Board questions set 3

Problem 1: Discrete Distributions

X	1	3	5	7
$F(X \leq x)$	0.5	0.75	0.9	1

(a) What is $P(X \leq 3)$?

(b) What is $P(X = 3)$?

Problem 2: Expectations

(a) Would you accept a gamble that offers a 10% chance to win €95 and a 90% chance of losing €5?

(b) Would you pay €5 to participate in a lottery that offers a 10% chance to win €100 and a 90% chance to win nothing?

Problem 3: Memorylessness

Assume that $X \sim \text{Geometric}(p)$. Show that the geometric distribution is memoryless (or stationary), i.e. show that

$$P(X = n + k | X \geq n) = P(X = k)$$

where $n, k > 0$.

Problem 4: Variance

X	1	2	3	4	5
$P(X = x)$	0.1	0.2	0.4	0.2	0.1

(a) Compute the variance and standard deviation $\sigma(X)$ of X.

(b) What are the variance and standard deviation of $\frac{X}{\sigma(X)}$?