Temă seminar 03

- relații de recurență -

- integrale -

1. $I_n = \int \ln^n x \, dx = \int \ln^n x \cdot 1 \, dx$

$(u^n)' = n \cdot u^{n-1}$

$(\ln x)' = \frac{1}{x}$

$\int dx = x$

$f(x) = \ln^n x \quad \rightarrow f'(x) = \frac{1}{x} \cdot n \cdot \ln x^{n-1}$

$g'(x) = 1 \quad \Rightarrow \int g'(x)\, dx = \int dx = x$

$I_n = \ln^n x \cdot x - \int \frac{1}{\cancel{x}} \cdot n \cdot \ln^{n-1} x \cdot \cancel{x} \, dx$

$I_n = \ln^n x \cdot x - n \underbrace{\int \ln^{n-1} x \, dx}_{I_{n-1}}$

$\boxed{I_n = \ln^n x \cdot x - n \cdot I_{n-1}}$ → calculați I_5

$I_5 = \ln^5 x \cdot x - 5 \cdot I_4$

$I_4 = \ln^4 x \cdot x - 4 \cdot I_3$

$I_3 = \ln^3 x \cdot x - 3 I_2$

$I_2 = \ln^2 x \cdot x - 2 I_1$

$I_1 = \ln x \cdot x - I_0$

$I_0 = x$

$(\ln x \cdot x)' = \frac{1}{x} \cdot x - \ln x \cdot x$

$\int \ln x \cdot 1 = \ln x \cdot x - \int \frac{1}{x} \cdot x$

$\int \ln x \cdot 1 = x \cdot \ln x - x$

$I_0 = x$

$I_1 = \ln x \cdot x - x$

$I_2 = x \ln^2 x - 2 I_1 = x \cdot \ln^2 x - (2x \ln x - 2x) = x \ln^2 x - 2x \ln x + 2x$

$I_3 = x \ln^3 x - 3 I_2 = x \cdot \ln^3 x - 3x \ln^2 x + 6x \ln x - 6x$

$I_4 = x \cdot \ln^4 x - 4 I_3 = x \cdot \ln^4 x - 4x \ln^3 x + 12x \ln^2 x - 24x \ln x + 24x$

$I_5 = x \cdot \ln^5 x - 5 I_4 \Rightarrow$

$I_5 = x \cdot \ln^5 x - 5x \cdot \ln^4 x + 20x \ln^3 x - 60x \ln^2 x + 120x \ln x - 120x$

$(\sqrt{u})' = \frac{1}{2\sqrt{u}} \cdot u'$

2) $I_n = \int \frac{x^n}{\sqrt{x^2-a^2}}\,dx = \int \frac{x^{n-1} \cdot (x \cdot 2)}{\sqrt{x^2-a^2}} \cdot \frac{1}{2}\,dx$

$= \int x^{n-1} \cdot \frac{(x^2-a^2)'}{2\sqrt{x^2-a^2}}\,dx = \int x^{n-1} \cdot \left(\sqrt{x^2-a^2}\right)' dx$

$f(x) = x^{n-1} \rightarrow f'(x) = (n-1)\, x^{n-2}$

$g'(x) = (\sqrt{x^2-a^2})' \rightarrow g(x) = \sqrt{x^2-a^2}$

$I_n = x^{n-1} \cdot \sqrt{x^2-a^2} - \int x^{n-2} (n-1) \cdot \sqrt{x^2-a^2}\,dx$

$I_n = x^{n-1} * \sqrt{x^2-a^2} - (n-1) \int x^{n-2} \frac{x^2-a^2}{\sqrt{x^2-a^2}}\,dx$

$\rightarrow I_{n-2}$

$I_n = x^{n-1}\sqrt{x^2-a^2} - (n-1) \int \frac{x^n}{\sqrt{x^2-a^2}}\,dx + (n-1) \int \frac{x^{n-2}\, a^2}{\sqrt{x^2-a^2}}\,dx$

$I_n = x^{n-1}\sqrt{x^2-a^2} - (n-1) \cdot I_n + (n-1) \cdot a^2 \cdot I_{n-2}$

~~I_0 = nu se poate~~

$\cancel{I_2 = x^1 \sqrt{x^2-a^2} - (n-1) I_2 + (n-1) a^2 \cdot I_1}$

$I_n = x^{n-1}\sqrt{x^2-a^2} - (n-1) \cdot I_n + (n-1) a^2\, I_{n-2}$

$I_n + (n-1) I_n = x^{n-1}\sqrt{x^2-a^2} + (n-1) a^2 \cdot I_{n-2}$

$(n+1-1) I_n = x^{n-1}\sqrt{x^2-a^2} + (n-1) a^2 \cdot I_{n-2}$

$n I_n = x^{n-1}\sqrt{x^2-a^2} + (n-1) a^2 \cdot I_{n-2}$

$I_n = \frac{x^{n-1}}{n}\sqrt{x^2-a^2} + \frac{n-1}{n} a^2 \cdot I_{n-2}$

$\sqrt{u}' = \frac{1}{2\sqrt{u}} \cdot u'$

3) $K_n = \int \frac{x^n}{\sqrt{a^2-x^2}}\,dx = \int x^{n-1} \frac{x \cdot 2 \cdot (-1)}{\sqrt{a^2-x^2}} \cdot \frac{-1}{2}\,dx$

$K_n = \int x^{n-1} \cdot \frac{(a^2-x^2)'}{2\sqrt{a^2-x^2}} \cdot (-1)\,dx = -\int x^{n-1} \cdot \frac{(a^2-x^2)'}{2\sqrt{a^2-x^2}}\,dx$

$K_n = -\int x^{n-1} (\sqrt{a^2-x^2})'\,dx$

Scanned with CamScanner

$$Kn = -\int x^{n-1}(\sqrt{a^2-x^2})dx$$

$$f(x) = x^{n-1} \rightarrow f'(x) = (n-1)x^{n-2}$$

$$g'(x) = (\sqrt{a^2-x^2})' \rightarrow g(x) = \sqrt{a^2-x^2}$$

$$Kn = -\left[x^{n-1}\cdot\sqrt{a^2-x^2} - \int (n-1)\cdot x^{n-2}\cdot\sqrt{a^2-x^2}\,dx\right]$$

$$Kn = -x^{n-1}\sqrt{a^2-x^2} + (n-1)\int x^{n-2}\cdot\sqrt{a^2-x^2}\,dx$$

$$Kn = -x^{n-1}\sqrt{a^2-x^2} + (n-1)\int x^{n-2}\cdot\frac{a^2-x^2}{\sqrt{a^2-x^2}}dx$$

$$Kn = -x^{n-1}\sqrt{a^2-x^2} + (n-1)\int x^{n-2}\cdot\frac{x^2-a^2}{\sqrt{a^2-x^2}}dx$$

$$Kn = -x^{n-1}\sqrt{a^2-x^2} + (n-1)\underbrace{\int\frac{x^n}{\sqrt{a^2-x^2}}}_{Kn} - \int\frac{a^2\,x^{n-2}}{\sqrt{a^2-x^2}}dx\cdot(n-1)$$

$Kn-2$

$$Kn = -x^{n-1}\sqrt{a^2-x^2} + (n-1)\cdot Kn - a^2\cdot Kn-2\cdot(n-1)$$

$$Kn-(n-1)Kn = -x^{n-1}\sqrt{a^2-x^2} - a^2\cdot Kn-2\cdot(n-1)$$

$$Kn(1-n+1) = -x^{n-1}\sqrt{a^2-x^2} - a^2(n-1)Kn-2$$

$$Kn(2-n) = -x^{n-1}\sqrt{a^2-x^2} - a^2(n-1)Kn-2$$

$$Kn[(-1)(n-2)] = (-1)\left[x^{n-1}\sqrt{a^2-x^2} - a^2(n-1)Kn-2\right]$$

$$Kn = \frac{\cancel{(-1)}}{\cancel{(-1)}(n-2)}\left[x^{n-1}\sqrt{a^2-x^2} - a^2(n-1)Kn-2\right]$$

$$Kn = \frac{1}{n-2}\left[x^{n-1}\sqrt{a^2-x^2} - a^2(n-1)Kn-2\right]$$

$$Kn = \frac{x^{n-1}}{n-2}\sqrt{a^2-x^2} - \frac{a^2(n-1)}{n-2}Kn-2$$