Temă - seminar 01

E1 / 180

a) $F(x) = 2x^3 - 4x^2 - 5x + 9$ , $x \in \mathbb{R}$

$F'(x) = 6x^2 - 8x - 5$

b) $F(x) = \sqrt[3]{x^2} + 4x^2\sqrt{x}$ , $x \in (0,\infty)$

$F(x) = x^{\frac{2}{3}} + 4x^2 \cdot x^{\frac{1}{2}}$

$F'(x) = \frac{2}{3}x^{\frac{2-3}{3}} + (4x^{\frac{5}{2}})'$

$F'(x) = \frac{2}{3}x^{-\frac{1}{3}} + 4 \cdot \frac{5}{2} \cdot x^{\frac{5}{2}-1}$

$F'(x) = \frac{2}{3} \cdot \sqrt[3]{x^{-1}} + 10x^{\frac{3}{2}}$

$F'(x) = \frac{2}{3}\sqrt[3]{\frac{1}{x}} + 10\sqrt[2]{x^3}$

$F(x)' = \frac{2}{3}\sqrt[3]{\frac{1}{x}} + 10x\sqrt{x}$

$(x^n)' = n \cdot x^{n-1}$

$(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

$(f \cdot g)' = f' \cdot g + f \cdot g'$

$(\sin x)' = \cos x$

$(\ln x)' = \frac{1}{x}$

$(c)' = 0$

c) $F(x) = x \cdot \sin x$ , $x \in \mathbb{R}$

$F'(x) = x' \cdot \sin x + x \cdot (\sin x)'$

$F'(x) = \sin x + x \cdot \cos x$

d) $F(x) = x(\ln x - 1)$, $x \in (0,\infty)$

$F'(x) = x'(\ln x - 1) + x(\ln x - 1)'$

$F'(x) = \ln x - 1 + x\left(\frac{1}{x} - 0\right)$

$F'(x) = \ln x - 1 + x \cdot \frac{1}{x} = \ln x - 1 + 1 = \ln x$

e) $F(x) = \frac{(x^3 - 2x)}{x+1}$ , $x \in (0,\infty)$

$F'(x) = \frac{(x^3-2x)' \cdot (x+1) - (x^3-2x) \cdot (x+1)'}{(x+1)^2}$

$F'(x) = \frac{(3x^2-2)(x+1) - (x^3-2x)}{(x+1)^2}$

$F'(x) = \frac{3x^3 + 3x^2 - \cancel{2x} - 2 - x^3 + \cancel{2x}}{(x+1)^2} = \frac{2x^3 + 3x^2 - 2}{(x+1)^2}$

f) $F(x) = e^x(x-1)+4$ , $x \in \mathbb{R}$

$F'(x) = (e^x)'(x-1) + e^x(x-1)'$

$F'(x) = e^x(x-1) + e^x$

$F'(x) = e^x(x-1+1) = x \cdot e^x + C$

$(e^x)' = e^x$

$(tg x)' = \frac{1}{\cos^2 x}$

g) $F(x) = tg^2 x + tg x$ , $x \in \left(0, \frac{\pi}{4}\right)$

$F(x) = tg x \cdot tg x + tg x$

$F(x) = tg x\,(tg x + 1)$

$F'(x) = (tg x)' \cdot (tg x + 1) + tg x \cdot (tg x + 1)'$

$F'(x) = \frac{1}{\cos^2 x}(tg x + 1) + tg x\left(\frac{1}{\cos^2 x} + 0\right)$

$F'(x) = \frac{tg x + 1}{\cos^2 x} + \frac{tg x}{\cos^2 x}$

$F'(x) = \frac{2 tg x + 1}{\cos^2 x} + C$

E2 / 180

$$F(x) = \begin{cases} \frac{2^x}{\ln 2} + x - \frac{2}{\ln 2}, & x \le 1 \\ \frac{x^2}{2} + 2x - \frac{3}{2}, & x > 1 \end{cases}$$

$(a^x)' = a^x \cdot \ln a$

$(x^n)' = n \cdot x^{n-1}$

$$F'(x) = \begin{cases} \frac{1}{\ln 2} \cdot (2^x)' + x' - \left(\frac{2}{\ln 2}\right)' \\ \frac{1}{2} \cdot (x^2)' + 2 \cdot x' - \left(\frac{3}{2}\right)' \end{cases}$$

$$F'(x) = \begin{cases} \frac{1}{\ln 2} \cdot 2^x \cdot \ln 2 + 1 \\ \frac{1}{2} \cdot 2x + 2 \end{cases}$$

$$F'(x) = \begin{cases} 2^x + 1, & x \le 1 \\ x + 2, & x > 1 \end{cases}$$

$$f(x) = \begin{cases} 2^x + 1, & x \le 1 \\ x + 2, & x > 1 \end{cases}$$

E

E3 / 180

$F_1(x) = \begin{cases} \frac{x^3}{3} + \frac{x^2}{2} + x + 1, & x \le 0 \\ e^x + 1, & x > 0 \end{cases}$

$(e^x)' = e^x$

$(x^n)' = n \cdot x^{n-1}$

$F_1(x)' = \begin{cases} \frac{1}{3} \cdot 3x^2 + \frac{1}{2} \cdot 2x + 1 \\ (e^x)' \end{cases}$

$F_1(x)' = \begin{cases} x^2 + x + 1, & x \le 0 \\ e^x, & x > 0 \end{cases}$

$F_2(x) = \begin{cases} \frac{x^3}{3} + \frac{x^2}{2} + x, & x \le 0 \\ e^x - 1, & x > 0 \end{cases}$

$f(x) = \begin{cases} e^x, & x > 0 \\ x^2 + x + 1, & x \le 0 \end{cases}$

$F_2'(x) = \begin{cases} \frac{1}{3} \cdot 3x^2 + \frac{1}{2} \cdot 2x + 1 \\ (e^x)' \end{cases}$

$F_2'(x) = \begin{cases} x^2 + x + 1, & x \le 0 \\ e^x, & x > 0 \end{cases}$

$F'(x) = f(x)$
↳ primitivă

E4 / 180

a) $f(x) = x^3 - 4x + x + 3$
↳ continuă (func. exponențială

$\int f(x)\,dx = \int x^3 - 4\int x + \int x + 3\int dx$

$\int f(x)\,dx = \frac{x^4}{4} - 4\frac{x^2}{2} + \frac{x^2}{2} + 3x$

$\int f(x)\,dx = \frac{x^4}{4} + \frac{(-4)x^2 + x^2}{2} + 3x$

$\int f(x)\,dx = \frac{x^4}{4} + \frac{(-3)x^2}{2} + 3x$

$F(x) = \frac{1}{4}x^4 - \frac{3}{2}x^2 + 3x + C$

$f(x) = x^3 - 4x^2 + x + 3$

↳ $F(x) = \frac{1}{4} \cdot x^4 - 4 \cdot \frac{1}{3}x^3 + \frac{1}{2} \cdot x^2 + 3x + C$

b) $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1 & x = 0 \end{cases}$

$\int f \cdot g' dx = f \cdot g - \int f' \cdot g \, dx$

$(\sin x)' = \cos x$

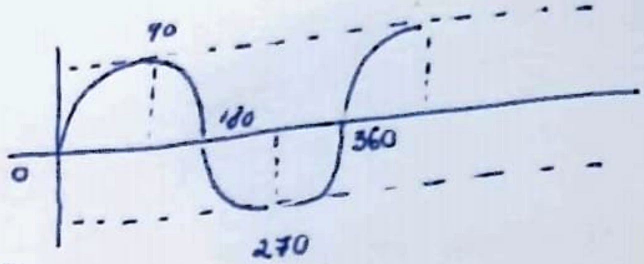
$\int dx = x + C$

$f(x)$ = continuă → admite primitive

| $x$ | 0 |
|---|---|
| $\frac{\sin x}{x}$ | / |
| 1 | 1 |

$\sin x \in [-1, 1]$

$\frac{\sin x}{x} \in \left[-\frac{1}{x}, \frac{1}{x}\right]$




dacă $x > 0 \to \frac{\sin x}{x} \in \left[-\frac{1}{x}, \frac{1}{x}\right]$

daca $x < 0 \to \frac{\sin x}{x} \in \left[\frac{-1}{-x}, \frac{1}{-x}\right]$

→ continuă

$\int \sin x \cdot \frac{1}{x} dx = \int \sin x \cdot (\ln x)' dx = \sin x \cdot \ln x - \int (\sin x)' \ln x$

$\int \sin x \cdot \frac{1}{x} dx = \sin x \cdot \ln x - \int \cos x \cdot \ln x$

$= \sin x \cdot \ln x - \left[\int \ln x \, (\sin x)' dx\right]$

$J = \sin x \cdot \ln x - \left[\ln x \cdot \sin x - \int \frac{1}{x} \cdot \sin x \, dx\right]$

$J = \sin x \ln x - \ln x \cdot \sin x + J$ → nu merge prin părți așa

$\int \left(\frac{\sin x}{x} + 1 - 1\right) dx = \int \frac{\sin x + 1}{x} - 1 \, dx = \int (\sin x + 1) \cdot \frac{1}{x} - \int x \, dx$

$\int (\sin x + 1) \cdot (\ln x)' dx - \int x \, dx$

$\int 1 dx = x + C$

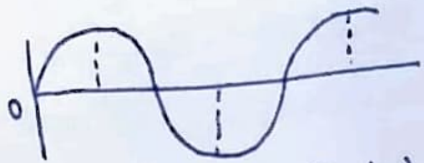
$F = x + C$

$F(x) = \begin{cases} x \neq 0 \to \\ x = 0 \to x + C \end{cases}$

c) $f(x) = \begin{cases} x^2 \cdot \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ $\quad (\sin u)' = -\cos u \cdot u'$

| $x$ | $- \; - \; - \; - \; 0 \; + \; + \; + \; + \; + \; +$ |
|---|---|
| $x^2$ | $+ \; + \; + \; + \; 0 \; + \; + \; + \; + \; + \; +$ |
| $\sin\frac{1}{x}$ | |
| $x^2 \cdot \sin\frac{1}{x}$ | / |
| 0 | 0 |

$\sin c \in [-1, 1]$



functia $x^2 \cdot \sin\frac{1}{x}$ este continua pe $\mathbb{R} / \{0\}$ dar pentru $x = 0$ functia e 0 (ca si caz particular)

$\hookrightarrow \int 0 \, dx = 0 \Rightarrow F(x) = 0 + C = C$

$\hookrightarrow \int x^2 \cdot \sin\frac{1}{x} \, dx \to \int \frac{(x^3)'}{2} \cdot \sin\frac{1}{x} \, dx = \frac{1}{2} \cdot \int (x^3)' \cdot \sin\frac{1}{x} \, dx$

$= \frac{1}{2}\left[ x^3 \cdot \sin\frac{1}{x} - \int \left[ \left(\sin\frac{1}{x}\right)' \cdot x^3 \right] dx \right]$ $\quad (x^r)' = r \cdot x^{r-1}$

$= \frac{1}{2}\left[ x^3 \cdot \sin\frac{1}{x} - \int \left[ \cos\frac{1}{x} \cdot \left(\frac{1}{x}\right)' \cdot x^3 \right] dx \right]$ $\quad \left(\sin\frac{1}{x}\right)' = \cos\frac{1}{x} \cdot \left(\frac{1}{x}\right)'$

$= \frac{1}{2}\left[ x^3 \sin\frac{1}{x} - \int \left[ \cos\frac{1}{x} \cdot (x^{-1})' \cdot x^3 \right] dx \right]$ $\quad \left(\frac{1}{x}\right)' = (x^{-1})' = -1 \cdot x^{-2} = \frac{-1}{x^2}$

$= \frac{1}{2}\left[ x^3 \cdot \sin\frac{1}{x} - \int \left[ \cos\frac{1}{x} \, (-1) \cdot x^{-1-1} \cdot x^3 \right] dx \right]$

$= \frac{1}{2}\left[ x^3 \cdot \sin\frac{1}{x} - \int \left( \cos\frac{1}{x} \, (-1) \, x^{-2} \cdot x^3 \right) dx \right]$

$= \frac{1}{2}\left[ x^3 \sin\frac{1}{x} + \int \left( \cos\frac{1}{x} \cdot x \right) dx \right]$

$= \frac{1}{2}\left[ x^3 \cdot \sin\frac{1}{x} + \int \left( \cos\frac{1}{x} \right) x \, dx \right]$

$\frac{1}{2} x^3 \sin\frac{1}{x} + \frac{1}{2} \int \cos\frac{1}{x} \cdot x \, dx$

E5/181

$f: \mathbb{R} \to \mathbb{R},\ f(x) = 3x^2 + 2x$

$\int f(x)\,dx = \int 3x^2\,dx + \int 2x\,dx$

$\int f(x)\,dx = 3 \cdot \frac{x^3}{3} + 2 \cdot \frac{x^2}{2} + C$

$F(x) = x^3 + x^2 + C$

$F(-1) = 2 \Rightarrow \quad F(-1) = (-1)^3 + (-1)^2 + C = 2$

$-1 + 1 + C = 2 \to C = 2$

$F(x) = x^3 + x^2 + 2$

$\int x^n = \frac{x^{n+1}}{n+1} \cdot C$

$(u^n)' = n u^{n-1}$

$(\ln u)' = \frac{1}{u} \cdot u'$

$(e^u)' = e^u \cdot u'$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

A1/181

a) $F(x) = x(\ln^2 x - \ln x^2 + 1)$, $x \in (0, \infty)$

$F'(x) = x' \cdot (\ln^2 x - \ln x^2 + 1) + x(\ln^2 x - \ln x^2 + 1)'$

$F'(x) = (\ln^2 x - \ln x^2 + 1) + x\left(2 \ln x \frac{1}{x} - \frac{1}{x^2} \cdot 2x\right)$

$F'(x) = \ln^2 x - \ln x^2 + 1 + x\left(\frac{2\ln x}{x} - \frac{2}{x}\right)$

$F'(x) = \ln^2 x - \ln x^2 + 1 + 2\ln x - 2$

$F'(x) = \ln^2 x - \ln x^2 + 2\ln x - 1$

b) $F(x) = e^{x+1}(x^2 - 4x)$

$F'(x) = (e^{x+1})'(x^2 - 4x) + e^{x+1}(x^2 - 4x)'$

$F'(x) = (x+1)' \cdot e^{x+1}(x^2 - 4x) + e^{x+1}(2x - 4)$

$F'(x) = e^{x+1}(x^2 - 4x) + e^{x+1}(2x - 4)$

$F'(x) = e^{x+1}(x^2 - 4x + 2x - 4)$

$F'(x) = e^{x+1}(x^2 - 2x - 4)$

c) $F(x) = 2x \sin x + 2\cos x - x^2$

$F'(x) = 2[x' \cdot \sin x + x(\sin x)'] + 2(\cos x)' - 2x$

$F'(x) = 2[\sin x + x\cos x] + (-2\sin x) - 2x$

$F'(x) = \cancel{2\sin x} + 2x\cos x - \cancel{2\sin x} - 2x$

$F'(x) = 2x(\cos x - 1)$

d) $F(x) = \frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\arcsin\frac{x}{3}$

$\sqrt{u}' = \frac{1}{2\sqrt{u}}\cdot u'$

$F'(x) = \frac{1}{2}\cdot(x\sqrt{9-x^2}) + \frac{9}{2}\cdot\arcsin\frac{x}{3}$

$(\arcsin u)' = \frac{1}{\sqrt{1-u^2}}\cdot u'$

$F'(x) = \frac{1}{2}\cdot\left[x'\sqrt{9-x^2} + x(\sqrt{9-x^2})'\right] + \frac{9}{2}\cdot\left(\arcsin\frac{x}{3}\right)'$

$F'(x) = \frac{1}{2}\cdot\left[\sqrt{9-x^2} + x\cdot\frac{1}{2\sqrt{9-x^2}}\cdot(9-x^2)'\right] + \frac{9}{2}\cdot\frac{1}{\sqrt{1-\frac{x^2}{9}}}\cdot\left(\frac{x}{3}\right)'$

$F'(x) = \frac{1}{2}\left[\sqrt{9-x^2} + \frac{x(-\cancel{2}x)}{\cancel{2}\sqrt{9-x^2}}\right] + \frac{9}{2}\cdot\frac{1}{\sqrt{\frac{9-x^2}{9}}}\cdot\frac{1}{3}$

$F'(x) = \frac{1}{2}\left[\frac{9-x^2-x^2}{\sqrt{9-x^2}}\right] + \frac{3}{2}\cdot\frac{1}{\frac{\sqrt{9-x^2}}{3}}$

$F'(x) = \frac{1}{2}\left[\frac{9-2x^2}{\sqrt{9-x^2}}\right] + \frac{3\cdot3}{2\sqrt{9-x^2}} = \frac{9-2x^2+9}{2\sqrt{9-x^2}} = \frac{18-2x^2}{2\sqrt{9-x^2}}$

$F'(x) = \frac{9-x^2}{\sqrt{9-x^2}} = \sqrt{9-x^2}$

$(\ln u)' = \frac{1}{u}\cdot u'$

$(\sqrt{u})' = \frac{1}{2\sqrt{u}}\cdot u'$

e) $F(x) = \frac{x}{2}\sqrt{x^2+1} + \frac{1}{2}\ln(x+\sqrt{x^2+1})$

$F'(x) = \frac{1}{2}\left(x\sqrt{x^2+1}\right)' + \frac{1}{2}\ln(x+\sqrt{x^2+1})'$

$F'(x) = \frac{1}{2}\left[x'\sqrt{x^2+1} + (x\sqrt{x^2+1}')\right] + \frac{1}{2}\cdot\frac{1}{x+\sqrt{x^2+1}}\cdot(x+\sqrt{x^2+1})'$

$F'(x) = \frac{1}{2}\left[\sqrt{x^2+1} + x\cdot\frac{1}{2\sqrt{x^2+1}}\cdot(x^2+1)'\right] + \frac{1}{2}\cdot\frac{1}{x+\sqrt{x^2+1}}\cdot\left(1+(\sqrt{x^2+1})'\right)$

$F'(x) = \frac{1}{2}\left[\sqrt{x^2+1} + \frac{x\cdot\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}\right] + \frac{1}{2}\cdot\frac{1}{x+\sqrt{x^2+1}}\cdot\left(1+\frac{1}{\cancel{2}\sqrt{x^2+1}}(\cancel{2}x)\right)$

$F'(x) = \frac{1}{2}\left(\frac{x^2+1+x^2}{\sqrt{x^2+1}}\right) + \frac{1}{2}\cdot\frac{1}{\cancel{x+\sqrt{x^2+1}}}\left(\frac{\cancel{\sqrt{x^2+1}+x}}{\sqrt{x^2+1}}\right)$

$F'(x) = \frac{1}{2}\cdot\frac{2x^2+1}{\sqrt{x^2+1}} + \frac{1}{2}\cdot\frac{1}{\sqrt{x^2+1}} = \frac{2x^2+2}{2\sqrt{x^2+1}}$

$F'(x) = \frac{\cancel{2}(x^2+1)}{\cancel{2}\sqrt{x^2+1}} = \sqrt{x^2+1}$

$F'(x) = \quad 1$

f) $F(x) = \dfrac{x^{n+1}}{n+1} \cdot \left(\ln x - \dfrac{1}{n+1}\right)$

$$F'(x) = \frac{1}{n+1} \cdot (x^{n+1})' \cdot \left(\ln x - \frac{1}{n+1}\right) + \frac{1}{n+1} \cdot x^{n+1} \cdot \left(\ln x - \frac{1}{n+1}\right)'$$

$$F'(x) = \frac{1}{\cancel{n+1}} \cdot \cancel{(n+1)} \cdot x^n \cdot \left(\ln x - \frac{1}{n+1}\right) + \frac{x^{n+1}}{n+1} \cdot \left(\frac{1}{x}\right)$$

$\sin(u)' = \cos u \cdot u'$

$\cos(u)' = -\sin u \cdot u'$

$$F'(x) = x^n \left(\ln x - \frac{1}{n+1}\right) + \frac{x^n}{n+1}$$

$$F'(x) = x^n \cdot \ln x - \cancel{\frac{x^n}{n+1}} + \cancel{\frac{x^n}{n+1}} = x^n \cdot \ln x$$

g) $F(x) = \dfrac{[\sin(\ln x) + \cos(\ln x)] \cdot x}{2}$

$$F'(x) = \frac{1}{2}\left([\sin(\ln x) + \cos(\ln x)]' \cdot x + (\sin(\ln x) + \cos(\ln x)) \cdot x'\right]$$

$$F'(x) = \frac{1}{2}\left[\left(\cos(\ln(x)) \cdot \frac{1}{x} - \sin(\ln x) \cdot \frac{1}{x}\right) \cdot x + \sin(\ln x) + \cos(\ln x)\right]$$

$$F'(x) = \frac{1}{2}\left[\left(\frac{\cos(\ln(x)) - \sin(\ln x)}{\cancel{x}} \cdot \cancel{x}\right) + \sin(\ln x) + \cos(\ln x)\right]$$

$$F'(x) = \frac{1}{2}\left(\cos(\ln x) - \cancel{\sin(\ln x)} + \cancel{\sin(\ln x)} + \cos(\ln x)\right)$$

$$F'(x) = \frac{1}{2} \cdot 2\cos(\ln x) = \cos(\ln x)$$

$(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}}$

$(\arcsin u)' = \dfrac{u'}{\sqrt{1-u^2}}$

h) $F(x) = \arcsin(2x\sqrt{1-x^2}) - 2\arcsin x$

$$F'(x) = \frac{1}{\sqrt{1-4x^2(1-x^2)}} \cdot (2x\sqrt{1-x^2})' - 2 \cdot \frac{1}{\sqrt{1-x^2}}$$

$$F'(x) = \frac{1}{\sqrt{1-4x^2+4x^4}} \cdot 2 \cdot \left[x'\sqrt{1-x^2} + x \cdot (\sqrt{1-x^2})'\right] - \frac{2}{\sqrt{1-x^2}}$$

$$F'(x) = \frac{1}{\sqrt{4x^4-4x^2+1}} \cdot 2\left[\sqrt{1-x^2} + x \cdot \frac{1}{2\sqrt{1-x^2}} \cdot (-2x)\right] - \frac{2}{\sqrt{1-x^2}}$$

$$F'(x) = \frac{1}{\sqrt{4x^4-4x^2+1}} \cdot 2 \cdot \left[\frac{1-x^2-x^2}{\sqrt{1-x^2}}\right] - \frac{2}{\sqrt{1-x^2}}$$

$$F'(x) = \frac{1}{\sqrt{4x^4-4x^2+1}} \cdot 2 \cdot \frac{1-2x^2}{\sqrt{1-x^2}} - \frac{2}{\sqrt{1-x^2}}$$

$$F'(x) = \frac{(2-4x^2) - 2\sqrt{4x^4-4x^2+1}}{\sqrt{4x^4-4x^2+1} \cdot \sqrt{1-x^2}}$$

$(\operatorname{arctg} u)' = \frac{1}{u^2+1} \cdot u'$

$(\ln u)' = \frac{1}{u} \cdot u'$

i) $F(x) = \frac{1}{3}\left(\ln \frac{x+1}{\sqrt{x^2-x+1}} + \frac{1}{\sqrt{3}} \cdot \operatorname{arctg} \frac{2x-1}{\sqrt{3}}\right)$

$$F'(x) = \frac{1}{3} \cdot \left[ \frac{1}{\frac{x+1}{\sqrt{x^2-x+1}}} \cdot \left(\frac{x+1}{\sqrt{x^2-x+1}}\right)' + \frac{1}{\sqrt{3}} \cdot \frac{1}{\left(\frac{2x-1}{\sqrt{3}}\right)^2+1} \cdot \left(\frac{2x-1}{\sqrt{3}}\right)' \right]$$

$$F'(x) = \frac{1}{3}\left[ \frac{\sqrt{x^2-x+1}}{x+1} \left( \frac{(x+1)' \cdot \sqrt{x^2-x+1} - \sqrt{x^2-x+1}' \cdot (x+1)}{(\sqrt{x^2-x+1})^2} + \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} \cdot \frac{2}{\frac{(2x-1)^2+3}{3}} \right.\right.$$

$$F'(x) = \frac{1}{3}\left[ \frac{\sqrt{x^2-x+1}}{x+1} \left( \frac{\sqrt{x^2-x+1} - \frac{1}{2\sqrt{x^2-x+1}}(2x-1)(x+1)}{(\sqrt{x^2-x+1})^2} + \frac{1}{\cancel{3}} \cdot 2 \cdot \frac{\cancel{3}}{4x^2-4x+1+3} \right.\right.$$

$$F'(x) = \frac{1}{3}\left[ \frac{\cancel{\sqrt{x^2-x+1}}}{x+1} \cdot \frac{2\sqrt{x^2-x+1}^2 - (2x-1)(x+1)}{2\cancel{\sqrt{x^2-x+1}}\,(\sqrt{x^2-x+1})^2} + \frac{2}{4(x^2-x+1)} \right]$$

$$F'(x) = \frac{1}{3}\left[ \frac{2(\sqrt{x^2-x+1})^2 - (2x^2+2x-x-1)}{(x+1)2(x^2-x+1)} + \frac{2}{4(x^2-x+1)} \right]$$

$$F'(x) = \frac{1}{3} \cdot \left( \frac{2(x^2-x+1) - 2x^2-x+1}{2(x+1)(x^2-x+1)} + \frac{2}{4(x^2-x+1)} \right)$$

$$F'(x) = \frac{1}{3}\left( \frac{\cancel{2x^2}-2x+2-\cancel{2x^2}-x+1}{2(x+1)(x^2-x+1)} + \frac{1}{2(x^2-x+1)} \right)$$

$$F'(x) = \frac{1}{3}\left( \frac{(-3x+3+x+1)}{2(x^2-x+1)(x+1)} \right)$$

$$F'(x) = \frac{1}{3} \frac{-2x+2}{2(x^2-x+1)(x+1)}$$

$$F'(x) = \frac{1}{3} \frac{-x+1}{(x+1)(x^2-x+1)}$$

$$F'(x) = \frac{1}{3} \cdot \frac{(1-x)}{(x+1)(x^2-x+1)} = \frac{1}{3} \frac{(1-x)}{(x+1)(-1)(x-x^2-1)}$$

$$F_1(x) = \frac{x}{2} + \frac{3}{16} \cdot \sin\left(\frac{4x}{3}\right) - \frac{3\sqrt{3}}{16} \cdot \cos\left(\frac{4x}{3}\right)$$

$$F_1(x)' = \frac{1}{2} + \frac{3}{16} \cdot \cos\left(\frac{4x}{3}\right) \cdot \left(\frac{4}{3}x\right)' - \frac{3\sqrt{3}}{16} \cdot \left(-\sin\frac{4x}{3} \cdot \frac{4}{3}x'\right)$$

$$F'(x) = \frac{1}{2} + \frac{\cancel{3}}{16} \cdot \frac{4}{\cancel{3}} \cos\frac{4x}{3} + \frac{\cancel{3}\sqrt{3}}{16} \cdot \frac{4}{\cancel{3}} \cdot \sin\frac{4x}{3}$$

$$F'(x) = \frac{1}{2} \cdot \frac{1}{4} \cos\frac{4x}{3} + \frac{\sqrt{3}}{4} \cdot \sin\frac{4x}{3}$$

$$F'(x) = \frac{1}{8} \cdot \cos\frac{4x}{3} + \frac{2\sqrt{3}}{8} \cdot \sin\frac{4x}{3}.$$

$$F'(x) = \frac{\cos\frac{4x}{3} + 2\sqrt{3} \cdot \sin\frac{4x}{3}}{8}$$

$$\cos^2\frac{4x}{3} + \sin^2\frac{4x}{3} = 1 \Rightarrow \cos^2\frac{4x}{3} = 1 - \sin^2\frac{4x}{3}$$

$$\hookrightarrow \sin\frac{4x}{3} = \sqrt{1 - \cos^2\frac{4x}{3}}$$

$$F_2(x) = \frac{x}{2} - \frac{3}{4} \sin\left(\frac{\pi}{6} - \frac{2x}{3}\right)$$

$$F_2'(x) = \frac{1}{2} - \frac{3}{4} \cdot \cos\left(\frac{\pi}{6} - \frac{2x}{3}\right) \cdot \left(-\frac{2}{3}\right)$$

$$F_2'(x) = \frac{1}{2} + \frac{\cancel{3} \cdot 2}{4 \cdot \cancel{3}} \cdot \cos\left(\frac{\pi}{6} - \frac{2x}{3}\right)$$

$$F_2'(x) = \frac{1}{2} + \frac{1}{2} \cos\left(\frac{\pi}{6} - \frac{2x}{3}\right)$$

$$F_2'(x) = \frac{1}{2}\left(1 + \cos\left(\frac{\pi}{6} - \frac{2x}{3}\right)\right)$$

$$F_3(x) = \frac{x}{2} - \frac{3}{8} \cos\left(\frac{\pi}{6} + \frac{4x}{3}\right)$$

$$F_3'(x) = \frac{1}{2} - \frac{3}{8}\left(-\sin\left(\frac{\pi}{6} + \frac{4x}{3}\right)\right) \cdot \left(\frac{\pi}{6} + \frac{4x}{3}\right)'$$

$$F_3'(x) = \frac{1}{2} + \frac{\cancel{3}}{8} \cdot \frac{4}{\cancel{3}} \cdot \sin\left(\frac{\pi}{6} + \frac{4x}{3}\right)$$

$$F_3'(x) = \frac{1}{2} + \frac{1}{2} \sin\left(\frac{\pi}{6} + \frac{4x}{3}\right)$$

$$F_3'(x) = \frac{1}{2}\left(1 + \sin\left(\frac{\pi}{6} + \frac{4x}{3}\right)\right)$$

a) $f(x) = \begin{cases} \dfrac{4x^5-5x^4+1}{(x-1)^2} & x<1 \\ 7x^2+4x-1 & x\geq 1 \end{cases}$

| $x$ | | 0 | 1 | |
|---|---|---|---|---|
| $\dfrac{4x^5-5x^4+1}{(x-1)^2}$ | $-\,-\,-\,-\,-$ | $+\,+\,+$ | / | $+\,+\,+\,+\,+\,+\,+$ |
| $7x^2+4x-1$ | $+\,+\,+$ | $-1$ | $+\,+\,+$ | $+\,+\,+\,+\,+\,+$ |

$(-1) \rightarrow \dfrac{-4-5+1}{4} = \dfrac{-8}{4} = -2 < 0$

$2 \rightarrow \dfrac{4\cdot 32 - 5\cdot 16+1}{1} = 128 - 80+1 > 0$

$-1 \rightarrow 7-4-1 = 7-5 = 2$

$-2 \rightarrow 28 - 16 - 1 = 11 > 0$

funcția e continuă $\rightarrow$ admite primitive

b) $f(x) = \begin{cases} \dfrac{e^{x^2}-1}{x^4+x^2} & x<0 \\ x^3-3x^2+1 & x\geq 0 \end{cases}$

| $x$ | | 0 | |
|---|---|---|---|
| $x^4$ | $+\,+\,+\,+\,+$ | $+\,+$ | $+\,+\,+\,+\,+\,+$ |
| $x^2$ | $+\,+\,+\,+\,+\,+$ | $+$ | $+\,+\,+\,+\,+\,+\,+$ |
| $x^4+x^2$ | $+\,+\,+\,+\,+\,+\,+$ | $+\,+$ | $+\,+\,+\,+\,+\,+\,+$ |
| $e^{x^2}$ | $+\,+\,+\,+\,+$ | $+$ | $+\,+\,+\,+\,+\,+$ |
| $e^{x^2}-1$ | $+\,+\,+\,+\,+$ | 0 | $+\,+\,+\,+\,+$ |

$\hookrightarrow$ continuă

| $x$ | | $-1$ | 0 | 1 | 2 | |
|---|---|---|---|---|---|---|
| $x^3-3x^2+1$ | $-\,-\,-$ | $-$ | 1 | $-1$ | $-3$ . . | $-\,-\,-$ |

continuă (funcție exponențială)

A4 / 182

$$f(x) = \begin{cases} e^{x+1}, & x \leq -1 \\ 2+x, & x > -1 \end{cases}$$

$\int x^n dx = \frac{x^{n+1}}{n+1}$

| $x$ | $-\infty$ | $-1$ | $\infty$ |
|---|---|---|---|
| $e^{x+1}$ | + + + + + | 1 | + + + + + + + |
| $2+x$ | - - - - - | 1 | + + + + + + + |

functia $f$ este continua pe $(-\infty, -1]$ $(-1, \infty) \Rightarrow$ continua pe $(-\infty, \infty) \Rightarrow$ admite primitive

$$\int_{x>-1} f(x)\,dx = \int (2+x)\,dx = \int 2\,dx + \int x\,dx = 2x + \frac{x^2}{2} + C$$

$F_1(x) = 2x + \frac{1}{2}x^2 + C$

$F_1(2) = \frac{3}{2} \Rightarrow 4 + \frac{1}{2} \cdot 4 + C = \frac{3}{2}$

$\frac{8+4+2C}{2} = \frac{3}{2}$

$12 + 2C = 3$

$2C = 3 - 12$

$C = -\frac{9}{2}$

V: $4 + \frac{1}{2} \cdot 4 - \frac{9}{2} = \frac{4}{2} + 4 - \frac{9}{2} = 4 + 2 - \frac{9}{2} = \frac{6 \cdot 2}{2} - \frac{9}{2} = \frac{12-9}{2} = \frac{3}{2}$

$F_1(x) = 2x + \frac{1}{2}x^2 - \frac{9}{2}$

A5 / 182

$$f(x) = \max\{1, x^2\} = \begin{cases} 1, & x = 0 \\ x^2, & x \neq 0, \quad x = \mathbb{R} \setminus \{0\} \end{cases}$$

| $x$ | 0 |
|---|---|
| 1 | 1 |
| $x^2$ | + + + + / + + + + + |

functia exponentiala ($x^2$) este continua $\Rightarrow$ functia admite primitive pe $\mathbb{R}$

functia e continua si in $x=0 \Rightarrow 1$

$F_1(x) = \int dx = x + C$

$F_2(x) = \int x^2 dx = \frac{x^3}{3} + C$

$\int dx = x + C$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$4F\left(-\frac{3}{2}\right) - 3F\left(\frac{1}{2}\right) = 3F(2)$

PASUL 1

↳ pentru $F_1(x) = x + C$

$4F_1\left(-\frac{3}{2}\right) - 3F_1\left(\frac{1}{2}\right) = 3F_1(2)$ !!

$F_1\left(-\frac{3}{2}\right) = \left(-\frac{3}{2}\right) + C$

$F_1\left(\frac{1}{2}\right) = \frac{1}{2} + C$

$F_1(2) = 2 + C$

$4 \cdot F_1\left(-\frac{3}{2}\right) - 3F_1\left(\frac{1}{2}\right) = 3F_1(2)$

$4\left[-\frac{3}{2} + C\right] - 3\left[\frac{1}{2} + C\right] = 3\left[2 + C\right)$

$-\frac{4 \cdot 3}{2} + 4C - \frac{3}{2} - 3C = 6 + 3C$

$-\frac{12}{2} - \frac{3}{2} + C = 6 + 3C$

$-\frac{15}{2} + C = 6 + 3C$

$C - 3C = 6 + \frac{15}{2} = \frac{12 + 15}{2}$

$-2C = \frac{27}{2}$

$-C = \frac{27}{2} \cdot \frac{1}{2}$

$C = \frac{-27}{4}$

$F_1(x) = x - \frac{27}{4}$

PASUL 2

↳ pentru $F_2(x)$

$4F_2\left(-\frac{3}{2}\right) - 3F_2\left(\frac{1}{2}\right) = 3F_2(2)$ !!

$F_2(x) = x^3 \cdot \frac{1}{3} + C$

$F_2(-\frac{3}{2}) = \frac{1}{\cancel{3}}\left(-\frac{\cancel{3}}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{-3}{2}\right) + C = C - \frac{9}{8}$

$F_2\left(\frac{1}{2}\right) = \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + C = \frac{1}{8}\cdot\frac{1}{3} + C = \frac{1}{24} + C$

$F_2(2) = \frac{8}{3} + C$

$4\cdot\left(C - \frac{9}{8}\right) - 3\left(\frac{1}{24} + C\right) = 3\left(\frac{8}{3} + C\right)$

$\underline{4C} - \frac{4\cdot 9}{8} - \frac{3}{24} \underline{- 3C} = \cancel{3}\cdot\frac{8}{\cancel{3}} + 3C$

$C - \frac{9}{2} - \frac{1}{8} = 8 + 3C$

$C - \frac{36}{8} - \frac{1}{8} - 3C = 8$

$-2C = 8 + \frac{36}{8} + \frac{1}{8}$

$-2C = \frac{64+36+1}{8}$

$-2C = \frac{100+1}{8} = \frac{101}{8}$

$C = -\frac{101}{8}\cdot\frac{1}{2} = -\frac{101}{16}$

$F_2(x) = \frac{x^3}{3} - \frac{101}{16}$

verificări

$4\left(-\frac{3}{2} - \frac{27}{4}\right) - 3\left(\frac{1}{2} - \frac{27}{4}\right) = 3\left(2 - \frac{27}{4}\right)$

$4\left(\frac{-6-27}{4}\right) - 3\left(\frac{2-27}{4}\right) = 3\left(\frac{8-27}{4}\right)$

$4\cdot\frac{-33}{4} - 3\cdot\frac{(-25)}{4} = 3\cdot\frac{-19}{4}$

$\frac{-132+75}{4} = \frac{-57}{4}$

$\frac{-57}{4} = \frac{-57}{4}$ A

A6/182

$a, b = ?$

$F:(0,\infty) \to \mathbb{R}$

$F(x) = \begin{cases} \ln^2 x, & x \in (0,e] \\ ax+b, & x \in (e,+\infty) \end{cases}$

$F(x)$ = primitivă $\Leftrightarrow F'(x) = f(x)$

$(u^n)' = n \cdot u^{n-1} \cdot u'$

$[f \cdot g]' = f' \cdot g + f \cdot g'$

$(\ln x)^n = \frac{n}{x} \cdot (\ln x)^{n-1}$

$F'(x) = \begin{cases} (\ln^2 x)', & x \in (0,e] \\ (ax+b)', & x \in (e,\infty) \end{cases}$

$[(\ln x)(\ln x)]' = (\ln x)' \cdot \ln x + \ln x (\ln x)' = 2 \cdot \ln x \cdot \frac{1}{x} = \frac{2}{x} \cdot \ln x$

$\boxed{f(x) = \frac{2}{x} \cdot \ln x}$

$(ax+b)' = a = f(x)$

$\cancel{a = \frac{2}{x} \ln x = 2 \cdot \frac{1}{x} \cdot \ln x}$

$b$ = constantă

$2 \cdot \int \frac{1}{x} \cdot \ln x = 2 \int (\ln x)' \cdot \ln x^1 = \frac{(\ln x)^2}{2} \cdot 2 = \ln x^2$

$\longrightarrow$ antiderivată = primitiva lui $f$

A7/182

$F(x) = \begin{cases} x^2 + ax + 3 & x \le 1 \\ \frac{3x+b}{x^2+2} & x > 1 \end{cases}$

$F_1'(x) = (x^2 + ax + 3)' = 2x + a$

$F_2'(x) = \frac{(3x+b)'(x^2+2) - (x^2+2)' \cdot (3x+b)}{(x^2+2)^2}$

$F_2'(x) = \frac{3x^2 + 6 - 2x(3x+b)}{(x^2+2)^2} = \frac{3x^2 + 6 - 6x^2 - 2xb}{(x^2+2)^2}$

$F_2'(x) = \frac{6 - 3x^2 - 2xb}{(x^2+2)^2}$

$f(x) = 2x + a$

$\int 2x + a = 2 \cdot \frac{x^2}{2} + ax + \mathcal{C} = x^2 + ax + \mathcal{C}$

$\hookrightarrow \mathcal{C} = 3$

$a \in \mathbb{R}$

$\hookrightarrow$ poate lua orice val.

/ 182

$f(x) = \frac{x-1}{\sqrt{x}}$

$F(x) = (ax+b)\sqrt{x}$

daca $F(x)$ primitivă a lui $f \Leftrightarrow F'(x) = f(x)$

$F'(x) = \left[(ax+b)\sqrt{x}\right]'$

$F'(x) = (ax+b)'\sqrt{x} + (ax+b)\sqrt{x}'$

$F'(x) = a\sqrt{x} + (ax+b)\cdot\frac{1}{2\sqrt{x}}$

$F'(x) = a\sqrt{x} + \frac{ax+b}{2\sqrt{x}}$

$F'(x) = \frac{2\sqrt{x}\cdot a\cdot\sqrt{x} + ax+b}{2\sqrt{x}}$

$F'(x) = \frac{2ax + ax + b}{2\sqrt{x}}$

$F'(x) = \frac{3ax+b}{2\sqrt{x}}$

$F'(x) = f(x) \Leftrightarrow \frac{3ax+b}{2\sqrt{x}} = \frac{x-1}{\sqrt{x}}$

$\cancel{\sqrt{x}}(3ax+b) = 2\cancel{\sqrt{x}}(x-1)$

$3ax+b = 2x-2$

$3ax+b = 2x-2$

$3ax = 2x \quad /:x$

$b = -2$

$3a = 2 \Rightarrow a = \frac{2}{3}$

$b = -2$

$F(x) = (ax+b)\sqrt{x}$

$F(x) = \left(\frac{2}{3}x-2\right)\sqrt{x}$

A9/182

$f, g : (0,\infty) \to \mathbb{R}$

$f(x) = \frac{x}{x+1} - \ln(x+1)$

$g(x) = \frac{1}{x}\left[c + bx + a\ln(x+1)\right]$

$h(x) = \frac{f(x)}{x^2}$

g - primitivă a lui h $\rightarrow g' = h$

$h(x) = \frac{1}{x^2} \cdot f(x) = \frac{1}{x^2}\left[\frac{x}{x+1} - \ln(x+1)\right]$

$(x^r)' = r \cdot x^{r-1}$

$h(x) = \frac{1}{x^2} \cdot \left[\frac{x}{x+1} - \frac{(x+1)\ln(x+1)}{(x+1)}\right]$

$x^{-1} = -x^{-2} = -\frac{1}{x^2}$

$h(x) = \frac{1}{x^2} \frac{x-(x+1)\ln(x+1)}{x+1}$

$(\ln u)' = \frac{1}{u} \cdot u'$

$h(x) = \frac{1}{x^2} \cdot \frac{x-(x+1)\ln(x+1)}{x+1}$

$g'(x) = \left(\frac{1}{x}\right)'\left[c + bx + a \cdot \ln(x+1)\right] + \frac{1}{x}\left[c + bx + a \cdot \ln(x+1)\right]'$

$g'(x) = \frac{-1}{x^2}\left[c + bx + a\ln(x+1)\right] + \frac{1}{x}\left[b + a \cdot \frac{1}{x+1}\right]$

$g'(x) = -\frac{1}{x^2}\left(c + bx + a\ln(x+1)\right) + \frac{1}{x}b + \frac{1}{x}\frac{a}{x+1}$

$g'(x) = -\frac{1}{x^2}c - \frac{1}{\cancel{x^2}} \cdot b\cancel{x} - \frac{1}{x^2} \cdot a \cdot \ln(x+1) + \frac{1}{x}b + \frac{1}{x} \cdot \frac{a}{x+1}$

$g'(x) = \frac{-c}{x^2} \cancel{- \frac{b}{x}} - \frac{a}{x^2} \cdot \ln(x+1) \cancel{+ \frac{b}{x}} + \frac{a}{x(x+1)}$

$g'(x) = \frac{1}{x^2}(-c - a\ln(x+1)) + \frac{a}{x(x+1)}$

$g'(x) = \frac{(x+1)(-c - a\ln(x+1)) + ax}{x^2(x+1)}$

$h(x) = \frac{x-(x+1)\ln(x+1)}{x^2(x+1)}$

$\frac{-cx - ax \cdot \ln(x+1) - c - a\ln(x+1) + ax}{x^2(x+1)} = \frac{x-(x+1)\ln(x+1)}{x^2(x+1)}$

$-cx - c + ax - a\ln(x+1) - ax\ln(x+1) = x - (x+1)\ln(x+1)$

$x(-c+a) - c - \underline{\ln(x+1)}\underbrace{(a+ax)} = x - \underbrace{(x+1)}\underline{\ln(x+1)}$

$a = 1 \rightarrow a + ax = 1 + x$

$x(-c+a) - c = x$

$x(-c+a) - c = \cancel{a}x \rightarrow x(-c+1) - c = x \rightarrow c = 0$

pentru $a=1$, $b \in \mathbb{R}$ și $c=0$ $\Rightarrow$ $g'(x) = h(x)$
↳ $g$ primitivă a lui $h$

**A 10 / 182**

$f : [0,3] \to \mathbb{R}$

$$f(x) = \begin{cases} x^2 + ax + b & , x \in [0,1] \\ 2x+1 & , x \in (1,2) \\ x + 3a & , x \in [2,3] \end{cases}$$

dacă $f$ continua pe $[0,3]$ $\Rightarrow$ admite primitive pe $[0,3]$

$$f'(x) = \begin{cases} 2x + a & , x \in [0,1] \\ 2 & , x \in (1,2) \\ 1 & , x \in [2,3] \end{cases}$$

AZUL CU $a \geq 0$

| $x$ | 0     1     2     3 |
|---|---|
| $x^2+ax+b$ | [+ + +] |
| $2x+1$ | (+ + + +) |
| $x+3a$ | [+ + + + +] |
| $2x+a$ | + + + + + + + + + + |
| 2 | ——— |
| 1 | ——— |

$x^2 > x$ $\Rightarrow$ $x^2 + ax + b > 0$
↳ functia continuă