

American Computer Science League

2021-2022 • Contest 4: Short Problems Solutions • Senior Division

1. Graph Theory

Consider the adjacency matrix raised to successive powers until at least one entry is more than 1.

C. 3

| Adjacency | Squared | Cubed |
|--|--|--|
| $\begin{vmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix}$ | $\begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{vmatrix}$ | $\begin{vmatrix} 0 & 2 & 2 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{vmatrix}$ |

An entry greater than 1 occurs in the cubed matrix.

2. Graph Theory

There are 10 cycles in the following directed graph:
ABA, ABCDEA, ABDEA, ADBA, ADEA, ACDBA,
ACDEA, AEA, BCDB, and BDB

D. 10

3. Digital Electronics

The Boolean expression that represents this digital circuit is:

A. $A B C$

$$\begin{aligned}
 \overline{A(A \oplus B + \overline{B C})} &= \overline{A(\overline{A B} + \overline{A B} + \overline{B} + \overline{C})} \\
 &= \overline{A A \overline{B} \overline{A B} B C} \\
 &= \overline{A B C (\overline{A} + B)(\overline{A} + \overline{B})} \\
 &= \overline{A B C (\overline{A A} + \overline{A B} + A B + B \overline{B})} \\
 &= \overline{A B C \overline{A B} + A B C} \\
 &= \overline{A B C}
 \end{aligned}$$

4. Digital Electronics

The Boolean expression representing this digital circuit is:

$$\begin{aligned}(A + B C) \oplus \bar{D} &= \overline{(A + B C) \oplus \bar{D}} \\(A + B C) \oplus \bar{D} &= \overline{A + B C} \bar{D} + (A + B C) \bar{\bar{D}} \\&= \bar{A} \bar{B} \bar{C} \bar{D} + A D + B C D \\&= \bar{A} (\bar{B} + \bar{C}) \bar{D} + A D + B C D \\&= \bar{A} \bar{B} \bar{D} + \bar{A} \bar{C} \bar{D} + A D + B C D\end{aligned}$$

If $A = 1$, then $0 + 0 + D + B C D = 1 \Rightarrow D = 1$ (1, *, *, 1)

If $A = 0$, then $\bar{B} \bar{D} + \bar{C} \bar{D} + B C D = 1$

If $B = 1$, then $0 + \bar{C} \bar{D} + C D = 1$

If $C = 1$, then $D = 1 \Rightarrow (0, 1, 1, 1)$

If $C = 0$, then $\bar{D} = 1 \Rightarrow (0, 1, 0, 0)$

If $B = 0$, then $\bar{D} + \bar{C} \bar{D} = 1 \Rightarrow (0, 0, *, 0)$

Therefore 8 ordered quadruples make it TRUE.

C. 8

5. Assembly Language

The program calculates $5! = 5*4*3*2*1 = 120$

D. 120