History of Logic and study of it's Timeline

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Abstract- Logic is the science or study of how to evaluate arguments and reasoning. Logic is studied within Philosophy, Mathematics and Computer Science. Aristotle is known as the father of logic. In India, study of logic started by Medhatithi Gautama.
The history of logic deals with the study of the development of the science of valid inference (logic). There are 3 periods of logic *Ancient*, *Medieval* and *Modern Logic*.
There are different Modern Logic Periods of the timeline viz. Embryonic Period, Algebraic Period, Logicist Period, Metamathematical Period and period after World War II.

INTRODUCTION

This research paper will help us to understand the different periods of logic, their importance, their advantages and disadvantages over other.

Logic is the science or study of how to evaluate arguments and reasoning. Logic is studied within Philosophy, Mathematics and Computer Science. Aristotle is known as the father of logic. In India, study of logic started by Medhatithi Gautama. There are different logic periods of the timeline viz. Embryonic Period, Algebraic Period, Logicist Period, Metamathematical Period and period after World War II.
The history of logic deals with the study of the development of the science of valid inference (logic). Formal logics developed in ancient times in India, China, and Greece.

ANCIENT LOGIC

Valid reasoning has been employed in all periods of human history. However, logic studies the principles of valid reasoning, inference and demonstration. It is probable that the idea of demonstrating a conclusion first arose in connection with geometry, which originally meant the same as "land measurement". The ancient Egyptians discovered geometry, including the formula for the volume of a truncated pyramid. Ancient Babylon was also skilled in mathematics. Esagil-kin-apli's medical Diagnostic Handbook in the 11th century BC was based on a logical set of axioms and assumptions, while Babylonian astronomers in the 8th and 7th centuries BC employed an internal logic within their predictive planetary systems, an important contribution to the philosophy of science.
Aristotle is the founder of logic as a truly formal discipline. His syllogistic, which can be viewed as a method of deductive inference, and his theory of the logical interrelation of affirmative and negative existential and universal statements were his keys of achievements. The logic of Aristotle is known as term-logic i.e. it relate words or terms.

PRE ARISTOTELIAN LOGIC

While the ancient Egyptians empirically discovered some truths of geometry, the great achievement of the ancient Greeks was to replace empirical methods by demonstrative proof. Both Thales and Pythagoras of the Pre-Socratic philosophers seem aware of geometry's methods.

The 3 basic principles of geometry are as follows:

1. Certain propositions must be accepted as true without demonstration; such a proposition is known as an axiom of geometry.
2. Every proposition that is not an axiom of geometry must be demonstrated as following from the axioms of geometry; such a demonstration is known as a proof or a "derivation" of the proposition.
3. The proof must be formal; that is, the derivation of the proposition must be independent of the particular subject matter in question.

Zeno (490 BC) and Socrates (470BC-399BC) both were famous for the ways in which they refused an opponent's view.

Zeno produced arguments that manifest variations of the pattern of the opponent's view.

Socratic refutation was a series of questions and answers in which opponents were led to a conclusion incompatible with their original argument based on their responses.

Finally, shaping of deduction and proof in Greek Mathematics that began in the later 5th century BC served as an inspiration for Aristotle's syllogistic.

ARISTOTELIAN LOGIC

In the history of logic, Aristotle is the first great logician. From the 4th to the 19th centuries CE, his logic was largely unrivalled. Aristotle distinguishes between things that have sentential unity ('a dog barks') and those that do not ('dog, barks'); the latter are dealt with in the categories (the title actually means 'predictions'). They have no truth value but signify one of the following: substance, quantity, quality, relation, location, time, position, possession, doing and undergoing.

Aristotle considers this classification to be one of linguistic expressions that can be predicted of something else or of kind of prediction.

POST ARISTOTELIAN LOGIC

Argument:

They are considered as a system of at least two premises and a conclusion. Every premise but the first is introduced by 'now' or 'but', and the conclusion by 'therefore'. An argument is valid if the conditional formed with the conjunction of its premises as antecedent and its conclusion as consequent is correct. An argument is sound, when in addition to being valid it has true premises.

If A, the B

But not the B

Therefore not the A

Syllogism

If a statement is either an indemonstrable or can be reduced to one, it is a syllogism.

As a result, syllogisms are a type of formally valid argument.

Stoic Logic

Stoics clearly recognized that legitimate claims are not syllogisms, but he believe that these could be converted into syllogisms in some way.

MEDIEVAL LOGIC

"Medieval logic" (also known as "Scholastic logic") generally means the form of Aristotelian logic developed in medieval Europe throughout roughly the period 1200–1600. For centuries after Stoic logic had been formulated, it was the dominant system of logic in the classical world. When the study of logic resumed after the Dark Ages, the main source was the work of the Christian philosopher Boethius, who was familiar with some of Aristotle's logic, but almost none of the work of the Stoics. Until the twelfth century, the only works of Aristotle available in the West were the Categories, On Interpretation, and Boethius's translation of the Isagoge of Porphyry (a commentary on the Categories). These works were known as the "Old Logic" (Logica Vetus or Ars Vetus). An important work in this tradition was the Logica Ingredientibus of Peter Abelard (1079–1142). His direct influence was small, but his influence through pupils such as John of Salisbury was great, and his method of applying rigorous logical analysis to theology shaped the way that theological criticism developed in the period that followed.

The period from the middle of the thirteenth to the middle of the fourteenth century was one of significant developments in logic, particularly in three areas which were original, with little foundation in the Aristotelian tradition that came before. These were:

- *The Theory of Supposition*

  Supposition theory deals with the way that predicates (e.g., 'man') range over a domain of individuals (e.g., all men). "The theory of supposition with the associated theories of copulatio (sign-capacity of adjectival terms), ampliatio (widening of referential domain), and distributio constitute one of the most original achievements of Western medieval logic".

- *The Theory of Syncategoremata*

  Syncategoremata are terms which are necessary for logic, but which, unlike categorematic terms, do not signify on their own behalf, but 'co-signify' with other words. Examples of syncategoremata are 'and', 'not', 'every', 'if', and so on.

- *The Theory of Consequences*

  A consequence is a hypothetical, conditional proposition: two propositions joined by the terms 'if ... then'. For example, 'if a man runs, then God exists'. A fully developed theory of consequences is given in Book III of William of Ockham's work Summa Logicae. There, Ockham distinguishes between 'material' and 'formal' consequences, which are roughly equivalent to the modern material implication and logical implication respectively.

MODERN LOGIC

Logic in the modern era has exhibited an extreme diversity, and has reflected clearly the surrounding political-intellectual turmoil, diminishing role of the Catholic church, Reformation and subsequent religious wars, scientific revolutions and growth of modern mathematics and the influence of the new world. The modern logic falls into roughly five periods:

- *Embryonic Period*

  Leibniz to 1847, notion of a logical calculus was developed but no schools were formed, and isolated periodic attempts were abandoned or went unnoticed.

- *Algebraic Period*

  Boole's Analysis in which there were more practitioners and a greater continuity of development with the starting of Venn's diagrams.

- *Logicist Period*

  The aim of the "logicist school" was to incorporate the logic of all mathematical and scientific discourse in a single unified system which, taking as a fundamental principle that all mathematical truths are logical, did not accept any non-logical terminology.

- *Metamathematical Period*

  1910-1930s, saw the development of metalogic, the finitist system, the non finitist system and the combination of logic and metalogic.

  Godel's incompleteness theorem was one of the greatest achievements in the history of logic and he developed the notion of set- theoretic constructability.

- *Period after WWII*

  Mathematical logic branched into four interrelated:

  i. Model Theory
  ii. Proof Theory
  iii. Computability Theory
  iv. Set Theory

  and its ideas and methods began to influence philosophy.

EMBRYONIC PERIOD

The work of logicians such as the Oxford Calculators led to a method of using letters instead of writing out logical calculations (calculationes) in words, a method used, for instance, in the Logica magna by Paul of Venice. Three hundred years after Llull, the English philosopher and logician Thomas Hobbes suggested that all logic and reasoning could be reduced to the mathematical operations of addition and subtraction. The same idea is found in the work of Leibniz, who had read both Llull and Hobbes, and who argued that logic can be represented through a combinatorial process or calculus. But, like Llull and Hobbes, he failed to develop a detailed or comprehensive system.

Leibniz says that ordinary languages are subject to "countless ambiguities" and are unsuited for a calculus, whose task is to expose mistakes in inference arising from the forms and structures of words

Sematic Validity

Gergonne (1816) said that reasoning does not have to be about objects about which one has perfectly clear ideas, because algebraic operations can be carried out without having any idea of the meaning of the symbols involved. Bolzano anticipated a fundamental idea of modern proof theory when he defined logical consequence or "deducibility" in terms of variables:

Hence I say that propositions M, N, O, … are deducible from propositions A, B, C, … with respect to variables i, j, … , if every class of ideas whose substitution for i, j, … makes all of A, B, C, … true, also makes all of M, N, O, … true. Occasionally, since it is customary, I shall say that propositions M, N, O, … follow, or can be inferred or derived, from A, B, C, … Propositions A, B, C, … I shall call the premises, M, N, O, … the conclusions.

Algebraic Period

Boole's system admits of two interpretations, in class logic, and propositional logic. Boole distinguished between "primary propositions" which are the subject of syllogistic theory, and "secondary propositions", which are the subject of propositional logic, and showed how under different "interpretations" the same algebraic system could represent both. An example of a primary proposition is "All inhabitants are either Europeans or Asiatics." An example of a secondary proposition is "Either all inhabitants are Europeans or they are all Asiatics". These are easily distinguished in modern predicate logic, where it is also possible to show that the first follows from the second, but it is a significant disadvantage that there is no way of representing this in the Boolean system.

In his Symbolic Logic (1881), John Venn used diagrams of overlapping areas to express Boolean relations between classes or truth-conditions of propositions.

The success of Boole's algebraic system suggested that all logic must be capable of algebraic representation, and there were attempts to express a logic of relations in such form, of which the most ambitious was Schröder's monumental Vorlesungen über die Algebra der Logik ("Lectures on the Algebra of Logic", vol iii 1895), although the original idea was again anticipated by Peirce.

Boole agreed with what Aristotle said; Boole's 'disagreements', if they might be called that, concern what Aristotle did not say. First, in the realm of foundations, Boole reduced the four propositional forms of Aristotelian logic to formulas in the form of equations — by itself a revolutionary idea. Second, in the realm of logic's problems, Boole's addition of equation solving to logic — another revolutionary idea — involved Boole's doctrine that Aristotle's rules of inference (the "perfect syllogisms") must be supplemented by rules for equation solving. Third, in the realm of applications, Boole's system could handle multi-term propositions and arguments whereas Aristotle could handle only two-termed subject-predicate propositions and arguments.

Logicist Period

Frege argued that the quantifier expression "all men" does not have the same logical or semantic form as "all men", and that the universal proposition "every A is B" is a complex proposition involving two functions, namely ' – is A' and ' – is B' such that whatever satisfies the first, also satisfies the second. In modern notation, this would be expressed as

$$\forall x(A(x) \rightarrow B(x))$$

In english, "for all x, if Ax then Bx". Thus only singular propositions are of subject-predicate form, and they are irreducibly singular, i.e. not reducible to a general proposition. Universal and particular propositions, by contrast, are not of simple subject-predicate form at all.

As Frege remarked in a critique of Boole's calculus:

"The real difference is that I avoid [the Boolean] division into two parts ... and give a homogeneous presentation of the lot. In Boole the two parts run alongside one another, so that one is like the mirror image of the other, but for that very reason stands in no organic relation to it"

Russell's Paradox

Frege's theory contained the axiom that for any formal criterion, there is a set of all objects that meet the criterion. Russell showed that a set containing exactly the sets that are not members of themselves would contradict its own definition (if it is not a member of itself, it is a member of itself, and if it is a member of itself, it is not).

Zermelo–Fraenkel set theory

One important method of resolving this paradox was proposed by Ernst Zermelo. Zermelo set theory was the first axiomatic set theory. It was developed into the now-canonical Zermelo–Fraenkel set theory (ZF). Russell's paradox symbolically is as follows:

$$Let\ R = \{x | x \notin x\}, then\ R \in R \leftrightarrow R \notin R$$

Metamathematical Period

Godel's Completeness Theorem

Work on metamathematics culminated in the work of Gödel, who in 1929 showed that a given first-order sentence is deducible if and only if it is logically valid – i.e. it is true in every structure for its language. This is known as Gödel's completeness theorem.

Godel's Incompleteness Theorem

2 results are known as Godel's Incompleteness Theorem which are as follows,

The first is that no consistent system of axioms whose theorems can be listed by an effective procedure such as an algorithm or computer program is capable of proving all facts about the natural numbers. For any such system, there will always be statements about the natural numbers that are true, but that are unprovable within the system.

The second is that if such a system is also capable of proving certain basic facts about the natural numbers, then the system cannot prove the consistency of the system itself.

Period after WWII

After World War II, mathematical logic branched into four inter-related but separate areas of research: model theory, proof theory, computability theory, and set theory.

Set Theory

In set theory, the method of forcing revolutionized the field by providing a robust method for constructing models and obtaining independence results. Paul Cohen introduced this method in 1963 to prove the independence of the continuum hypothesis and the axiom of choice from Zermelo–Fraenkel set theory.

Computability Theory

Computability theory had its roots in the work of Turing, Church, Kleene, and Post in the 1930s and 40s. It developed into a study of abstract computability, which became known as recursion theory.

The fields of constructive analysis and computable analysis were developed to study the effective content of classical mathematical theorems; these in turn inspired the program of reverse mathematics.

Model Theory

Model theory applies the methods of mathematical logic to study models of particular mathematical theories.

Proof Theory

In proof theory, the relationship between classical mathematics and intuitionistic mathematics was clarified via tools such as the realizability method invented by Georg Kreisel and Gödel's Dialectica interpretation.

Indian Logic

The Nasadiya Sukta of the Rigveda (RV 10.129) contains ontological speculation in terms of various logical divisions that were later recast formally as the four circles of catuskoti: "A", "not A", "A and 'not A'", and "not A and not not A".

Medhatithi Gautama (c. 6th century BCE) founded the anviksiki school of logic. The Mahabharata (12.173.45), around the 4th century BCE to 4th century CE, refers to the anviksiki and tarka schools of logic.

Pāṇini (5th century BC) developed a form of logic (to which Boolean logic has some similarities) for his formulation of Sanskrit grammar.

Logic is described by Chanakya (350-283 BC) in his Arthashastra as an independent field of inquiry.

The Jains have doctrines of relativity used for logic and reasoning:

- *Anekāntavāda*

  The theory of relative pluralism or manifoldness
- *Syādvāda*

  The theory of conditioned predication
- *Nayavāda*

  The theory of partial standpoints

Comparative study

MODERN LOGIC PERIOD

The logic gets improved day by day so we cannot judge any period as better or worse than other because every period has its own significance and every period contribute in the development of logic.

However, we can discuss where the logic fails to prove and we need other logic to overcome that problem.

Disadvantages of

- *Embryonic Period*

  From Embryonic period we get to know the method of using letters instead of writing out logical calculations.

  Failure of this period

  Ordinary languages are subject to "countless ambiguities" and are unsuited for a calculus, whose task is to expose mistakes in inference arising from the forms and structures of words.

- *Algebraic Period*

  Symbolic Logic Period

  From Algebraic period we get to know about Venn's diagrams which was one of the most successful theory in that time. Venn diagrams are based on Boolean algebra.

  Failure of this period

  The defects in Boole's system (such as the use of the letter v for existential propositions) were all remedied by his followers.

  This failure was later solved by Jevons as he suggested a symbol to signify exclusive or.

- *Logicist Period*

  In this period we don't get any disadvantage but we get a breakthrough in logical world when German Mathematician Gottlob Frege questioned the traditional logic which states that

  'Sentences with a proper name subject must be regarded as universal in character'

  So after this question or we can say a failure in traditional logic we get to know new term called quantifier which changed the world of logic.

General Comparison

Now we talk about general failure of each of it that is Embryonic Period, Algebraic Period and Logicist Period.

- *Embryonic Period*
  - Fails because of use of ordinary languages.
  - Fails because it's not quantitative in nature.
  - Difficult to solve if we take heavy data or simply many propositions are there.

- *Algebraic Period*

  Boolean Algebra

  - Difficult to find out the quantitative data from it.
  - Sometimes difficult to remember the Boolean algebraic equations.
  - Confusion between symbols and alphabets may take place.

- *Logicist Period*

  Real Algebra

  - Difficult to understand and learn.

CASE STUDY

MODERN LOGIC PERIOD

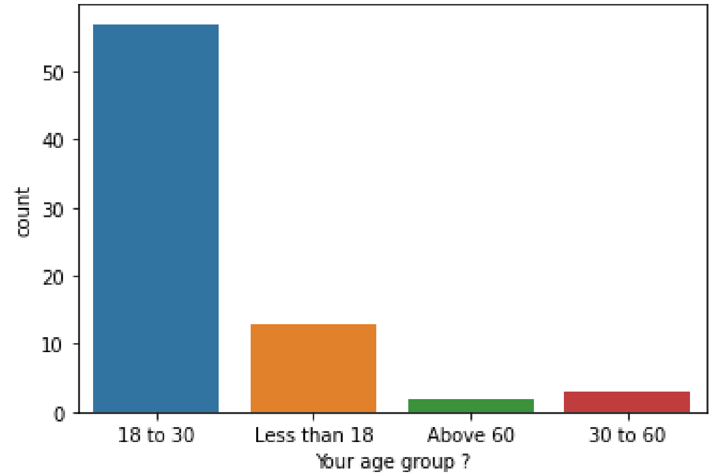
We did a real case study in which we asked some people of different ages about some questions on Logical Reasoning. Which period did they prefer for solving questions.

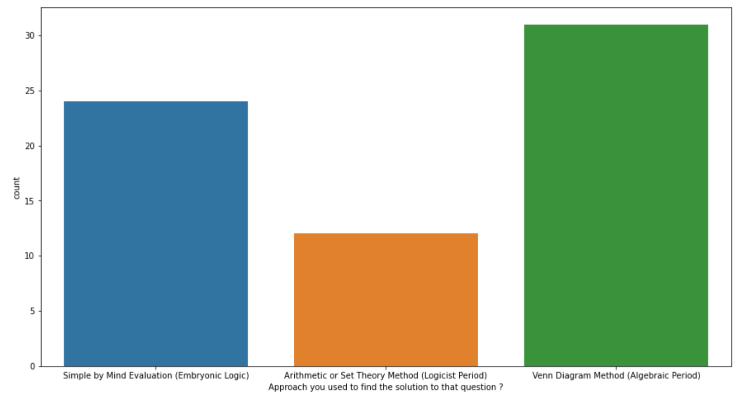
All of the case study done through google forms only. After this case study we do a simple data analysis on that data using python and google colab to get some outcomes from the data.

Some data analyzed graph is given below

1. Bar graph for age limit of persons filled the form.

2. Approach used by persons to solve a question.

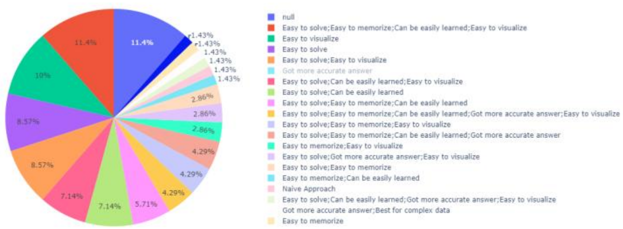




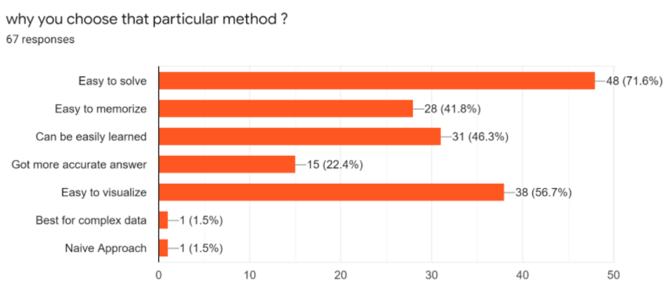
From the 1st bar plot its clear that persons that filled the forms are maximum youth.

From 2nd bar plot we can see that Algebraic period logic is most preferable choice followed by Embryonic period logic and Logistic period logic is least preferable.

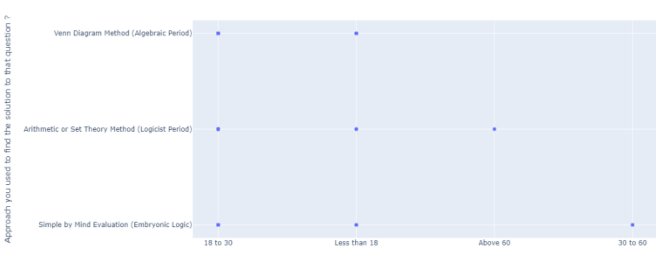
These are basic observations which we see. Now, we go into depth of the case study.



From this pie chart we see the reasons why one choose a specific period over another for calculations.



Here is the graph obtained from google forms. We see that most of the student go for "Easy to solve" and "Easy to visualize" part and least number of student go for the "Accuracy of Results".



This is the scatter plot of Age limit v/s Logic period.

So all of the 3 periods are chosen by someone in all age groups except persons age above 30.

For age group above 30, we can't say anything because we have very limited data of the specified age group.

This is the overall observations in a single histogram we get from the people.

So from this case study it is clear that there are so many students who solve their questions with Embryonic period logic which may give good or bad results as it totally depends on language. Some human error may occur in that case.

Algebraic period is the most taken period by the people as it is best for visualization and have lesser chance of error than Embryonic period.

There are least number of students who used the Logicist period method which is taken as the best among 3 for accurate results.

Conclusions

We successfully studied and understood the history of logic and its timeline. We concluded our Research project with giving our outcomes as

a) All periods are equally important and we can’t say that this one is better than that other because every period have its own significance in the history of logic.

b) Out of Logicist, Embryonic, and Algebraic periods, we see
Logicist period is best for accurate results.
Algebraic period is best for visualizing the data and interpretation.

c) Every period have its own advantages and disadvantages. As far as disadvantages remain there, a scope of advancement in the world of logic exists.

USEFUL LINKS

Google form used for data collection:
https://forms.gle/88FNiEnjFFH6b6MW8

Github repository for the python code and data file of the graph (Python code):
https://github.com/Aneeshcoder/FEC_32

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