CONFIDENTIAL

Set β solutions

Academic Team

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February 8, 2023

Problem 1. Multiple Choiced Questions

1. Suppose the Earth's orbit has an eccentricity of 0.0167 and a semi-major axis of 1 AU. Further, suppose the Sun has a mass of 2×10^{30} kg. What is the ratio of angular momentum at perihelion and aphelion?

   (a) e
   (b) $\frac{1}{e}$
   (c) $\boxed{1}$
   (d) $\frac{e+1}{e-1}$

2. A double star is observed from Earth to have a period of 1.2 days. It turns out that the star system is moving away from Earth at a speed of 500km/s. The actual orbital period of the binary star is thus–

   (a) 1.198 day
   (b) $\boxed{1.2}$ day
   (c) 1.204 day
   (d) 2.408 days

3. In 1924, Edwin Hubble measured the distance to the Andromeda nebula, which firmly established it as a separate galaxy outside our own Milky Way. What did he measure to allow him to calculate the distance of Andromeda?

   (a) $\boxed{\text{Cepheid Variable}}$
   (b) Galactic redshift
   (c) Gamma-ray bursts
   (d) Rotational Doppler shift (Tully-Fisher relation)

4. How much a magnitude 2 star is brighter than a magnitude 6 star?

   (a) 27.71
   (b) $\boxed{39.81}$
   (c) 4.00
   (d) 12.67

5. Our Sun is mid-temperature G-2 type star. Arrange the following spectral classes from highest to lowest surface temperature:

   (a) $\boxed{\text{A,F,G,K,M}}$
   (b) F,A,G,M,K
   (c) A,F,K,G,M
   (d) K,A,F,M,G

Idea 1. For the **MCQ (5)** use the pnemonic: *Oh Be A Fine Girl, Kiss Me.* There is nothing chauvinistic about this because you are using this for educational purposes.

Problem 2. Dimmed Sun

Sun has been enclosed by a spherical gas cloud. As a result, its apparent magnitude is increased by $+1^m$.

(a) Find the change in flux received from the sun.

(b) Find the temperature of the gas cloud.

Solution 1. (a) Distance from Sun,

$$D = 1.5 \times 10^{11}\ m$$

Luminosity of Sun,

$$L = 3.8 \times 10^{26}\ W$$

Apparent flux on Earth,

$$F = \frac{L}{4\pi D^2} = 1343\ \frac{W}{m^2}$$

The apparent magnitude of Sun,

$$m_{sun} = -26.74$$

From here, considering $m_{sun} = m$ and the new magnitude to be m',

$$m' - m = -2.5 \log\left(\frac{F'}{F}\right)$$

Solving for F' gives,

$$F' = 534.7\ \frac{W}{m^2}$$

The change is thus,

$$\boxed{\Delta F = F' - F = 808.3 \frac{W}{m^2}}$$

(b) Start by Luminosity,

$$L = 4\pi R^2 \sigma T^4$$

Here, $R = d$,

$$\frac{L}{4\pi d^2} = \frac{4\pi R^2}{4\pi d^2}\sigma T^4$$

From here,

$$\Delta F = \sigma T^4$$

$$808.3 = \sigma T^4$$

From here,

$$\therefore T = \boxed{345.5\ K = 72^\circ\ C}$$

□

Problem 3. Astronomer Duo

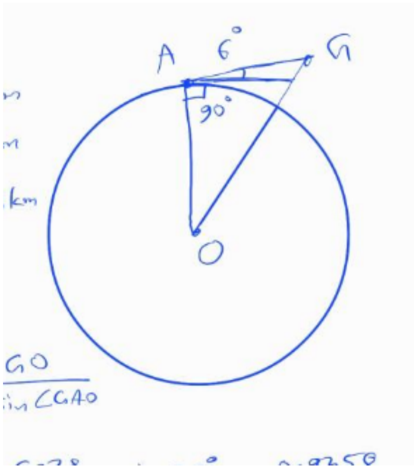
Raji and Turja are two friends living in two different cities within the same longitude but different latitudes. There is a satellite X which has a height of 400 km from the earth's surface. The latitude and longitude of Turja's city is 30° N and 30° E The coordinates of Paris is 49° N and 2° E

(a) When satellite X is at zenith in Turja's sky, Raji observes that the satellite has an azimuth 0° of and an altitude of 6°. What is the latitude of Raji?

(b) Determine the distance between Paris and Turja's city.

(c) It is given that the satellite also goes over Paris. When the satellite is at the zenith in Paris' sky, from Turja's location. Find the azimuth of the satellite.

Solution 2. (a) Using raw angle chasing and following the diagram (note that R_e is Earth Radius),



$$\angle GAO = 96^\circ$$
$$GO = R_e + 400\ km = 6777\ km$$
$$AO = R_e = 6378\ km$$

Using Sine Law,

$$\frac{AO}{\sin \angle AGO} = \frac{GO}{\sin \angle GAO}$$

We have to solve for $\angle AGO$,

$$\sin \angle AGO = \frac{6378}{6778} \times \sin 96^\circ = 0.9358$$

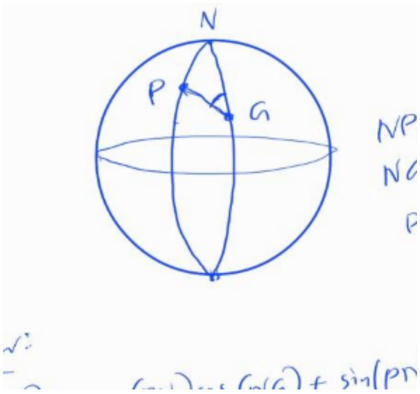
Using this,

$$\angle AGO = \sin^{-1}(0.9358) = 69.36^\circ$$
$$\therefore \angle AOG = 14.64^\circ$$

Latitude of Raji is,

$$30^\circ - 14.64^\circ = \boxed{15.36^\circ\ N}$$

(b) At first we will find the follwing angles from the diagram,



$$NP = 41^\circ$$
$$NG = 60^\circ$$
$$PNG = 28^\circ$$

We will crunch the cosine law for spheres here now,

$$\cos(PG) = \cos(PN)\cos(NG) + \sin(PN)\sin(NG)\cos(PNG) = 0.879$$

$$\therefore PG = 28.478^\circ$$

From there the distance, using the definition of nautical mile,

$$D = 28.47^\circ \times 60 \text{ Nautical Miles} = \boxed{3164.5\ km}$$

(c) From the solution (b),

$$PG = 28.478^\circ$$

Using the Sine Law,

$$\frac{\sin(\angle NGP)}{\sin(PN)} = \frac{\sin(\angle PNG)}{\sin(PG)}$$

Plugging in the values we found in the previous section and we get,

$$\angle NGP = 40.24^\circ$$

Hence the Azimuth is,

$$360^\circ - 40.24^\circ = \boxed{319.76^\circ}$$

Problem Author: Rafi Mahmud

□

Problem 4. Sending satellites to Mars
Mars and Earth revolves around the sun at a distance of 1.5 AU and 1 AU respectively. Artificial satellites were sent to mars for information transfer. But due to the great distance from earth, the communication is done between 2 quadrature of Mars.

(a) Find the phase angle of mars at quadrature

(b) How long can communication be done? Remember, satellites can be sent only between two quadratures

(c) When Mars was at eastern quadrature, a satellite was sent in a direction on a linear path so that it reaches mars when mars is on western quadrature. Ignore the gravity of sun on the satellite. What was the magnitude of velocity of the satellite with respect to earth?

Solution 3. (a) The phase angle of Mars during Quadrature is given by,

$$\boxed{\sin^{-1}\left(\frac{R_E}{R_M}\right) = 41.85^\circ}$$

(b) Period of Earth is 1 year. Call the time taken Y. Then, angular frequency of earth is $360^\circ/Y$. Period of Mars is $1.84Y$. This gives us the angular frequency to be $360^\circ/1.84Y = 195.65^\circ/Y$.

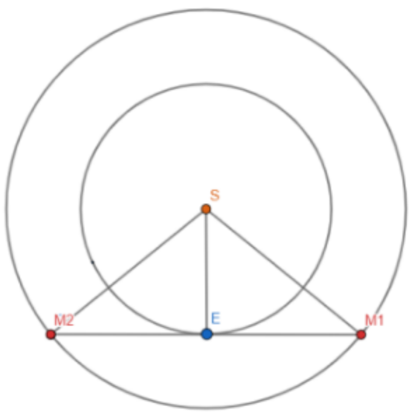
The difference of the frequency is the relative angular frequency, which is $164.35^\circ/Y$. Now we wil consider the Quadrature measurement. The angular distance between two quadrature is,

$$2\cos^{-1}\left(\frac{R_E}{R_M}\right) = 96.38^\circ$$

From here, using $\omega = \frac{\theta}{T}$,

$$T = \frac{96.38}{164.35} = 0.59Y = 215.35 \text{ Days}$$

(c) Please refer to the diagram.



Duration between Quadrature is $0.59Y$. In the figure, M_1 is the Eastern Quadrature and M_2 is the Western Quadrature. When Mars was at M_1, the satellite has to go to the EM_2 direction. We can calculate the EM_2 by,

$$EM_2 = \sqrt{1.5^2 - 1^2}$$

This gives us, $EM_2 = 1.12$ AU. The distance has to be covered within $0.59Y$, for which, the relative velocity is,

$$v = \frac{1.12 \text{ AU}}{0.59Y} = 8.97 \frac{km}{s}$$

The above velocity v is the velocity of satellite *relative to Sun*. Earth's velocity being $29.78 \frac{km}{s}$, the relative velocity will be $29.78 - 8.97 = 20.81$, in the direction of EM_1, or the opposite direction.

$$\boxed{v_{rel} \rightarrow \vec{v}_{rel} = 20.81 \frac{km}{s} \text{ along } \mathbf{EM_1}}$$

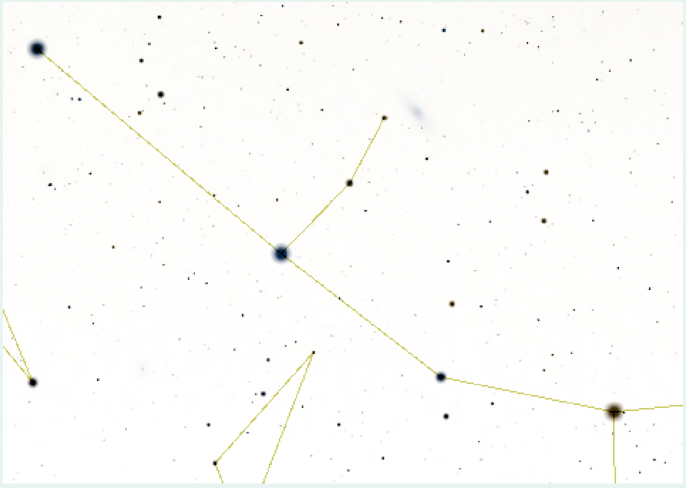
Problem Author: Arman Hassan

□

Idea 2. WARNING: A similar approach gives the answer $\approx 18 \frac{km}{s}$ with an acceptable range $\pm 2 \sim 3$. Answers near the value should be assumed to be correct

Observation Solution

Problem 5. (OBSERVATION 01): Name the galaxy in the following image.



(a) Triangulum galaxy

(b) $\boxed{\text{Andromeda Galaxy}}$

(c) Sombrero galaxy

(d) Milky way galaxy

Problem 6. (OBSERVATION 02): Name the famous asterism showed in the following image.

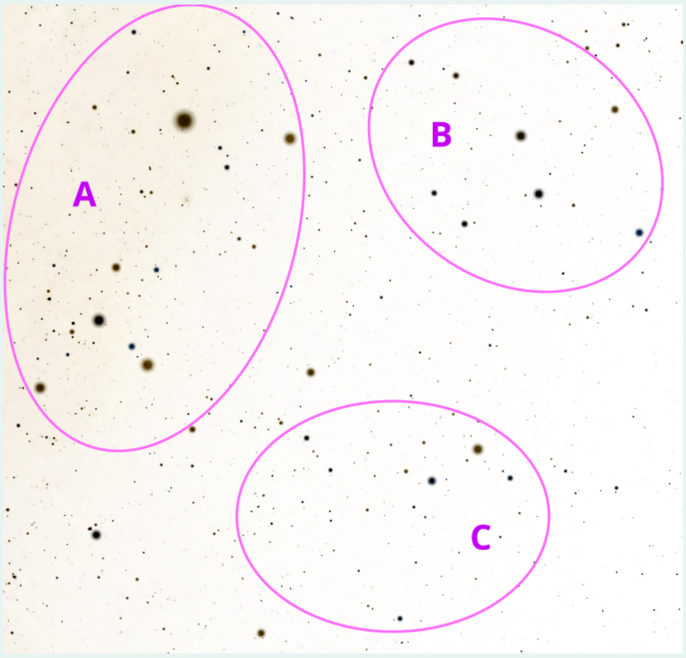
(a) Summer triangle

(b) Winter rhombus

(c) $\boxed{\text{Winter Hexagon}}$

(d) The great square

Problem 7. (OBSERVATION 03): Name the following constellations labeled as A, B, and C



A	*Canis Major*
B	*Lepus*
C	*Columba*