Board questions set 6

Problem 1: Geometric EM

You are given a mixture model with mixture components c_1 and c_2 which are linked to geometric distributions with parameters

$$\theta_{c_1}^{(0)} = 0.2, \qquad \text{and} \qquad \theta_{c_2}^{(0)} = 0.6. \tag{1}$$

Both geometric distributions are of the form $p(X = k) = \theta(1-\theta)^k$. Assume that the latent variables Y are identically and independently distributed and that $P(Y = c_1|\Theta = \theta^{(0)}) = 0.2$. You observe the data set

$$x_1^4 = (0, 2, 2, 3).$$

(a) What is the (marginal) log-likelihood of this data set under the model? Feel free to use calculators. Also, it might be helpful to collect all relevant probabilities in a table.

(b) Find the most likely mixture component for each data point.

(c) Perform one EM iteration.

(d) Compute the marginal log-likelihood of the data with the updated parameters. The new value should be higher than the one computed in the beginning.