Tema curs 04

a) $y' - \frac{4y}{x} = x\sqrt{x}$; $y \geq 0$ și $x \neq 0$

ecuație neomogenă $y' + P(x)\cdot y + Q(x) = 0$

$y' + \left(-\frac{4}{x}\right)y - x\sqrt{x} = 0$

$P(x) = -\frac{4}{x}$ $\quad Q(x) = -x\sqrt{x}$

ecuația omogenă $y' + P(x)\cdot y = 0$

$y' + \left(-\frac{4}{x}\right)y = 0$

$y' - \frac{4}{x}\cdot y = 0 \rightarrow y' = \frac{4}{x}y \,\Big|\cdot \frac{1}{y}$; p.p. $y \neq 0$

$\Rightarrow \frac{y'}{y} = \frac{4}{x}$ | integrăm

$\Rightarrow \int \frac{y'}{y}dx = 4\int \frac{1}{x}dx + \ln C$ (așa e în curs)

$\ln y = 4\cdot \ln(x) + \ln C$

$\ln y = \ln(x^4\cdot C)$

$y = C\cdot x^4$ soluția ec. omogenă

$y(x) = C(x)\cdot x^4$

$y'(x) = C'(x)\cdot x^4 + 4x^3 C(x)$

înlocuim în ec neomogenă $y' - \frac{4}{x}y - x\sqrt{x} = 0$

$C'(x)\cdot x^4 + \cancel{4x^3 C(x)} - \cancel{\frac{4}{x}\cdot x^4 \cdot C(x)} - x\sqrt{x} = 0$

$x^4\cdot C'(x) - x\sqrt{x} = 0$

$C'(x) = \frac{x\sqrt{x}}{x^4} = \frac{\sqrt{x}}{x^3} = x^{\frac{1}{2}-3} = x^{-\frac{5}{2}}$

$\Rightarrow C(x) = \int x^{-\frac{5}{2}}dx + K = \frac{x^{-\frac{5}{2}+\frac{2}{2}}}{-\frac{5}{2}+\frac{2}{2}} + K = \frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} + K = -\frac{2}{3}\cdot\frac{1}{\sqrt{x^3}} + K = \frac{-2}{3}\cdot\frac{1}{x\sqrt{x}}$ $+K$

$y(x) = C(x)\cdot x^4$

$y(x) = \left(-\frac{2}{3}\cdot\frac{1}{x\sqrt{x}} + K\right)x^4 = K\cdot x^4 - \frac{2}{3}\cdot\frac{x^3}{\sqrt{x}} = K\cdot x^4 - \frac{2}{3}x^{\frac{6-1}{2}} = K\cdot x^4 - \frac{2}{3}x^{\frac{5}{2}}$ $+K$

~~$y(x) = -\frac{2}{3}\cdot\sqrt{x^5} = -\frac{2}{3}\cdot x^2\sqrt{x}$~~

$y(x) = \underbrace{K\cdot x^4}_{y_0} - \underbrace{\frac{2}{3}x^2\sqrt{x}}_{y_p}$

b) $y' = \frac{y}{x} - 2xy^2$

Obs. $y=0$ soluție
Pp. $y \neq 0$

Sol Cauchy $y(1)=1$

ecuație Bernoullie $y' + P(x)y \pm Q(x)y^{\alpha}$ cu $\alpha \neq 1$ și $\alpha \neq 0$

$y' - \frac{y}{x} = -2xy^2 \;\Big|\cdot \frac{1}{y^2}$, $\alpha=2$ împărțim la $\frac{1}{y^{\alpha}}$

$$\frac{y'}{y^2} - \frac{1}{x}\cdot\frac{y}{y^2} = -2x$$

$$\frac{y'}{y^2} - \frac{1}{x}\cdot\frac{1}{y} = -2x$$

notăm $z = \frac{1}{y^{\alpha-1}}$, $\alpha = 2$

$\Rightarrow z = \frac{1}{y}$

* $\boxed{z = \frac{1}{y}}$

calculam z'

$$z' = \left(\frac{1}{y}\right)' = (y^{-1})' = -1\cdot y^{-2}\cdot y' = -\frac{1}{y^2}\cdot y'$$

$\boxed{z' = \frac{-y'}{y^2}} \Rightarrow \frac{y'}{y^2} = -z'$ calculam $\frac{y'}{y^{\alpha}}$ în funcție de z'

avem ecuatia ~~neomogena~~ $\frac{y'}{y^2} - \frac{1}{x}\cdot\frac{1}{y} + 2x = 0$

înlocuim cu z' si z (dupa caz)

$$-z' - \frac{1}{x}\cdot z + 2x = 0 \;\Big|(-1)$$

$\boxed{z' + \frac{1}{x}z - 2x = 0}$ $y' + P(x)y + Q(x) = 0$

↳ am obț. o ecuație neomogenă

ecuație omogena $y' + P(x)y = 0$

$$z' + \frac{1}{x}\cdot z = 0$$

$$z' = -\frac{1}{x}\cdot z \;\Big|\cdot\frac{1}{z}$$

$$\frac{z'}{z} = -\frac{1}{x} \;\Big|\text{ integrăm}$$

$$\int \frac{z'}{z}\,dx = -\int\frac{1}{x}\,dx + \ln C$$

$$\ln z = -\ln x + \ln C$$

$$\ln z = \ln\frac{C}{x} \Rightarrow \boxed{z = \frac{C}{x}}$$

$\boxed{z(x) = \frac{C(x)}{x}}$

$$z'(x) = \frac{C'(x)\cdot x - x'\cdot C(x)}{x^2} = \frac{C'(x)}{x} - \frac{C(x)}{x^2}$$

înlocuim în ec. neomogenă

Lipsește sol. pb. Cauchy!

$$\frac{C'(x)}{x} - \frac{C(x)}{x^2} + \frac{1}{x}\cdot\frac{C(x)}{x} - 2x = 0$$

$$\frac{C'(x)}{x} - \cancel{\frac{C(x)}{x^2}} + \cancel{\frac{C(x)}{x^2}} - 2x = 0$$

$$\frac{C'(x)}{x} = 2x \;/\cdot x$$

$$C'(x) = 2x^2 \;/\text{integrăm}$$

$$\int C'(x)\,dx = 2\int x^2 dx$$

$$C(x) = 2\cdot\frac{x^3}{3} + K = \frac{2}{3}\cdot x^3 + K$$

$z = \frac{1}{y} \Rightarrow$

$$\frac{1}{y} = \frac{2}{3}x^2 + K$$

$$\frac{1}{y} = \frac{2x^2+3K}{3}$$

$$(3K+2x^2)y = 3$$

$$y = \frac{3}{2x^2+3K}$$

$$z(x) = \frac{C(x)}{x} = \frac{1}{x}\cdot\left(\frac{2}{3}\cdot x^3 + K\right) = x^2\cdot\frac{2}{3} + K\cdot\frac{1}{x}$$ soluție implicită

$$z(x) = \frac{2}{3}x^2 + \frac{K}{x} = \frac{K}{x} + \frac{2}{3}x^2 = \frac{3K+2x^3}{3x}$$

$$\frac{1}{y} = \frac{3K+2x^3}{3x} \Rightarrow y = \frac{3x}{3K+2x^3}$$ soluții sub forma implicită

c) $xy' + y = -x^2y^2 \;\Big|\cdot\frac{1}{x}$ ecuația Bernoullie

$y' + \frac{1}{x}\cdot y = -x\cdot y^2 \;\Big|\cdot\frac{1}{y^2}$

Obs $y=0$ soluție

$y' + P(x)y = Q(x)\cdot y^\alpha$ cu $\alpha \neq \{0,1\}$

Pp $y \neq 0$

$$\frac{y'}{y^2} + \frac{1}{x}\cdot\frac{1}{y} = -x$$

→ împărțim la y^α

$$\boxed{\text{Sol Cauchy } y(1)=1}$$

$$\boxed{\text{notam } z = \frac{1}{y}}$$

notam $z = \frac{1}{y^{\alpha-1}}$

$$z' = (y^{-1})' = -\frac{1}{y^2}\cdot y' = \frac{-y'}{y^2}$$

calculam z'

$$\frac{y'}{y^2} = -z'$$

calculam $\frac{y'}{y^\alpha}$ în functie de z'

înlocuim în ecuația $\frac{y'}{y^2} + \frac{1}{x}\cdot\frac{1}{y} + x = 0$

$$-z' + \frac{1}{x}z + x = 0 \;/(-1)$$

$$z' + \left(-\frac{1}{x}\right)z - x = 0$$ ecuatie neomogena $y' + P(x)y + Q(x) = 0$

ecuatia omogenă $y' + P(x)y = 0$

avem $z' + \left(-\frac{1}{x}\right)z = 0$ ecuatie omogenă asociată ec. neomogene

$$z' = \frac{1}{x}z \quad \Big| \cdot \frac{1}{z}$$

$$\frac{z'}{z} = \frac{1}{x} \quad \text{integrăm}$$

$$\int \frac{z'}{z}dx = \int \frac{1}{x}dx$$

$$\ln(z) = \ln(x) + \ln C$$

$$\ln z = \ln(x \cdot C)$$

$$\boxed{z = x \cdot C} \Rightarrow \boxed{z(x) = x \cdot C(x)}$$

sol. ec. omogenă

calculăm $z'(x)$

$$z'(x) = x' \cdot C(x) + x \cdot C'(x) = C(x) + x \cdot C'(x)$$

înlocuim în ecuatia neomogenă $z' - \frac{1}{x} \cdot z - x = 0$

$$C(x) + x \cdot C'(x) - \frac{1}{x} \cdot x \cdot C(x) - x = 0$$

$$\cancel{C(x)} + x\,C'(x) - \cancel{C(x)} - x = 0$$

$$x\,C'(x) - x = 0$$

$$C'(x) = \frac{x}{x} = 1 \quad \Big|\ \text{integrau}$$

$$C(x) = \int dx$$

$$C(x) = x + K \Rightarrow z(x) = x \cdot C(x)$$

$$\hookrightarrow z(x) = x \cdot (x + K)$$

$$z = \frac{1}{y} \Rightarrow \frac{1}{y} = x\overbrace{(x+K)}^{C(x)} \Rightarrow y \cdot x(x+K) = 1$$

$$\hookrightarrow y = \frac{1}{x(x+K)} \quad \text{solutie implicită}$$

$$y(1) = 1 \rightarrow \frac{1}{1(1+K)} = 1 \Rightarrow \frac{1}{1+K} = 1 \rightarrow K = 0 \Longrightarrow y = x^2 \text{ este sol a pb CAUCHY}$$

d) $2x^2y' - 4xy = y^2 \quad \Big| : 2x^2$ ec. Bernoulli $y' + P(x)y = Q(x)y^{\alpha}$

$$2x^2y' - 4xy = y^2 \quad \Big| \cdot \frac{1}{y^2} \quad \text{împărtim la } y^{\alpha}$$

Sol Cauchy $y(1) = 1$

$$2x^2 \cdot \frac{y'}{y^2} - 4x \cdot \frac{1}{y} = 1 \quad \Big| \cdot \frac{1}{x^2}$$

$$2\frac{y'}{y^2} - \frac{4}{x} \cdot \frac{1}{y} = \frac{1}{x^2} \quad \Big| \cdot \frac{1}{2}$$

$$y' - 2\frac{y}{x} = \frac{1}{2x^2} \cdot y^2 \ \Big| : y^2 \Rightarrow \frac{y'}{y^2} - \frac{2}{x} \cdot \frac{1}{y} - \frac{1}{2x^2} = 0$$

$C'(x) = -\frac{1}{2}$

$\frac{y'}{y^2} - \frac{2}{x} \cdot \frac{1}{y} = \frac{1}{2x^2}$

ec. neliniara $y' + P(x)y + Q(x) = 0$

notăm $\boxed{z = \frac{1}{y}}$ $\rightarrow z'(= -1 \cdot \frac{1}{y^2} \cdot y' = -\frac{y'}{y^2}$

$\hookrightarrow \boxed{\frac{y'}{y^2} = -z'}$

înlocuim cu z' si z după caz

$-z' - \frac{2}{x} \cdot z = \frac{1}{2x^2}$

$-z' - \frac{2}{x} z - \frac{1}{2x^2} = 0 \;\Big|(-1)$

$z' + \frac{2}{x} z + \frac{1}{2x^2} = 0 \Rightarrow$ obt ec. neomogenă

ec. omogenă $y' + P(x)y = 0$

$\Rightarrow z' + \frac{2}{x} z = 0 \Rightarrow z' = -\frac{2}{x} z \;\Big| \cdot \frac{1}{z} \Rightarrow \frac{z'}{z} = -\frac{2}{x}$

$\longrightarrow$ integrând obtinem $\int \frac{z'}{z} dx = -2 \int \frac{1}{x} dx$

$\ln z = -2 \ln x + \ln C$

$\ln z = \ln x^{-2} + \ln C$

$\ln z = \ln \frac{C}{x^2} \Rightarrow \boxed{z = \frac{C}{x^2}}$ → sol ec. omogenă

$z(x) = \frac{C(x)}{x^2}$ calculam z'

$z'(x) = \frac{C'(x) \cdot x^2 - 2x \cdot C(x)}{x^4} = \frac{1}{x^2} \cdot C'(x) - \frac{2}{x^3} \cdot C(x)$

înlocuim în ecuatia neomogenă

$\frac{1}{x^2} \cdot C'(x) - \frac{2}{x^3} C(x) + \frac{2}{x} \cdot \frac{C(x)}{x^2} = -\frac{1}{2x^2}$

$\frac{1}{x^2} \cdot C'(x) - \cancel{\frac{2}{x^3} C(x)} + \cancel{\frac{2}{x^3} C(x)} + \frac{1}{2x^2} = 0$

$\frac{1}{x^2} C'(x) = -\frac{1}{2} \cdot \frac{1}{x^2}$

$\frac{C'(x)}{x^2} = \frac{-1}{2x^2} \;\Big| \cdot x^2$

$C'(x) = -\frac{1}{2}$

$C(x) = -\int \frac{1}{2} dx = -\frac{1}{2}x + K$

$z = \frac{C(x)}{x^2} = \frac{1}{x^2} \cdot \left(-\frac{1}{2}x + \frac{2K}{2}\right)$

$z = \frac{C(x)}{x^2} = \frac{1}{x^2} \cdot \frac{2K-x}{2}$

$$\left.\begin{array}{l} z = \frac{2K-x}{2x^2} \\ z = \frac{1}{y} \end{array}\right\} \frac{1}{y} = \frac{2K-x}{2x^2} \rightarrow y(2K-x) = 2x^2$$

$y = \frac{2x^2}{2K-x}$ sol. implicită

$y(1) = 1 \rightarrow \frac{2}{2K-1} = 1 \Rightarrow 2 = 2K-1 \Rightarrow 2K = 3 \rightarrow K = \frac{3}{2}$

$\Rightarrow y = \frac{2x^2}{3-x}$ este sol. a probl. Cauchy

e) $y' + \frac{2}{x}y = \frac{\ln x}{y^3}$ ec Bernoulli

$y' + P(x)y = Q(x)y^{\alpha}$

$y' + \frac{2}{x}y = \ln x \cdot y^{-3} \,|\cdot y^3$

$y^3 \cdot y' + \frac{2}{x} \cdot y^4 = \ln x$

$\frac{y'}{y^{-3}} + \frac{2}{x} \cdot \frac{1}{y^{-4}} = \ln x$

notam $\boxed{z = \frac{1}{y^{-4}}}$ calculam z'

$z' = (y^4)' = 4 \cdot y^3 \cdot y' = 4 \cdot \frac{y'}{y^{-3}}$

$\boxed{z' = 4 \cdot \frac{y'}{y^{-3}}} \rightarrow \frac{y'}{y^{-3}} = \frac{z'}{4}$

înlocuim în ecuație

$\frac{z'}{4} + \frac{2}{x} \cdot z - \ln x = 0 \,|\cdot 4$ ec neomogena $y' + P(x)y + Q(x) = 0$

$z' + \frac{8}{x} \cdot z - 4\ln x = 0$

ec. omogena $y' + P(x)y = 0$

$$z' + \frac{8}{x} z = 0$$

$$z' = -\frac{8}{x} \cdot z \rightarrow \frac{z'}{z} = \frac{-8}{x}$$

$$\int \frac{z'}{z} dx = -8 \int \frac{1}{x} dx$$

$$\ln z = -8 \ln x + \ln C$$

$$\ln z = \ln x^{-8} \cdot C$$

$$\boxed{z = \frac{C}{x^8}}$$ ← ec. sol. omogenă

$$\boxed{z(x) = \frac{C(x)}{x^8}}$$

$$z'(x) = \frac{C'(x) \cdot x^8 - 8x^7 \cdot C(x)}{x^{8 \cdot 2}} = \frac{C'(x)}{x^8} - \frac{8}{x^9} C(x)$$

înlocuim în ec. neomogenă

$$\frac{C'(x)}{x^8} - \cancel{\frac{8}{x^9} C(x)} + \cancel{\frac{8}{x} \cdot \frac{C(x)}{x^8}} - 4 \ln x = 0$$

$$\int x^8 \ln x \, dx = \frac{x^9}{9} \ln x - \int \frac{x^{9}}{9} \cdot \frac{1}{x} dx = \frac{x^9}{9} \ln x - \frac{1}{9} \int x^8 dx = \frac{x^9}{9}\left(\ln x - \frac{x}{9}\right) + \mathcal{C}$$

$$\frac{C'(x)}{x^8} - 4 \ln x = 0$$

$$\frac{C'(x)}{x^8} = 4 \ln x$$

$$C'(x) = x^8 \cdot 4 \cdot \ln x$$

$$C(x) = 4 \int x^8 \cdot \ln x \, dx + k = 4\left(\frac{1}{9} x^9 \ln x - \frac{1}{99} \cdot x^{11}\right) + K$$?

$$I = \int x^8 \cdot \ln x \, dx = \ln x \cdot \frac{1}{9} \cdot x^9 - \int x \frac{1}{9} \cdot x^9 dx$$

$$\frac{1}{x} = (\ln x)'$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$f = \ln x \rightarrow f' = x$$

$$g' = x^8 \rightarrow g = \int x^8 dx = \frac{x^9}{9}$$

$$I = \frac{1}{9} x^9 \cdot \ln x - \frac{1}{9} \int x^{10} dx$$

$$I = \frac{1}{9} x^9 \cdot \ln x - \frac{1}{9} \cdot \frac{1}{11} \cdot x^{11}$$

$$C(x) = \frac{4}{9}x^9 \ln x - \frac{4}{99}x^{11} + K = \frac{44x^9\ln x - 4x^{11} + 99K}{99}$$

$$\left.\begin{aligned} z &= \frac{C(x)}{x^8} \\ z &= y^4 \end{aligned}\right\} y^4 = \frac{C(x)}{x^8}$$

$$y^4 = \frac{1}{x^8} \cdot \frac{44x^9\ln x - 4x^{11} + 99K}{99}$$

$$y^4 = \frac{1}{\cancel{x^8}} \cdot \frac{\cancel{x^8}\left(44x\ln x - 4x^3 + \frac{99K}{x^8}\right)}{99}$$

???

$$y^4 = \frac{44x\ln x - 4x^3 + \frac{99K}{x^8}}{99}$$

$$y = \sqrt[4]{\frac{44x\ln x - 4x^3 + \frac{99K}{x^8}}{99}}$$

f) $y' + y \cdot \operatorname{tg} x = y^2 \mid \frac{1}{y^2}$

ec Bernoulli $y' + P(x)y = Q(x) \cdot y^\alpha$

$$\frac{y'}{y^2} + \operatorname{tg} x \cdot \frac{1}{y} = 1$$

notam $z = \frac{1}{y^{\alpha-1}}$

$$\boxed{z = \frac{1}{y}} \quad \boxed{z' = -\frac{y'}{y^2}} \rightarrow \frac{y'}{y^2} = -z'$$

înlocuind $\Rightarrow -z' + \operatorname{tg} x \cdot z = 1$

ec. neomogenă
$y' + P(x)y + Q(x) = 0$

$$-z' + \operatorname{tg} x \cdot z - 1 = 0 \mid (-1)$$

$$z' - z \cdot \operatorname{tg} x + 1 = 0$$

ec. omogenă
$y' + P(x)y = 0$

$$z' - z\operatorname{tg} x = 0$$

$\int \operatorname{tg} x\, dx = -\ln|\cos x| + C$

$$z' = z\operatorname{tg} x$$

$$\frac{z'}{z} = \operatorname{tg} x \rightarrow \int \frac{z'}{z}dx = \int \operatorname{tg} x\, dx$$

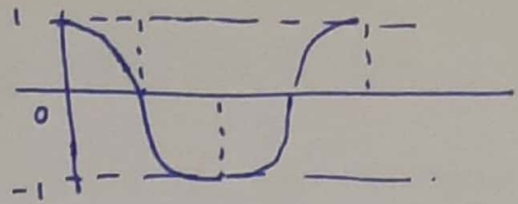
De ce?

$$\ln z = \ln C - \ln|\cos x|$$

$$\ln z = \ln \frac{C}{|\cos x|}$$

in locuind =>

$$\boxed{z = \frac{C}{|\cos x|}}$$



$\forall x \quad \cos x \in [-1, 1]$

$\Rightarrow |\cos x| \in [0, 1]$

$\cos x' = -\sin x$

$$z(x) = \frac{C(x)}{|\cos x|} \rightarrow z' = \frac{C'(x)\,|\cos x| + |\sin x| \cdot C(x)}{(\cos x)^2}$$

înlocuind $\rightarrow$ $\frac{C'(x)}{|\cos x|} + \frac{\sin x}{|\cos x|^2} \cdot C(x) - \frac{C(x)}{|\cos x|} \cdot tg\,x + 1 = 0$

$$\frac{C'(x)}{\cos x} + \frac{\sin x}{(\cos x)^2} C(x) - \frac{C(x)}{\cos x} \cdot \frac{\sin x}{\cos x} + 1 = 0$$

$$\frac{C'(x)}{\cos x} + \cancel{\frac{\sin x}{\cos^2 x} C(x)} - \cancel{\frac{\sin x}{\cos^2 x} \cdot C(x)} + 1 = 0$$

$$\frac{C'(x)}{\cos x} = 1 \rightarrow C'(x) = \cos x$$

$$C(x) = \int \cos x \, dx$$

$$C(x) = \sin x + K$$

$$z = \frac{C}{\cos x} = \frac{\sin x + K}{\cos x}$$

$$z = \frac{1}{y}$$

$$\left.\right\} \frac{1}{y} = \frac{\sin x + K}{\cos x}$$

$$\Rightarrow y = \frac{\cos x}{\sin x + K} \rightarrow \text{sol. implicită}$$

g) $x y' - y = 3 x y^3 \,/ \cdot \frac{1}{y^3}$

ec Bernoulli $y' + P(x) y = Q(x) y^{\alpha}$

ec neomogen $y' + P(x) y + Q(x) = 0$

ec omogen $y' + P(x) = 0$

$$x \cdot \frac{y'}{y^3} - \frac{1}{y^2} = 3x \,/ \cdot \frac{1}{x}$$

$$\frac{y'}{y^3} - \frac{1}{x} \cdot \frac{1}{y^2} = 3$$

$$\boxed{z = \frac{1}{y^2}} \Rightarrow z' = -2 \cdot \frac{1}{y^3} \cdot y' \Rightarrow \boxed{z' = \frac{-2y'}{y^3}}$$

înlocuind $\Rightarrow$ $-\frac{1}{2}\cdot z' - \frac{1}{x}\cdot z = 3$

$\frac{y'}{y^3} = \frac{z'}{-2}$

$\hookrightarrow -\frac{1}{2}z' - \frac{1}{x}\cdot z - 3 = 0 \,\big|\cdot(-1)$

$\frac{1}{2}z' + \frac{1}{x}z + 3 = 0 \,\big/\cdot 2$ ec. neomogenă

~~ec. omogenă $\frac{1}{2}z' + \frac{1}{x}z = 0$~~

$z' + \frac{2}{x}z + 6 = 0$

ec omogena $\quad z' + \frac{2}{x}z = 0$

$z' = -\frac{2}{x}z \;\big|\cdot\frac{1}{z}$

$\frac{z'}{z} = -2\cdot\frac{1}{x}$

$\Rightarrow \int\frac{z'}{z}dx = -2\int\frac{1}{x}dx$

$\Rightarrow \ln z = -2\ln x + \ln C$

$\ln z = \ln x^{-2} + \ln C$

$\ln z = \ln\frac{C}{x^2}$

$z = \frac{C}{x^2}$ $\rightarrow$ sol. ec. omogenă

$z(x) = \frac{C(x)}{x^2}$

$z'(x) = \frac{C'(x)\cdot x^2 - 2x\,C(x)}{x^4} = \frac{C'(x)}{x^2} - \frac{2}{x^3}C(x)$

înlocuind $\Rightarrow$ $\frac{C'(x)}{x^2} - \cancel{\frac{2}{x^3}C(x)} + \cancel{\frac{2}{x}\cdot\frac{C(x)}{x^2}} + 6 = 0$

$\frac{C'(x)}{x^2} = -6 \rightarrow C'(x) = -6x^2$

$C(x) = -6\int x^2dx$

$C(x) = -6\cdot\frac{x^3}{3} + k$

$C(x) = -2x^3 + k$

$z(x) = \frac{C(x)}{x^2} = \frac{-2x^3+k}{x^2} = -2x + \frac{k}{x^2}$

$$z = \frac{1}{y^2} \Rightarrow \frac{-2x^3+K}{x^2} = \frac{1}{y^2}$$

$$\Rightarrow y^2(-2x^3+K) = x^2$$

$$y^2 = \frac{x^2}{-2x^3+K}$$

$$y = \pm\sqrt{\frac{x^2}{-2x^3+K}}$$

↳ sol. implicită