When Class Imbalance Shifts the Decision Boundary: A Short Proof

Boammani Aser Lompo

October 7, 2025

1 Setup

Let $Y \in \{0,1\}$ with true prior $\pi = P(Y = 1)$ and feature vector $X \in \mathcal{X}$. Let $p(x \mid y)$ denote class-conditional densities and $p(x) = \sum_y p(x \mid y)P(Y = y)$. Suppose we train on an imbalanced sample with empirical prior $\hat{\pi}$ (typically $\hat{\pi} \neq \pi$).

2 Bayes decision rule

Under 0–1 loss with equal costs, the Bayes classifier predicts 1 iff

$$\log \frac{p(x \mid 1)}{p(x \mid 0)} \; \geq \; \log \frac{1-\pi}{\pi}.$$

Equivalently, $p(Y=1 \mid x) \geq \frac{1}{2}$.

Proposition 1 (Imbalanced-sample boundary shift)**.** *If a learner minimizes empirical* 0–1 *risk on a sample whose class prior is* $\hat{\pi}$*, then as* $n \to \infty$ *it converges to the decision rule*

$$\log \frac{p(x \mid 1)}{p(x \mid 0)} \; \geq \; \log \frac{1-\hat{\pi}}{\hat{\pi}}.$$

If $\hat{\pi} \neq \pi$*, the decision threshold differs from the true Bayes threshold by*

$$\Delta \; = \; \log \frac{\pi}{1-\pi} \; - \; \log \frac{\hat{\pi}}{1-\hat{\pi}},$$

biasing decisions toward the majority class of the training sample.

Proof. Minimizing empirical 0–1 risk on the sample is equivalent to using the sample distribution as the data-generating law; the Bayes optimal under that law uses prior $\hat{\pi}$. Therefore the induced threshold is $\log \frac{1-\hat{\pi}}{\hat{\pi}}$. Subtract the true Bayes threshold $\log \frac{1-\pi}{\pi}$ to obtain Δ. $\square$

3 Cross-entropy and prior correction

Under cross-entropy, logistic models learn $p_{\text{train}}(Y = 1 \mid x)$ with prior $\hat{\pi}$. If deployment prior π differs, correct via

$$\operatorname{logit} p_{\text{test}}(1 \mid x) = \operatorname{logit} p_{\text{train}}(1 \mid x) + \log \frac{\pi/(1-\pi)}{\hat{\pi}/(1-\hat{\pi})}.$$

Alternatively, use class-weighted losses with $w_y \propto 1/\hat{\pi}_y$ or resampling to neutralize the sample prior.

4 Practical notes

State assumptions (no covariate shift, equal costs), clarify what “bias” means (boundary shift vs calibration vs minority metrics), and mention regularization/data scarcity effects.

5 Conclusion

Imbalance per se shifts the decision rule when the sample prior differs from deployment prior. The effect can be neutralized with weighting, resampling, calibrated thresholds, or prior-shift correction.