) i_{pq} + (V_q - V_p) i_{qp}$$
$$V_k i_k = \frac{\left(V_p - V_q\right) i_{pq} + (V_q - V_p) i_{qp}}{2}$$

Equation (iv) can be written for every branch of the network. Assuming n branches, generalisation yields,

$$\sum_{k=1}^{n} V_k i_k = \frac{1}{2} \sum_{p=1}^{n} \sum_{q=1}^{n} \left(V_p - V_q\right) i_{pq} = \frac{1}{2} \sum_{p=1}^{n} V_p \sum_{q=1}^{n} i_{pq} - \frac{1}{2} \sum_{q=1}^{n} V_q \sum_{p=1}^{n} i_{pq}$$

However, following Kirchhoff's current laws, the algebraic sum of currents at each node is equal to zero.

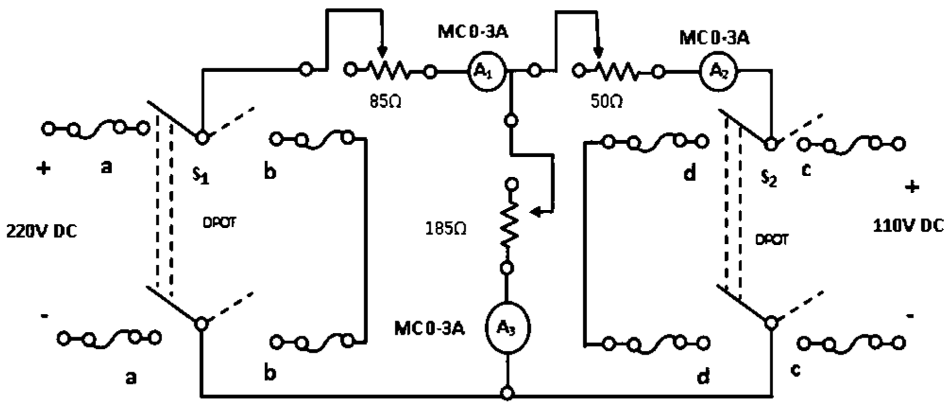
$$\sum_{p=1}^{n} i_{pq} = \sum_{q=1}^{n} i_{pq} = 0$$

Therefore we can clearly see,

$$\sum_{k=1}^{n} V_k i_k = 0$$

This equations shows that the sum of power delivered to a closed network is zero. This proves Tellegen's theorem and also validates the conservation of power in any electrical network. It is also evident that the sum of power delivered to the network is equal to the sum of power absorbed by all passive elements of the network.

Procedure:



1. Connect the circuit as shown in the circuit diagram above, keeping the switches open and resistance at their maximum positions.

2. Case 1: In presence of both the sources Select switch of S1 to Power and S2 to Power and switch on the supply to get the ammeter readings. Observe the power supplied in +ve and power dissipated in -ve by the elements and voltage source for this condition.

3. Case 2: In presence of V1 only select switch of S1 to Power and S2 to short and switch on the supply. Read the corresponding power values as done in the above case.

4. Case 3: In presence of V2 only select switch of S1 to Short and S2 to switch on the supply. Read the corresponding power values. Calculate the power consumed or delivered by each element for each case and check if power absorbed = power delivered. This proves the Tellegen's theorem.

Observations:

Sl no.	In presence of both V_1 and V_2 All values are in Watt					In presence of V_1 only All values are in Watt				In presence of V_2 only All values are in Watt			
	P_1(by R_1)	P_2(by R_2)	P_3(by R_3)	Pv_1(by V_1)	Pv_2(by V_2)	P_1(by R_1)	P_2(by R_2)	P_3(by R_3)	Pv_1(by V_1)	P_1(by R_1)	P_2(by R_2)	P_3(by R_3)	Pv_2(by V_2)
1	-1.85	-20.2	-61.9	20.45	63.63	-46.4	-33.4	-22.3	102.2	-29.7	-105.	-9.91	145.4
2	-0.20	-14.8	-30.3	4.545	40.90	-20.6	-14.8	-9.91	45.45	-16.7	-59.5	-5.57	81.81
3	-0.26	-12.6	-25.0	4.225	33.80	-17.1	-9.72	-6.94	33.80	-13.1	-44.6	-5.62	63.38
4	-0.61	-17.7	-32.7	6.382	44.68	-22.0	-9.05	-7.24	38.29	-15.2	-52.1	-9.16	76.59
5	-0.55	-59.8	-55.4	-10.5	126.3	-27.1	-26.5	-19.9	73.68	-35.4	-166.	-8.86	210.5

Calculations:

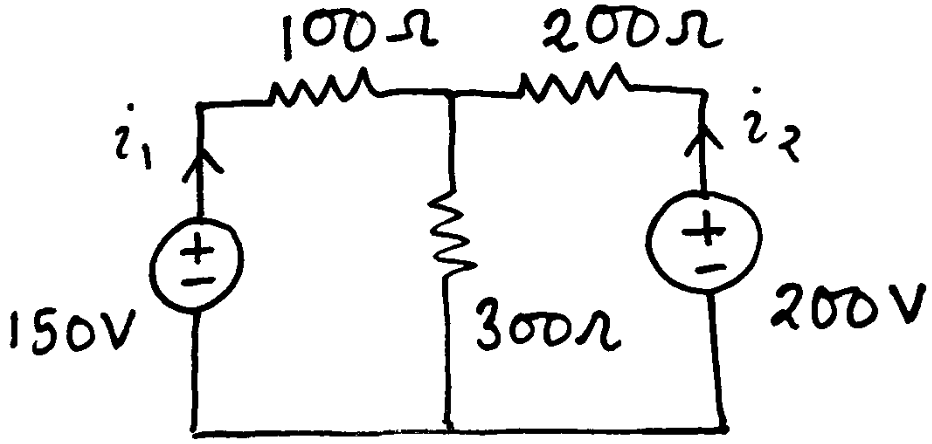
1st observation:

$$V_1 = 150V\,, V_2 = 200V\,, R_1 = 100\ ohm\,, R_2 = 200\ ohm\,, R_3 = 300\ ohm$$

Case I:

When both of the sources are present

$$100i_1 + 300(i_1 + i_2) = 150 == 8i_1 + 6i_2 = 3$$
$$200i_2 + 300(i_1 + i_2) = 200 == 3i_1 + 5i_2 = 2$$



Solving these equations we get,

$i_1 = \frac{3}{22} A = 0.136364\ A$

$i_2 = \frac{7}{22} A = 0.318182\ A$

$P(V_1) = i_1 V_1 = 20.4546\ W$

$P(V_2) = i_2 V_2 = 63.6364\ W$

$P(R_1) = -i_1^2 R_1 = -0.136364 * 0.136364 * 100 = -1.8595140496\ W$

$P(R_2) = -i_2^2 R_2 = -0.318182 * 0.318182 * 200 = -20.2479570248\ W$

$P(R_3) = -(i_1 + i_2)^2 R_3 = -0.454546 * 0.454546 * 300 = -61.9836198348\ W$

$\sum V_k i_k = P(V_1) + P(V_2) + P(R_1) + P(R_2) + P(R_3)$

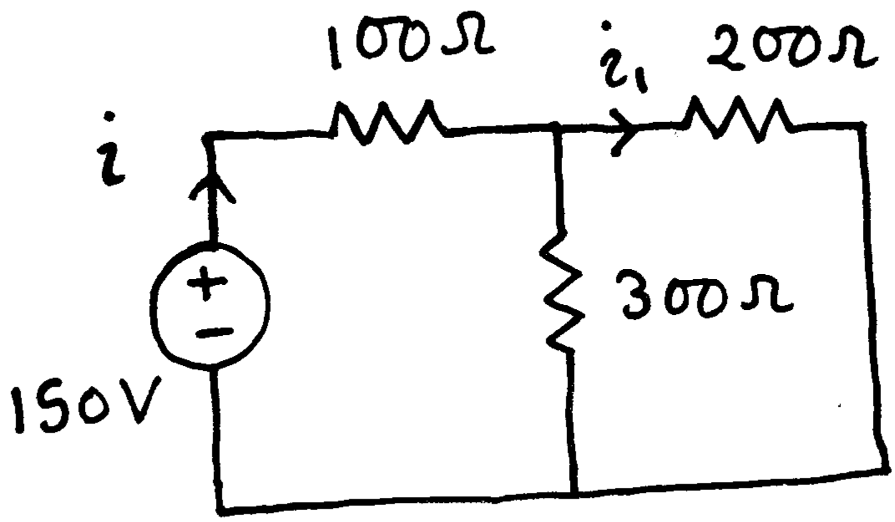
$\sum V_k i_k = 20.4546 + 63.6364 - 61.9836198348 - 1.8595140496 - 20.2479570248 = 0$

Case II:

When only V_1 is present

$$i = \frac{150}{100 + \frac{200 * 300}{200 + 300}} = \frac{150}{100 + 120} = \frac{15}{22} = 0.68182\ A$$

$$i_1 = i * \frac{300}{300 + 200} = \frac{15}{22} * \frac{3}{5} = \frac{9}{22} = 0.40909\ A$$



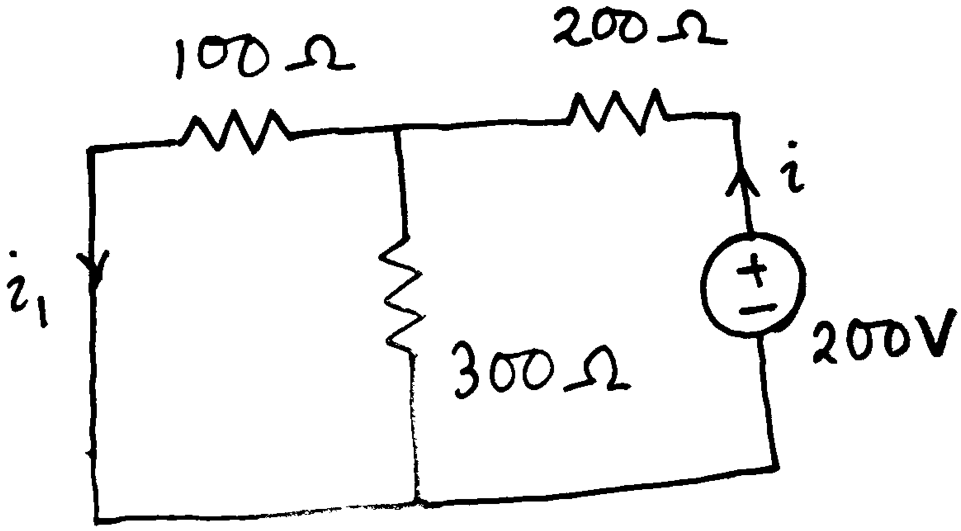
$P(V_1) = iV_1 = 102.273\ W$
$P(R_1) = -i^2R_1 = -0.68182 * 0.68182 * 100 = -46.48785124\ W$
$P(R_2) = -{i_1}^2R_2 = -0.40909 * 0.40909 * 200 = -33.47092562\ W$
$P(R_3) = -(i - i_1)^2R_1 = -0.27273 * 0.27273 * 300 = -22.31449587\ W$

$\sum V_k i_k = P(V_1) + P(R_1) + P(R_2) + P(R_3)$
$\sum V_k i_k = 102.273 - 46.48785124 - 33.47092562 - 22.31449587 = 0$

Case III:

When only V_2 is present

$$i = \frac{200}{200 + \frac{100 * 300}{100 + 300}} = \frac{200}{200 + 75} = \frac{8}{11} = 0.727273\ A$$
$$i_1 = i * \frac{300}{300 + 100} = \frac{8}{11} * \frac{3}{4} = \frac{6}{11} = 0.545455\ A$$



$P(V_2) = iV_2 = 145.4546\ W$
$P(R_2) = -i^2R_2 = -0.727273 * 0.727273 * 200 = -105.7852033058\ W$
$P(R_1) = -{i_1}^2R_1 = -0.545455 * 0.545455 * 100 = -29.7521157025\ W$
$P(R_3) = -(i - i_1)^2R_3 = -0.181818 * 0.181818 * 300 = -9.9173355372\ W$

$\sum V_k i_k = P(V_2) + P(R_1) + P(R_2) + P(R_3)$
$\sum V_k i_k = 145.4546 - 105.7852033058 - 29.7521157025 - 9.91733553372 = 0$

As In all of the 3 cases,
$\sum V_k i_k = 0$
So, Tellegen's Theorem is verified for the 1st observation.
Similarly we can verify Tellegen's theorem for all of the other observations.

Results:

We have successfully verified Tellegen's Theorem.
All of the observations are verified.