MAT436 Lecture Notes

Arky!! :3c

'25 Fall Semester

Contents

§1 Day 1: Inaudible Lecture (Sep. 3, 2025)

Attendance won't be taken every lecture. This class is glorified linear algebra. Otherwise, incomprehensible lecture (professor speaks quieter than the air conditioner). This class is truly a test if you have tinnitus or not.

Throughout this class, we will use H to refer to a Hilbert space, U a unitary operator (where $U^* = U^{-1}$), and $T \in B(H)$ a bounded operator on H. In particular, we have that $(UTU^{-1})^* = UT^*U^{-1}$.

Exercise from class. Let $S \subseteq H$. If S is a closed linear subspace, show that $S^{\perp\perp} = S$ and that S admits an orthogonal complement. Is this a sufficient and necessary condition?

§2 Day 2: (Sep. 5, 2025)

Exercise 2.1. How many diagonals are there in a parallelepiped in an arbitrary vector space?

When we started talking about the projection theorem, which is one of the properties of Hilbert spaces, i.e., for every closed linear subspace, there is a complementary linear subspace where the intersection is 0 and the sum is the whole space.

Any Hilbert space has an orthonormal basis.

Exercise 2.2. Hilbert spaces can be vector spaces. Given a Hilbert space basis, if it is not finite dimensional, then this cannot be a vector space basis.

Let H be a hilbert space, and let K be a closed subspace of H. We call $K^\perp$ the *orthogonal complement*, consisting of the vectors perpendicular to all vectors in K. Is it true that $H = K + K^\perp$? Note that even though $K \cap K^\perp = \{0\}$, we do not use the direct sum here; we refer to $+$ as the internal direct sum. $\oplus$ is reserved for external direct sums, if we wish to create a third Hilbert space from two separate Hilbert spaces.

Let H be a Hilbert space, and let S be a closed and convex subset of H, and let $x \in H$. There exists a unique closest point of S to x. It can be proven using the parallelogram law. Similarly, given $H = K + K^\perp$ and $x \in H$, there exists a closest point to x in K, which we shall call y. Then we have that $x - y \in K^\perp$.