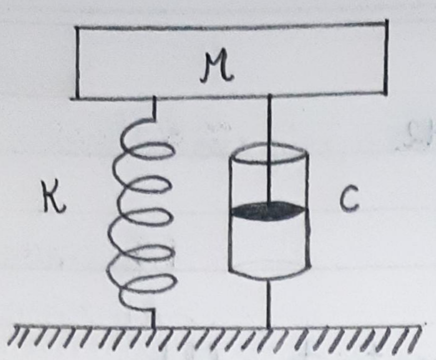

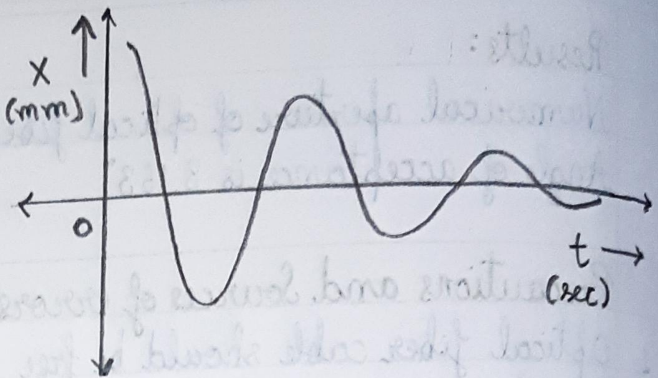

Observations :

S.no.	x_0 (mm)	M (kg)	K (N/m)	t (s)	C_c (Ns/m)	C (Ns/m)	ω_n (rad/s)	ω_d (rad/s)	Amplitude & time	
									1^{st} pt.	2^{nd} pt.
1.	7	7	113	8	56.2496	9	4.017817	3.9660549	(1.6, 2.52)	(3.2, 0.91)
2.	5	8	147	11	68.5857	13	4.2866070	4.2089005	(1.5, 1.49)	(3.0, 0.44)
3.	4	6	191	12	67.7053	11	5.6421036	5.5671406	(1.1, 1.4)	(2.3, 0.49)

Calculations :

for 1^{st} wave,,

pts are (1.6, 2.52) & (3.2, 0.91)

Time period, $T_1 = 3.2 - 1.6 = 1.6\,s$

Logarithmic decrement, $\delta_1 = \ln \frac{2.52}{0.91} \simeq 1.01857$

$$\sqrt{4\pi^2 + \delta_1^2} = \sqrt{4 \times (3.14)^2 + (1.01857)^2} = 6.36521$$

Damping factor, $\xi_1 = \frac{\delta_1}{\sqrt{4\pi^2 + \delta_1^2}} = \frac{1.01857}{6.36521} = 0.16002143$

$$C_1 = 2\xi_1\sqrt{K_1 M_1} = 2 \times 0.16002143 \sqrt{113 \times 7} = 9.0011165 \text{ Ns/m}$$

Experiment 7.

Aim:
To determine the damping coefficient and damped and undamped natural frequency of an under damped single degree of freedom system from its response to an initial displacement.

Apparatus:
Spring mass damper system at bottom.

Theory:
In a usual spring mass damper arrangement, the spring & damper are joined together & connected to the ground & rigid mass M is hanged to spring in a vertical direction.
Since there is only one mass & one probable direction of motion (vertical motion), the system is a single degree of freedom system. The damping will be assumed to be viscous ie. resistance of the motion is proportional to velocity of mass of the system.
Because of resistance to motion, energy is continuesly dissipated and free vibrations of such systems comes to halt after some time. This is called the damped vibrations & such systems are called damped systems.
When the resistance is offered by a damping element proportional to the velocity of mass of system, it is termed as viscous damping.
System without damping elements are undamped system.
Let this type of SDOF system be acted upon by a harmonic force ie value of force varies wrt time following the equation,,
$f = F_0 \sin \omega t$ or, $f = F_0 \cos \omega t$

$$\omega_{n_1} = \frac{\sqrt{4\pi^2+\delta_1^2}}{T_1} = \frac{6.36521}{1.6} = 3.97825625 \text{ rad s}^{-1}$$

$$\omega_{d_1} = \omega_n\sqrt{1-\xi_1^2} = 3.97825625\sqrt{1-(0.16002143)^2} = 3.92699 \text{ rad s}^{-1}$$

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for 2^{nd} wave,

pts are (1.5, 1.49) & (3, 0.44)

Time period, $T_2 = 3-1.5 = 1.5$ s

Logarithmic decrement, $\delta_2 = \ln\frac{1.49}{0.44} \simeq 1.2197567$

$$\sqrt{4\pi^2+\delta_2^2} = \sqrt{4\times(3.14)^2+(1.2197567)^2} = 6.4005$$

Damping factor, $\xi_2 = \frac{1.2197567}{6.4005} = 0.1905721$

$$C_2 = 2\times 0.1905721\sqrt{147\times 8} = 13.0705233 \text{ Ns/m}$$

$$\omega_{n_2} = \frac{6.4005}{1.5} = 4.267 \text{ rad s}^{-1}$$

$$\omega_{d_2} = 4.267\sqrt{1-(0.1905721)^2} = 4.18879 \text{ rad s}^{-1}$$

---

for 3^{rd} wave,

pts are (1.1, 1.4) & (2.3, 0.49)

Time period, $T_3 = 2.3-1.1 = 1.2$ s

Logarithmic decrement, $\delta_3 = \ln\frac{1.4}{0.49} \simeq 1.049822$

$$\sqrt{4\pi^2+\delta_3^2} = \sqrt{4\times(3.14)^2+(1.049822)^2} = 6.3703$$

Damping factor, $\xi_3 = \frac{1.049822}{6.3703} = 0.1648$

$$C_3 = 2\times 0.1648\sqrt{191\times 6} = 11.157824 \text{ Ns/m}$$

$$\omega_{n_3} = \frac{6.3703}{1.2} = 5.3085833 \text{ rad s}^{-1}$$

$$\omega_{d_3} = 5.3085833\sqrt{1-(0.1648)^2} = 5.2359877 \text{ rad s}^{-1}$$

Formula used:

$$\omega_n = \sqrt{\frac{K}{M}} \quad , \quad \omega_n = \frac{\sqrt{4\pi^2 + \delta^2}}{T}$$

where ω_n is natural frequency in radians per second.

Damping natural frequency, $\omega_d = \omega_n \sqrt{1 - \xi^2}$

Damping factor, $\xi = \frac{C}{C_c} = \frac{C}{2\sqrt{KM}}$

for under damped ie. $\xi << 1$, $\xi \approx \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$

Logarithmic decrement, $\delta = \log_e \frac{A_1}{A_2}$

where A_1 is amplitude of one cycle

A_2 is amplitude of consecutively next cycle.

Procedure:

1. Use the simulator & enter the values X, K, M & C by provided numerical values. Click run botton like releasing the mass that was presses through X m.
2. Observe X v/s t ie. amplitude is exponentially decreasing with t. Time taken to complete 1 cycle ie. time period T is constant.
3. Measure time period T.
4. Calculate logarithmic decrement (δ) by measuring A_1 & A_2.
5. Substitute values of δ & T & find ω_d & ξ.
6. Repeat the above mentioned steps further.
7. Use the values of ω_d & ξ & find ω_n & C.

Error estimation:

S.no.	% error in C	% error in ω_n	% error in ω_d
for (i)	$\frac{9.0011165 - 9}{9} \times 100$ $= 0.0124\%$	$\frac{(4.01781746 - 3.97825625) \times 100}{4.01781746}$ $= 0.98464428\%$	$\frac{(3.966054946 - 3.92699) \times 100}{3.966054946}$ $= 0.98498247\%$
for (ii)	$\frac{13.0705233 - 13}{13} \times 100$ $= 0.5425\%$	$\frac{(4.286607050 - 4.267) \times 100}{4.286607050}$ $= 0.45740255\%$	$\frac{(4.208900539 - 4.18879) \times 100}{4.208900539}$ $= 0.47780979\%$
for (iii)	$\frac{11.157824 - 11}{11} \times 100$ $= 1.43476\%$	$\frac{(5.642103627 - 5.30858333) \times 100}{5.642103627}$ $= 5.91127558\%$	$\frac{(5.567140698 - 5.2359877) \times 100}{5.567140698}$ $= 5.94834972\%$
Average errors	$\frac{1.43476 + 0.5425 + 0.0124}{3}$ $= 0.66322\%$	$\frac{0.98464428 + 0.45740255 + 5.91127558}{3}$ $= 2.45110747\%$	$\frac{0.98498247 + 0.47780979 + 5.94834972}{3}$ $= 2.47038066\%$

Results:

Damping Coefficient (C)

for 1st case, $C_1 = 9.0011165$ Ns/m ≈ 9 Ns/m

2nd case, $C_2 = 13.0705233$ Ns/m ≈ 13 Ns/m

3rd case, $C_3 = 11.157824$ Ns/m ≈ 11.16 Ns/m

Natural frequency (ω_n)

for 1st case, $\omega_{n_1} = 3.97825625$ rad $s^{-1} \approx 3.98$ rad s^{-1}

2nd case, $\omega_{n_2} = 4.267$ rad s^{-1}

3rd case, $\omega_{n_3} = 5.3085833$ rad $s^{-1} \approx 5.31$ rad s^{-1}

Damping frequency (ω_d)

for 1st case, $\omega_{d_1} = 3.92699$ rad $s^{-1} \approx 3.93$ rad s^{-1}

2nd case, $\omega_{d_2} = 4.18879$ rad $s^{-1} \approx 4.19$ rad s^{-1}

3rd case, $\omega_{d_3} = 5.2359877$ rad $s^{-1} \approx 5.24$ rad s^{-1}

Errors in estimating (avg)

(i) Damping Coefficient, $\% e_c = 0.66322\% \approx 0.66\%$

(ii) Natural frequency, $\% e_{\omega_n} = 2.45110747\% \approx 2.45\%$

(iii) Damping frequency, $\% e_{\omega_d} = 2.47038066\% \approx 2.47\%$

Precautions & Sources of error:

1. Changing the value of coefficient of damping on screen, it will change the value of damping factor.
2. The rate of decay of amplitude increases with an increase in the value of damping factor.