Bangladesh Olympiad on Astronomy and Astrophysics

FORMULA BOOK

Instructions

- THIS BOOK CONTAINS GENERAL FORMULAS USED DURING MATH PROBLEM SOLVING.
- IT'S AN EXCLUSIVE PREPARATION MATERIAL FOR BDOAA
- **DON'T** USE THIS BOOK FOR BUSINESS PURPOSE
-

Fahim Rajit Hossain
Academic Coordinator
BDOAA
farahoshwadhin.13@gmail.com

Md. Mahmudunnobe
Academic Head
BDOAA
mahmud.nobe@minerva.kgi.edu

1 Useful Constant

Table 1: Physical and orbital characteristics of selected bodies in the Solar System

Property	Sun ☉	Mercury ☿	Venus ♀	Earth ⊕	Moon ☾	Mars ♂	Jupier ♃	Saturn ♄	Uranus □	Neptune ♆
Mass m / Kg	1.989×10^{30}	3.302×10^{23}	4.868×10^{24}	5.972×10^{24}	7.348×10^{22}	6.419×10^{23}	1.899×10^{27}	5.685×10^{26}	$8.681 \times 10^{2}5$	1.024×10^{26}
Radius R / m	6.963×10^{8}	2.439×10^{6}	6.051×10^{6}	6.370×10^{6}	1.738×10^{6}	3.396×10^{6}	7.149×10^{7}	6.027×10^{7}	2.556×10^{7}	2.476×10^{7}
Orbital Semi-Major axis, a / m	-	5.791×10^{10}	1.82×10^{11}	1.496×10^{11}	3.843×10^{8}	2.279×10^{11}	7.785×10^{11}	1.433×10^{12}	2.877×10^{12}	4.503×10^{12}
Orbital period T	-	87.97 days	224.70 days	365.24 days	27.322 days (sidereal) 29.531 days (synodic)	683.97 days	11.86 years	29.46 years	84.32 years	164.79 years
Orbital Eccentricity ϵ	-	0.205	0.0067	0.0167	0.0549	0.0933	0.0488	0.0557	0.0444	0.0112

Table 2: Commonly used fundamental constants and unit denitions

Units and Physical Quantities	Universal Constants
1 *Astronomical Unit* (*AU*) = $1.495978770700 \times 10^{11} m$	Planck's Constant $h = 6.62606957 \times 10^{-34} m^2\ kg\ s^{-1}$
1 *Light year* (*ly*) = $c \times 1\ year = 9.4605284 \times 10^{15}\ m$	Reduced Planck's Constant $\hbar = \frac{h}{2\pi}$
1 *parsec* (*pc*) = $3.261663344\ ly$	Gravitational Constant $G = 6.67384 \times 10^{-11} N\ m^2\ kg^{-2}$
1 *electron − volt* (*eV*) = $1.60217657 \times 10^{-19}\ J$	Speed of Light $c = 2.99792458 \times 10^{8}\ m\ s^{-1}$
Avogadro's Number $N_A = 6.0221413 \times 10^{23}$	Boltzmann's Constant $k_B = 1.3806488 \times 10^{-23}\ J\ K^{-1}$
Average Solar Luminosity, $L_\odot = 3.846 \times 10^{26}\ W$	Coulomb Constant $k_e = \frac{1}{4\pi\epsilon_0} = 8.98755179 \times 10^{9}\ N\ m^2\ C^{-2}$
Average Solar Temperature, $T_\odot = 5778\ K$	Stefan-Boltzmann Constant $\sigma = 5.67 \times 10^{-8}\ W\ m^2\ K^{-4}$
Atomic mass unit, $u = 1.660539 \times 10^{-27}\ kg$	Electronic charge $q_e = 1.602 \times 10^{-19}\ C$
Proton Mass = $1.672622 \times 10^{-27}\ kg = 1.007276u$	Fine structure constant $\alpha = \frac{k_e (q_e)^2}{\hbar c} \approx \frac{1}{137}$
Neutron Mass = $1.674927 \times 10^{-27}\ kg = 1.008665u$	Wien's Displacement Constant $b = 2.89776829 \times 10^{-3}\ m\ K$
Electron Mass = $9.10938 \times 10^{-31}\ kg$	Hubble Constant $H_0 = 67.80 \pm 0.77\ km\ s^{-1}\ Mpc^{-1}$ (as of 03/13)

2 Useful Formula

Table 3: Mathematical Formulae

Description	Formula
Arc length on a circle is proportional to circular angle in radians	$s = r\theta$ (Give the circumference when $\theta = 2\pi$)
Law of sines	$\frac{sin\,A}{a} = \frac{sin\,B}{b} = \frac{sin\,C}{c} = 2R$ (on a plane) $\frac{sin\,A}{sina} = \frac{sin\,B}{sinb} = \frac{sin\,C}{sinc}$ (on a sphere)
Law of cosines	$c^2 = a^2 + b^2 - 2ab\ cosC$ (on a plane) $cos\ c = cos\,a\ cos\,b + sin\,a\ sin\,b\ cos\,C$ (on a sphere)
Small-angle approximations ($x \ll 1$, x in radians)	$sin\,x \approx x$ $cos\,x \approx 1 - \frac{x^2}{2}$ $tan\,x \approx x$
First-order binomial expansion	$(1+x)^y \approx 1 + xy$
Dot product of vectors	$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = \|\vec{a}\|\|\vec{b}\|\,cos\theta$ $(\vec{a} + \vec{b}) \cdot \vec{c} = \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c}$ $\|\vec{a}\| = \sqrt{\vec{a} \cdot \vec{a}}$
Cross product of vectors	$\|\vec{a} \times \vec{b}\| = \|\vec{a}\|\|\vec{b}\|\ sin\theta$ $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$ $(\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \times \vec{c})$ $= (\vec{b} \times \vec{c}) \cdot \vec{a}$ $(\vec{a} + \vec{b}) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$ $\widehat{x} \times \widehat{y} = \widehat{z}$

Table 4: Classical Astrophysics

Description	Formula
Kinetic Energy	$E_{kin} = \frac{1}{2}mv^2$
Newton's Universal Law of Gravitation	$\overrightarrow{F} = -\frac{Gm_1m_2}{r^2}\widehat{r}$
Gravitational Potential Energy	$E_{pot} = -\frac{Gm_1m_2}{r}$
Gravitational binding energy of a uniform sphere	$U = -\frac{3}{5}\frac{GM^2}{R}$
Roche Limit for a small, rigid body of density ρ_2 approaching a larger body of density ρ_1 and radius R	$d_{Roche} = 1.26R \times (\frac{\rho_1}{\rho_2})^{-\frac{1}{3}}$
Angular Velocity ω and angular momentum l	$v = r\omega;\ \omega = 2\pi f = \frac{2\pi}{T};\ l = I\omega = mr^2\omega$ (for orbiting bodies)
Centripetal acceleration and force	$a_c = \omega^2 r = \frac{v^2}{r};\ F_c = ma_c$
Kepler's 3rd Law	$T^2 = \frac{4\pi^2}{G(m_1 + m_2)}a^3$
Hydrostatic Equilibrium	$\frac{dP}{dR} = -\rho_r\frac{GM_r}{R^2}$
Quantisation of energy-momentum	$E = hf = \hbar\omega;\ p = \frac{h}{\lambda} = \hbar k$
Planck's Law for intensity per unit frequency	$I_f = \frac{2\pi hf^3}{c^2}\frac{1}{e^{\frac{hf}{kt}} - 1}$
Stefan-Boltzmann Law	$L = 4\pi R^2\sigma T^4$
Wien's Displacement Law	$\lambda_{max} = \frac{b}{T}$

Table 5: Relativistic Expressions

Description	Formula
Lorentz Factor	$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$
Velocity Addition	$u' = \frac{u + v}{1 + \frac{uv}{c^2}}$
Relativistic Doppler Effect	$f_{observed} = f_{source}\sqrt{\frac{c+v}{c-v}}$
Relativistic Redshift	$z = \sqrt{\frac{c+v}{c-v}} - 1 \approx \frac{v}{c}$
Schwarzschild Radius	$r_s = \frac{2GM}{c^2}$
Redshift	$z = \frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}}$

Table 6: Practical Astronomy

Description	Formula
Keplerian orbital ellipse as a function of angular deviation from periapsis	$r = \frac{a(1 - \epsilon^2)}{1 + \epsilon cos\phi}$
Orbital Eccentricity in terms of other parameters	$\epsilon = \frac{a - r_{periapsis}}{a} = \frac{r_{apoapsis} - a}{a} = \frac{r_a - r_p}{r_a + r_p}$
Rayleigh resolution criterion with aperture diameter D	$sin\Delta\phi_{min} = 1.220\frac{\lambda}{D}$
Rocket Equation	$\Delta v = v_{exh}\ log_e\frac{m_i}{m_j}$

Table 7: Distance Determination and Some Empirical Results

Description	Formula
Pogson's Law	$m_1 - m_2 = -2.5\, log(\frac{B_1}{B_2})$
Absolute Bolometric Magnitude	$M_{bol} = -2.5\, log\frac{L}{L_\odot} + 4.7554$
Distance modulus: difference between apparent and absolute magnitude	$m - M = 5\, log\frac{d}{10\, pc}$
Relationship between Luminosity and Absolute Magnitude	$\frac{L_1}{L_2} = 10^{\frac{M_2 - M_1}{2.5}}$
Determining distance d in parsecs using an ob-served parallax p in arcseconds	$D \approx \frac{1}{p}$
Color Indices	$B - V = M_B - M_V = m_B - m_V$ $= 2.5\, log(\frac{f(V)}{f(B)})$
Period-Luminosity relationship for Cepheid variable stars, with period P in days	$M = -2.76\, log_{10}\, P - 1.4$
Absolute magnitude of RR Lyrae stars	$M \sim 0.75$
Absolute magnitude of Type Ia supernovae (at peak)	$M \sim -19.3$
Tully-Fisher Relation	$L \propto V^4$
Mass-Luminosity Relation for Main Sequence stars	$L \propto M^{3.5}$
Hubble's Law	$v = H_0 d$