American Computer Science League

2021-2022 ● Contest 1: Short Problems ● Senior Division

sr soln

1. Computer Number Systems

$2122_{10} = 8 * 16^2 + 4 * 16^1 + 10 * 16^0$
$= 84A_{16}$

The next year with all the same hex digits is 888_{16}
$888_{16} = 8 * 16^2 + 8 * 16^1 + 8 * 16^0 = 2184$

So 2184 - 2122 = 62

B. 62

2. Computer Number Systems

Fibonacci sequence 0, 1, 1, 2, 3, 5, 8, 13, … in base 10
Fibonacci sequence in hexadecimal is:
0, 1, 1, 2, 3, 5, 8, D, 15, 22, 37, 59, 90, ...

1st: 1 from 0
Next: 90 from 144

D. 144

3. Recursive Functions

$f(1) = 1$
$f(2) = f(1) + 3 * 2 - 1 = 1 + 6 - 2 = 5$
$f(3) = f(2) + 3 * 3 - 2 = 5 + 9 - 2 = 12$
$f(4) = f(3) + 3 * 4 - 2 = 12 + 12 - 2 = 22$
At this point the process could continue to:
$f(10) = f(9) + 3 * 10 - 2 = 145$
or analyze the sequence of numbers: 1, 5, 12, 22,... . Each time the difference between terms increases by 3. These are the pentagonal numbers.

D. 145

4. Recursive Number Systems

$f(100,36) = f([100/2],[36/2]) + 2 = f(50,18) + 2 = -136 + 2$
$f(50,18) = f([50/2],[18/2]) + 2 = f(25,9) + 2 = -138 + 2 =$
$f(25,9) = f([25/2],[9/2]) + 2 = f(12,4) + 2 = -140 + 2 =$
$f(12,4) = f(2 * 12, 4 - 3) + 1 = f(24,1) + 1 = -141 + 1 =$
$f(24,1) = f(2 * 24, 1 - 3) + 1 = f(48,-2) + 1 = -142 + 1 =$
$f(48,-2) = 48(-2) - 48 - (-2) = -142$

$f(36,100) = f([36/2],[100/2]) + 2 = f(18,50) + 2 = 7 + 2 = 9$
$f(18,50) = f([18/2],[50/2]) + 2 = f(9,25) + 2 = 5 + 2 = 7$
$f(9,25) = f([9/2],[25/2) + 2 = f(4,12) + 2 = 3 + 2 = 5$
$f(4,12) = f(2 * 4, 12 - 3) + 1 = f(8,9) + 1 = 2 + 1 = 3$
$f(8,9) = f([8/2],[9/2]) + 2 = f(4,4) + 1 = 0 + 2 = 2$
$f(4,4) = f(2 * 4, 4 - 3) + 1 = f(8,1) + 1 = -1 + 1 = 0$
$f(8,1) = 8(1) - 8 - 1 = -1$

So $f(100,36) - f(36,100) = -134 - 9 = -143$

E. -143

5. What Does This Program Do? (Branching)

The following table can be used to trace the program:

a	b	c	x
1	2	3	0
1	2	3	1
1	2	3	2
1	2	3	2
1	2	3	5

B. 5