LLMs and Economics Research

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A simple model of the economics of LLMs

Let's start with a simple model of LLMs (so fresh we don't even have a draft for you!)

Then let's see how that model maps to current LLM capabilities

And finally, best research use cases given all

Basic Model of LLM Decision-Making

- N: Number of possible solutions
- c_0 : Cost for a human to verify if a solution is correct
- C: Cost for a human to write a proof
- Payoffs:
 - Correct choice: H
 - Incorrect choice: L
- Assumption:

$$c_0 \leq C \leq rac{N}{2}c_0$$

Proving is harder than verifying one guess but easier than verifying all guesses.

Statistical and Generative Decision-Making

- c_1 : Cost to use the Statistical Model S
- c_2 : Cost to use the Generative Model G
- Statistical Model (S):
 - Accuracy (p): Fraction of boxes for which S can reveal whether they are H or L.
 - ullet The boxes S can investigate are an **ex-ante known subset** of all ideas.
- Generative Model (G):
 - Accuracy (q): Probability that G correctly predicts the box containing H.
 - Assumption: q > p.
- Outside Option: Payoff of zero (doing nothing).

Manual Search

Value Function - Construct a Proof:

$$V_{\text{Manual, A}} = H - C$$

A proof is constructed at a one-time cost C.

Value Function - Sequential Search:

$$V_{ ext{Manual, B}} = H - rac{N+1}{2} c_0$$

Sequential search continues until H is found, with the expected cost decreasing as boxes are searched.

Optimal Behavior

- 1. Construct a proof when $C < rac{N+1}{2}c_0$ and C < H.
- 2. Sequentially search when $C>rac{N+1}{2}c_0$ and $rac{N+1}{2}c_0< H$.

Otherwise, do nothing and earn 0.

Total Payoff with Statistical Model

The total expected payoff when using the statistical model (cost c_1) is:

$$V_{
m Optimal} = p(H-c_1) + (1-p)V_{
m Continuation},$$

where $V_{\text{Continuation}}$ is:

$$\max \left\{ H-C,\, H-rac{(1-p)N+1}{2}c_0,\, 0
ight\}.$$

Strategy Insight

If c_1 is low enough, the optimal strategy is to **first use the statistical search**. Then, upon failure, **manually search** with a higher chance of verification compared to skipping the statistical search.

Total Payoff with Generative Model ${\cal G}$

The total expected payoff when using the generative model (cost c_2) and verifying its prediction is:

$$V_{G,\mathrm{verify}} = q(H-c_2-c_0) + (1-q)V_{G,\mathrm{verify, fail}},$$

where $V_{G,\mathrm{verify,\,fail}}$ is the optimal continuation payoff after failure, given by:

$$\max \left\{ L - c_2 - c_0, H - c_2 - c_0 - C, H - c_2 - \frac{N}{2}c_0, 0 \right\}.$$

Insights

- 1. If L (loss from an incorrect choice) is **sufficiently high**, it is always optimal to verify the prediction.
- 2. If the **verification cost** c_0 is too high, you **do not use the generative model at all**. Instead, you switch to either a **statistical model** (if its cost is sufficiently low) or proceed with **pure manual work** (constructing a proof or sequential search).
- 3. The continuation options balance the trade-offs among verification, proof construction, or sequential search to maximize the payoff.

Summing up:

Generative AI most useful when either verification (c_0) is cheap or inaccuracy (H-L) is not too costly

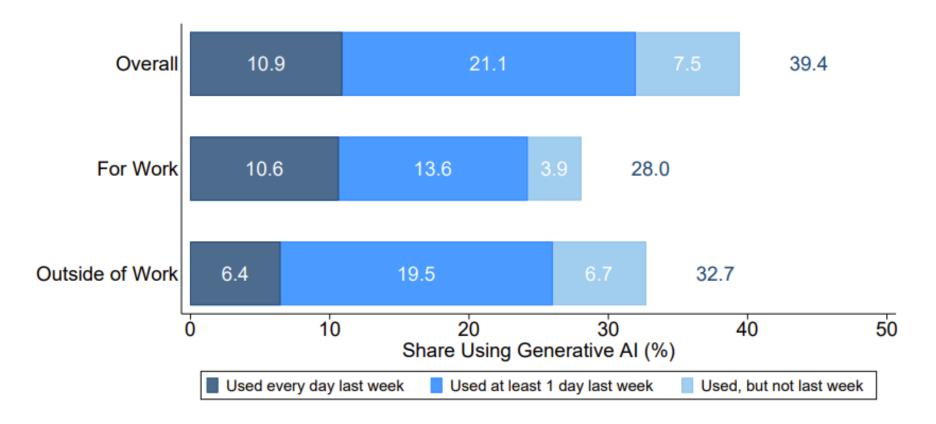
Only some problems have these characteristics!

Summing up:

Generative AI most useful when either verification (c_0) is cheap or inaccuracy (H-L) is not too costly

O-Ring problems, agentic tasks, adversarial tasks: cost of inaccuracy higher, only use when verification is very cheap

Figure 2: Share of Working Age Adults Using Generative AI



Notes: The figure shows the share of respondents who use AI for work, outside of work, and overall (either for work or outside of work). Intensity of use is broken down into every day last week (dark blue), at least one day but not every day last week (medium blue), and not last week (light blue). Data source is the August 2024 wave of the RPS, ages 18-64. The "For Work" sample is employed individuals (N = 3216); the other bars include all respondents (N = 4682).

How do they work?

Take a huge corpus of written language.

Predict the distribution of next token (word).

(Ok, technically it uses multibeam search to predict next many words)

Add a little bit of noise to not always select the top option. *This actually works ok!*

"The Jagged Frontier"

explain the nature of quantum entanglement using formal math but using language that is accessible to a smart high schooler

You likely do not know what tasks AI is good or bad at.



Quantum entanglement is a phenomenon where two or more particles become linked in such a way that the state of one particle directly influences the state of the other, no matter the distance between them. To understand this formally, we'll use some basic concepts from linear algebra and probability, focusing on vectors, matrices, and complex numbers.

1. Quantum States and Qubits

In quantum mechanics, the state of a single particle, or **qubit**, can be represented as a **vector** in a two-dimensional complex vector space. We denote the two possible states of a qubit as:

$$|0
angle = egin{pmatrix} 1 \ 0 \end{pmatrix}, \quad |1
angle = egin{pmatrix} 0 \ 1 \end{pmatrix}$$

These vectors are called **basis states**. A general quantum state for a qubit can be any linear combination (or **superposition**) of these basis states, written as:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where α and β are complex numbers that satisfy $|\alpha|^2+|\beta|^2=1$, ensuring that the total probability is 1.

"The Jagged Frontier"

You likely do not know what tasks AI is good or bad at.

ChatGPT 4o V

is 9.9 or 9.11 bigger?



9.11 is bigger than 9.9.

how many r's are there in strawberry



There are two "r"s in "strawberry."

Where Al is most useful right now

Brainstorming

Cost of verification much lower than creation

One shot: Claude 3.5 Sonnet (new), GPT-4o. Involving deductions: GPT o1

Document search (e.g., NotebookLM) Cost of inaccuracy reasonably low

how many companies participated and how were they selected?

Selection and Participation of Startups

- A total of **26 early-stage**, **actively-hiring startups** participated in the primary RCT, focused on a custom job board, out of 183 firms contacted from a leading science-based entrepreneurship program (SEP). 1 2
- The 183 firms contacted were part of the SEP cohort from a particular location. (2)
- Firms self-selected into the RCT by responding to an email asking if they wished to participate. 2
- The email mentioned that experimentation may occur but did not provide specifics. (2)
- A key motivation for SEP to participate was the belief that their best firms faced hiring difficulties, possibly due to challenges for workers in assessing firm quality. 3
- The 26 participating firms represented 14% of the contacted firms. 2
- Analysis suggests that participating firms are broadly representative of SEP firms. 2
- The relatively low participation rate (14%) is attributed to the fact that many startups are not actively hiring at any given time.



And if you know how to code? (Cursor)

Cost of verification relatively low

```
what does this function do?
  ~- ∉ quick question
def detect_language(text):
   try:
       # Detect the Language
       language = detect(text)
 what does this function do?
                         Follow-up instructions... Ctrl+Shift+K
  Ctrl+⟨∑ Reject
   Detects the language of input text and returns both the full language name and confidence probabilities.
   Args:
        text (str): The text to detect the language of
    Returns:
        tuple: (str, list) containing:
            - Full language name (e.g. "English", "Spanish", etc)
            - List of language detection probabilities
```

Summarize an Econ Paper

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Entrepreneurial Migration

Kevin A. Bryan, Jorge Guzman

Research Question: What factors influence the migration patterns of high-potential startups across U.S. cities?

Why Do We Care: Understanding entrepreneurial migration is essential for policymakers and economists as it impacts regional economic growth and productivity. Startups are pivotal for job creation and innovation, but their tendency to migrate can lead to geographic imbalances in economic activity and resource allocation. Identifying the factors that attract or repel startups helps in crafting effective urban policies and improving economic ecosystems that promote entrepreneurial activities.

What Did We Do: Using a dataset covering over 400,000 high-potential startups registered in Delaware between 1988 and 2014, we analyzed cross-state migration patterns utilizing revealed preference methods. We identified 6.6% of startups that migrated across state borders within their first five years. The study applied econometric techniques to capture the average utility of various cities for startups, considering multiple socioeconomic variables and controlling for biases in startup movement data.

What Did We Find: The analysis revealed that startup hubs like Silicon Valley and Boston lose more startups than they gain, with Dallas, Phoenix, and Austin being the most attractive for migrating startups. The probability of migration decreased with the age of the firm, with younger startups (0-2 years) more likely to move to ecosystems rich in resources, while older firms (3-5 years) preferred areas with lower taxes. Our revealed preference estimates showed that the utility of cities for moving startups significantly diverged from the utility associated with startup births, as cities with higher births did not necessarily attract movers.

Empirical

PRIMARY DATA

- Dataset of 400,000 high-potential startups registered in Delaware from 1988 to 2014 across 36 jurisdictions (35 states plus D.C.)
- Migration rate of 6.6% within five years for startups born during this period, based on business registration records.

ASSUMPTIONS

- Startups' migration decisions are influenced by both pecuniary and non-pecuniary factors.
- The revealed preference model assumes that firms choose locations based on immediate expected utility and face a fixed cost of moving.
- Idiosyncratic factors do not correlate across cities, ensuring unique utility perceptions for each city.

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The Allocation of Decision Authority to Human and Artificial Intelligence

Susan C. Athey, Kevin A. Bryan, Joshua S. Gans

Research Question: How does the allocation of decision authority between humans and AI affect human effort and organizational outcomes?

Why Do We Care: Understanding the dynamics of decision authority between human agents and AI is crucial as AI adoption often influences human behavior in the workplace. This interaction shapes how decisions are made, especially in settings where human judgment and experience play significant roles alongside AI capabilities. A deeper analysis of these interactions can help organizations optimize their use of AI, enhancing productivity while mitigating potential issues of reduced human engagement and effort.

What Did We Do: Employing a theoretical model based on the framework developed by previous researchers, we analyzed the decision authority allocation between a principal overseeing human and AI agents. The model incorporated parameters such as human effort, AI ability, and the effectiveness of each decision-making party in real-world scenarios, with key variables defined as payoff structures, agent effort levels, and probabilities of learning outcomes.

What Did We Find: Proposition 1 establishes that a principal will always prefer an AI with higher prediction ability if certain conditions regarding parameter alignment are met, specifically if performance is low or if human effort responsiveness is limited. Additionally, it was found that human effort decreases as AI performance improves, suggesting a strategic substitution effect. The further implication indicates that a principal might actually choose a less capable AI to sustain higher effort levels from human agents, highlighting the complex trade-offs in decision authority allocation.

Theory

PRIMARY DATA

· No primary empirical data.

ASSUMPTIONS

- The principal and agents have clearly defined payoffs related to project selection.
- The performance of the AI and the human agent is governed by congruence parameters that lie within a specified range.
- Human effort is assumed to be responsive to decision authority, impacting learning outcomes in project payoffs.

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Classification tasks

Cost of inaccuracy reasonably low

βarr2-GFP trafficking was visualized in transfected HEK-293 cells on a heated (37° C.) microscope stage as described in Barak, L. S., Ferguson, S. S., Zhang, J., and Caron, M. G. (1997) J. Biol. Chem., 272:27497-27500. Images were collected sequentially using single line excitation (488 nm) with a Zeiss laser scanning confocal microscope (LSM-510). For experiments assessing βarr2-GFP trafficking after agonist removal, cells were washed as described above to remove agonist and returned to a 37° C. incubator for 60 min. Colocalization of βarr2-GFP with rhodamine-labeled receptors was performed on transfected cells pre-incubated in serum-free medium containing a rhodamine-conjugated anti-HA 12CA5 mouse monoclonal antibody (1:100) for 45 min. at 37° C. Cells were then washed three times with serum-free medium, treated with the appropriate agonist at 37° C. for 30 min, and imaged by confocal microscopy. βarr2-GFP and rhodamine-labeled receptor fluorescence were performed using dual excitation (488, 568 nm) and emission (515–540 nm, GFP: 590–610 nm, rhodamine) filter sets.

We used 495 tokens

Tool/technique: The inventor cited the article by Barak et al. as a tool or tech nique to help create the invention. The article describes a method for visualizing βarr2-GFP trafficking in transfected HEK-293 cells using a heated microscope stage and a laser scanning confocal microscope. The inventor likely used this me thod to visualize the trafficking of βarr2-GFP in their own experiments and to confirm the effectiveness of their invention.

We used 485 tokens

Tool/technique: The inventor cited the article by Barak et al. as a tool or tech nique to help create the invention. The article describes a method for visualizing βarr2-GFP trafficking in transfected HEK-293 cells using a heated microscope stage and a laser scanning confocal microscope. The inventor likely used this me thod to visualize the trafficking of βarr2-GFP in their own experiments and to help develop their invention.

We used 724 tokens

Tool/technique. The Barak et al. article describes the methodology used to visua lize βarr2-GFP trafficking in transfected HEK-293 cells, which is a technique us ed to create the invention.

Document cleanup Cost of inaccuracy reasonably low



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Figure 1

Name: Asger Aaboe

Education: PhD in history of mathematics

from Brown University

Work Experience:

Visiting Lecturer of Math at Washington
University (1947, 1948)

University (1947-1948)

Adjunct Professor at Birkerod State School in Denmark (1948-1952)

•••

Memberships: Royal Danish Society History of Mathematics and Mathematical Astronomy

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- 1) If p_i are independent and search is simultaneous, optimal search opens all boxes p_j such that $\prod_{i:i< j} (1-p_i)p_jR \geq c$, where boxes are ordered by size. We therefore find the prize with probability $1-\prod_{i\leq j*}(1-p_i)$ where j* is the smallest box j such that $\prod_{i:i< j}(1-p_i)p_jR \geq c$. This is a direct implication of the marginal improvement algorithm in Chade and Smith [2006].⁶
- 2) If p_i are independent and search is sequential, optimal search opens boxes in order of p_i until the prize is found, or until there exists no box such that $p_i R \geq c$. We therefore find the prize with probability $1 \prod_{i:p_i R \geq c} (1 p_i)$ (Weitzman [1979]). Note that sequential search opens weakly more boxes than simultaneous search.

Consider rst three results which are either known in the literature, or straightforward implications of known results.

1) If pi are independent and search is simultaneous, optimal search opens all

boxes pj such that Q

i:i<j $(1-pi)pjR \ge c$, where boxes are ordered by size. We therefore nd the prize with probability 1-

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i≤j∗

(1 - pi) where j* is the smallest box j such

that Q

i:i<j $(1 - pi)pjR \ge c$. This is a direct implication of the marginal improvement

algorithm in Chade and Smith [2006].6

2) If pi are independent and search is sequential, optimal search opens boxes in

order of pi until the prize is found, or until there exists no box such that piR ≥ c. We

therefore nd the prize with probability 1 -

Q

i:piR≥c

(1 – pi) (Weitzman [1979]).7 Note

that sequential search opens weakly more boxes than simultaneous search.

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If \$p_i\$ are independent and search is sequential, optimal search opens boxes in order of \$p_i\$ until the prize is found, or until there exists no box such that \$p_iR \geq c\$. We therefore find the prize with probability \$1 - \prod_{i:p_iR\geq c}(1 - p_i)\$ (Weitzman [1979]). Note that sequential search opens weakly more boxes than simultaneous search.

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Ok…let's just go completely crazy here

Ready?

- 1) If p_i are independent and search is simultaneous, optimal search opens all boxes p_j such that $\prod_{i:i< j} (1-p_i)p_jR \geq c$, where boxes are ordered by size. We therefore find the prize with probability $1-\prod_{i\leq j*}(1-p_i)$ where j* is the smallest box j such that $\prod_{i:i< j}(1-p_i)p_jR \geq c$. This is a direct implication of the marginal improvement algorithm in Chade and Smith [2006].
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Ok…let's just go completely crazy here

Let's just take a blurry picture of this text and use a modern multimodal model...

- 1) If p_i are independent and search is simultaneous, optimal search opens all boxes p_j such that $\prod_{i:i < j} (1-p_i)p_j R \ge c$, where boxes are ordered by size. We therefore find the prize with probability $1 \prod_{i \le j*} (1-p_i)$ where j* is the smallest box j such that $\prod_{i:i < j} (1-p_i)p_j R \ge c$. This is a direct implication of the marginal improvement algorithm in Chade and Smith [2006].
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- 2) If \$p_i\$ are independent and search is \textit{sequential}, optimal search opens boxes in order of \$p_i\$ until the prize is found, or until there exists no box such that \$p_iR \geq c\$. We therefore find the prize with probability \$1 \prod {i:p_iR\geq c} (1 p_i)\$ (Weitzman [1979])\footnote{Weitzman, M. L. (1979). Optimal search for the best alternative. \textit{Econometrica}, 47(3), 641-654.}. Note that sequential search opens weakly more boxes than simultaneous search.

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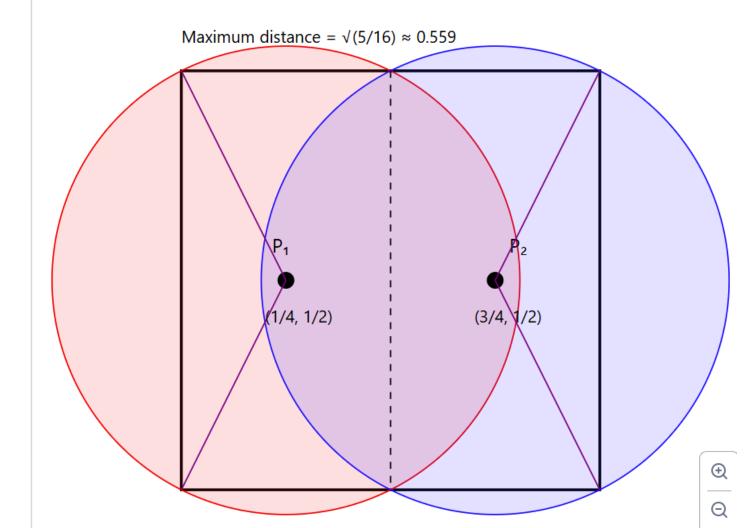
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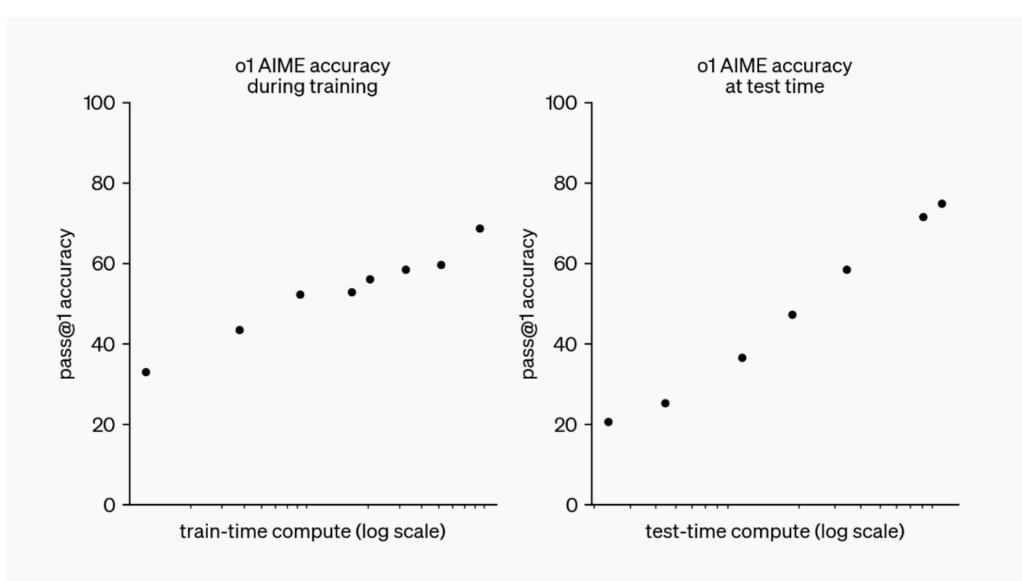
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Mathematical derivation improving quickly (o1-pro is best but expensive; Gemini 2.0 and Claude 3.5 Sonnet (new) are good)

your problem: choose two points in or on the unit square such that the maximal distance from any point in or on the unit square to the nearest of your chosen two points is minimized. Prove it. Draw a picture of what the solution looks like.



Looking forward



o1 performance smoothly improves with both train-time and test-time compute

A simple model of the economics of LLMs

Generative AI: more accurate prediction on wider set of tasks, but under 'jagged frontier' where hard to know when accurate

Rapidly improving, so moving target

Use when *inaccuracy* less costly or *verification* cheap relative to production