Capitol 4

Ex 4.2.1

$$A = \begin{pmatrix} 5 & -3 & 2 \\ 6 & -4 & 4 \\ 4 & -4 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 5-\lambda & -3 & 2 \\ 6 & -4-\lambda & 4 \\ 4 & -4 & 5-\lambda \end{pmatrix} \rightarrow \begin{pmatrix} 2-\lambda & -3 & 2 \\ 2-\lambda & -4-\lambda & 4 \\ 0 & -4 & 5-\lambda \end{pmatrix} \rightarrow$$

$$\rightarrow (2-\lambda) \cdot \begin{pmatrix} 1 & -3 & 2 \\ 0 & -4-\lambda & 4 \\ 0 & -4 & 5-\lambda \end{pmatrix} \rightarrow$$

$$\rightarrow (2-\lambda)\left[(-4-\lambda)(5-\lambda)+8\right] \rightarrow$$

$$\rightarrow (2-\lambda)(\lambda-1)(\lambda-3)$$

$\lambda_1 = 2 \quad \lambda_2 = 1 \quad \lambda_3 = 3$

$\lambda_2 = 1$

$$\left(\begin{array}{ccc|c} 4 & -3 & 2 & 0 \\ 6 & -5 & 4 & 0 \\ 4 & -4 & 4 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 4 & -3 & 2 & 0 \\ 0 & -2 & 4 & 0 \\ 0 & -4 & 8 & 0 \end{array}\right) \to$$

$$\left(\begin{array}{ccc|c} 4 & -3 & 2 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & -1 & 2 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} -4 & 0 & 4 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \to$$

$x_3 = \alpha \quad x_1 = \alpha \quad x_2 = 2\alpha$

$v_2 = (\alpha, 2\alpha, \alpha) = \alpha(1,2,1) \; ; \alpha \neq 0$

$v_2 = (1,2,1)$

$\lambda_1 = 2$

$$\left(\begin{array}{ccc|c} 3 & -3 & 2 & 0 \\ 6 & -6 & 4 & 0 \\ 4 & -4 & 3 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 3 & -3 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \to$$

$$\left(\begin{array}{ccc|c} 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \quad \begin{array}{l} 3x_1 = 3x_2 \\ x_2 = \alpha \\ x_3 = 0 \end{array} \left\} \begin{array}{l} x_1 = \alpha \\ x_2 = \alpha \\ x_3 = 0 \end{array}\right.$$

$v_1 = (\alpha, \alpha, 0) \Rightarrow \alpha(1, 1, 0), \ \alpha \neq 0$

$v_1 = (1, 1, 0)$

$\lambda_3 = 3$

$$\begin{pmatrix} 2 & -3 & 2 \\ 6 & -7 & 4 \\ 4 & -4 & 2 \end{pmatrix} \to \left(\begin{array}{ccc|c} 2 & -3 & 2 & 0 \\ 0 & 4 & -4 & 0 \\ 0 & 4 & -4 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 2 & -3 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 2 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 = \alpha/2 \quad x_2 = \alpha \quad x_3 = \alpha$

$v_3 = \left(\frac{\alpha}{2}, \alpha, \alpha\right) \to \alpha\left(\frac{1}{2}, 1, 1\right) \to (2, 1, 1)$

$v_3 = (2, 1, 1)$

Ex. 4.22

$$A = \begin{pmatrix} 2 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 0 \end{pmatrix} \to \begin{pmatrix} 2-\lambda & -2 & 0 \\ -2 & 1-\lambda & -2 \\ 0 & -2 & -\lambda \end{pmatrix} \to$$

$$\Rightarrow \left[(2-\lambda)(1-\lambda)\cdot-\lambda\right] + 0 + 0 - 0 - \left[-2\cdot-2\cdot-\lambda\right] -$$

$$\left[(2-\lambda)\cdot-2\cdot-2\right] \to$$

$$\to \left[(2-2\lambda-\lambda+\lambda^2)\cdot-\lambda\right] + 4\lambda - \left[8-4\lambda\right] \to$$

$$\to \left[(2-3\lambda+\lambda^2)\cdot-\lambda\right] + 8\lambda - 8 \to$$

$$\to (-2\lambda+3\lambda^2-\lambda^3) - 8 + 8\lambda \to$$

$$\to -\lambda^3+3\lambda^2+6\lambda-8$$

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

λ	-1	3	6	-8
1	-1	2	8	0

$\lambda_1 = 1$

$(\lambda-1)(-\lambda^2+2\lambda+8)$

$\lambda_2 = 4 \quad \lambda_3 = -2$

$\lambda_1 \neq \lambda_2 \neq \lambda_3 \to$ Diagonalizabila

$\lambda_1 = 1$

$$\begin{pmatrix} 1 & -2 & 0 \\ -2 & 0 & -2 \\ 0 & -2 & -1 \end{pmatrix} \qquad \begin{cases} x_1 - 2x_2 = 0 \\ -2x_1 - 2x_3 = 0 \\ -2x_2 - x_3 = 0 \end{cases}$$

$$\begin{cases} x_1 = 2x_2 \\ -4x_2 - 2x_3 = 0 \\ -2x_2 - x_3 = 0 \end{cases} \qquad \begin{cases} x_1 = 2x_2 \\ x_3 = -2x_2 \\ 0 = 0 \end{cases} \qquad \begin{matrix} x_1 = 2\alpha \\ x_3 = -2\alpha \\ x_2 = \alpha \end{matrix}$$

$v_1 = (2, 1, -2)$

$\lambda_2 = 4$

$$\begin{pmatrix} -2 & -2 & 0 \\ -2 & -3 & -2 \\ 0 & -2 & -4 \end{pmatrix} \quad \begin{cases} -2x_1 - 2x_2 = 0 \\ -2x_1 - 3x_2 - 2x_3 = 0 \\ -2x_2 - 4x_3 = 0 \end{cases}$$

$$\begin{cases} x_1 = -x_2 \\ x_2 - 2x_3 = 0 \\ -2x_2 - 4x_3 = 0 \end{cases} \quad \begin{cases} x_1 = -x_2 \\ x_2 = -2x_3 \\ 0 = 0 \end{cases} \quad \begin{aligned} x_1 &= +2\alpha \\ x_2 &= -2\alpha \\ x_3 &= \alpha \end{aligned}$$

$v_2 = (-2, 2, 1)$

$\lambda_3 = -2$

$$\begin{pmatrix} 4 & -2 & 0 \\ -2 & 3 & -2 \\ 0 & -2 & 2 \end{pmatrix} \quad \begin{cases} 4x_1 - 2x_2 = 0 \\ -2x_1 + 3x_2 - 2x_3 = 0 \\ -2x_2 + 2x_3 = 0 \end{cases}$$

$$\begin{cases} x_3 = 2x_1 \\ 0 = 0 \\ x_2 = x_3 \end{cases} \quad \begin{aligned} x_3 &= 2\alpha \\ x_1 &= \alpha \\ x_2 &= 2\alpha \end{aligned} \quad v_3 = (1, 2, 2)$$

$$\begin{pmatrix} 2 & +2 & 1 \\ 1 & -2 & 2 \\ -2 & 1 & 2 \end{pmatrix} = C \qquad C^{-1} = \begin{pmatrix} 2/9 & 1/9 & -2/9 \\ 2/9 & -2/9 & 1/9 \\ 2/9 & 2/9 & 2/9 \end{pmatrix}$$

$$D^n = C \cdot A^n \cdot C^{-1} \rightarrow A^n = C^{-1} \cdot D^n \cdot C$$

$$A^{50} = \begin{bmatrix} 2 & 2 & 1 \\ 1 & -2 & 2 \\ -2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{100} & 0 \\ 0 & 0 & 2^{50} \end{bmatrix} \cdot C^{-1}$$

Ex 4.3.14

$$A = \begin{pmatrix} 5 & 2 & -3 \\ 6 & 4 & -4 \\ 4 & 5 & -4 \end{pmatrix}$$

$$\begin{pmatrix} 5-\lambda & 2 & -3 \\ 6 & 4-\lambda & -4 \\ 4 & 5 & -4-\lambda \end{pmatrix} \rightarrow [(5-\lambda)(4-\lambda)(-4-\lambda)] - 32 - 9\ldots$$

$$-[-3(4-\lambda)+4] - [2 \cdot 6 \cdot (-4-\lambda)]$$

$$-[(5-\lambda) \cdot -4 \cdot 5] =$$

$$\rightarrow [(5-\lambda)(16+4\lambda+4\lambda+\lambda^2)] - 12\ldots$$

$$-[\cancel{-12}\ 48+12\lambda] - [-48-12\ldots$$

$$-[-100+20\lambda] \rightarrow$$

$$\rightarrow (80+20\lambda+20\lambda+5\lambda^2$$
$$-16\lambda-4\lambda^2-4\lambda^2-\lambda^3)$$
$$-12\lambda+100-20\lambda =$$

$$\rightarrow -\lambda^3 - 3\lambda^2 + 4\lambda + 58$$

$[(5-z)(4-z)(-4-z)] - 12z - [-3(4-z)4] +$

$-[2\cdot 6\cdot(-4-z)] - [(5-z)\cdot 4\cdot 5] =$

$= [(5-z)(-16+z^2)] - 12z - [-48+12z] +$

$-[-48-12z] - [-100+20z] =$

$= [-80+5z^2+16z-z^3] - 12z + 48 - \cancel{12z} +$

$+48 + \cancel{12z} + 100 - 20z =$

$= -z^3 + 5z^2 - 4z - 6$

$$\begin{array}{c|cccc} z & -1 & 5 & -4 & -6 \\ \hline 1 & -1 & 4 & 0 & \\ 3 & -3 & 2 & -6 & 0 \end{array}$$

$z_1 = 3$

$(z-3)(-3z^2+2z-6)$

$z_{1,2} = -2 \pm \sqrt{4 -}$

$$\begin{array}{c|cccc} z & -1 & 5 & -4 & -6 \\ \hline 3 & -1 & 2 & +2 & 0 \end{array}$$

$-(z-3)(1z^2-2z-2)$

$z_{1,2} = \dfrac{2 \pm \sqrt{4+8}}{2} = \dfrac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$

$\left.\begin{array}{l} z_1 = 3 \\ z_2 = 1+\sqrt{3} \\ z_3 = 1-\sqrt{3} \end{array}\right\} \neq$ D I A G O N.

Ex. 4.3.16

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{pmatrix} =$$

$$= -\lambda^3 + 1 + 1 + \lambda + \lambda + \lambda =$$

$$= -\lambda^3 + 3\lambda + 2$$

λ	-1	0	3	2
1	-1	-1	2	4
-1	-1	1	2	0

$\lambda_1 = -1$

$\lambda_2 = 2$

$\lambda_3 = -1$

$$-(\lambda - 1)(\lambda^2 - \lambda - 2)$$

$$\lambda_{1,2} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2}$$

$\lambda_1, \lambda_3 = -1$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 = -x_2 - x_3$

$x_2 = \alpha$

$x_3 = \beta$

$$\to \begin{pmatrix} -\alpha - \beta \\ \alpha \\ \beta \end{pmatrix} \to \alpha \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \beta \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$v_1 = (-1, 1, 0) \quad v_3 = (-1, 0, 1)$

$$D = \begin{pmatrix} -1 & & 0 \\ & 2 & \\ 0 & & -1 \end{pmatrix}$$

$\lambda_2 = 2$

$$\begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix} \to \left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 0 & 3 & -3 & 0 \\ 0 & -3 & 3 & 0 \end{array}\right) \to$$

$$\to \left(\begin{array}{ccc|c} -2 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 = \alpha \quad x_2 = \alpha \quad x_3 = \alpha$

$v_2 = (1, 1, 1)$

$$C = \begin{pmatrix} -1 & 1 & -1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \qquad C^{-1} = \begin{pmatrix} -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \end{pmatrix}$$

$$A^k = C^{-1} \cdot D^k \cdot C$$

$$A^k = \begin{pmatrix} -1 & 1 & -1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1^k & 0 & 0 \\ 0 & 2^k & 0 \\ 0 & 0 & -1^k \end{pmatrix} \begin{pmatrix} -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \end{pmatrix}$$

E. 4.3.17

b)

$$A = \begin{pmatrix} -1 & 0 & -3 \\ 3 & 2 & 3 \\ -3 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} -1-\lambda & 0 & -3 \\ 3 & 2-\lambda & 3 \\ -3 & 0 & -1-\lambda \end{pmatrix} \quad \begin{aligned} &[(-\lambda-1)(2-\lambda)(-1-\lambda)] + \\ &-[9\cdot(2-\lambda)] = \\ &= [(\lambda^2+1+2\lambda)(2-\lambda)] - 18 + 9\lambda \end{aligned}$$

$= -\lambda^3 + 12\lambda - 16$

$$\begin{array}{c|cccc} \lambda & +1 & 0 & -12 & -16 \\ \hline -4 & +1 & -4 & +4 & 0 \end{array}$$

$\lambda_1 = -4$

$\lambda_2 = 2$

$\lambda_3 = 2$

$\cdot(\lambda+4)(\lambda^2 - 4\lambda + 4)$

$$\lambda_{2,3} = \frac{+4 \pm \sqrt{16-16}}{2} = \frac{+4}{2} = +2$$

$\lambda_1 = -4$

$$\begin{pmatrix} 3 & 0 & -3 \\ 3 & 6 & 3 \\ -3 & 0 & 3 \end{pmatrix} \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 1 & 2 & 1 & 0 \\ -1 & 0 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 = \alpha$ $\quad x_2 = -\alpha$ $\quad x_3 = \alpha$

$v_1 = (1, -1, 1)$

$\lambda_{2,3} = +2$

$$\begin{pmatrix} -3 & 0 & -3 \\ 3 & 0 & +3 \\ -3 & 0 & -3 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$X_1 = -\beta$

$X_2 = \alpha$

$X_3 = \beta$

$$v = (-\beta, \alpha, \beta) \to \alpha \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \beta$$

$v_2 = (0, 1, 0) \quad v_3 = (-1, 0, 1)$

$$D = \begin{pmatrix} -4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$C^{-1} = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 1/2 & 1 & 1/2 \\ -1/2 & 0 & 1/2 \end{pmatrix} \quad A^k = C^{-1} \cdot D^k \cdot C$$

c)

$$A = \begin{bmatrix} 2 & 2 & 2 & -2 \\ 2 & 2 & -2 & 2 \\ 2 & -2 & 2 & 2 \\ -2 & 2 & 2 & 2 \end{bmatrix}$$

$$\begin{pmatrix} 2-\lambda & 2 & 2 & -2 \\ 2 & 2-\lambda & -2 & 2 \\ 2 & -2 & 2-\lambda & 2 \\ -2 & 2 & 2 & 2-\lambda \end{pmatrix} \rightarrow \lambda^4 - 8\lambda^3 + 128\lambda - 256$$

$$\cancel{\begin{array}{c|cccc} \lambda & 1 & -8 & 128 & -256 \\ \hline +4 & 1 & -4 & 176 & \end{array}}$$

$$\begin{array}{c|ccccc} \lambda & 1 & -8 & 0 & 128 & -256 \\ \hline 4 & 1 & -4 & -16 & +64 & 0 \\ -4 & 1 & -8 & 16 & 0 & \\ +4 & 1 & -4 & 0 & & \\ 4 & 1 & 0 & & & \end{array}$$

$\lambda_1 = 4$

$\lambda_2 = -4$

$\lambda_3 = 4$

$\lambda_4 = 4$

$\lambda_1, \lambda_3, \lambda_4 = 4$

$$\begin{bmatrix} -2 & 2 & 2 & -2 \\ 2 & -2 & -2 & 2 \\ 2 & -2 & -2 & 2 \\ -2 & 2 & 2 & -2 \end{bmatrix} \rightarrow \left[\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = \alpha + \beta - \gamma$ $\quad x_2 = \alpha \quad x_3 = \beta \quad x_4 = \gamma$

$$v = \begin{pmatrix} \alpha + \beta - \gamma \\ \alpha \\ \beta \\ \gamma \end{pmatrix} \rightarrow \alpha \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$v_1 = (1, 1, 0, 0)$ $\quad v_3 = (1, 0, 1, 0)$ $\quad v_4 = (-1, 0, 0, 1)$

$\lambda_2 = -4$

$$\begin{bmatrix} 6 & 2 & 2 & -2 \\ 2 & 6 & -2 & 2 \\ 2 & -2 & 6 & 2 \\ -2 & 2 & 2 & 6 \end{bmatrix} \rightarrow \left[\begin{array}{cccc|c} 3 & 1 & 1 & -1 & 0 \\ 1 & 3 & -1 & 1 & 0 \\ 1 & -1 & 3 & 1 & 0 \\ -1 & 1 & 1 & 3 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 3 & 1 & 1 & -1 & 0 \\ 0 & 8 & -4 & 4 & 0 \\ 0 & -4 & 8 & 4 & 0 \\ 0 & 4 & 4 & 8 & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 3 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 1 & 0 \\ 0 & -1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 6 & 0 & 3 & -3 & 0 \\ 0 & 2 & -1 & 1 & 0 \\ 0 & 0 & 3 & 3 & 0 \\ 0 & 0 & 3 & 3 & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 2 & 0 & 1 & -1 & 0 \\ 0 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 2 & 0 & 0 & -2 & 0 \\ 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{array}{l} x_1 = 2 \\ x_2 = -2 \\ x_3 = -2 \\ x_4 = 2 \end{array}$$

$$v_2 = (1, -1, -1, 1)$$

$$D = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \qquad C = \begin{bmatrix} v_1 & v_2 & v_3 & v_4 \end{bmatrix}$$

$$C^{-1} = \begin{bmatrix} 1/4 & 3/4 & -1/4 & 1/4 \\ 1/4 & -1/4 & -1/4 & 1/4 \\ 1/4 & -1/4 & 3/4 & 1/4 \\ -1/4 & 1/4 & 1/4 & 1/4 \end{bmatrix}$$

$$A^k = C^{-1} \cdot D^k \cdot C$$

Ex 4.3.18

$$A = \begin{bmatrix} 11 & -5 & 5 \\ -5 & 3 & -3 \\ 5 & -3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 11-\lambda & -5 & 5 \\ -5 & 3-\lambda & -3 \\ 5 & -3 & 3-\lambda \end{bmatrix} = -\lambda^3 + 17\lambda^2 - 16\lambda$$

$$= \lambda^3 - 17\lambda^2 + 16\lambda$$

$$\lambda \cdot (\lambda^2 - 17\lambda + 16)$$

λ	1	-17	$+16$
$+1$	1	-16	0
16	1	0	

$\lambda_1 = 0$

$\lambda_2 = 1$

$\lambda_3 = 16$

$\lambda_1 = 0$

$$\left(\begin{array}{ccc|c} 11 & -5 & 5 & 0 \\ -5 & 3 & -3 & 0 \\ 5 & -3 & 3 & 0 \end{array}\right) \rightarrow$$

$$\rightarrow \left(\begin{array}{ccc|c} 11 & -5 & 5 & 0 \\ 0 & 8 & -8 & 0 \\ 0 & -8 & 8 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 11 & -5 & 5 & \\ 0 & 1 & -1 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 11 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \begin{array}{l} X_1 = 0 \\ X_2 = \alpha \\ X_3 = \alpha \end{array} \quad v_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$\lambda_2 = 1$

$$\left(\begin{array}{ccc|c} \overset{2}{\cancel{10}} & \overset{-1}{\cancel{-5}} & \overset{1}{\cancel{5}} & 0 \\ -5 & 2 & -3 & 0 \\ 5 & -3 & 2 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 2 & -1 & 1 & 0 \\ 0 & +1 & +1 & 0 \\ 0 & +1 & +1 & 0 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 2 & 0 & 2 & 0 \\ 0 & 1 & +1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \begin{array}{l} x_1 = -\alpha \\ x_2 = -\alpha \\ x_3 = \alpha \end{array} \quad v_2 = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

$\lambda_3 = 16$

$$\left(\begin{array}{ccc|c} -5 & -5 & 5 & 0 \\ -5 & -13 & -3 & 0 \\ 5 & -3 & -13 & 0 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ -5 & -13 & -3 & 0 \\ 5 & -3 & -13 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -8 & -8 & 0 \\ 0 & -8 & -8 & 0 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 = 2\alpha \quad x_2 = -\alpha \quad x_3 = \alpha$

$v_3 = (2, -1, 1)$

Diagonalizabila

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 16 \end{pmatrix} \quad C = \begin{pmatrix} 0 & -1 & 2 \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix}$$