Advanced Algorithms
Homework 1: Preliminaries, Divide and Conquer
CS 611, Spring 2020

Name: Kunjal Panchal
Student ID: 32126469
Email: kpanchal@umass.edu

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1 Problem 1: Summation Appendix

1.1 (a) A.1-1

$$\sum_{k=1}^{n}(2k-1) = \sum_{k=1}^{n}(2k) - \sum_{k=1}^{n} 1 \tag{1}$$

$$= 2\sum_{k=1}^{n} k - \sum_{k=1}^{n} 1 \quad \text{according to linearity rule} \sum_{k=1}^{n}(ca_k + b_k) = c\sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k \tag{2}$$

$$= 2\left[\frac{1}{2}n(n+1)\right] - \sum_{k=1}^{n} 1 \quad \text{according to arithmetic series rule} \sum_{k=1}^{n} k = \frac{1}{2}n(n+1) \tag{3}$$

$$= 2 \cdot \frac{1}{2}n(n+1) - n \quad \text{according to constant rule} \sum_{k=1}^{n} 1 = 1 + \cdots + 1(\text{n times}) = n \tag{4}$$

$$= n(n+1) - n \quad \text{simplification} \tag{5}$$

$$= n^2 + n - n \tag{6}$$

$$= n^2 \tag{7}$$

$$\boxed{\therefore \sum_{k=1}^{n}(2k-1) = n^2}$$

1.2 (b) A.1-7

$$\prod_{k=1}^{n} 2 \cdot 4^k = 2 \cdot 4^1 \; \cdot \; 2 \cdot 4^2 \; \cdot \; 2 \cdot 4^3 \; \ldots \; 2 \cdot 4^n \quad \therefore \text{2s will be multiplied } n \text{ times} \tag{8}$$

$$= 2^n \cdot 4^1 \; \cdot \; 4^2 \; \cdot \; 4^3 \; \ldots \; 4^n \quad \therefore \text{we can take sum of powers of 4s} \tag{9}$$

$$= 2^n \cdot 4^{(1+2+3+\cdots+n)} \quad \text{the powers are in arithmetic series} \tag{10}$$

$$= 2^n \cdot 4^{\sum_{j=1}^{n} j} \tag{11}$$

$$= 2^n \cdot 4^{\frac{n(n+1)}{2}} \tag{12}$$

$$= 2^n \cdot 2^{\left[2 \cdot \frac{n(n+1)}{2}\right]} \quad 4 = 2^2;\ x^y = z \text{ and } z^b \text{ then } z^b = x^{y \cdot b} \tag{13}$$

$$= 2^n \cdot 2^{n(n+1)} \quad \text{simplification} \tag{14}$$

$$= 2^{n+n(n+1)} \tag{15}$$

$$= 2^{n^2+2n} \tag{16}$$

$$= 2^{n^2} \cdot 2^{2n} \tag{17}$$

$$= 2^{n^2} \cdot 4^n \tag{18}$$

$$\tag{19}$$

$$\boxed{\therefore \prod_{k=1}^{n} 2 \cdot 4^k = 2^{n^2} \cdot 4^n}$$

1.3 (c) A.2-4

$\sum_{k=1}^{n} k^3$ is a monotonically increasing function, we can approximate it by integrals:

$$\int_0^n x^3 dx \leq \sum_{k=1}^{n} k^3 \leq \int_1^{n+1} x^3 dx$$

1.3.1 Lower Bound

For a lower bound, we can obtain

$$\sum_{k=1}^{n} k^3 \geq \int_0^n x^3 dx \quad (20)$$

$$= \frac{x^4}{4}\bigg|_0^n \quad (21)$$

$$= \frac{n^4}{4} \quad (22)$$

1.3.2 Upper Bound

For an upper bound, we can obtain

$$\sum_{k=1}^{n} k^3 \leq \int_1^{n+1} x^3 dx \quad (23)$$

$$= \frac{x^4}{4}\bigg|_1^{n+1} \quad (24)$$

$$= \frac{(n+1)^4 - 1}{4} \quad (25)$$