Tema curs 03

a) $3y'(x^2-1)-2xy=0$ → neomogenă

aducem la o formă simplif.

$y'+P(x)y+Q(x)=0$

dacă $Q(x)=0$ →
devine omogen

$3y'(x^2-1)-2xy=0 \mid : (x^2-1) \neq 0$

$$3y' - \frac{2xy}{x^2-1} = 0 \mid : 3$$

$$y' - \frac{2xy}{x^2-1}\cdot\frac{1}{3} = 0$$

$$y' - \frac{2}{3}\frac{xy}{x^2-1} = 0$$

$$y' - \underline{\frac{2}{3}\cdot\frac{x}{x^2-1}}\cdot y = 0$$

$$y' + \frac{(-2)}{3}\frac{x}{x^2-1}y = 0$$

$$P(x) = -\frac{2}{3}\frac{x}{x^2-1}$$

$$y' = \frac{2}{3}\cdot\frac{x}{x^2-1}y$$

$$\frac{y'}{y} = \frac{2}{3}\cdot\frac{x}{x^2-1}$$

$$\int\frac{y'}{y}dx = \frac{2}{3}\int\frac{x}{x^2-1}dx$$

$$\ln y = \frac{2}{3}\int\frac{1}{2}\cdot\frac{(x^2-1)'}{x^2-1}dx$$

$$\ln y = \frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{2}}\cdot\ln(x^2-1)+\ln C$$

$$\ln y = \frac{1}{3}\ln(x^2-1)+\ln C$$

Aceasta este soluția!

$$\ln y = \ln(x^2-1)^{\frac{1}{3}}+\ln C$$

$y = (x^2-1)^{\frac{1}{3}}\cdot C$ → solutie ec. omogenă

Fie $C = C(x) \Rightarrow y(x) = (x^2-1)^{\frac{1}{3}} \cdot C(x)$

calculam $y'(x) = C'(x) \cdot (x^2-1)^{\frac{1}{3}} + \left[(x^2-1)^{\frac{1}{3}}\right]' \cdot C(x)$

$(u^n)' = n \cdot u^{n-1} \cdot u'$

$y'(x) = C'(x) \cdot (x^2-1)^{\frac{1}{3}} + \frac{1}{3}\left[(x^2-1)^{\frac{1}{3}-1}\right] \cdot (x^2-1)' \cdot C(x)$

$y'(x) = C'(x) \cdot (x^2-1)^{\frac{1}{3}} + \frac{1}{3}(2x) \cdot (x^2-1)^{-\frac{2}{3}} C(x)$

$y'(x) = C'(x)(x^2-1)^{\frac{1}{3}} + C(x) \cdot \frac{1}{3} \cdot 2x \cdot (x^2-1)^{-\frac{2}{3}}$

$y'(x) = C'(x)\sqrt[3]{x^2-1} + \frac{2}{3} C(x) \cdot x \sqrt[3]{(x^2-1)^2}$

$C'(x)\sqrt[3]{x^2-1} + \frac{2}{3} C(x) \cdot x \sqrt[3]{(x^2-1)^2} + \frac{(-2)}{3} \cdot \frac{x}{x^2-1} \cdot (x^2-1)^{\frac{1}{3}} \cdot C(x) = 0$

$C'(x)\sqrt[3]{x^2-1} + \frac{2}{3} C(x) \cdot x \sqrt[3]{(x^2-1)^2} - \frac{2}{3} x \cdot (x^2-1)^{\frac{1}{3}-1} \cdot C(x) = 0$

$C'(x)\sqrt[3]{x^2-1} + \cancel{\frac{2}{3} C(x) \cdot x \sqrt[3]{(x^2-1)^2}} - \cancel{\frac{2}{3} \cdot x \cdot C(x) \cdot \sqrt[3]{(x^2-1)^2}} = 0$

$C'(x)\sqrt[3]{x^2-1} = 0 \quad | : \sqrt{x^2-1}$

$C'(x) = 0$

$C(x) = \int 0 \, dx = C$

$y = (x^2-1)^{\frac{1}{3}} \cdot C(x)$

$y = \underbrace{(x^2-1)^{\frac{1}{3}} \cdot C}_{\text{solutie pt omogen}}$ $\rightarrow y = (x^2-1)^{\frac{1}{3}} \cdot C$ soluție

b) $y' + 2xy - x^3 = 0 \rightarrow$ neomogen

$y' + 2x \cdot y + (-x^3) = 0$ $\qquad y' + P(x) \cdot y + Q(x) = 0$

$\hookrightarrow P(x) = 2x$

$Q(x) = (-x^3)$

ecuatia omogena $P(x)y + y' = 0$

$y' + 2xy = 0$

$y' = -2xy \quad | \cdot \frac{1}{y}$

$$\frac{y'}{y} = -2x \Leftrightarrow \int \frac{y'}{y}\,dx = \int -2x\,dx$$

$$\ln(y) = -2\int x\,dx + \ln C \qquad \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\ln(y) = -\cancel{2}\cdot\frac{x^2}{\cancel{2}} + \ln C \qquad (e^u)' = e^u \cdot u'$$

$$\ln(y) = -x^2 + \cancel{c}\ln C \Rightarrow y = e^{-x^2+\ln C} = e^{-x^2}\cdot e^{\ln C} = C\cdot e^{-x^2}$$

$$\ln(y) = C - x^2$$

Aşa e în curs!

$$y_0 = e^{C-x^2} \rightarrow \text{soluţie : ecuaţie omogenă}$$

De refăcut!

Fie $C = C(x) \rightarrow y = e^{C(x)-x^2}$

$$y' = e^{C(x)-x^2}\cdot(C(x)-x^2)'$$

$$y' = e^{C(x)-x^2}\cdot[C(x)' - 2x]$$

înlocuim în ecuaţia neomogenă

$$y' + 2xy - x^3 = 0$$

$$e^{C(x)-x^2}(C(x)' - 2x) + 2x\cdot e^{C(x)-x^2} - x^3 = 0$$

$$C(x)'\cdot e^{C(x)-x^2} - \cancel{2x\cdot e^{C(x)-x^2}} + \cancel{2x\cdot e^{C(x)-x^2}} - x^3 = 0$$

$$C(x)'\cdot e^{C(x)-x^2} - x^3 = 0 \;\big|+x^3$$

$$C(x)'\cdot e^{C(x)-x^2} = x^3 \;\big|:\left(e^{C(x)-x^2}\right)$$

$$C(x)' = \frac{x^3}{e^{C(x)-x^2}}$$

$$C(x) = \int \frac{x^3}{e^{C(x)-x^2}}\,dx$$

$$y = e^{C-x^2} = e^{\int \frac{x^3}{e^{C(x)-x^2}}dx - x^2}$$

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