1) $f(x) = x^3 - x^2 - x + 1$ | Newton

$x_0 = -0{,}2$; $x_0 = -0{,}3$

$\varepsilon = 0{,}01$

$f'(x) = 3x^2 - 2x - 1$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \iff x_1 = -0{,}2 - \frac{(-0{,}2)^3 - (0{,}2)^2 - (-0{,}2) + 1}{3 \cdot (-0{,}2)^2 - 2 \cdot (-0{,}2) - 1} =$

$= -0{,}2 - \left(-\frac{12}{5}\right) = -0{,}2 + 2{,}4 = 2{,}2$

$|x_1 - x_0| = 2{,}2 + 0{,}2 = 2{,}4 > \varepsilon$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \iff x_2 = 2{,}2 - \frac{2{,}2^3 - 2{,}2^2 - 2{,}2 + 1}{3 \cdot (2{,}2)^2 - 2 \cdot 2{,}2 - 1} =$

$= 2{,}2 - \frac{48}{95} = 1{,}6947$

$|x_2 - x_1| = |1{,}6947 - 2{,}2| = 0{,}5053 > \varepsilon$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \iff x_3 = 1{,}6947 - \frac{1{,}3004}{4{,}2266} = 1{,}3870$

$|x_3 - x_2| = |1{,}3870 - 1{,}6947| = 0{,}3077 > \varepsilon$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} \iff x_4 = 1,3870 - \frac{0,3574}{1,9973} = 1,2080$$

$$|x_4 - x_3| = |1,2080 - 1,3870| = 0,1790 > \varepsilon$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} \iff x_5 = 1,2080 - \frac{0,0955}{0,9617} = 1,1087$$

$$|x_5 - x_4| = |1,1087 - 1,2080| = 0,0993 > \varepsilon$$

$$x_6 = x_5 - \frac{f(x_5)}{f'(x_5)} \iff x_6 = 1,1087 - \frac{0,0249}{0,4702} = 1,0557$$

$$|x_6 - x_5| = |1,0557 - 1,1087| = 0,0530 > \varepsilon$$

$$x_7 = 1,0282$$

$$|x_7 - x_6| = |1,0282 - 1,0557| = 0,0274 > \varepsilon$$

$$x_8 = 1,0142$$

$$|x_8 - x_7| = |1,0142 - 1,0282| = 0,1401 > \varepsilon$$

$$\underline{x_9 = 1,0071}$$

$$|x_9 - x_7| = |1,0071 - 1,0142| = \underline{0,0071 < \varepsilon}$$

Pentru $x_0 = -0,2$ soluția este: 1,0071 (9 i)

--- Pentru $x_0 = -0,3$ soluția este: 1,0069 (13 i)

2) $f(x) = 8x^3 + x^2 + 8x - 3$ | Secanta

$x_0 = 0{,}0$; $x_1 = 0{,}6$

$\varepsilon = 0{,}01$

$$x_2 = \frac{x_0 \cdot f(x_1) - x_1 \cdot f(x_0)}{f(x_1) - f(x_0)} \Leftrightarrow x_2 = \frac{0{,}0 \cdot 3{,}888 - 0{,}6 \cdot (-3)}{3{,}888 - (-3)} = 0{,}2613$$

$$|x_2 - x_1| = |0{,}2613 - 0{,}6| = 0{,}3387 > \varepsilon$$

$$x_3 = \frac{x_1 \cdot f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)} \Leftrightarrow x_3 = \frac{0{,}6 \cdot (-0{,}6985) - 0{,}2613 \cdot 3{,}888}{-0{,}6985 - 3{,}888} =$$

$$= 0{,}3128$$

$$|x_3 - x_2| = |0{,}3128 - 0{,}2613| = 0{,}0515 > \varepsilon$$

$$x_4 = \frac{x_2 \cdot f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)} \Leftrightarrow x_4 = \frac{0{,}2613 \cdot (-0{,}1549) - 0{,}3128 \cdot (-0{,}6985)}{-0{,}1549 - (-0{,}6985)} =$$

$$= 0{,}3274 > \varepsilon$$

$$|x_4 - x_2| = |0{,}3274 - 0{,}3128| = 0{,}0146 > \varepsilon$$

$$x_5 = \frac{x_3 \cdot f(x_4) - x_4 \cdot f(x_3)}{f(x_4) - f(x_3)} \Leftrightarrow$$

$$\Leftrightarrow x_5 = \frac{0{,}3128 \cdot 0{,}0041 - 0{,}3274 \cdot (-0{,}1549)}{-0{,}0041 - (-0{,}1549)} = 0{,}3206$$

$$|x_5 - x_4| = |0{,}3206 - 0{,}3274| = 0{,}0068 < \varepsilon$$

Pentru $x_0 = 0{,}0$ și $x_1 = 0{,}6$ soluția este: $0{,}3206$ (5 i)

3) $f(x) = x^3 - 4x + 2$ | Bisecție

$a = 0$; $b = 1$ => $l = 0$; $u = 1$

$\varepsilon = 0,01$

① $x_m = \frac{l+u}{2} \Leftrightarrow x_m = \frac{0+1}{2} = 0,5$

$|f(x_m)| = |0,5^3 - 4 \cdot 0,5 + 2| = 0,125 > \varepsilon$

$f(x_m) \cdot f(l) = 0,125 \cdot (0^3 - 4 \cdot 0 + 2) = 0,25 > 0 \Rightarrow$

$\Rightarrow l = x_m = 0,5$

II $x_m = \frac{l+u}{2} \Leftrightarrow x_m = \frac{0,5+1}{2} = 0,75$

$|f(x_m)| = |0,75^3 - 4 \cdot 0,75 + 2| = 0,5781 > \varepsilon$

$f(x_m) \cdot f(l) = -0,5781 \cdot (0,5^3 - 4 \cdot 0,5 + 2) =$

$= -0,0722 < 0 \Rightarrow u = x_m = 0,75$

III $x_m = \frac{l+u}{2} \Leftrightarrow x_m = \frac{0,5+0,75}{2} = 0,625$

$|f(x_m)| = |0,625^3 - 4 \cdot 0,625 + 2| = 0,2558 > \varepsilon$

$f(x_m) \cdot f(l) = -0,2558 \cdot (0,5^3 - 4 \cdot 0,5 + 2) =$

$= -0,0319 < 0 \Rightarrow u = x_m = 0,625$

IV $x_m = \frac{l+u}{2} \Leftrightarrow x_m = \frac{0,5+0,625}{2} = 0,5625$

$|f(x_m)| = |0,5625^3 - 4 \cdot 0,5625 + 2| = 0,0720 > \varepsilon$

$f(x_m) \cdot f(l) = -0,0720 \cdot (0,5^3 - 4 \cdot 0,5 + 2) =$

$= -0,009 < 0 \Rightarrow u = x_m = 0,5625$

(v) $x_m = \frac{l+u}{2} \Leftrightarrow x_m = \frac{0,5+0,5625}{2} = 0,5312$

$|f(x_m)| = |0,5312^3 - 4\cdot 0,5312 + 2| = 0,1573 > \varepsilon$

$f(x_m)\cdot f(l) = 0,0250 \cdot (0,5^3 - 4\cdot 0,5 + 2) =$

$= 0,0031 > 0 \Rightarrow l = x_m = 0,5312$

$x_m = \frac{l+u}{2} \Leftrightarrow x_m = \frac{0,5312+0,5625}{2} = 0,54685$

$|f(x_m)| = |0,54685^3 - 4\cdot 0,54685 + 2| = 0,0238 > \varepsilon$

$f(x_m)\cdot f(l) = -0,0238 \cdot (0,5312^3 - 4\cdot 0,5312 + 2) =$

$= -0,0005 < 0 \Rightarrow u = x_m = 0,54685$

--- Pentru $a=0$ și $b=1$ soluția este: $0,5391$ (8 i)
unde $|f(x_m)| \leq \varepsilon$

4) $f(x) = x^3 - 4x + 2$ | Newton – Raphson

$x_1 = 1;$

$f'(x) = 3x^2 - 4$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{-1}{-1} = 0$

5) $f(x) = x^3 - 4x + 2$ | Secantă

$x_0 = 0;\ x_1 = 1$

$x_2 = \frac{x_0\cdot f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \frac{\cancel{0}\cdot(1^3 - 4\cdot 1 + 2) - 1\cdot(0^3 - 4\cdot 0 + 2)}{(1^3 - 4\cdot 1 + 2) - (0^3 - 4\cdot 0 + 2)} =$

$= \frac{-2}{-3} = 0,6$

$x_3 = \frac{x_1\cdot f(x_2) - x_2\cdot f(x_1)}{f(x_2) - f(x_1)} = \frac{1\cdot(0,6^3 - 4\cdot 0,6 + 2) - 0,6\cdot(1^3 - 4\cdot 1 + 2)}{(0,6^3 - 4\cdot 0,6 + 2) - (1^3 - 4\cdot 1 + 2)} =$

$= \frac{26}{51} = 0,5098$