

MAT482 Lecture Notes

ARKY!! :3C

'24 Fall Semester

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§1 Day 1: Introduction to Class (Sep. 3, 2024)

Class administration notes;

- Prof. Ila (she prefers to be called Ila) will be in Montreal once in a while.
- Masks should be worn if attending lectures in person.
- All reference material for the class can be found on [here](#), or in the UofT library.
- This class will be held in a more experimental teaching style; specifically with the Tuesday discussions.
- Prof. Ila prefers to be contacted on Zulip instead of mail.

To start, this class is on arithmetic statistics, which studies “arithmetic objects.” Examples of such objects interesting from a number theory perspective include

- Fields, specifically finite extensions of \mathbb{Q} (number fields),
- Binary quadratic forms, i.e. $f(x, y) = ax^2 + bxy + cy^2$,
- Varieties over \mathbb{Z} , i.e. zero sets of polynomials with integer coefficients,
- Ideal class groups,
- Primes.

Composition laws can be described as equipping a set with group operations; for example, let us consider the $\mathrm{SL}_2(\mathbb{Z})$ -equivalence classes of binary quadratic forms; given $\gamma \in \mathrm{SL}_2(\mathbb{Z})$, we have $\gamma f(x, y) = f((x, y)\gamma)$.

Exercise 1.1. If $\gamma \in \mathrm{SL}_2(\mathbb{Z})$, prove that $\mathrm{disc}(f(x, y)) = \mathrm{disc}(\gamma \cdot f(x, y))$. Specifically, the discriminant of $\mathrm{disc}(ax^2 + bxy + cy^2) = b^2 - 4ac$.

Exercise 1.2. Any polynomial Δ in a, b, c satisfying $\Delta(f(x, y)) = \Delta(\gamma \cdot f(x, y))$ is a multiple of the discriminant, or is constant.