American Computer Science League

2021-2022 ● Contest 1: Short Problems ● Senior Division

1. Computer Number Systems

Starting with the year 2122_{10} converted to hex, in how many years (in base10) will the year first be represented by a hexadecimal number where all the digits are the same?

A. 22
B. 62
C. 260
D. 406
E. 608

2. Computer Number Systems

The Fibonacci sequence {0, 1, 1, 2, 3, 5, 8, 13, …} is formed by adding the two previous terms: 5 + 8 = 13. Which number in the sequence when converted to hexadecimal would have the 2nd occurrence of the substring "0"?

A. 34
B. 55
C. 89
D. 144
E. 233

3. Recursive Functions

Find $f(10)$, given

$$f(x) = \begin{cases} 1 & if\ x = 1 \\ f(x-1) + 3x - 2 & if\ x > 1 \end{cases}$$

A. 70
B. 92
C. 117
D. 145
E. 176

4. Recursive Number Systems

Evaluate $f(100, 36) - f(36, 100)$, given

$$f(x.y) = \begin{cases} f\left(\left[\frac{x}{2}\right], \left[\frac{y}{2}\right]\right) + 2 & if\ xy \geq 50 \\ f(2x, y-3) + 1 & if\ 10 < xy < 50 \\ xy - x - y & if\ xy \leq 10 \end{cases}$$

where [z] returns the greatest integer less than or equal to z

A. 128
B. -134
C. -1
D. -125
E. -143

5. What Does This Program Do? (Branching)

What is the final value of x after this program is executed?

```
a = 1: b = 2: c = 3: x = 0
if a < b then
   x = x + a
else
   x = x + b
end if
if b > c && a <= b then
   x = x + c
else
   x = x + a
end if
if !(a < b && a < c) then
   x = x + a
end if
if b < c || c < a then
   x = x + c
end if
output x
```

A. 4
B. 5
C. 6
D. 7
E. 8