Temă curs05

Exemplul 01

$$B = \{\log_e x ,\ x \ln x\}$$

a) verif independența y_1 și y_2 $W(y_1, y_2) \neq 0$

$$y_1 = \ln x$$
$$y_2 = x \ln x$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \ln x & x \ln x \\ \frac{1}{x} & \ln x + 1 \end{vmatrix}$$

$$W = \ln^2 x + \ln x - \ln x = \ln^2 x \neq 0$$

b) ec. dif care le admite ca sist. fundamental $W(y, y_1, y_2) = 0$

$$W(y, y_1, y_2) = 0$$

$$W(y, y_1, y_2) = \begin{vmatrix} y & y_1 & y_2 \\ y' & y_1' & y_2' \\ y'' & y_1'' & y_2'' \end{vmatrix} = \begin{vmatrix} y & \ln x & x \ln x \\ y' & \frac{1}{x} & \ln x + 1 \\ y'' & -\frac{1}{x^2} & \frac{1}{x} \end{vmatrix}$$

$$W(y, y_1, y_2) = 0 \rightarrow \begin{vmatrix} y & \ln x & x \ln x \\ y' & \frac{1}{x} & \ln x + 1 \\ y'' & -\frac{1}{x^2} & \frac{1}{x} \end{vmatrix} = 0$$

$$y(-1)^2 \begin{vmatrix} \frac{1}{x} & \ln x + 1 \\ -\frac{1}{x^2} & \frac{1}{x} \end{vmatrix} + y'(-1)^3 \begin{vmatrix} \ln x & x \ln x \\ -\frac{1}{x^2} & \frac{1}{x} \end{vmatrix} + y''(-1)^4 \begin{vmatrix} \ln x & x \ln x \\ \frac{1}{x} & \ln x + 1 \end{vmatrix} = 0$$

$$y\left(\frac{1}{x^2} + \frac{1}{x^2}(\ln x + 1)\right) - y'\left(\frac{\ln x}{x} + \frac{\ln x}{x}\right) + y''\left(\ln^2 x + \cancel{\ln x} - \cancel{\ln x}\right) = 0$$

$$\boxed{\ln^2 x = 2 \ln x}$$

$$y\left(\frac{1 + \ln x + 1}{x^2}\right) - y' \cdot \frac{2 \ln x}{x} + y'' \ln^2 x = 0$$

$$y'' \ln^2 x - y' \cdot \frac{2 \ln x}{x} + y \cdot \frac{2 \cdot \ln x}{x^2} = 0$$

Exemplul 02

$\mathcal{B} = \{e^{x}, e^{-x}, e^{2x}\}$

a) verif independentă $y_1, y_2, y_3 \to W(y_1, y_2, y_3) \neq 0$

$$W = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix} = \begin{vmatrix} e^{x} & e^{-x} & e^{2x} \\ e^{x} & -e^{-x} & 2\cdot e^{2x} \\ e^{x} & e^{-x} & 4\cdot e^{2x} \end{vmatrix}$$

$(e^{x})' = e^{x}$

$(e^{u})' = e^{u}\cdot u'$

$(e^{x})' = e^{x} \to (e^{x})' = e^{x}$

$(e^{-x})' = e^{-x}\cdot(-x)' = -e^{-x} \Rightarrow (-e^{-x})' = e^{-x}$

$e^{2x} = e^{2x}\cdot(2x)' = 2\cdot e^{2x}$

$(2e^{2x})' = 2\cdot 2\, e^{2x}$

$$W = e^{x}\cdot(-e^{-x})\cdot 4\cdot e^{2x} + e^{x}\cdot e^{-x}\cdot e^{2x} + e^{x}\cdot e^{-x}\cdot 2e^{2x}$$
$$- e^{x}\cdot(-e^{-x})\cdot e^{2x} - e^{-x}\cdot 2e^{2x}\cdot e^{x} - e^{x}\cdot e^{-x}\cdot 4\cdot e^{2x}$$

$$W(y_1, y_2, y_3) = \frac{-\cancel{e^{x}}}{\cancel{e^{x}}}\cdot 4\cdot e^{2x} + \frac{\cancel{e^{x}}}{\cancel{e^{x}}}\cdot e^{2x} + \frac{\cancel{e^{x}}}{\cancel{e^{x}}}\cdot 2\cdot e^{2x}$$
$$+ \frac{\cancel{e^{x}}}{\cancel{e^{x}}}\cdot e^{2x} - \frac{\cancel{e^{x}}}{\cancel{e^{x}}}\cdot 2e^{2x} - \frac{\cancel{e^{x}}}{\cancel{e^{x}}}\cdot 4\cdot e^{2x}$$

$$W(y_1, y_2, y_3) = -4\cdot e^{2x} + e^{2x} + \cancel{2e^{2x}} + e^{2x} - \cancel{2e^{2x}} - 4\cdot e^{2x}$$
$$= -8e^{2x} + 2e^{2x} = -6e^{2x}$$

$-6e^{2x} \neq 0 \quad \forall x$

$\Rightarrow y_1, y_2$ si y_3 independente

b) $W(y, y_1, y_2, y_3) = 0$

$$W(y, y_1, y_2, y_3) = \begin{vmatrix} y & e^{x} & e^{-x} & e^{2x} \\ y' & e^{x} & -e^{-x} & 2e^{2x} \\ y'' & e^{x} & e^{-x} & 4\cdot e^{2x} \\ y''' & e^{x} & -e^{-x} & 8\cdot e^{2x} \end{vmatrix} = 0$$

$$y(-1)^{2}\underbrace{\begin{vmatrix} e^{x} & -e^{-x} & 2e^{2x} \\ e^{x} & e^{-x} & 4\cdot e^{2x} \\ e^{x} & -e^{-x} & 8\cdot e^{2x} \end{vmatrix}}_{D_1} + y'(-1)^{1+2}\underbrace{\begin{vmatrix} e^{x} & e^{-x} & e^{2x} \\ e^{x} & e^{-x} & 4\cdot e^{2x} \\ e^{x} & -e^{-x} & 8\cdot e^{2x} \end{vmatrix}}_{D_2}$$

$$+\, y''(-1)^{1+3}\underbrace{\begin{vmatrix} e^{x} & e^{-x} & e^{2x} \\ e^{x} & -e^{-x} & 2e^{2x} \\ e^{x} & -e^{-x} & 8e^{2x} \end{vmatrix}}_{D_3} + y'''(-1)^{1+4}\underbrace{\begin{vmatrix} e^{x} & e^{-x} & e^{2x} \\ e^{x} & -e^{-x} & 2e^{2x} \\ e^{x} & e^{-x} & 4\cdot e^{2x} \end{vmatrix}}_{D_4} = 0$$

avem $y D_1 - y' D_2 + y'' D_3 - y''' D_4 = 0$

$$D_1 = \cancel{e^{x}}\cdot\frac{1}{\cancel{e^{x}}}\cdot 8\cdot e^{2x} + \cancel{e^{x}}\cdot\frac{(-1)}{\cancel{e^{x}}}\cdot 2\cdot e^{2x} + \frac{(-1)}{\cancel{e^{x}}}\cdot 4e^{2x}\cdot\cancel{e^{x}} - \frac{\cancel{e^{x}}}{\cancel{e^{x}}}\cdot 2e^{2x}$$

$$- 4e^{2x}\cdot\frac{(-1)}{\cancel{e^{x}}}\cdot\cancel{e^{x}} - \cancel{e^{x}}\cdot\frac{(-1)}{\cancel{e^{x}}}\cdot 8e^{2x}$$

$$D_1 = \cancel{8\cdot e^{2x}} - 2e^{2x} - \cancel{4e^{2x}} - 2e^{2x} + \cancel{4e^{2x}} \cancel{+8e^{2x}}$$

De ce?

?

$$\boxed{D_1 = -4e^{2x}}$$

$D_1 = 12e^{2x}$

$$D_2 = \frac{\cancel{e^{x}}}{\cancel{e^{+x}}}\cdot 8\cdot e^{2x} + \cancel{e^{x}}\cdot\frac{(-1)}{\cancel{e^{x}}}\cdot e^{2x} + \frac{1}{\cancel{e^{x}}}\cdot 4\cdot e^{2x}\cdot\cancel{e^{x}} - \cancel{e^{x}}\cdot\frac{1}{\cancel{e^{x}}}\cdot e^{2x} -$$

$$- 4\cdot e^{2x}\cdot\frac{(-1)}{\cancel{e^{+x}}}\cdot\cancel{e^{x}} - \frac{e^{x}}{e^{x}}\cdot 8\cdot e^{2x}$$

$$D_2 = 8\cdot\cancel{e^{2x}} - e^{2x} + 4e^{2x} - e^{2x} + 4e^{2x} - \cancel{8e^{2x}}$$

$$\boxed{D_2 = 6e^{2x}}$$

$$D_3 = \cancel{e^{x}}\cdot\frac{(-1)}{\cancel{e^{x}}}\cdot 8e^{2x} + \cancel{e^{x}}\cdot\frac{(-1)}{\cancel{e^{x}}}\cdot e^{2x} + \frac{1}{\cancel{e^{x}}}\cdot 2e^{2x}\cdot\cancel{e^{x}} - \cancel{e^{x}}\cdot\frac{(-1)}{\cancel{e^{x}}}\cdot e^{2x}$$

$$- \cancel{e^{x}}\cdot\frac{(-1)}{\cancel{e^{x}}}\cdot 2e^{2x} - \frac{\cancel{e^{x}}}{\cancel{e^{x}}}\cdot 8e^{2x}$$

$$D_3 = -8e^{2x} \cancel{-e^{2x}} + 2e^{2x} \cancel{+e^{2x}} + 2e^{2x} - 8e^{2x}$$

?

$$\boxed{D_3 = -14e^{2x}}$$

$D_3 = -12e^{2x}$

$$D_4 = \cancel{e^x}\cdot\frac{(-1)}{\cancel{e^x}}\cdot 4e^{2x} + \frac{\cancel{e^x}}{\cancel{e^x}}\cdot e^{2x} + \cancel{e^x}\cdot 2\cdot e^{2x}\cdot\frac{1}{\cancel{e^x}} - \cancel{e^x}\frac{(-1)}{\cancel{e^x}}\cdot e^{2x} -$$

$$- \frac{1}{\cancel{e^x}}\cdot 2e^{2x}\cdot\cancel{e^x} - \frac{\cancel{e^x}}{\cancel{e^x}}\cdot 4e^{2x}$$

$$D_4 = \underline{-4e^{2x}}\ \underline{+e^{2x}} + 2e^{2x}\ \underline{+e^{2x}} - 2e^{2x}\ \underline{-4e^{2x}}$$

$$\boxed{D_4 = -6e^{2x}} \checkmark$$

$$\to \boxed{yD_1 - y'D_2 + y''D_3 - y'''D_4 = 0}$$

$$+y\cancel{4}12e^{2x} - y'\cdot 6e^{2x} - y''\cancel{14}12e^{2x} + y'''6e^{2x} = 0 \quad |:\cancel{2}\ 6e^{2x} \neq 0$$

$$-2y\cdot e^{2x} - 3y'\cdot e^{2x} - 7y''\cdot e^{2x} + 3y'''\cdot e^{2x} = 0 \quad |: e^{2x}$$

$$-2y - 3y' - 7y'' + 3y''' = 0 \quad |(-1)$$

$$2y + 3y' + 7y'' - 3y''' = 0$$

$$2y - y' - 2y'' + y''' = 0$$

Exemplul 03

$B = \{\sin x, \cos x\}$ a) verif independenta y_1, y_2

$$W(y_1, y_2) = \begin{vmatrix} \sin x & \cos x \\ \sin'(x) & \cos' x \end{vmatrix} = \begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix}$$

$$W(y_1, y_2) = -\sin^2 x - \cos^2 x = -(\sin^2 x + \cos^2 x) = -1 \neq 0$$

b) $W(y, y_1, y_2) = 0$

$$W(y, y_1, y_2) = \begin{vmatrix} y & \sin x & \cos x \\ y' & \cos x & -\sin x \\ y'' & -\sin x & -\cos x \end{vmatrix} = 0$$

$$y(-1)^2 \begin{vmatrix} \cos x & -\sin x \\ -\sin x & -\cos x \end{vmatrix} + y'(-1)^3 \begin{vmatrix} \sin x & \cos x \\ -\sin x & -\cos x \end{vmatrix} + y''(-1)^4 \begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix} = 0$$

$$y(\underbrace{-\cos^2 x - \sin^2 x}_{-1}) - y'(\underbrace{-\sin x\cos x + \sin x\cos x}_{0}) + y''(\underbrace{-\sin^2 x - \cos^2 x}_{-1}) = 0$$

$$-y + (-1)y'' = 0 \Rightarrow y'' + y = 0 \checkmark$$