Advanced Algorithms
Homework 1: Preliminaries, Divide and Conquer
CS 611, Spring 2020

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Contents

1 Problem 4: Solving Recursions

1.1 (a) Recurrence Tree Method

Figure 1 shows the recurrence for $T(n) = T(n/3) + T(2n/3) + cn$.

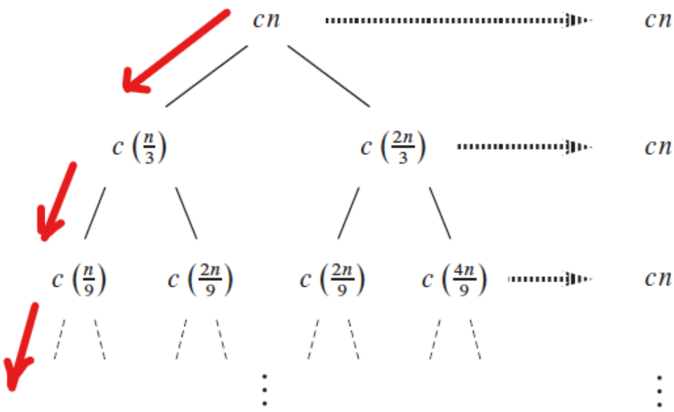

Figure 1: A recursion tree for the recurrence $T(n) = T(n/3)+T(2n/3)+cn$

When we add the values across the levels of the recursion tree, we get a value of cn for every level.

The shortest simple path from root to a leaf is $n \to (1/3)n \to (1/3)^2 n \to \cdots \to 1$. Since $(1/3)^k n = 1$ when $k = \log_3 n$, the height of the tree is $\log_3 n$.

Let's consider the cost at the leaves. If this recursion tree were a complete binary tree of height $\log_3 n$, there would be $2^{\log_3 n} = n^{\log_3 2}$ leaves. Since the cost of each leaf is a constant, the total cost of all leaves would then be

$$cn(\log_3 n + 1) \geq cn \log_3 n = \frac{c}{\log 3} n \log n = \Omega(n \log n)$$

1.2 (b) Master Method

We first recall that Strassen's algorithm had 7 subproblems, each of size $n/2 \times n/2$ and the divide and combine steps together took $\Theta(n^2)$ time. This resulted in running time of $\Theta(n^{\log_2 7})$.

Here, our runtime goal is $T(n) = aT(n/4) + \Theta(n^2)$.
Thus, according to master method,

$$\begin{aligned} a &= a \\ b &= 4 \\ f(n) &= \Theta(n^2) \\ \therefore n^{\log_b a} &= n^{\log_4 a} \end{aligned}$$

We need to make the term $n^{\log_4 a}$ have smaller asymptotic value than $n^{\log_2 7}$.

$$\begin{aligned} n^{\log_2 7} &> n^{\log_4 a} \\ \log_2 7 &> \log_4 a \\ \frac{\log 7}{\log 2} &> \frac{\log a}{\log 4} \\ \frac{\log 7}{\log 2} &> \frac{\log a}{\log 2^2} \\ \frac{\log 7}{\log 2} &> \frac{\log a}{2 \log 2} \\ 2 \log 7 &> \log a \\ \log 7^2 &> \log a \\ \log 49 &> \log a \\ 49 &> a \end{aligned}$$

Hence, the largest integer value of a for which Professor Caesar's algorithm would be asymptotically faster than Strassen's algorithm is $\boxed{a = 48}$.

Moreover, $2 < \log_4(48)$ so there exists $\epsilon > 0$ such that $n^2 < n^{log_4(48) - \epsilon}$.