Advanced Algorithms
Homework 1: Preliminaries, Divide and Conquer
CS 611, Spring 2020

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1 Problem 3: Strassen's Algorithm

1.1 (a) Matrix Multiplication

1.1.1 Multiplying $kn \times n$ matrix with $n \times kn$ matrix

The resultant matrix will be $kn \times kn$. This gives k^2 multiplications of $n \times n$ matrices.

There will be 7 recursive multiplicative calls as per Strassen's method. The matrix is divided with a 2×2 grid. We are dividing $kn \times n$ matrix into $\frac{kn}{2} \times \frac{n}{2}$ matrix.

For a $n \times n$ matrix, the run-time would be $\Theta(n^{\log_2 7})$ and the extra k^2 multiplications are to be accounted for too, thus, making the runtime $\boxed{\Theta(k^2 n^{\log_2 7})}$.

Note that this k^2 is because for each $c_{ij} = \sum_{m=1}^{n} a_{im} b_{mj}$, there will be k inner and outer loops, $1 \leq i, j \leq k$.

1.1.2 Multiplying $n \times kn$ matrix with $kn \times n$ matrix

The resultant matrix will be $n \times n$. This gives k multiplications of $n \times n$ matrices.

There will be 7 recursive multiplicative calls as per Strassen's method. The matrix is divided with a 2×2 grid. We are dividing $n \times kn$ matrix into $\frac{n}{2} \times \frac{kn}{2}$ matrix.

For a $n \times n$ matrix, the run-time would be $\Theta(n^{\log_2 7})$ and the extra k multiplications are to be accounted for too, thus, making the runtime $\boxed{\Theta(kn^{\log_2 7})}$.

Note that this k is because for each $c_{ij} = \sum_{m=1}^{k} a_{im} b_{mj}$, there will be k innermost loops, $1 \leq m \leq k$.

1.2 (b) Complex Number Multiplication

First Complex Number: $a + bi$
Second Complex Number: $c + di$
We need to achieve $(a + bi)(c + di)$, which is $(ac - bd) + (ad + bc)i$.

Suppose, we calculate:

$$\boxed{P_1 = (a + b)(c + d)}$$

$$\boxed{P_2 = ac}$$

$$\boxed{P_3 = bd}$$

To achieve the **real** part of the result,
we can calculate $P_2 - P_3$.

$$P_2 - P_3 = ac - bd$$

To achieve the **imaginary** part of the result,
we can calculate $P_1 - P_2 - P_3$.

$$P_1 - P_2 - P_3 = ac + ad + bc + bd - ac - bd = ad + bc$$

Thus, there are 3 multiplications and 3 subtractions.