Tema la Fundamentele algebrice ale informaticii

Sisteme (n,n) 5.2.14

a) $\begin{cases} 3x - y + z = 4 \\ x + y - 2x = -2 \\ -x + y + z = 2 \end{cases}$

Scriem matricea extinsă a sistemului și o aducem la forma triunghiulară

$$\left(\begin{array}{ccc|c} 3 & -1 & 1 & 4 \\ 1 & 1 & -2 & -2 \\ -1 & 1 & 1 & 2 \end{array}\right) \xrightarrow{L_2 \to L_2 + L_3} \left(\begin{array}{ccc|c} 3 & -1 & 1 & 4 \\ 0 & 2 & -1 & 0 \\ -1 & 1 & 1 & 2 \end{array}\right) \xrightarrow{L_3 \to L_1 + 3L_3} \left(\begin{array}{ccc|c} 3 & -1 & 1 & 4 \\ 0 & 2 & -1 & 0 \\ 0 & 2 & 4 & 6 \end{array}\right)$$

$$\xrightarrow{L_3 \to L_3 - L_2} \left(\begin{array}{ccc|c} 3 & -1 & 1 & 4 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 5 & 10 \end{array}\right) \Rightarrow$$ Obținem sistemul $\begin{cases} 3x - y + z = 4 \\ 2y - z = 0 \\ 5z = 10 \Rightarrow z = \frac{10}{5} = 2 \end{cases} \Rightarrow$

$\Rightarrow 2y - 2 = 0 \Rightarrow 2y = 2 \Rightarrow y = 1$

$S = \begin{cases} x = 1 \\ y = 1 \\ z = 2 \end{cases}$

$3x - 1 + 2 = 4 \Rightarrow 3x = 4 - 1 \Rightarrow x = \frac{3}{3} = 1$

b) $\begin{cases} x + y + z + t = 2 \\ 2y + 2z + t = 2 \\ -2x + 2y - t = 2 \\ 3x + y - z = 2 \end{cases}$

Scriem matricea extinsă

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 2 & 2 & 1 & 2 \\ -2 & 2 & 0 & -1 & 2 \\ 3 & 1 & -1 & 0 & 2 \end{array}\right) \xrightarrow[L_3 \to L_3 + 2L_1]{L_4 \leftrightarrow L_1} \left(\begin{array}{cccc|c} 3 & 1 & -1 & 0 & 2 \\ 0 & 2 & 2 & 1 & 2 \\ 0 & 4 & 2 & 1 & 6 \\ 1 & 1 & 1 & 1 & 2 \end{array}\right)$$

$$\xrightarrow{L_4 \to 3L_4 - L_1} \left(\begin{array}{cccc|c} 3 & 1 & -1 & 0 & 2 \\ 0 & 2 & 2 & 1 & 2 \\ 0 & 4 & 2 & 1 & 6 \\ 0 & 2 & 4 & 3 & 4 \end{array}\right) \xrightarrow[L_4 \to L_4 - L_2]{L_3 \to L_3 - 2L_2} \left(\begin{array}{cccc|c} 3 & 1 & -1 & 0 & 2 \\ 0 & 2 & 2 & 1 & 2 \\ 0 & 0 & -2 & -1 & 2 \\ 0 & 0 & 2 & 2 & 2 \end{array}\right) \xrightarrow{L_4 \to L_4 + L_3} \left(\begin{array}{cccc|c} 3 & 1 & -1 & 0 & 2 \\ 0 & 2 & 2 & 1 & 2 \\ 0 & 0 & -2 & -1 & 2 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right)$$

Obținem sistemul:

$\begin{cases} 3x + y - z = 2 \\ 2y + 2z + t = 2 \\ -2z - t = 2 \\ t = 4 \end{cases} \Rightarrow -2z = 2 + 4 \Rightarrow z = -\frac{6}{2} = -3$

1

$$2y+2(-3)+4=2 \Rightarrow 2y-6+4=2 \Rightarrow 2y=4 \Rightarrow \underline{\underline{y=2}}$$

$$3x+2+3=2 \Rightarrow 3x=2-5 \Rightarrow x=-\frac{3}{3}=\underline{\underline{-1}}$$

$$S=\begin{cases} x=-1 \\ y=2 \\ z=-3 \\ t=4 \end{cases}$$

c) $\begin{cases} 2x+3y+4z=16 \\ 5x-8y+2z=1 \\ 3x-y-2z=5 \end{cases}$ Scriem matricea extinsă $\left(\begin{array}{ccc|c} 2 & 3 & 4 & 16 \\ 5 & -8 & 2 & 1 \\ 3 & -1 & -2 & 5 \end{array}\right) \xrightarrow{L_2 \leftrightarrow L_1}$

$$\left(\begin{array}{ccc|c} 5 & -8 & 2 & 1 \\ 2 & 3 & 4 & 16 \\ 3 & -1 & -2 & 5 \end{array}\right) \xrightarrow{L_2 \to L_2 - \frac{2L_3}{3}} \left(\begin{array}{ccc|c} 5 & -8 & 2 & 1 \\ 0 & \cancel{31}\,\frac{11}{3} & \frac{16}{3} & -\frac{17}{3} \\ 3 & -1 & -2 & 5 \end{array}\right) \xrightarrow{L_3 \to L_3 - \frac{3L_1}{5}}$$

$$\rightarrow \left(\begin{array}{ccc|c} 5 & -8 & 2 & 1 \\ 0 & \frac{11}{3} & \frac{16}{3} & -\frac{17}{3} \\ 0 & \frac{19}{5} & -\frac{16}{5} & \frac{22}{5} \end{array}\right) \rightarrow \begin{pmatrix} 5 & -8 & 2 & 1 \\ 0 & 33 & 48 & -51 \\ 0 & 95 & -80 & 110 \end{pmatrix}$$

d) $\begin{cases} x+y+z+t=5 \\ x-2y+z-t=4 \\ 3x-y-z-t=7 \\ 2x+y+z-t=9 \end{cases} \Rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 1 & -2 & 1 & -1 & 4 \\ 3 & -1 & -1 & -1 & 7 \\ 2 & 1 & 1 & -1 & 9 \end{array}\right) \xrightarrow{L_1 \leftrightarrow L_3} \left(\begin{array}{cccc|c} 3 & -1 & -1 & -1 & 7 \\ 1 & -2 & 1 & -1 & 4 \\ 1 & 1 & 1 & 1 & 5 \\ 2 & 1 & 1 & -1 & 9 \end{array}\right)$

$$\xrightarrow{L_2 \to L_2 - L_3} \left(\begin{array}{cccc|c} 3 & -1 & -1 & -1 & 7 \\ 0 & -3 & 0 & -2 & -1 \\ 1 & 1 & 1 & 1 & 5 \\ 2 & 1 & 1 & -1 & 9 \end{array}\right) \xrightarrow{L_3 - 3L_1} \quad \xrightarrow[L_3 - 3L_1]{L_2 - L_1} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & -3 & 0 & -2 & -1 \\ 0 & -4 & -4 & -4 & -8 \\ 2 & 1 & 1 & -1 & 9 \end{array}\right) \xrightarrow{L_4 - 2L_1} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & -3 & 0 & -2 & -1 \\ 0 & -4 & -4 & -4 & -8 \\ 0 & -1 & -1 & -3 & -1 \end{array}\right)$$

$$\xrightarrow{\cancel{L_3 \to L_3 + 4L_4}} \begin{pmatrix} 1 & 1 & 1 & 1 & 5 \\ 0 & -3 & 0 & -2 & -1 \\ 0 & -8 & -8 & -16 & -12 \end{pmatrix} \xrightarrow{L_3 \to L_3 - 4L_4} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & -3 & 0 & -2 & -1 \\ 0 & 0 & 0 & 8 & 12 \\ 0 & -1 & -1 & -3 & -1 \end{array}\right) \xrightarrow{L_4 \to 3L_4 + L_2}$$

Tema la Fundamentele algebrice ale informaticii

Cap 1 ~~Rang~~ Matricei, sisteme de ecuații algebrice liniare

5.2.13 - Rang

Să determinăm rangul matricei $A = \begin{pmatrix} 2 & 4 & 3 & 5 \\ 1 & 2 & 1 & 2 \\ 3 & 1 & 5 & 3 \\ -1 & 5 & 2 & 8 \end{pmatrix}$

$$\begin{pmatrix} 2 & 4 & 3 & 5 \\ 1 & 2 & 1 & 2 \\ 3 & 1 & 5 & 3 \\ -1 & 5 & 2 & 8 \end{pmatrix} \begin{matrix} L_1 \to L_1 + L_4 \\ L_2 \to L_2 + L_4 \\ L_3 \to L_3 + 3L_4 \end{matrix} \Rightarrow \begin{pmatrix} 1 & 9 & 5 & 13 \\ 0 & 7 & 3 & 10 \\ 0 & 16 & 11 & 27 \\ -1 & 5 & 2 & 8 \end{pmatrix} \xrightarrow{L_4 \to L_4 + L_1} \begin{pmatrix} 1 & 9 & 5 & 13 \\ 0 & 7 & 3 & 10 \\ 0 & 16 & 11 & 27 \\ 0 & 14 & 7 & 21 \end{pmatrix}$$

$$\begin{matrix} C_2 \to C_2 - 9C_1 \\ C_3 \to C_3 - 5C_1 \\ C_4 \to C_4 - 13C_1 \end{matrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 7 & 3 & 10 \\ 0 & 16 & 11 & 27 \\ 0 & 14 & 7 & 21 \end{pmatrix} \begin{matrix} L_2 \to L_2/7 \\ L_3 \to L_3 - 16L_2 \\ L_4 \to L_4 - 14L_2 \end{matrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3/7 & 10/7 \\ 0 & 0 & 29/7 & 29/7 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{matrix} C_3 \to C_3 - 3/7 C_2 \\ C_4 \to C_4 - 10/7 C_2 \end{matrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 29/7 & 29/7 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{matrix} L_3 \to 7/29 L_3 \\ L_4 \to L_4 - L_3 \end{matrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{C_4 \to C_4 - C_3} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow$$

rang $A = 3$

5.3.17 Să se calculeze inversele matricelor:

a) $\begin{pmatrix} 2 & -1 \\ 5 & 3 \end{pmatrix}$ $\det A = \begin{vmatrix} 2 & -1 \\ 5 & 3 \end{vmatrix} = 6 - (-1)5 = 6 + 5 = 11 \neq 0 \Rightarrow \exists A^{-1}$

$A^{t} = \begin{pmatrix} 2 & 5 \\ -1 & 3 \end{pmatrix}$

$$\left.\begin{matrix} a_{11} = (-1)^{1+1} \cdot 3 = 3 \\ a_{12} = (-1)^{1+2} \cdot (-1) = 1 \\ a_{21} = (-1)^{2+1} \cdot 5 = -5 \\ a_{22} = (-1)^{2+2} \cdot 2 = 2 \end{matrix}\right\} A^{*} = \begin{pmatrix} 3 & 1 \\ -5 & 2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{11}\cdot\begin{pmatrix}3 & 1\\ -5 & 2\end{pmatrix} = \begin{pmatrix}3/11 & 1/11\\ -5/11 & 2/11\end{pmatrix}$$

b) $\begin{pmatrix}2 & 3 & 4\\ 0 & 1 & 1\\ 2 & 2 & -1\end{pmatrix}$ $\quad \det A = 2\cdot 1(-1) + \underbrace{0\cdot 2\cdot 4}_{0} + 2\cdot 3\cdot 1 - 4\cdot 1\cdot 2 - 1\cdot 2\cdot 2 -$

$\begin{matrix}2 & 3 & 4\\ 0 & 1 & 1\end{matrix}$ $\quad -(-1)\cdot 3\cdot 0$

$$\det A = -2+6-8-4 = -8 \neq 0 \Rightarrow \exists A^{-1}$$

$$A^t = \begin{pmatrix}2 & 0 & 2\\ 3 & 1 & 2\\ 4 & 1 & -1\end{pmatrix}$$

A^*: $a_{11} = (-1)^{1+1}\begin{vmatrix}1 & 2\\ 1 & -1\end{vmatrix} = -3$

$a_{12} = (-1)^{1+2}\begin{vmatrix}3 & 2\\ 4 & -1\end{vmatrix} = 1$

$a_{13} = (-1)^{1+3}\begin{vmatrix}3 & 1\\ 4 & 1\end{vmatrix} = -1$

$a_{21} = (-1)^{2+1}\begin{vmatrix}0 & 2\\ 1 & -1\end{vmatrix} = 2$

$a_{22} = (-1)^{2+1}\begin{vmatrix}2 & 2\\ 4 & -1\end{vmatrix} = 10$

$a_{23} = (-1)^{2+2}\begin{vmatrix}2 & 0\\ 4 & 1\end{vmatrix} = -2$

$a_{31} = (-1)^{3+1}\begin{vmatrix}0 & 2\\ 1 & 2\end{vmatrix} = -2$

$a_{32} = (-1)^{3+2}\begin{vmatrix}2 & 2\\ 3 & 2\end{vmatrix} = 2$

$a_{33} = (-1)^{3+3}\begin{vmatrix}2 & 0\\ 3 & 1\end{vmatrix} = 2$

$$A^* = \begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix} = \begin{pmatrix}-3 & 11 & -1\\ 2 & -10 & -2\\ -2 & 2 & 2\end{pmatrix}$$

$$A^{-1} = \frac{1}{-8}\begin{pmatrix}-3 & 11 & -1\\ 2 & -10 & -2\\ -2 & 2 & 2\end{pmatrix} = \begin{pmatrix}3/8 & -11/8 & 1/8\\ -1/4 & 5/4 & 1/4\\ 1/4 & -1/4 & -1/4\end{pmatrix}$$

5.3.18| Să se rezolve ecuațiile

a) $AX = B$ și $YA = B$ unde $A = \begin{pmatrix}1 & 0 & -1\\ 0 & 3 & 1\\ -1 & 2 & 2\end{pmatrix}$ și $B = \begin{pmatrix}1 & 0 & 2\\ -1 & 1 & 1\\ 0 & 1 & -3\end{pmatrix}$

$$\det A = \begin{vmatrix}1 & 0 & -1\\ 0 & 3 & 1\\ -1 & 2 & 2\end{vmatrix} = 6-3-2 = 1 \neq 0 \Rightarrow \exists A^{-1}$$

$A^t \begin{pmatrix} 1 & 0 & -1 \\ 0 & 3 & 2 \\ -1 & 1 & 2 \end{pmatrix}$ $\qquad A^* = \begin{pmatrix} a_{11} & -a_{12} & a_{13} \\ -a_{21} & a_{22} & -a_{23} \\ a_{31} & -a_{32} & a_{33} \end{pmatrix}$

$a_{11} = \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} = 4 \quad a_{21} = 4 \quad a_{31} = 3$

$a_{12} = 2 \quad a_{22} = 1 \quad a_{32} = 2 \Rightarrow A^* = \begin{pmatrix} 4 & -2 & 3 \\ -4 & 1 & 1 \\ 3 & 2 & 3 \end{pmatrix}$

$a_{13} = 3 \quad a_{23} = 1 \quad a_{33} = 3$

$\cancel{a_{32}}$

$$\Rightarrow X = A^{-1} \cdot B = \begin{pmatrix} 4 & -2 & 3 \\ -4 & 1 & 1 \\ 3 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 0 & 1 & -3 \end{pmatrix} \Rightarrow$$

$$\Rightarrow X = \begin{pmatrix} 4+2 & -2+3 & 4-2-9 \\ -4-1 & 1+1 & -8+1-3 \\ 3-2 & 2+3 & 6+2-9 \end{pmatrix} \Rightarrow X = \begin{pmatrix} 6 & 1 & -7 \\ -5 & 2 & -10 \\ 1 & 5 & -1 \end{pmatrix}$$

$YA = B \Rightarrow Y = B \cdot A^{-1}$

$$Y = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 0 & 1 & -3 \end{pmatrix} \begin{pmatrix} 4 & -2 & 3 \\ -4 & 1 & 1 \\ 3 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 10 & 2 & 9 \\ -5 & 5 & 1 \\ -13 & -5 & -8 \end{pmatrix}$$

b) $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \cdot X \cdot \begin{pmatrix} -3 & 2 \\ 5 & -3 \end{pmatrix} = \begin{pmatrix} -2 & 4 \\ 3 & -1 \end{pmatrix}$

$A X \cdot B = C \Rightarrow X = A^{-1} \cdot C \cdot B^{-1}$

$A^{-1} = \frac{1}{\det A} \cdot A^*$

$\det A = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 4 - 3 = 1 \neq 0 \Rightarrow \exists A^{-1}$

$A^t = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} \Rightarrow A^* = \begin{vmatrix} 2 & -1 \\ -3 & 2 \end{vmatrix} \Rightarrow A^{-1} = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$

$\det B = \begin{vmatrix} -3 & 2 \\ 5 & -3 \end{vmatrix} = 9 - 10 = -1 \neq 0 \Rightarrow \exists B^{-1}$

$B^t = \begin{pmatrix} -3 & 5 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix} \Rightarrow X = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} -2 & 4 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix} \Rightarrow$

$X = \begin{pmatrix} 2-2+3 & -1+4+2 \\ -3+3+5 & 2-1+3 \end{pmatrix} \Rightarrow X = \begin{pmatrix} 3 & 5 \\ 5 & 4 \end{pmatrix}$

5.3.27) Să se calc A^{-1} pt. unui matrice fol. Gauss

a) $A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & -4 & -5 \\ 3 & 5 & 6 \end{pmatrix}$ $\det A = \begin{vmatrix} 1 & 2 & 3 \\ -2 & -4 & -5 \\ 3 & 5 & 6 \end{vmatrix}$ $= \cancel{-24} - 30 - 30 + 36 + 25 + \cancel{+24} = 1 \neq 0 \Rightarrow \exists A^{-1}$

$\begin{matrix} 1 & 2 & 3 \\ -2 & -4 & -5 \end{matrix}$

~~$A^{-1} = \frac{1}{\det A} A^* = \frac{1}{1}\Big(-3$~~

$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ -2 & -4 & -5 & 0 & 1 & 0 \\ 3 & 5 & 6 & 0 & 0 & 1 \end{array}\right] \xrightarrow{L_2 \to L_2 + 2L_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 \\ 3 & 5 & 6 & 0 & 0 & 1 \end{array}\right] \xrightarrow{L_3 \to L_3 - 3L_1}$

$\to \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & -1 & -3 & -3 & 0 & 1 \end{array}\right] \xrightarrow{L_2 \leftrightarrow L_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -3 & -3 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 & 0 \end{array}\right] \xrightarrow{L_1 \to L_1 + 2L_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & -5 & 0 & 2 \\ 0 & -1 & 3 & -3 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 & 0 \end{array}\right]$

$\xrightarrow{L_1 \to L_1 + 3L_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 2 \\ 0 & -1 & 3 & -3 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 & 0 \end{array}\right] \xrightarrow{L_2 \to L_2 - 3L_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 2 \\ 0 & -1 & 0 & -9 & -3 & 1 \\ 0 & 0 & 1 & 2 & 1 & 0 \end{array}\right] \Rightarrow$

$A^{-1} = \begin{pmatrix} 1 & 3 & 2 \\ -9 & -3 & 1 \\ 2 & 1 & 0 \end{pmatrix}$

5.3.27

b) $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix}$ $\det A = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{vmatrix} = 1+2+0-2+1 = 2 \neq 0$
$\begin{matrix} 1 & 1 & -1 \\ 2 & 1 & 0 \end{matrix}$
$\Rightarrow \exists A^{-1}$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 3 & 5 & 6 & 0 & 0 & 1 \end{array}\right] \xrightarrow{L_2 \to L_2 - 2L_1} \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & -2 \\ 3 & 5 & 6 & 0 & 0 & 1 \end{array}\right] \xrightarrow{L_3 \to L_3 - 3L_1}$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & -2 \\ 0 & 2 & 9 & -3 & 0 & 1 \end{array}\right] \xrightarrow{L_3 = L_3 + 2L_2} \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & -2 \\ 0 & 0 & 13 & -3 & 2 & -1 \end{array}\right] \xrightarrow{L_3/13}$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 1 & -3/13 & 2/13 & -1/13 \end{array}\right] \xrightarrow{L_1 \to L_1 + L_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 1 & 2 \\ 0 & -1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 1 & -3/13 & 2/13 & -1/13 \end{array}\right] \xrightarrow{L_1 \to L_1 - L_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 16/13 & 11/13 & 27/13 \\ 0 & -1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 1 & -3/13 & 2/13 & -1/13 \end{array}\right] \xrightarrow{L_2 = L_2 - 2L_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 16/13 & 11/13 & 27/13 \\ 0 & -1 & 0 & 3/13 & 11/13 & 27/13 \\ 0 & 0 & 1 & -3/13 & 2/13 & -1/13 \end{array}\right] \xrightarrow{*-1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 16/13 & 11/13 & 27/13 \\ 0 & 1 & 0 & -3/13 & -11/13 & -27/13 \\ 0 & 0 & 1 & -3/13 & 2/13 & -1/13 \end{array}\right] \Rightarrow A^{-1} = \frac{1}{13}\begin{pmatrix} 16 & 11 & 27 \\ -3 & -11 & -27 \\ -3 & 2 & -1 \end{pmatrix}$$

5.3.27

c) $A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \\ 2 & -1 & 3 \end{pmatrix}$ $\det. A = \begin{vmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \\ 2 & -1 & 3 \end{vmatrix} = 9 - 2 + 3 + 1 - 12 = -1 \neq 0$
$\begin{matrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{matrix}$
$\Rightarrow \exists A^{-1}$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 & 1 & 0 \\ 2 & -1 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{L_3 \to L_3 - 2L_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & -3 & -2 & 0 & 1 \end{array}\right] \xrightarrow{L_2 \leftrightarrow L_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 0 & 1 \\ 0 & 3 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow[L_3 = L_3 - 3L_2]{L_1 \to L_1 + L_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 1 & -3 & -2 & 0 & 1 \\ 0 & 0 & 9 & 6 & 1 & -3 \end{array}\right] \xrightarrow{L_3/9} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 1 & -3 & -2 & 0 & 1 \\ 0 & 0 & 1 & 6/9 & 1/9 & -3/9 \end{array}\right] \xrightarrow{L_1 = L_1 + L_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3/9 & 1/9 & 6/9 \\ 0 & 1 & -3 & -2 & 0 & 1 \\ 0 & 0 & 1 & 6/9 & 1/9 & -3/9 \end{array}\right] \xrightarrow{L_2 + 3L_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3/9 & 1/9 & 6/9 \\ 0 & 1 & 0 & 1 & 3/9 & 0 \\ 0 & 0 & 1 & 6/9 & 1/9 & -3/9 \end{array}\right] \Rightarrow A^{-1} = \begin{pmatrix} -3/9 & 1/9 & 2/3 \\ 1 & 1/3 & 0 \\ 2/3 & 1/9 & -1/3 \end{pmatrix}$$

5.3.19 Să se aducă matricile la forma diagonală și să se deducă rangul

a)

$$\begin{pmatrix} 1 & 0 & 2 \\ -2 & 0 & 1 \\ -4 & 0 & 2 \end{pmatrix} \xrightarrow{L_2 - \left(\frac{2}{\lambda-1}\right)L_1} \begin{pmatrix} -\lambda+1 & 0 & 2 \\ 0 & \lambda & \frac{\lambda-5}{\lambda-2} \\ -4 & 0 & -\lambda+2 \end{pmatrix} \xrightarrow{L_3 - \left(\frac{4}{\lambda-1}\right)L_1} \begin{pmatrix} -\lambda+1 & 0 & 2 \\ 0 & -\lambda & 1 \\ 0 & 0 & \frac{-\lambda^2+3\lambda-10}{\lambda-1} \end{pmatrix}$$

$$a_{21} = -2 - \frac{-2}{-\lambda+1} \cdot (-\lambda+1) = 0$$

$$a_{33} = -\lambda+2 - \frac{-4}{-\lambda+1} \cdot 2 = \frac{-\lambda^2+3\lambda-10}{\lambda-1}$$

$$a_{23} = 1 - \frac{-2}{-\lambda+1} \cdot 2 = \frac{\lambda-5}{\lambda-2}$$

$$a_{31} = -4 - \frac{-4}{-\lambda+1} \cdot (-\lambda+1) = 0$$

$$a_{32} = 0 - \frac{-4}{-\lambda+1} \cdot 0 = 0$$

$$\begin{vmatrix} -\lambda+1 & 0 & 2 \\ -2 & -\lambda & 1 \\ -4 & 0 & -\lambda+2 \end{vmatrix} = \begin{vmatrix} -\lambda+1 & 0 & 2 \\ 0 & -\lambda & \frac{\lambda-5}{\lambda-1} \\ 0 & 0 & \frac{-\lambda^2+3\lambda-10}{\lambda-1} \end{vmatrix} = (-\lambda+1)(-\lambda)\left(\frac{-\lambda^2+3\lambda-10}{\lambda-1}\right) =$$

$$= -\lambda^3 + 3\lambda^2 - 10\lambda$$

Rangul (Metoda Gauss)

$$A = \begin{pmatrix} 1 & 0 & 2 \\ -2 & 0 & 1 \\ -4 & 0 & 2 \end{pmatrix} \xrightarrow{L_2 + 2L_1} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & 5 \\ -4 & 0 & 2 \end{pmatrix} \xrightarrow{L_3 + 4L_1} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & 5 \\ 0 & 0 & 10 \end{pmatrix} \xrightarrow{L_3 - 2L_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow$ rang $A = 2$

b)

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 3 & 0 & 1 \\ 2 & -2 & 0 \end{pmatrix} \xrightarrow{L_2 - 3L_1} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 3 & 1 \\ 2 & -2 & 0 \end{pmatrix} \xrightarrow{L_3 - 2L_1} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

rang $A = 2$

5.3.32| Să se rezolve ec. matriceală

$AX = B$

a) $A = \begin{pmatrix} 2 & -1 & 3 \\ 1 & 2 & -1 \\ -2 & 1 & -2 \end{pmatrix}, B = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

$\det. A = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 2 & -1 \\ -2 & 1 & -2 \end{vmatrix} = -8+3-2+12 \cancel{-2+2} = 5 \neq 0 \Rightarrow \exists A^{-1}$

$\begin{matrix} 2 & -1 & 3 \\ 1 & 2 & -1 \end{matrix}$

$A \cdot X = B \Rightarrow X = A^{-1} \cdot B$

$A^T = \begin{pmatrix} \cancel{2\ 3\ 1} \\ 2 & -1 & 3 \\ 1 & 2 & -1 \\ -2 & 1 & -2 \end{pmatrix}$

$\cancel{A^{-1} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \end{pmatrix}}$

$A^{(-1)} = \frac{1}{|A|} \cdot a^T = \frac{1}{|A|} \cdot \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}$

$a_{11} = (-1)^{(1+1)} \begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} = 1 \cdot (2(-2)+1) = -3$

$a_{12} = -1^{(1+2)} \begin{vmatrix} 1 & -1 \\ -2 & -2 \end{vmatrix} = -1(-2-2) = 4$

$a_{13} = -1^{(1+3)} \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 1(1+4) = 5$

$a_{21} = -1^{(2+1)} \begin{vmatrix} -1 & 3 \\ 1 & -2 \end{vmatrix} = -1(2-3) = 1$

$a_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 3 \\ -2 & -2 \end{vmatrix} = 1(-4+6) = 2$

$a_{23} = -1^{(2+3)} \begin{vmatrix} 2 & -1 \\ -2 & 1 \end{vmatrix} = -1(2-2) = 0$

$a_{31} = -1^{3+1} \begin{vmatrix} -1 & 3 \\ 2 & -1 \end{vmatrix} = 1(1-6) = -5$

$a_{32} = -1^{(3+2)} \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = -1(-2-3) = 5$

$a_{33} = -1^{(3+3)} \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 1(4+1) = 5$

$\Rightarrow A^{(-1)} = \frac{1}{|A|} \cdot a^T = \frac{1}{5} \begin{pmatrix} -3 & 1 & -5 \\ 4 & 2 & 5 \\ 5 & 0 & 5 \end{pmatrix} = \begin{pmatrix} -3/5 & \frac{1}{5} & -1 \\ 4/5 & 2/5 & 1 \\ 1 & 0 & 1 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} 1 & -4 & -6 & -11 \\ 0 & 28 & 32 & 56 \\ 0 & 11 & 16 & 38 \end{array}\right) \xrightarrow{L_3 - \frac{1}{2}L_2} \left(\begin{array}{ccc|c} 1 & -4 & -6 & -11 \\ 0 & 28 & 32 & 56 \\ 0 & -3 & 0 & 38 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -6 & -4 & -11 \\ 0 & 32 & 28 & 56 \\ 0 & 0 & -3 & 38 \end{array}\right)$$

Obtinem sistemul

$$\begin{cases} x - 4y - 6z = -11 \\ 28y + 32z = 56 \\ -3z = 38 \Rightarrow z = \frac{38}{3} \end{cases}$$

$$X = A^{(-1)} \cdot B \Rightarrow X = \begin{pmatrix} -\frac{3}{5} & \frac{1}{5} & -1 \\ 4/5 & 2/5 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -\frac{3}{5} + 2\frac{1}{5} + 1 \\ 4/5 + 2\frac{2}{5} + 1 \\ 1 + 0 - 1 \end{pmatrix} \Rightarrow$$

$$X = \begin{pmatrix} \frac{4}{5} \\ 3/5 \\ 0 \end{pmatrix}$$

5.3.32 b) $A = \begin{pmatrix} 2 & 5 & 3 \\ 4 & -6 & -3 \\ 6 & 10 & -10 \end{pmatrix}$, $B = \begin{pmatrix} 17 \\ 0 \\ 8 \end{pmatrix}$

$$\det A = \begin{vmatrix} 2 & 5 & 3 \\ 4 & -6 & -3 \\ 6 & 10 & -10 \end{vmatrix} = 120 - 90 + 120 + 108 + 60 + 200 = 518 \neq 0 \Rightarrow \exists A^{-1}$$

$$A^{(-1)} = \frac{1}{|A|} \cdot \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} = \frac{1}{518} \begin{pmatrix} 90 & 80 & 3 \\ 22 & -38 & 18 \\ 76 & 10 & -32 \end{pmatrix} = \begin{pmatrix} 45/259 & \frac{40}{259} & \frac{3}{518} \\ \frac{11}{259} & \frac{-19}{259} & \frac{9}{259} \\ \frac{38}{259} & \frac{5}{259} & \frac{-16}{259} \end{pmatrix}$$

$$a_{11} = -1^{(1+1)} \begin{vmatrix} -6 & -3 \\ 10 & -10 \end{vmatrix} = 1(60 + 30) = 90 \qquad a_{22} = -1^{2+2} \begin{vmatrix} 2 & 3 \\ 6 & -10 \end{vmatrix} = 1(-20 - 18) = -38$$

$$a_{12} = -1^{(1+2)} \begin{vmatrix} 4 & -3 \\ 6 & -10 \end{vmatrix} = -1(-40 + 18) = 22 \qquad a_{23} = -1^{2+3} \begin{vmatrix} 2 & 5 \\ 6 & 10 \end{vmatrix} = -1(20 - 30) = 10$$

$$a_{31} = -1^{3+1} \begin{vmatrix} 5 & 3 \\ -6 & -3 \end{vmatrix} = 1(-15 + 18) = 3$$

$$a_{13} = -1^{(1+3)} \begin{vmatrix} 4 & -6 \\ 6 & 10 \end{vmatrix} = 1(40 + 36) = 76 \qquad a_{32} = -1^{3+2} \begin{vmatrix} 2 & 3 \\ 4 & -3 \end{vmatrix} = -1(-6 - 12) = 18$$

$$a_{21} = -1^{(2+1)} \begin{vmatrix} 5 & 3 \\ 10 & -10 \end{vmatrix} = -1(-50 - 30) = 80 \qquad a_{33} = -1^{3+3} \begin{vmatrix} 2 & 5 \\ 4 & -6 \end{vmatrix} = 1(-12 - 20) = -32$$

$$A x = B \Rightarrow x = A^{(-1)} B$$

$$\Rightarrow x = \begin{pmatrix} 45/259 & 40/259 & 3/518 \\ 11/259 & -19/259 & 9/259 \\ 38/259 & 5/259 & -16/259 \end{pmatrix} \cdot \begin{pmatrix} 17 \\ 0 \\ 8 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{45}{259} 17 + 0 + 8 \frac{3}{518} \\ 17 \frac{11}{259} + 0 + 8 \frac{9}{259} \\ 17 \frac{38}{259} + 0 + 8 \frac{-16}{259} \end{pmatrix} = \begin{pmatrix} \frac{765}{259} + \frac{24}{518} \\ \frac{187}{259} + \frac{72}{259} \\ \frac{646}{259} + \frac{-128}{259} \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$$

5.2.18 Să se rezolve prin met. lui Gauss sistemul

$$\begin{cases} 3x_1 - 5x_2 + 2x_3 + 4x_4 = 2 \\ 7x_1 - 4x_2 + x_3 + 4x_4 = 5 \\ 5x_1 + 7x_2 - 4x_3 - 6x_4 = 3 \end{cases}$$

$$\left(\begin{array}{cccc|c} 3 & -5 & 2 & 4 & 2 \\ 7 & -4 & 1 & 3 & 5 \\ 5 & 7 & -4 & -6 & 3 \end{array}\right) \to \left(\begin{array}{cccc|c} 3 & -5 & 2 & 4 & 2 \\ 0 & \frac{23}{3} & -\frac{11}{3} & -\frac{19}{3} & \frac{1}{3} \\ 0 & \frac{46}{3} & -\frac{22}{3} & -\frac{38}{3} & -\frac{1}{3} \end{array}\right) \to$$

$$\left(\begin{array}{cccc|c} 3 & -5 & 2 & 4 & 2 \\ 0 & \frac{23}{3} & -\frac{11}{3} & -\frac{19}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & -1 \end{array}\right) \Rightarrow \text{Sistemul este incompatibil}$$