American Computer Science League

2021-2022 ● Contest 3: Short Problems Solutions ● Senior Division

1. Boolean Algebra

$$\overline{A\,(B\ +\ \overline{C})}\ +\ \overline{A}\,\overline{B}(A\ +\ C) = \overline{A} + \overline{B\ +\ \overline{C}} + \overline{A}\,\overline{B}\,A\ +\ \overline{A}\,\overline{B}C$$
$$= \overline{A} + \overline{B}C\ +\ 0\ +\ \overline{A}\,\overline{B}C$$
$$= \overline{A}(1\ +\ \overline{B}\,C)\ +\ \overline{B}C$$
$$= \overline{A} + \overline{B}C$$

D. $\overline{A} + \overline{B}C$

2. Boolean Algebra

$$(A \oplus (B\ +\ A)\,\overline{C})\ +\ \overline{A}\,\overline{B}\,C = \overline{A(B\ +\ A)\,\overline{C}} + \overline{A}((B\ +\ A)\,\overline{C})\ +\ \overline{A}\,\overline{B}C$$
$$= A(\overline{(A\ +\ B)}\ +\ C)\ +\ \overline{A}B\overline{C}\ +\ A\overline{A}\,\overline{C} + \overline{A}\,\overline{B}\,C$$
$$= A\overline{A}\,\overline{B} +\ AC\ +\ \overline{A}B\overline{C} + 0\ +\ \overline{A}\,\overline{B}C$$
$$= 0\ +\ AC\ +\ \overline{A}B\overline{C} + \overline{A}\,\overline{B}C$$

If $A\ =\ 1$, then $0\ +\ C\ +\ 0\ +\ 0\ =\ C\ =\ 1$ TRUE for (1, *, 1)

If $A\ =\ 0$, then $0\ +\ 0\ +\ B\overline{C} + \overline{B}C\ =\ 1$

If $B\ =\ 1$, then $\overline{C}\ +\ 0\ =\ \overline{C}\ =\ 1$ TRUE for (0, 1, 0)

If $B\ =\ 0$, then $0\ +\ C\ =\ C\ =\ 1$ TRUE for (0, 0, 1)

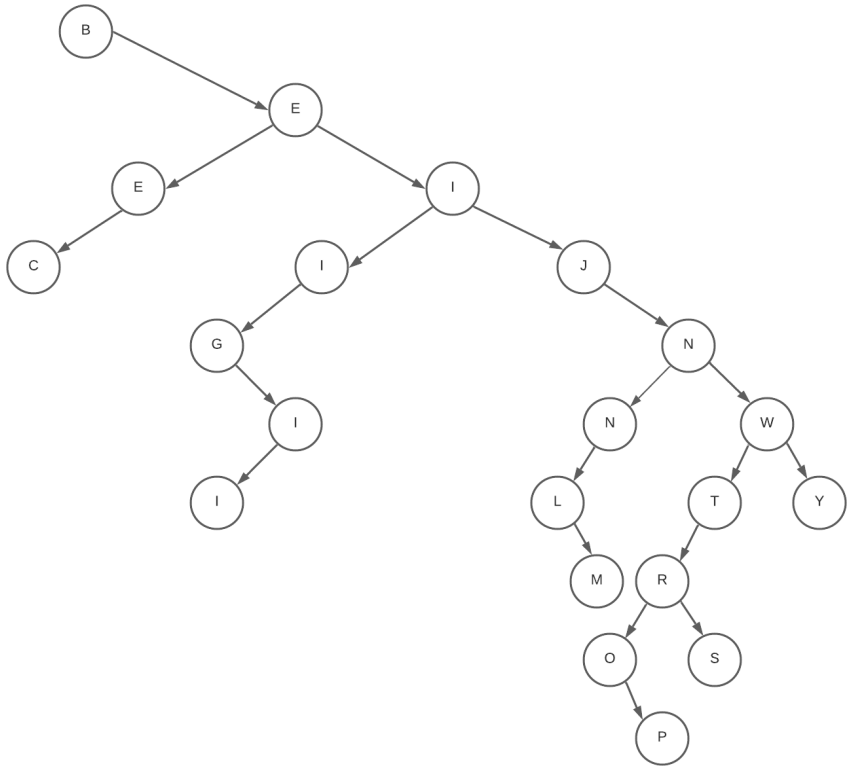
Therefore 4 ordered triples make it TRUE.

D. 4

3. Data Structures

The binary search tree for BEIJINGWINTEROLYMPICS is:

C. 100



The internal path length is:

1*1 + 2*2 + 3*3 + 2*4 + 3*5 + 4*6 + 2*7 + 2*8 + 1*9
= 1 + 4 + 9 + 8 + 15 + 24 + 14 + 16 + 9
= 100

4. Data Structures

The stack is constructed using LIFO as follows:

C, CA, C, CS, CSS, CSSI, CSS, CS, CSO, CSOP,
CSO, CSOE, CSOEI, CSOE, CSOEA, CSOE, CSO

D. O

5. FSAs and Regular Expressions

[ACSL]*[^aeiou]*21.2

C. c, e

a. Ab212 Fails - needs a character between 1 and 2 b. ACSL21 Fails - missing a character and 2 at the end c. ACSL442122 Valid d. cmptr212 Fails - needs a character between 1 and 2 e. A202122 Valid f. pgrmmng21 Fails - missing a character and 2 at the end	