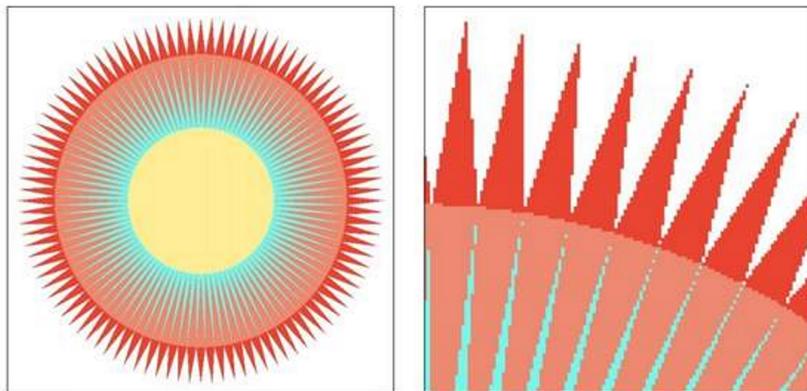


# Lecture 06 Rasterization 2 (Antialiasing and Z-buffering)

2020年12月26日 17:27

## 走样与反走样

### Aliasing



Is this the best we can do?

Slide courtesy of Prof. Ren Ng, UC Berkeley

CS180, Winter 2020

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## Moiré Patterns in Imaging



Skip odd rows and columns

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## Wagon Wheel Illusion (False Motion)



采用上一讲的光栅化方法，会出现走样 (Aliasing) 现象

采样中出现的误差，错误，或不准确，称作Artifacts

例如：锯齿，摩尔纹，旋转车轮错觉等

## Sampling Artifacts in Computer Graphics

Artifacts due to sampling - "Aliasing"

- Jaggies – sampling in space
- Moire – undersampling images
- Wagon wheel effect – sampling in time
- [Many more] ...

Behind the Aliasing Artifacts

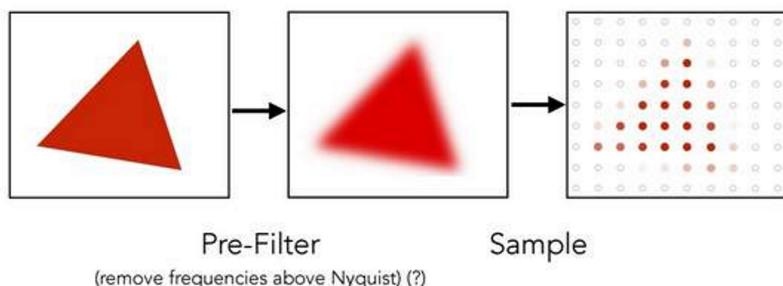
- Signals are changing **too fast** (high frequency),  
but **sampled too slowly**

这些artifacts背后的原因：

信号变化速度太快 (高频率)

但采样速度相对太慢

## Rasterization: Antialiased Sampling



Note antialiased edges in rasterized triangle  
where pixel values take intermediate values

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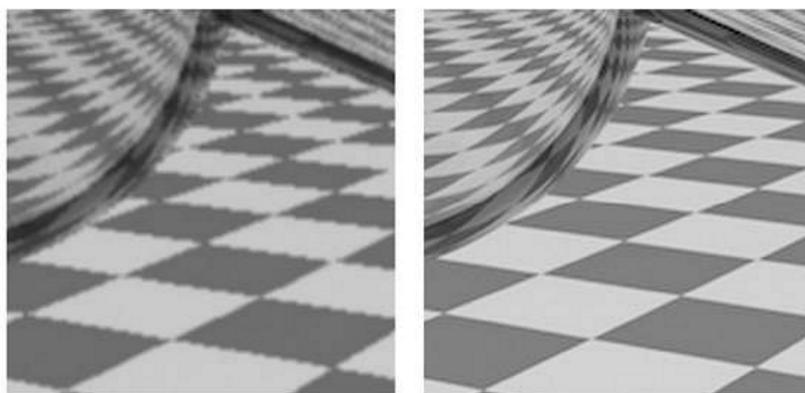
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一种反走样的思想：先模糊，再采样

模糊的部分，可以看到光栅化以后像素颜色取了个中间值 (intermediate values)

## Antialiasing vs Blurred Aliasing



(Sample then filter, WRONG!)

(Filter then sample)

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注意：如果先采样，再模糊，不能得到反走样的结果，而是得到blurred alias，模糊的锯齿

But why?

1. Why undersampling introduces aliasing?
2. Why pre-filtering then sampling can do antialiasing?

Let's dig into fundamental reasons

And look at how to implement antialiased rasterization

思考背后原理：

- (1) 为何欠采样会导致走样?
- (2) 为何pre-filtering (预滤波, 前置滤波) 可以反走样?

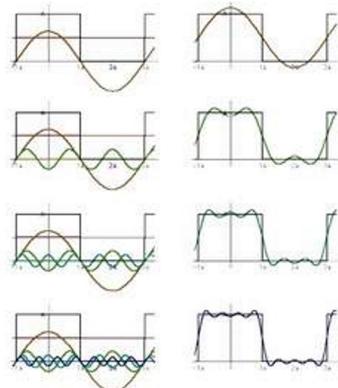
## 频域 frequency domain

Fourier Transform

Represent a function as a weighted sum of sines and cosines



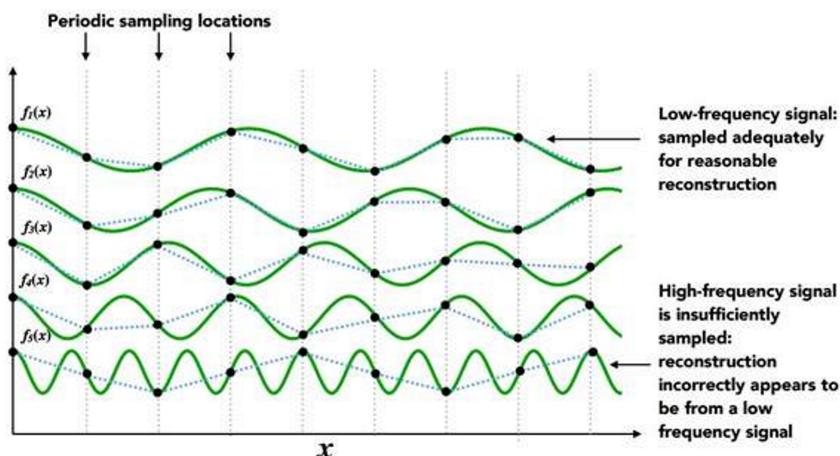
Joseph Fourier 1768 - 1830



$$f(x) = \frac{A}{2} + \frac{2A \cos(t\omega)}{\pi} - \frac{2A \cos(3t\omega)}{3\pi} + \frac{2A \cos(5t\omega)}{5\pi} - \frac{2A \cos(7t\omega)}{7\pi} + \dots$$

傅立叶变换：任何一个周期函数，都能用sin, cos及常数项表示

## Higher Frequencies Need Faster Sampling



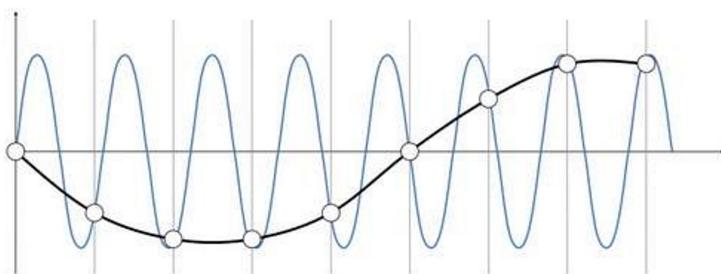
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一个三角函数它有一定的频率，采样也需要有一个相应的频率，否则就容易失真。

## Undersampling Creates Frequency Aliases



High-frequency signal is insufficiently sampled: samples erroneously appear to be from a low-frequency signal

Two frequencies that are indistinguishable at a given sampling rate are called "aliases"

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走样：假设有两个完全不同的函数（蓝色线和黑色线），都在空心点采样，采样出来的结果都是黑色线，也就是两个不同的函数采样出来的结果是一样的，这就是“走样”。

傅里叶变换：可以把函数从时域变换到频域。时域的卷积等于频域的乘积

## Filter Out Low Frequencies Only (Edges)



High-pass filter

高通滤波：保留图像边界；

## Filter Out High Frequencies (Blur)



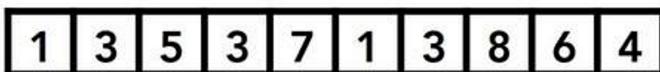
Low-pass filter

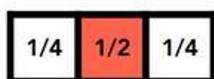
低通滤波：模糊图像边界。 (边界：指图像中颜色发生剧烈变化的地方)

## Filtering = Convolution (= Averaging)

滤波=卷积 (=求平均)

### Convolution

Signal 

Filter 

$$3 \times (1/4) + 5 \times (1/2) + 3 \times (1/4) = 4$$

Result 

卷积

(不严谨的说法) 按照一定的卷积核 (权重) 求某个点位周围带权的均值

# Convolution Theorem

Convolution in the spatial domain is **equal to** multiplication in the frequency domain, and vice versa

Option 1:

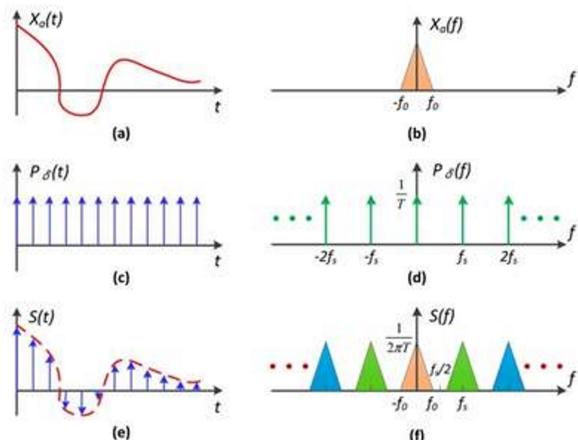
- Filter by convolution in the spatial domain

Option 2:

- Transform to frequency domain (Fourier transform)
- Multiply by Fourier transform of convolution kernel
- Transform back to spatial domain (inverse Fourier)

空间域的卷积相当于频域的乘积，反之亦然

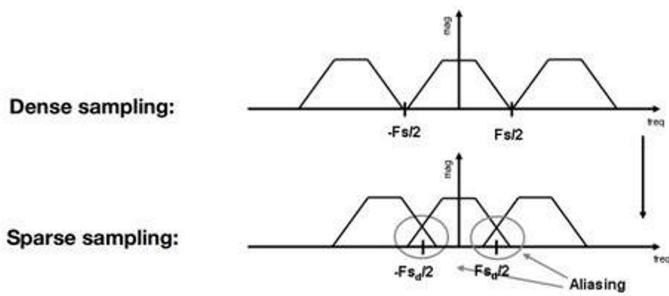
## Sampling = Repeating Frequency Contents



[https://www.researchgate.net/figure/The-evolution-of-sampling-theorem-a-The-time-domain-of-the-band-limited-signal-and-b-fig5\\_3015566095](https://www.researchgate.net/figure/The-evolution-of-sampling-theorem-a-The-time-domain-of-the-band-limited-signal-and-b-fig5_3015566095)

采样=重复频域的内容

# Aliasing = Mixed Frequency Contents



走样=弄混了频率内容

## How Can We Reduce Aliasing Error?

### Option 1: Increase sampling rate

- Essentially increasing the distance between replicas in the Fourier domain
- Higher resolution displays, sensors, framebuffers...
- But: costly & may need very high resolution

### Option 2: Antialiasing

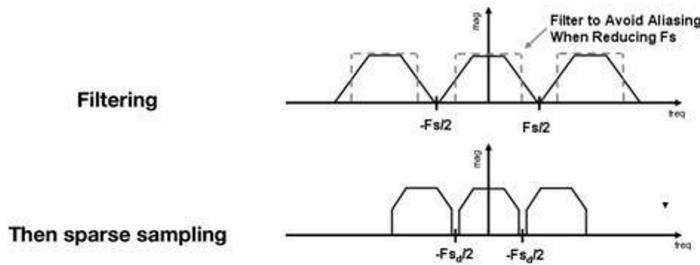
- Making Fourier contents “narrower” before repeating
- i.e. **Filtering out high frequencies before sampling**

如何减少走样误差?

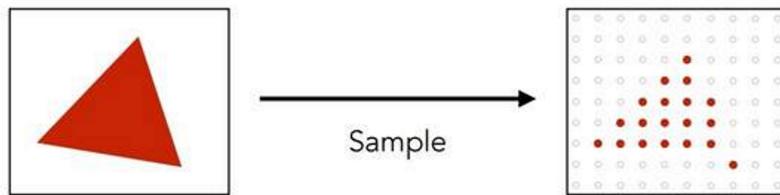
增加采样率: 终极方法, 但开销很大

反走样: **先做模糊 (低通滤波) , 再做采样**

Antialiasing = Limiting, then repeating



## Regular Sampling

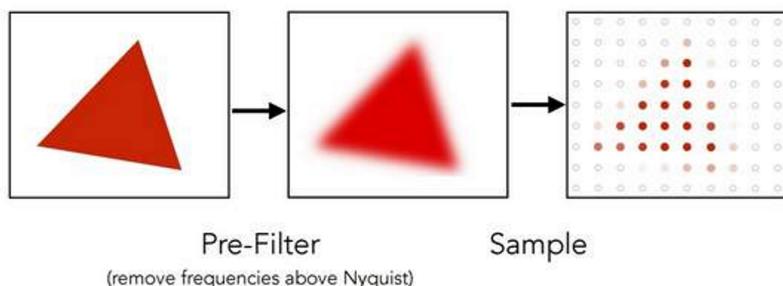


Note jaggies in rasterized triangle  
where pixel values are pure red or white

普通采样：

像素点非红即白，因此造成锯齿

## Antialiased Sampling



Note antialiased edges in rasterized triangle  
where pixel values take intermediate values

反走样后的采样：

先经过预过滤，再采样

边缘取了周围的像素点颜色的中间值 (intermediate values)

## Antialiasing By Averaging Values in Pixel Area

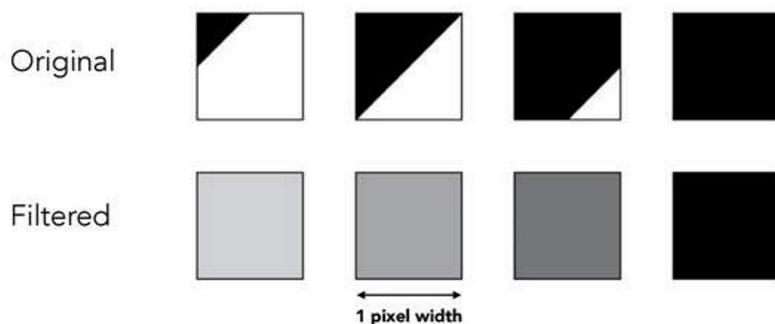
Solution:

- **Convolve**  $f(x,y)$  by a 1-pixel box-blur
  - Recall: convolving = filtering = averaging
- **Then sample** at every pixel's center

先做模糊操作（卷积），再在每个像素中心采样

## Antialiasing by Computing Average Pixel Value

In rasterizing one triangle, the average value inside a pixel area of  $f(x,y) = \text{inside}(\text{triangle},x,y)$  is equal to the area of the pixel covered by the triangle.



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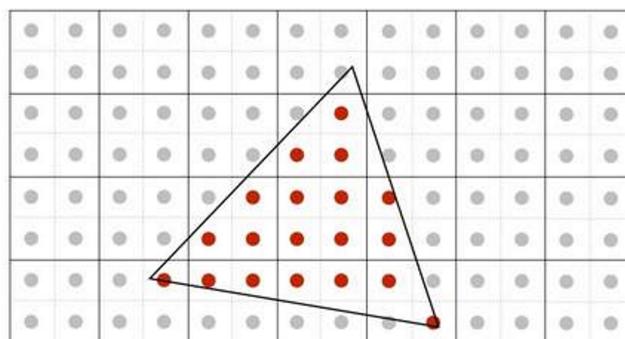
通过计算平均像素点值的方法实现反走样

在对三角形光栅化时，像素点区域内部的颜色平均值等于该像素点被三角形覆盖的面积

## 通过多重采样进行反走样 (MSAA)

### Supersampling: Step 1

Take  $N \times N$  samples in each pixel.



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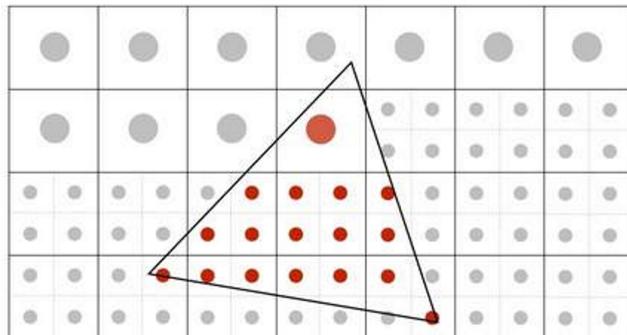
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首先，在每个像素点内，取 $N \times N$ 个小的“像素”（采样点），对每个小采样点判断它们是不是在图形内

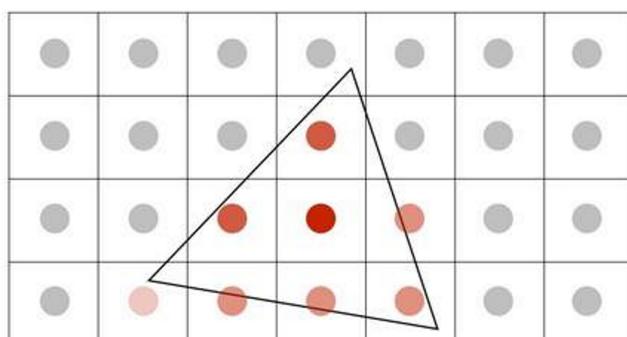
## Supersampling: Step 2

Average the NxN samples “inside” each pixel.



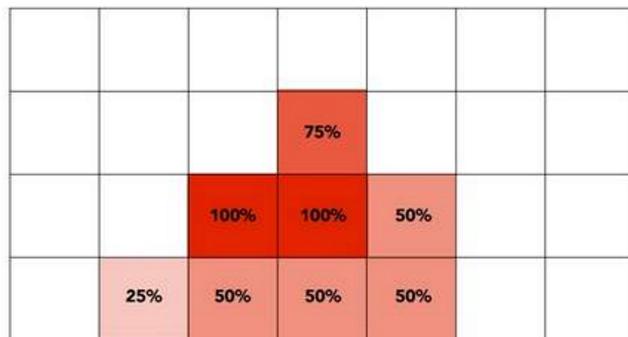
## Supersampling: Step 2

Average the NxN samples “inside” each pixel.



## Supersampling: Result

This is the corresponding signal emitted by the display



求每个小像素判断结果的平均值, 在图形内的小像素越多, 则颜色越深, 否则越浅。

MSAA并不是提升了屏幕分辨率解决了锯齿问题, 而是增加采样点而得到近似的三角形覆盖  
开销: 增加了计算量

## 其他AA方法

FXAA (Fast Approximate AA) (配置要求低, 效果一般)

后期做图像处理, 把有锯齿的边界找到, 换成没有锯齿的边界

TAA (Temporal AA) (配置要求高, 效果好)

找上一帧的信息。对相邻两帧, 找某个像素内不同位置的点, 判断像素是否在三角形内

超采样 (DLSS) :Deep Learning Super Sampling

小图拉大, 用深度学习 “猜” 出缺失的细节