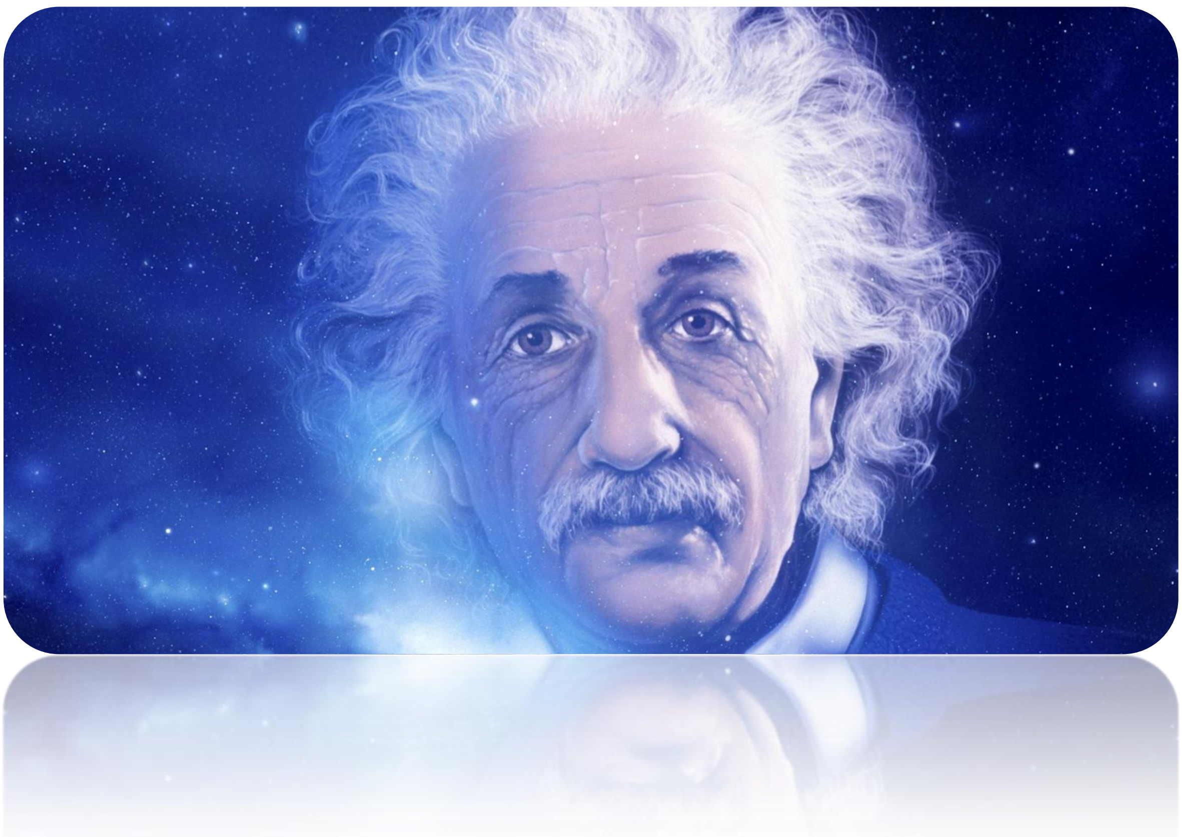
Project Report -

Relativistic Mass

Submitted by -

Aneesh panchal
2k20/a6/56



Submitted to –

DEPARTMENT OF APPLIED PHYSICS,
DELHI TECHNOLOGICAL UNIVERSITY

CONTENT:

i. Abstract
ii. Introduction to Mass
iii. Validity of Law of Conservation of Energy
iv. Validity of Law of Conservation of Momentum
v. Validity of Law of Conservation of Mass
vi. Einstein's Mass Energy Equivalence
vii. Energy Momentum Relation
viii. Applications of Mass Energy Equivalence
ix. Einstein's Radiation Beam Box Experiment (Updated Version)
x. Einstein's Relative Mass formula for Photons
xi. Conclusion

ABSTRACT:

In physics, mass–energy equivalence is the relationship between mass and energy in a system's rest frame, where the two values differ only by a constant and the units of measurement. The principle is described by Albert Einstein's famous formula:

Mass Energy Relation:

$$\boldsymbol{E = mc^2}$$

Here, **E** is energy of a particle in its rest frame

m is mass of the particle (Relativistic Mass)

c is the speed of light (**c = 3x10^8 m/s**)

Let us check the validity of this Mass energy equivalence and the concept of Relativistic Mass.

INTRODUCTION TO MASS:

The word mass have two meanings in special relativity: invariant mass (also called rest mass) is an invariant quantity which is the same for all observers in all reference frames; while the relativistic mass is dependent on the velocity of the observer. Mass is generally denoted by **m**. SI unit is Kilograms (**kg**).

- ## Rest Mass:

  The term mass in special relativity usually refers to the rest mass of the object, which is the Newtonian mass as measured by an observer moving along with the object. The invariant mass is another name for the rest mass of single particles.

  This mass will remain constant at any surrounding conditions.

  Rest mass is generally denoted by **m** or, $\boldsymbol{m_o}$

- ## Relativistic Mass:

  The relativistic mass is the sum total quantity of energy in a body or system (divided by $\boldsymbol{c^2}$). Thus, the mass in the formula:

  $\boldsymbol{E = m_{rel}c^2}$

  is the Relativistic mass.

  Relativistic mass is generally denoted by $\boldsymbol{m_{rel}}$

  For a particle of finite rest mass m moving at a speed relative to the observer, one finds,

  $$\boldsymbol{m_{rel} = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} = \Gamma m_o}$$

  Relativistic Mass is frame dependent while Rest Mass is invariant.

Validity of Law of Conservation of Energy:

The Conservation of Energy is a universal principle in physics and holds for any interaction.

According to Law of Conservation of Energy,

Energy can neither be created nor be destroyed it can only converted from one form of energy to another form of energy.

Forms of energy are Kinetic energy, Potential energy, Vibrational energy, Thermal energy, Radiant energy etc.

In Einstein's Energy Mass Equivalence,

Mass is also taken as a form of energy.

Mass of a particle is converted into other forms of energy such as kinetic energy, thermal energy, radiant energy. Similarly, kinetic or radiant energy can be used to create particles that have mass, always conserving the total energy.

So, the Total Energy always remains constant.

Thus, **Law of Conservation of Energy holds good for Relativistic Mass also in isolated systems**.

Validity of Law of Conservation of Momentum:

The Conservation of Momentum is a universal principle in physics and holds for any interaction.

According to Law of Conservation of Momentum,

Total momentum of the particles before collision (Total initial momentum) is equal to the total momentum of the particles after collision (Total final momentum).

According to 1st Posulate of Special Theory of Relativity,

The laws of physics all take same identical form for all frames of reference in uniform relative motion i.e. for all the inertial frames of references.

So, According to Newton's Second Law of Motion,

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{p} = m\frac{d\vec{x}}{dt}$$

But after Lorentz Transformations, $dt \rightarrow dT$

$$\vec{p} = m_o\frac{d\vec{x}}{dT} = m_o\frac{d\vec{x}}{dt}\frac{dt}{dT}$$

$$\vec{p} = \Gamma m_o\vec{v}$$

So Law of Conservation of Momentum is always applicable.

Thus, **Law of Conservation of Momentum holds good for Relativistic Mass also in isolated systems**.

Validity of Law of Conservation of Mass:

Classical Conservation of Mass is somehow violated in Relativity.

According to Classical Conservation of Mass,

Total mass of a body always remains constant or conserved.

But in relativity mass changes with the velocity of the body and it also relates with energy of the body.

According to Classical Theory, Mass is constant and will not change its value.

According to Modern Relativity Theory, Relativistic Mass is conserved as Total Energy remains conserved and relationship between mass and energy is given by Mass Energy Relation i.e. In relativity mass is taken as a form of energy or, mass and energy are interchangeable.

So "Law of Conservation of Mass" is nothing but we can say just another name of "Law of Conservation of Energy".

In general,

For isolated systems and single observers,

Relativistic mass is conserved (each observer sees it constant over time), but is not invariant (that is, different observers see different values).

Invariant mass (Rest mass), however, is both conserved and invariant (all single observers see the same value, which does not change over time).

Thus, Law of Conservation of Mass simply Law of Conservation of Energy holds good for Relativistic Mass also in isolated systems.

EINSTEIN'S MASS ENERGY EQUIVALENCE:

In physics, mass–energy equivalence is the relationship between mass and energy in a system's rest frame, where the two values differ only by a constant and the units of measurement. The principle is described by Albert Einstein's famous formula:

Mass Energy Relation:

$$\boldsymbol{E = mc^2}$$

Here, **E** is energy of a particle in its rest frame

m is mass of the particle (Relativistic Mass)

c is the speed of light (**c = 3x10^8 m/s**)

The rest mass is a fundamental physical property that remains independent of momentum, even at extreme speeds approaching the speed of light (i.e., its value is the same in all inertial frames of reference).

Massless particles such as photons have zero invariant mass, but massless free particles have both momentum and energy. The equivalence principle implies that when energy is lost in chemical reactions, nuclear reactions, and other energy transformations, the system will also lose a corresponding amount of mass.

In relativity, all the energy that moves with an object (i.e., the energy as measured in the object's rest frame) contributes to the total mass of the body, which measures how much it resists acceleration.

If an isolated box of ideal mirrors could contain light, the individually massless photons would contribute to the total mass of the Description box, by the amount equal to their energy divided by $\boldsymbol{c^2}$.

For an observer in the rest frame, removing energy is the same as removing mass and the formula $\boldsymbol{m = \frac{E}{c^2}}$ indicates how much mass is lost when energy is removed.

In the same way, when any energy is added to an isolated system, the increase in the mass is equal to the added energy divided by $\boldsymbol{c^2}$.

ENERGY MOMENTUM RELATION:

Unlike a system's energy in an inertial frame, the relativistic energy (E_r) of a system depends on both the rest mass (m_o) and the total momentum of the system.

$E_r = \sqrt{(m_o c^2)^2 + (pc)^2}$

This equation is called the Energy Momentum Relation.

It reduces to $E = m_o c^2$ when the momentum term is zero ($p = 0$).

For photons rest mass is zero ($m_o = 0$), the equation reduces to $E_r = pc$.

According to Mass Energy Equivalence,

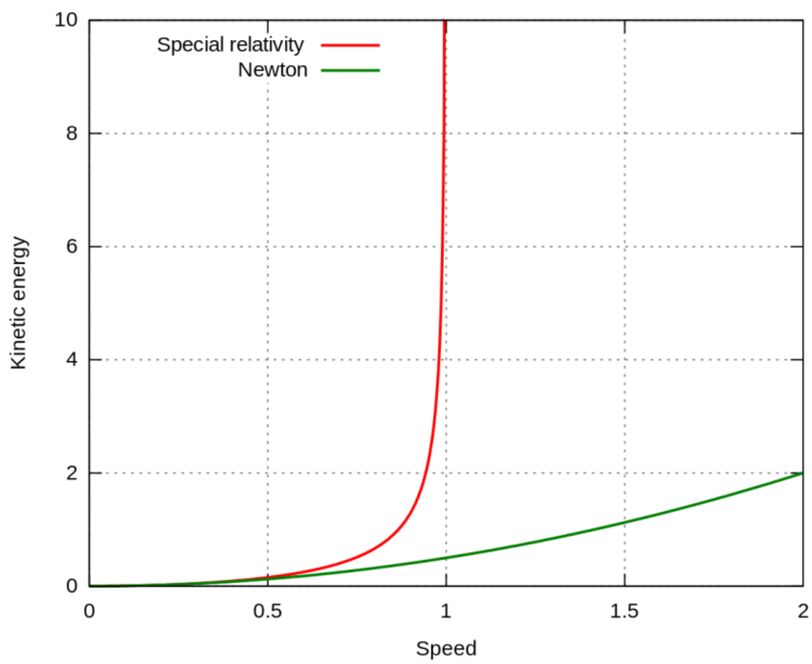
$E = \Gamma m_o c^2$

$E = m_o c^2 \left[1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots\right]$

For speed **v<<<c**,

$E \rightarrow m_o c^2 + \frac{1}{2} m_o v^2$

Where m_o is rest mass of body and **v** is velocity of body.



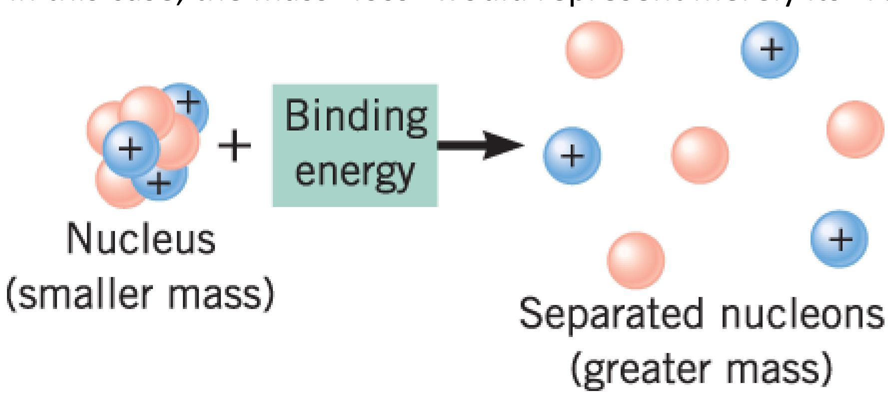
APPLICATIONS OF MASS ENERGY EQUIVALENCE

(at v equivalent to c):

- The nuclear binding energy is the minimum energy that is required to disassemble the nucleus of an atom into its component parts. **The mass of an atom is less than the sum of the masses of its constituents** due to the attraction of the strong nuclear force. The difference between the two masses is called the **mass defect** and is related to the binding energy through Einstein's formula. The principle is used in modelling nuclear fission reactions and it implies a great amount of energy can be released by the nuclear fission chain reactions used in both nuclear weapons and nuclear power.

- **A stick of dynamite in theory weighs a little bit more than the fragments after the explosion**. In this case the mass difference is the energy and heat that is released when the dynamite explodes.

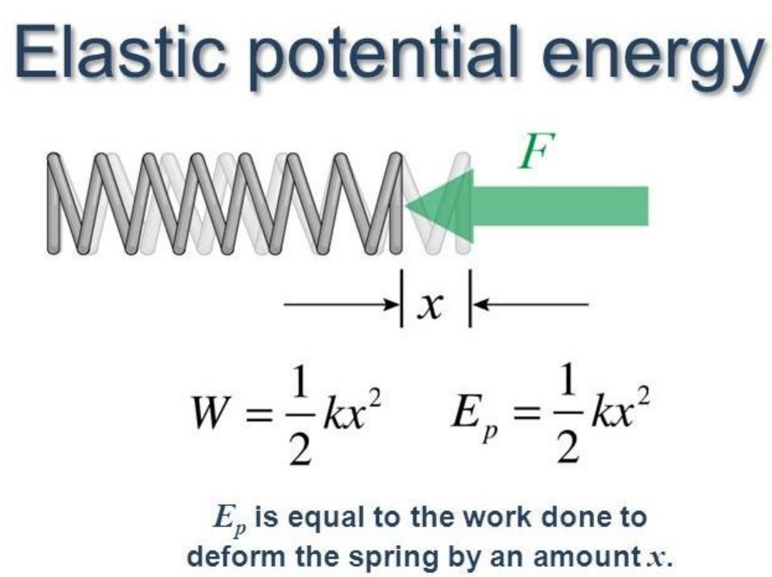
- If a transparent window passing only electromagnetic radiation were opened in such an ideal box after the explosion, and a beam of X-rays and other lower-energy light allowed to escape the box, it would eventually be found to weigh less than it had before the explosion. This weight loss and mass loss would happen as the box was cooled by this process, to room temperature. However, any surrounding mass that absorbed the X-rays (and other "heat") would gain this gram of mass from the resulting heating. Thus, in this case, the mass "loss" would represent merely its “relocation”.



Applications of Mass Energy Equivalence
(at v<<<c):

- **A spring's mass increases whenever it is put into compression or tension.**
  Its added mass arises from the added potential energy stored within it, which is bound in the stretched chemical (electron) bonds linking the atoms within the spring.
  Let spring with spring constant **k** compress to a distance **x**,
  According to Einstein's Mass energy Equivalence
  $\boldsymbol{E = mc^2}$
  Differentiating both sides we get,
  $\boldsymbol{\Delta E = \Delta m.c^2}$
  Change (increase) in energy of spring mass system,
  $\boldsymbol{E_s = \frac{1}{2}kx^2 = \Delta E}$
  Change (increase) in mass of the spring,
  $\boldsymbol{\Delta m = \frac{1}{2}k\frac{x^2}{c^2}}$

  When value of **|x|<<<|c|**,
  So change in mass is negligible but is there.



- **Raising the temperature of an object (increasing its heat energy) increases its mass.**

  For example, consider the world's primary mass standard for the kilogram, made of platinum and iridium. If its temperature is allowed to change by 1°C, its mass changes by 1.5 picograms (1 pg = 1 $\times 10^{-12}$g).

  Let energy of the body increases by $\boldsymbol{E} \rightarrow \boldsymbol{E} + \boldsymbol{\Delta E}$ when its temperature rises by $\boldsymbol{T} \rightarrow \boldsymbol{T} + \boldsymbol{\Delta T}$,

  Change in Energy of the body is $\boldsymbol{\Delta E}$,

  According to Einstein's Mass energy Equivalence

  $\boldsymbol{E = mc^2}$

  Differentiating both sides we get,

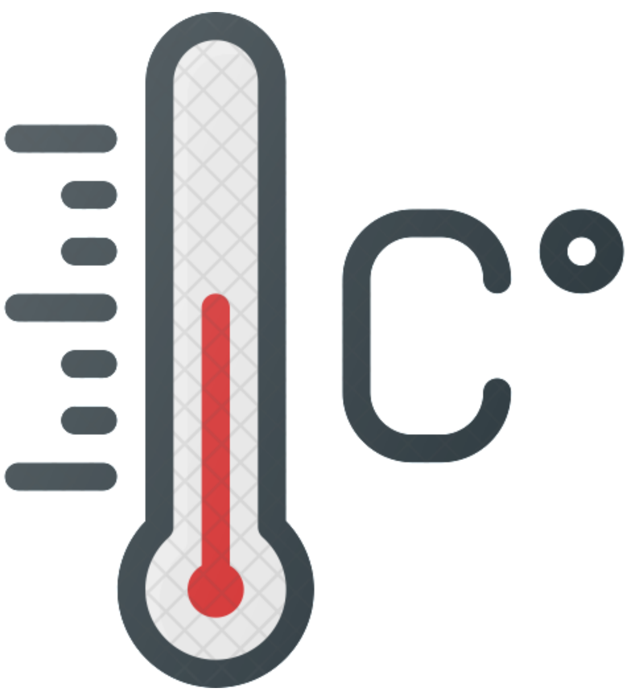
  $\boldsymbol{\Delta E = \Delta m . c^2}$

  Change (increase) in mass of the body,

  $\boldsymbol{\Delta m = \frac{\Delta E}{c^2}}$

  When change in temperature is very small,

  Then change in mass is negligible but is there.



- **A spinning ball weighs more than a ball that is not spinning.** Its increase of mass is exactly the equivalent of the mass of energy of rotation, which is itself the sum of the kinetic energies of all the moving parts of the ball.
  For example, the Earth itself is more massive due to its rotation, than it would be with no rotation. The rotational energy of the earth is greater than 10^{24} Joules, which is over 10^{7} kg.
  Let a body starts rotating with angular velocity **ω** and moment of inertia **I**,
  Rotational energy of the body, $\boldsymbol{E_{rot} = \frac{1}{2} I\omega^2}$
  According to Einstein's Mass energy Equivalence
  $\boldsymbol{E = mc^2}$
  Differentiating both sides we get,
  $\boldsymbol{\Delta E = \Delta m. c^2}$
  Change (increase) in energy of the body,
  $\boldsymbol{\Delta E = E_{rot}}$
  Change (increase) in mass of the body,
  $\boldsymbol{\Delta m = \frac{E_{rot}}{c^2} = \frac{1}{2} I \frac{\omega^2}{c^2}}$

  When **|ω|<<<|c|**,
  So change in mass is negligible but is there.

EINSTEIN'S RADIATION BEAM BOX EXPERIMENT (UPDATED VERSION):

Let us consider laser gun in place of radiation beam box and laser pulse in place of Einstein's radiation beam pulse.

A laser gun firing a laser pulse, however, the barrel of the gun is sealed by an impenetrable lead stopper.

The length of the barrel is represented by **l**.

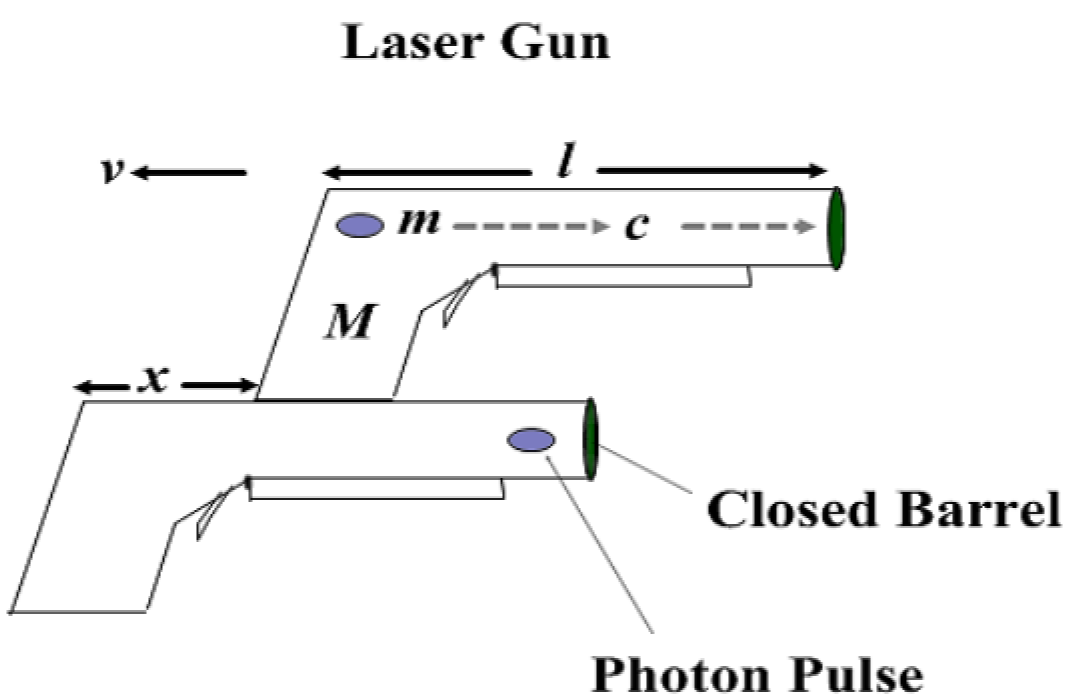
The mass of the laser gun is represented by **M**.

The 'mass' of the laser pulse is **m**.

The speed of the laser pulse (light) is **c**.

When the gun is fired it recoils and its recoiling velocity is represented by **v**.

When the gun recoils it moves back a distance, and this recoil distance is represented by **x**.



Einstein's radiation box (Laser gun) thought experiment the recoil velocity of the laser gun is expressed as:

$$v = \frac{dx}{dt}$$

$$dx = vdt$$

The work and/or energy (**E**) performed on the laser gun by the photons of the laser pulse is: $E = Fx$

$$dE = Fdx$$

Here,

F is the force that causes the recoil of the gun,

dE is the change in the gun's energy due to the change in the energy of the moving photon pulse.

Eliminating **dx** and **F** we get:

$$dE = Fvdt$$

Where, $dx = vdt$

$$dE = d(Mv)v$$

As we know $F = \frac{d(Mv)}{dt}$

$$dE = (Mdv + vdM)v$$

$$dE = vMdv + v^2dM$$

From Einstein's Mass Energy Equivalence,

$$E = mc^2$$

Differentiating both sides we get,

$$dE = c^2dm$$

From above equations we get,

$$c^2dm = Mvdv + v^2dM$$

For solving this Einstein consider $\boldsymbol{dm = dM}$.

- But this is impossible for our consideration as it would require the laser gun recoiling a distance equal to the distance travelled by photon pulse i.e. the impossibility of $\boldsymbol{x = L}$.

After its implication we get,

$$c^2 dm = Mvdv + v^2 dm$$

$$(c^2 - v^2)dm = mvdv$$

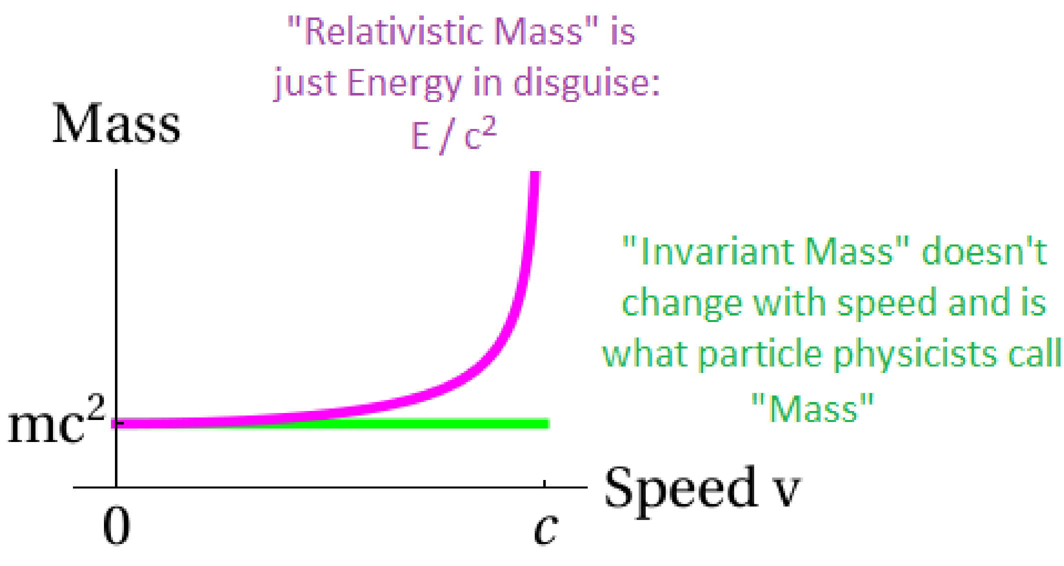
$$\frac{dm}{m} = \frac{vdv}{(c^2 - v^2)}$$

Integrating this equation for $\boldsymbol{m}$ $(\boldsymbol{m_o \to m})$ and for $\boldsymbol{v}$ $(\boldsymbol{0 \to v})$,

$$\int_{m_o}^{m} \frac{dm}{m} = \int_{0}^{v} \frac{v}{(c^2 - v^2)} dv$$

After integration we get,

Relative mass, $\boldsymbol{m_r = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} = \Gamma m_o}$



- But here one of the most basic requirement for The Special Theory of Relativity is that for this experiment Einstein consider the movement of the particle to be constant that is velocity is constant or without acceleration. This implies as **velocity is constant then change in velocity is zero**, $\boldsymbol{dv = 0}$

  Now taking the integral part we mentioned above,

$$\int_{m_o}^{m} \frac{dm}{m} = \int_{0}^{v} \frac{v}{(c^2 - v^2)} dv = 0$$

  After solving the above equation we get,

  $\boldsymbol{log\left(\frac{m}{m_o}\right) = 0}$

  After solving this we get,

  Relative mass, $\boldsymbol{m_r = m_o \neq \Gamma m_o}$

  This clearly shows that formula for relativistic mass given by Einstein is incorrect.

EINSTEIN'S RELATIVE MASS FORMULA FOR PHOTONS:

Einstein assumed that,

- **Rest mass of photon is zero.**
- **Photon moves with velocity equal to the speed of light.**

Using Relative Mass formula given by Einstein,

Relative mass or Moving mass, $\boldsymbol{m_r} = \frac{\boldsymbol{m_o}}{\sqrt{1-\frac{v^2}{c^2}}}$

As $\boldsymbol{m_o} = \boldsymbol{0}$ and $\boldsymbol{v} = \boldsymbol{c}$,

These values will give,

Relative mass or Moving mass, $\boldsymbol{m_r} = \frac{\boldsymbol{0}}{\boldsymbol{0}}$ or, indeterminate form

This denies both of the assumptions of Einstein.

If we consider Einstein's Relative Mass formula to be correct then,

These **Einstein's assumptions** can be resolved or **modified** by saying,

- **Rest Mass of photon is not zero but is very small that it seems to be zero.**
- **Photon moves with very high speed (just approaching to speed of light) but not equal to the speed of light.**

After these assumptions we can say that Photons are particles with rest mass $\boldsymbol{m_o} \to \boldsymbol{0}$ and moves with velocity $\boldsymbol{v} \to \boldsymbol{c}$.

But,

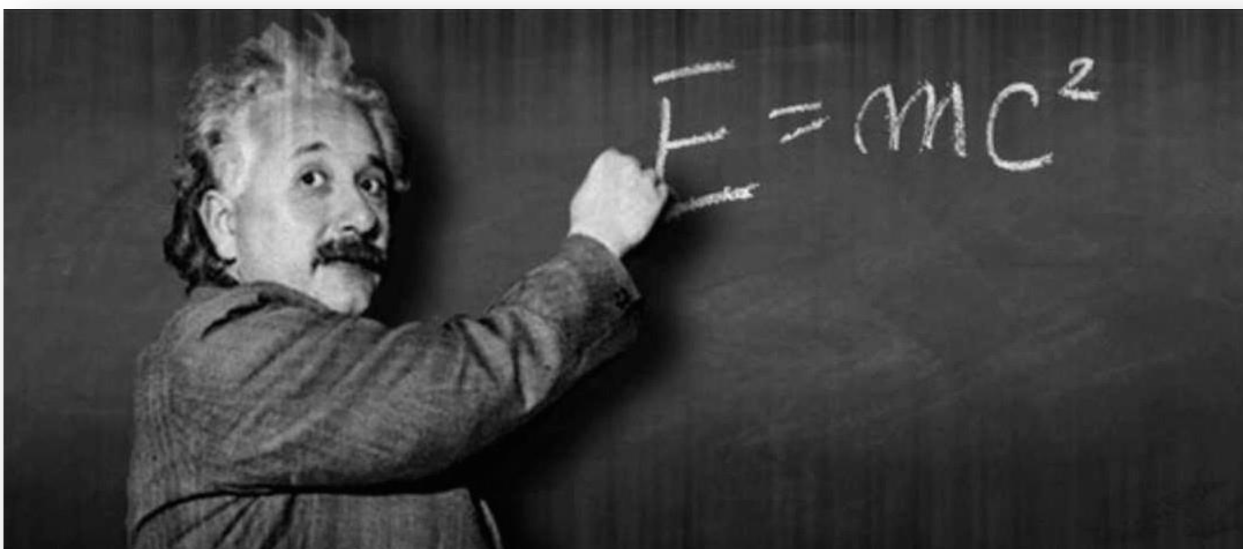
This denies the basic definition of photons given by Einstein that **Light moves as small packets of energy** which are known as **photons**.

So we conclude here that,

Einstein's Relative Mass formula is incorrect.

CONCLUSION:

- Mass is relative only in isolated systems.
- Law of conservation of mass is just another form/name of Law of conservation of energy and is always valid.
- Relativistic Mass formula is invalid due to some conditions.
    - Recoiling distance of gun is never equal to distance travelled by the photon pulse.
    - Change in velocity is considered as zero as we assumed non accelerated linear motion.
- Photons moves with speed equal to speed of light and have rest mass equal to zero which then gives Relative Mass of photons as indeterminate form. It shows <u>Einstein's Relativistic Mass formula is incorrect.</u>



REFERENCE:

- Talal Al-Ameen, THE CONTROVERSIAL ISSUE OF RELATIVISTIC MASS AND ITS IMPACT BEYOND SCIENCE, Proceedings of INTCESS 2019- 6th International Conference on Education and Social Sciences, 4-6 February 2019- Dubai, U.A.E.
- en.wikipedia.org