Temă curs 2

1) $\left(\frac{1}{y}-\frac{1}{x^2}\right)dx+\left(\frac{1}{x}-\frac{x}{y^2}\right)dy=0$

PAS 1

$$P(x,y)=\frac{1}{y}-\frac{y}{x^2} \Rightarrow \frac{\partial P}{\partial y}=-\frac{1}{y^2}-\frac{1}{x^2}$$
$$\parallel$$
$$Q(x,y)=\frac{1}{x}-\frac{x}{y^2} \Rightarrow \frac{\partial Q}{\partial x}=-\frac{1}{x^2}-\frac{1}{x^2}$$

PAS 2

$$\int_{x_0}^{x_0}\left(\frac{1}{y_0}-\frac{y_0}{t^2}\right)dt+\int_{y_0}^{y_0}\left(\frac{1}{x}-\frac{x}{t^2}\right)dt=$$

$$\Rightarrow \int_{x_0}^{x}\frac{1}{y_0}\,dt-\int_{x_0}^{x}\frac{y_0}{t^2}\,dt+\int_{y_0}^{y}\frac{1}{x}\,dt-\int\frac{x}{t^2}\,dt=$$

$$\Rightarrow \frac{1}{y_0}\int_{x_0}^{x}dt-y_0\int_{x_0}^{x}t^{-2}\,dt+\frac{1}{x}\int_{y_0}^{y}dt-x\int_{y_0}^{y}\frac{1}{t^2}\,dt=$$

$$=\frac{1}{y_0}\cdot t\Big|_{x_0}^{x}-y_0\cdot\frac{1}{t}\Big|_{x_0}^{x}+\frac{1}{x}\cdot t\Big|_{y_0}^{y}-x\cdot-\frac{1}{t}\Big|_{y_0}^{y}=$$

$$=\cancel{\frac{x}{y_0}}-\frac{x_0}{y_0}+\cancel{\frac{y_0}{x}}-\frac{y_0}{x_0}+\frac{y}{x}-\cancel{\frac{y_0}{x}}+\frac{x}{y}-\cancel{\frac{x}{y_0}}=$$

$$=\underbrace{\frac{y}{x}+\frac{x}{y}}_{F(x,y)}-\underbrace{\left(\frac{x_0}{y_0}+\frac{y_0}{x_0}\right)}_{F(x_0,y_0)}$$

$$\Rightarrow F(x,y)=C \rightarrow \frac{y}{x}+\frac{x}{y}=C$$

2) $\left(-\frac{1}{x}+y+\frac{y}{x^2+y^2}\right)dx+\left(\frac{1}{y}+x-\frac{x}{x^2+y^2}\right)dy=0$

$P(x,y)=-\frac{1}{x}+y+\frac{y}{x^2+y^2} \to \frac{\partial P}{\partial y}= \quad +1+\left(\frac{y}{x^2+y^2}\right)'= \quad +1+\frac{x^2+y^2-y\cdot(x^2+y^2)'}{(x^2+y^2)^2}$ *

$Q(x,y)=\frac{1}{y}+x-\frac{x^2}{x^2+y^2} \to \frac{\partial Q}{\partial x}=1-\frac{x^2+y^2-x\cdot(2x)}{(x^2+y^2)^2}=$ ◎

$* = \quad +1+\frac{x^2+y^2-y\cdot(+2y)}{(x^2+y^2)^2}=- \quad +1+\frac{x^2+y^2-2y^2}{(x^2+y^2)^2}= \quad +1+\frac{x^2-y^2}{(x^2+y^2)^2}=$

◎ $= +1-\frac{x^2+y^2-2x^2}{(x^2+y^2)^2}=+1-\frac{-x^2+y^2}{(x^2+y^2)}=1+\frac{x^2-y^2}{(x^2+y^2)}$ $\qquad \frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$

PAS 2

$$\int_{x_0}^{x}\left(-\frac{1}{t}+y_0+\frac{y_0}{t^2+y_0^2}\right)dt+\int_{y_0}^{y}\left(\frac{1}{t}+x-\frac{x}{x^2+t^2}\right)dt=$$

$$\Rightarrow -\int_{x_0}^{x}\frac{1}{t}\,dt+\int_{x_0}^{x}y_0\,dt+\int_{x_0}^{x}\frac{y_0}{t^2+y_0^2}\,dt+\int_{y_0}^{y}\frac{1}{t}\,dt+\int_{y_0}^{y}x\,dt-\int_{y_0}^{y}\frac{x}{x^2+t^2}\,dt=$$

$$\Rightarrow -\ln t\Big|_{x_0}^{x}+y_0\cdot t\Big|_{x_0}^{x}+\ln t\Big|_{y_0}^{y}+x\cdot t\Big|_{y_0}^{y}+\cancel{y_0}\cdot\frac{1}{\cancel{y_0}}\operatorname{arctg}\frac{t}{y_0}\Big|_{x_0}^{x}+\cancel{x}\cdot\frac{1}{\cancel{x}}\operatorname{arctg}\frac{t}{x}\Big|_{y_0}^{y}$$

$\Rightarrow -\ln x+\ln x_0+y_0x-y_0x_0+\ln y-\ln y_0+xy-xy_0+\operatorname{arctg}\frac{x}{y_0}-\operatorname{arctg}\frac{x_0}{y_0}+$
$+\operatorname{arctg}\frac{y}{x}-\operatorname{arctg}\frac{y_0}{x}=$

$\Rightarrow -\ln x+\ln y+xy-\operatorname{arctg}\frac{y}{x}-(-\ln x_0+\ln y_0+y_0x_0+$ $\operatorname{arctg}x+\operatorname{arctg}\frac{1}{x}=\frac{\pi}{2}$
$+\operatorname{arctg}\frac{y_0}{x_0})+\frac{\pi}{2}$

$\Rightarrow -\ln x+\ln x_0+\cancel{y_0}x-y_0x_0+\ln y-\ln y_0+xy-x\cancel{y_0}+\operatorname{arctg}\frac{x}{y_0}$
$-\operatorname{arctg}\frac{x_0}{y_0}+\operatorname{arctg}\frac{x}{y}-\operatorname{arctg}\frac{y}{x}=$

$\Rightarrow -\ln x+\ln y+xy-\operatorname{arctg}\frac{x}{y}-$
$(-\ln x_0+\ln y_0+x_0y_0-\operatorname{arctg}\frac{y_0}{x_0})-\operatorname{arctg}\frac{y_0}{x_0}-\operatorname{arctg}\frac{x_0}{y_0}+$
$\operatorname{arctg}\frac{x}{y_0}+\operatorname{arctg}\frac{y_0}{x}$

$\Rightarrow F(x,y)-F(x_0,y_0)-\frac{\pi}{2}+\frac{\pi}{2} \to F(x,y)-F(x_0,y_0)$

$-\ln x+\ln y+xy-\operatorname{arctg}\frac{x}{y}=C$

3) $x(y^2+1)\,dx + \left(x^2y + \frac{1}{\sqrt{1-y^2}}\right)dy = 0$

PAS 1

$$P(x,y) = xy^2 + x = \frac{\partial P}{\partial y} = 2xy$$
$$Q(x,y) = x^2y + \frac{1}{\sqrt{1-y^2}} = \frac{\partial Q}{\partial x} = 2xy$$
$$\rightarrow \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

PAS 2

$$\int_{x_0}^{x} t(y_0^2+1)\,dt + \int_{y_0}^{y} x^2t + \frac{1}{\sqrt{1-t^2}}\,dt =$$

$$(y_0^2+1)\int_{y_0}^{x} t\,dt + x^2\int_{y_0}^{y} t\,dt + \int_{y_0}^{y} \frac{1}{\sqrt{1-t^2}}\,dt =$$

$$(y_0^2+1)\cdot\frac{t^2}{2}\Big|_{x_0}^{x} + x^2\cdot\frac{t^2}{2}\Big|_{y_0}^{y} + \arcsin t\Big|_{y_0}^{y} =$$

$$(y_0^2+1)\cdot\left(\frac{x^2}{2} - \frac{x_0^2}{2}\right) + x^2\cdot\left(\frac{y^2}{2} - \frac{y_0^2}{2}\right) + \arcsin y - \arcsin y_0$$

$$\cancel{\frac{x^2y_0^2}{2}} - \frac{x_0^2y_0^2}{2} + \frac{x^2}{2} - \frac{x_0^2}{2} + \frac{x^2y^2}{2} - \cancel{\frac{x^2y_0^2}{2}} + \arcsin y - \arcsin y_0 =$$

$$= \underbrace{\frac{x^2}{2} + \frac{x^2y^2}{2} + \arcsin y}_{F(x,y)} - \underbrace{\left(\frac{x_0^2}{2} + \frac{x^2y_0^2}{2} + \arcsin y_0\right)}_{F(x_0,y_0)} \Rightarrow \frac{x^2}{2}(y^2+1) + \arcsin y = C$$

4) $\text{arctg}\frac{y}{x}\,dx + \frac{1}{2}\ln(x^2+y^2)\,dy$

$$P(x,y) = \text{arctg}\frac{y}{x} = \frac{\partial P}{\partial y} = \frac{1}{\left(\frac{y}{x}\right)^2+1}\cdot\frac{1}{x} = \frac{x}{x^2+y^2}$$
$$Q(x,y) = \frac{1}{2}\cdot\ln(x^2+y^2) = \frac{1}{2}\cdot\frac{\partial Q}{\partial y} = \frac{1}{\cancel{2}}\cdot\frac{1}{x^2+y^2}\cdot\cancel{2}x = \frac{x}{x^2+y^2}$$
$$\rightarrow \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$