Ora suplimentară 12.11.2021

Exercitiul 1:

se considera $\sum_{1}^{\infty} \frac{x^n}{n}$

$x + \frac{x^2}{2} + \frac{x^3}{3} + \dots + \frac{x^n}{n} + \dots$ $\qquad a_n = \frac{1}{n}$

$$\rho = \left|\frac{a_n}{a_{n+1}}\right| = \left|\frac{\frac{1}{n}}{\frac{1}{n+1}}\right| = \lim_{n\to\infty}\left|\frac{n+1}{n}\right| = 1$$

$$I = (-1, 1)$$

convergenta la capetele lui I

$x = 1$ seria $1 + \frac{1}{2} + \dots + \frac{1}{n} + \dots$

serie armonica divergentă

$x = -1$ seria $-1 + \frac{1}{2} - \frac{1}{3} + \dots + \frac{(-1)^n}{n} + \dots$

$\Rightarrow A = [-1, 1)$ serie alternata convergentă

$-1\left(1 - \frac{1}{2} + \frac{1}{3} + \dots + \frac{(-1)^{n-1}}{n} + \dots\right)$ serie alternata cu semn schimbat.

$= -\ln 2$

fie $S(x)$ suma seriei $S(x) = x + \frac{x^2}{2} + \dots + \frac{(x)^n}{n} + \dots$

$x \in [-1, 1)$

$S'(x) = 1 + x + x^2 + \dots + x^{n-1} + \dots$

prog geo rația $r = x$ convergentă pe $x \in (-1, 1)$

$$S'(x) = \lim_{n\to\infty}\left(1 + x + x^2 + \dots + x^{n-1}\right)$$

$$= \lim_{n\to\infty} \frac{1 - x^n}{1 - x} = \frac{1}{1 - x}$$

$S(x) = \int \frac{1}{1-x} dx = -\ln|1-x| = -\ln(1-x) = \ln(1-x)^{-1} = \ln\left(\frac{1}{1-x}\right)$

$-\ln(1-x) = x + \frac{x^2}{2} + \dots + \frac{x^n}{n} + \dots \quad x \in [-1, 1)$

$\Rightarrow 1 - \frac{1}{2} + \frac{1}{3} + \dots + \frac{(-1)^{n-1}}{n} + \dots = \ln 2$

daca $x \in [-1, 1) \rightarrow -x \in (-1, 1)$

înlocuim pe x cu $-x$ și $\Rightarrow$

$-\ln(1+x) = -x + \frac{x^2}{2} - \frac{x^3}{3} + \dots + \frac{(-x)^n}{n} + \dots \quad | \cdot (-1)$

$$(+)\begin{cases} \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots + \frac{(-1)^{n+1} x^n}{n} \quad x \in (-1, 1] \\ -\ln(1-x) = x + \frac{x^2}{2} - \frac{x^3}{3} + \dots + \frac{(-1)^n x^n}{n} + \dots \end{cases}$$

$\ln(1+x) - \ln(1-x) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \dots + \frac{x^{2n+1}}{2n+1} + \dots\right)$

$\frac{1}{2} \ln\left(\frac{1+x}{1-x}\right) = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots + \frac{x^{2n+1}}{2n+1} + \dots$

$|x| < 1$

seria permite calculul logaritmului natural oricărui nr pozitiv.

fie $\frac{1+x}{1-x} = y \Rightarrow 1+x = y(1-x)$

$1 + x = y - yx$

$1 - y = -yx - x \quad | \cdot (-1)$

$x(1+y) = y - 1$

$x = \frac{y-1}{y+1}$

dacă $y > 0 \Rightarrow |x| < 1$.

$\ln y = 2\left(\frac{y-1}{y+1} + \frac{1}{3}\left(\frac{y-1}{y+1}\right)^3 + \frac{1}{5}\left(\frac{y-1}{y+1}\right)^5 + \dots + \frac{1}{2n+1}\left(\frac{y-1}{y+1}\right)^{2n+1} + \dots\right)$

Seria binomială

$(1+x)^n = 1 + C_n^1 x + C_n^2 \cdot x^2 + \dots + C_n^k x^k + \dots + C_n^n x^n$

$C_n^k = \frac{n!}{(n-k)!\,k!} = \frac{A_n^k}{P_n}$

fie seria de puteri ale lui x:

$$1+Kx+\frac{K(K-1)}{1\cdot 2}x^2+\dots+\frac{K(K-1)\dots(K-n+1)}{n!}x^n+\dots$$

$K\in\mathbb{R}$ un nr real oarecare

$$\rho=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|$$

$$=\lim_{n\to\infty}\left|\frac{\frac{K(K-1)\dots(K-n+1)}{n!}}{\frac{K(K-1)\dots(K-n)}{(n+1)!}}\right|$$

$$=\lim_{n\to\infty}\left|\frac{n+1}{K-n}\right|=1$$

$$\Rightarrow x\in(-1,1)=J$$

studiem suma seriei pe J

= J - interval de convergență)

funcția $f(x)$ pt care se arată că verifică ecuația

$(1+x)\cdot f'(x)=K\cdot f(x)$ pt $|x|<1$

$$\Rightarrow \frac{f'(x)}{f(x)}=\frac{K}{1+x}\Rightarrow$$

$$\int\frac{f'(x)}{f(x)}=\int\frac{K}{1+x}dx$$

$$\ln(f(x))=K\cdot\ln(1+x)+C$$

$$\ln(f(x))=\ln(1+x)^K+C$$

$$f(x)=e^{[\ln(1+x)^K+C]}=(1+x)^K\cdot e^C$$

$$\boxed{e^{\ln a}=a}$$

pt $x=0$ $e^C=1\Rightarrow C=0$

$$(1+x)^K=1+K\cdot x+\frac{K(K-1)}{2!}x^2+\dots+\frac{K(K-1)\dots(K-n+1)}{n!}x^n+\dots$$

$|x|<1$

are o infinitate de termeni

notam simbolic $C_K^n = \dfrac{K(K-1)\dots(K-n+1)}{n!}$

seria binomiala se scrie astfel $(1+x)^K = 1 + C_n^1 x + C_n^2 x^2 + \dots + C_n^n x^n$

este generalizare a binomului lui Newton valabilă

pt $\forall K \in \mathbb{R}$ si pentru $|x| < 1$

pt diverse valori ale lui K avem:

$K = \frac{1}{2} \Rightarrow f(x) = (1+x)^{\frac{1}{2}} = \sqrt{1+x}$, $x \geq -1$ $f: [-1, \infty) \to \mathbb{R}$

$$C_{\frac{1}{2}}^n = \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\dots\left(\frac{1}{2}-n+1\right)}{n!} = \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\dots\left(-\frac{2n-3}{2}\right)}{n!}$$

$$= \frac{(-1)^{n-1} \cdot 1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-3)}{2^n \cdot n!} \Rightarrow$$

$$K = \frac{1}{2} \Rightarrow \sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{2^2 \cdot 2!} + \frac{1 \cdot 3\, x^3}{2^3 \cdot 3!} + \dots + \frac{(-1)^{n-1} \cdot 1 \cdot 3 \dots (2n-3) \cdot x^n}{2^n \cdot n!}$$

$$K = -\frac{1}{2} : \frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2} + \frac{1 \cdot 3\, x^2}{2^2\, 2!} - \frac{1 \cdot 3 \cdot 5\, x^3}{2^3\, 3!} + \dots (-1)^n \frac{1 \cdot 3 \dots (2n-1)\, x^n}{2^n \cdot n!}$$

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