Capitol 2

Ex 2.2.1 | a

$v_1=(2,1,3,1)$ $v_2=(1,2,0,1)$ $v_3=(-1,1,-3,0)$

$\alpha_1 \cdot v_1 + \alpha_2 \cdot v_2 + \alpha_3 v_3 = (0,0,0,0)$

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 0 \\ 1 & 2 & 1 & 0 \\ 3 & 0 & -3 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 2 & 1 & -1 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & -3 & -3 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ /:3 \\ /:-3 \\ \equiv \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 2 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

rang=2

$\alpha_1 = \alpha_3$ $\alpha_2 = -\alpha_3$ $\alpha_3 = \lambda \in \mathbb{R}$

$v_1 - v_2 + v_3 = 0$ ✓

b)

$v_1 = (8 - t + t^2)$ $v_2 = (2 - t + 3t^2)$ $v_3 = (1 + t + t^2)$

$$\left[\begin{array}{ccc|c} 8 & 2 & 1 & 0 \\ -1 & -1 & 1 & 0 \\ 7 & 3 & 1 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 8 & 2 & 1 & 0 \\ 0 & -6 & 9 & 0 \\ 0 & 10 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 8 & 2 & 1 & 0 \\ 0 & 2 & -3 & 0 \\ 0 & 10 & 1 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 16 & 0 & 8 & 0 \\ 0 & 2 & -3 & 0 \\ 0 & 0 & 32 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 0 \\ 0 & 2 & -3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

rank = 3

vectorii v_1, v_2, v_3 sunt liniar independenți

d) $v_1=(2,0,1,3,-1)$ $v_2=(1,1,0,-1,1)$ $v_3=(0,-2,1,5,-3)$
$v_4=(1,-3,2,9,-5)$

$$\left[\begin{array}{cccc|c} 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & -3 & 0 \\ 1 & 0 & 1 & 2 & 0 \\ 3 & -1 & 5 & 9 & 0 \\ -1 & 1 & -3 & -5 & 0 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{cccc|c} 2 & 1 & 0 & 1 & 0 \\ 0 & 2 & -4 & -6 & 0 \\ 0 & -1 & 2 & +3 & 0 \\ 0 & -5 & 10 & 15 & 0 \\ 0 & 3 & -6 & -9 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & -3 & 0 \\ 0 & 1 & -2 & -3 & 0 \\ 0 & \overset{1}{\cancel{-5}} & \overset{-2}{\cancel{10}} & \overset{-3}{\cancel{16}} & 0 \\ 0 & 1 & -2 & -3 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 2 & 0 & 2 & 3 & 0 \\ 0 & 1 & -2 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

rank = 2

$2\alpha_1 = -2\lambda - 3\gamma$

$\alpha_2 = 2\lambda + 3\gamma$

$\alpha_3 = \lambda \in R \quad \alpha_4 = \gamma \in R$

vectorii sunt L.D.

Ex 2.3.6

$v_1 = (1, 2, -1, 0)$ $v_2 = (1, -1, 1, 1)$ $v_3 = (-1, 2, 1, 1)$

$v_4 = (-1, -1, 0, 1)$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0 \\ 2 & -1 & 2 & -1 & 0 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0 \\ 0 & -3 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} -3 & 0 & 3 & 2 & 0 \\ 0 & -3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & -4 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} -3 & 0 & 3 & 2 & 0 \\ 0 & -3 & 0 & 1 & 0 \\ 0 & 0 & -3 & -4 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 9 & 0 & 0 & 6 & 0 \\ 0 & 9 & 0 & -3 & 0 \\ 0 & 0 & -3 & -4 & 0 \\ 0 & 0 & 0 & -3 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 3 & 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & -1 & 0 \\ 0 & 0 & -3 & -4 & 0 \\ 0 & 0 & 0 & +1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

rank = 4

vectorii sunt

L.I

Ex 2.3.24 {o să folosesc ~~algori~~ promul în C locul de mine}

a)

$$\begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 5 & -1 & 0 & 0 \\ 2 & 6 & 3 & 4 & 0 \\ 2 & 6 & 4 & -3 & 0 \\ 1 & 5 & 0 & -1 & 0 \end{array}\right] \to \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \text{rank}=4$$

vectorii sunt L.I

c)

$$\left[\begin{array}{cccc|c} 2 & 1 & 3 & 2 & 0 \\ 0 & +2 & 1 & 4 & 0 \\ 4 & -2 & 3 & 9 & 0 \\ 2 & -3 & 4 & 5 & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array}$$

rank=4

vectorii sunt L.I

Ex 2.2.3

$v_1=(1,1,2,1)$ $v_2=(1,-1,0,1)$ $v_3=(0,0,-1,1)$
$v_4=(1,2,2,0)$

$x=(1,1,1,1)$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & -1 & 0 & 2 \\ 2 & 0 & -1 & 2 \\ 1 & 1 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & -2 & 0 & 1 \\ 0 & -2 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \to \begin{bmatrix} -2 & 0 & 0 & -3 \\ 0 & -2 & 0 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & -2 & 2 \end{bmatrix} \to \begin{bmatrix} +2 & 0 & 0 & 3 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 & -3 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ rang = 4

$v_{1,2,3,4}$ sunt LI formează o bază

$\mathcal{B}=\{v_1, v_2, v_3, v_4\}$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 1 \\ 1 & -1 & 0 & 2 & 1 \\ 2 & 0 & -1 & 2 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{array}\right]$$

$$\left.\begin{array}{l} \alpha_1=0{,}25 \\ \alpha_2=0{,}25 \\ \alpha_3=0{,}50 \\ \alpha_4=0{,}50 \end{array}\right\} \text{coordonatele lui } x \text{ în } \mathcal{B}$$

Ex 2.2.4

~~$v_1=(1,-2,3,1)$~~ $v_1=(1,2,-4,3,1)$ $v_2=(2,5,-3,4,8)$
$v_3=(6,17,-7,10,22)$ $v_4=(1,3,-3,2,0)$

$$\begin{bmatrix} 1 & 2 & 6 & 1 \\ 2 & 5 & 17 & 3 \\ -4 & -3 & -7 & -3 \\ 3 & 4 & 10 & 2 \\ 1 & 8 & 22 & 0 \end{bmatrix} \rightarrow \text{rank} = 3$$

$B = \{v_1, v_2, v_3\}$

$$v_4 = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 6 & 1 \\ 2 & 5 & 17 & 3 \\ -4 & -3 & -7 & -3 \\ 3 & 4 & 10 & 2 \\ 1 & 8 & 22 & 0 \end{array}\right] \rightarrow \begin{array}{ll} \alpha_1 = 1 & \{1\} \\ \alpha_2 = -1{,}50 & \{-\frac{3}{2}\} \\ \alpha_3 = 0{,}50 & \{\frac{1}{2}\} \end{array}$$

$$(1,3,-5,2,0) = (1,2,-4,3,1) - \frac{3}{2}(2,5,-3,4,8) + \frac{1}{2}(6,17,-7,10,22)$$

Ex 2.2.12 Folosesc programul

$v_1 = (2,1,11,2)$ $v_2 = (1,0,4,-1)$ $v_3 = (2,1,5,6)$
$v_4 = (11,4,56,5)$

Calculăm Rangul

$$A = \begin{array}{c} \begin{bmatrix} 2 & 1 & 2 & 11 \\ 1 & 0 & 1 & 4 \\ 11 & 4 & 5 & 56 \\ 2 & -1 & 6 & 5 \end{bmatrix} \\ \begin{matrix} v_1 & v_2 & v_3 & v_4 \end{matrix} \end{array}$$

rank = 3

$\rightarrow B = \{v_1, v_2, v_3\}$

$\hookrightarrow \mathrm{Dim}(A) = 3$

Ex 2.2.15 Am folosit programul C

$\mathcal{B}_1 = \; v_1 = (1,0,1) \;\; v_2 = (0,1,1) \;\; v_3 = (1,1,1)$

$\mathcal{B}_2 = \; u_1 = (1,1,0) \;\; u_2 = (-1,0,0) \;\; u_3 = (0,0,1)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\text{Gauss}} T = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 1 \\ 2 & -1 & -1 \end{bmatrix}$$

$$\underbrace{}_{\mathcal{B}_1} \quad \underbrace{}_{\mathcal{B}_2}$$

Ex 2.3.13

$v_1 = \{1, \lambda, \lambda+2\}$ $v_2 = \{1, \lambda^2, -\lambda\}$ $v_3 = \{\lambda, \lambda-1, \lambda^2\}$

$$\begin{bmatrix} 1 & 1 & \lambda \\ \lambda & \lambda^2 & \lambda-1 \\ \lambda+2 & -\lambda & \lambda^2 \end{bmatrix}$$

Să fie o bază?

$\lambda \to$ rank $= 3$?

$$\begin{bmatrix} 1 & 1 & \lambda \\ 0 & \lambda^2-\lambda & -\lambda^2+\lambda-1 \\ 0 & -2\lambda-2 & -2\lambda \end{bmatrix}$$

1. $\lambda \neq 0$ & $\lambda \neq 1$

$$\begin{bmatrix} \lambda^2-\lambda & 0 & \lambda^3-\lambda+1 \\ 0 & \lambda^2-\lambda & -\lambda^2+\lambda-1 \\ 0 & 0 & -4\lambda^3+2\lambda^2-2 \end{bmatrix}$$

1.1. $\lambda \neq 0, \lambda \neq 1, -4\lambda^3+2\lambda^2-\lambda \neq 0$

$$\left.\begin{matrix} -4\lambda^3+2\lambda^2-\lambda & 0 & 0 \\ 0 & -4\lambda^3+2\lambda^2-\lambda & 0 \\ 0 & 0 & -4\lambda^3+2\lambda^2-\lambda \end{matrix}\right|$$

rank $=3$

$\{v_1, v_2, v_3\} = \mathcal{B}$

1.2 $\lambda \neq 0 \ \lambda \neq 1, -4\lambda^3+2\lambda^2-\lambda = 0$

$$\left.\begin{matrix} \lambda^2-\lambda & 0 & 0 \\ 0 & \lambda^2-\lambda & 0 \\ 0 & 0 & 0 \end{matrix}\right|$$

rank $=2$

$\{v_1, v_2\} = \mathcal{B}$

2 $\lambda^2-\lambda=0$

$$\begin{bmatrix} 1 & 1 & \lambda \\ 0 & 0 & -\lambda^2+\lambda-1 \\ 0 & -2\lambda-2 & -2\lambda \end{bmatrix} \to \begin{bmatrix} 1 & 1 & \lambda \\ 0 & -2\lambda-2 & -2\lambda \\ 0 & 0 & -\lambda^2+\lambda-1 \end{bmatrix}$$

$$\begin{bmatrix} 2\lambda^3+\lambda & 0 & 0 \\ 0 & 2\lambda^3+\lambda & 0 \\ 0 & 0 & 2\lambda^3+\lambda \end{bmatrix} \to$$

$2\lambda^3+\lambda=0$

↳ rank $=0$

$2\lambda^3+\lambda \neq 0$

rank $=3$

$\{v_1, v_2, v_3\} = \mathcal{B}$

Ex 2.3.14

$v_1 = (1, \lambda, 0) \quad v_2 = (\lambda, 1, 1) \quad v_3 = (1, 0, \lambda)$

$$\begin{bmatrix} 1 & \lambda & 1 \\ \lambda & 1 & 0 \\ 0 & 1 & -\lambda \end{bmatrix}$$

$$\begin{bmatrix} 1 & \lambda & 1 \\ 0 & 1-\lambda^2 & -\lambda \\ 0 & 1 & -\lambda \end{bmatrix}$$

1. $\lambda \neq 1$, $\lambda \neq -1$

$$\begin{bmatrix} 1-\lambda^2 & 0 & 1 \\ 0 & 1-\lambda^2 & -\lambda \\ 0 & 0 & \lambda^3 \end{bmatrix}$$

1.1 $\lambda \neq \pm 1$, $\lambda \neq 0$ rank $= 3$ $\{v_1, v_2, v_3\} = \beta$

1.2 $\lambda \neq \pm 1$, $\lambda = 0$ rank $= 2$ $\{v_1, v_2\} = \beta$

2, $\lambda = \pm 1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & -\lambda \\ 0 & 1 & -\lambda \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \lambda & 1 \\ 0 & 1 & -\lambda \\ 0 & 0 & -\lambda \end{bmatrix}$$

2.1 $\lambda \neq \pm 1$, $\lambda \neq 0$ rank $= 3$ $\{v_1, v_2, v_3\} = \mathbb{R}^3$

2.2 $\lambda = \pm 1$, $\lambda \neq 0$ rank $= 2$ $\{v_1, v_2\} = \mathbb{R}^3$

Ex 2.3.17

$v_1=\{1,1,1,1\}$ $v_2=\{1,1,-1,-1\}$ $v_3=\{1,-1,1,-1\}$ $v_4=\{1,-1,-1$

$v_5=\{1,2,1,1\}$

$$A=\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \rightarrow \text{rank}=4$$

$\hookrightarrow$ A este o bază

$A=\{v_1, v_2, v_3, v_4\}$

coordonate lui v_5 în A

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 2 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & 1 \end{array}\right]$$

v_5 în A

$\hookrightarrow v_5=\left(\frac{4}{5}, -\frac{1}{4}, \frac{1}{4}, -\frac{1}{4}\right)$

Ex 2.3.22

$x_1=(1,3,5)$ $x_2=(6,3,2)$ $x_3=(3,1,0)$

$$B=\begin{bmatrix} 1 & 6 & 3 \\ 3 & 3 & 1 \\ 5 & 2 & 0 \end{bmatrix} \rightarrow \text{rank}=3$$

↳ B este o bază

$B=\{x_1, x_2, x_3\}$

coordonatele în B lui $x=(1,1,1)$

$$\left[\begin{array}{ccc|c} 1 & 6 & 3 & 1 \\ 3 & 3 & 1 & 1 \\ 5 & 2 & 0 & 1 \end{array}\right] \rightarrow$$

x în B

↳ $x=(1,-2,4)$

coordonatele în B lui $y=(2,2,2)$

$$\left[\begin{array}{ccc|c} 1 & 6 & 3 & 2 \\ 3 & 3 & 1 & 2 \\ 5 & 2 & 0 & 2 \end{array}\right] \rightarrow$$

y în B

↳ $y=(2,-4,8)$

Ex 2.3.23

$B=\{(m,1,1),(1,m,1),(1,1,m+1)\}$

$$\begin{pmatrix} m & 1 & 1 \\ 1 & m & 1 \\ 1 & 1 & m+1 \end{pmatrix} \to \begin{pmatrix} m & 1 & 1 \\ 0 & m^2-1 & m-1 \\ 0 & m-1 & m^2+m \end{pmatrix}$$

1. $m \neq \pm 1$

$$\begin{matrix} m^3-m & 0 & m^2-1 \\ 0 & m^2-1 & m-1 \\ 0 & 0 & m\cdot(m^5-m^4-2m^3+2m^2+m-1) \end{matrix}$$

1.1 $m\neq 0$, $m \neq \pm 1$, $m^5-m^4-2m^3+2m^2+m-1 \neq 0$

rank $=3 \to B$ este o bază

1.2 $(m=0 \ || \ m^5-m^4-2m^3+2m^2+m-1=0)$, $m \neq \pm 1$

rank $=2 \to B$ nu este o bază

2. $m=-1$

↳ rank $=3$ B este o bază

3. $m=1$

$$\begin{pmatrix} m & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} \to \begin{array}{c} \begin{matrix} v_1 & v_3 & v_2 \end{matrix} \\ \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

rank = 2 → B nu este o bază

Ex 2.2.6

$$\underbrace{\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 3 & 1 \end{bmatrix}}_{B^1} \underbrace{\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}}_{B^2} \to T = \begin{bmatrix} 1 & 3 & 1 \\ 1 & -1 & -1 \\ -2 & 1 & 2 \end{bmatrix}$$

coordonatele lui $v=(1,-1,2)$ în B^2

Ca să găsesc coordonatele lui v în B^2 calculez mai întâi ~~coordonatele~~ matricea $S \leftarrow T^{-1}$, și apoi $T^{-1} \cdot \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$

$$S = \begin{bmatrix} \frac{1}{2} & \frac{5}{2} & 1 \\ 0 & -2 & -1 \\ \frac{1}{2} & \frac{7}{2} & 2 \end{bmatrix}$$

$v_{B_2} = (0,0,1)$

$$v_{\text{in } B_2} \to S \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} - \frac{5}{2} + 2 \\ 2 - 2 \\ \frac{1}{2} - \frac{7}{2} + 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Ex 2.3.31

$B_1 = e_1 = \{1,1,1\}$ $e_2 = \{1,1,0\}$ $e_3 = \{1,0,0\}$

$B_2 = f_1 = \{0,0,1\}$ $f_2 = \{3,-1,2\}$ $f_3 = \{1,2,5\}$

$v_{B_1} = (1,-1,0)$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 3 & 1 \\ 1 & 1 & 0 & 0 & -1 & 2 \\ \underbrace{1 \quad 0 \quad 0}_{B_1} & & & \underbrace{1 \quad 2 \quad 5}_{B_2} & & \end{array}\right] \rightarrow T = \begin{bmatrix} 1 & 2 & 5 \\ -1 & -3 & -3 \\ 0 & 4 & -1 \end{bmatrix}$$

Calculer S $\{T^{-1}\}$

$$S = \begin{bmatrix} -15/7 & -22/7 & -9/7 \\ 1/7 & 1/7 & 2/7 \\ 4/7 & 4/7 & 1/7 \end{bmatrix}$$

$v_{B_2} = (-1,2,4)$

$$v_{B_2} = S \cdot \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix}$$