**BASIC PROBABILITY: THEORY**
Master of Logic, University of Amsterdam, 2018 — 
TEACHERS Alexandre Cremers and Jakub Dotlačil TA Bas Cornelissen

# Board questions set 4

## Problem 1: Variances

**(a)** Prove that if $X \sim \text{Bernoulli}(p)$, then $\text{Var}(X) = p(1-p)$.

**(b)** Prove that if $X \sim \text{Bin}(n,p)$, then $\text{Var}(X) = np(1-p)$.

**(c)** Suppose $X_1, X_2, \ldots, X_n$ are independent and all have the same standard deviation $\sigma$. Let $\overline{X}$ be the average of $X_1, X_2, \ldots, X_n$. What is the standard deviation of $\overline{X}$? What does this mean?

## Problem 2: Covariance

**(a)** Flip a fair coin 3 times. Let $X$ be the number of heads in the first 2 flips and let $Y$ be the number of heads in the last 2 flips. Give a table describing the joint distribution of $X$ and $Y$ and directly compute $\text{Cov}(X,Y)$.

**(b)** Let $X_1, X_2, X_3$ be the results of the three fair coin flips and let $X$ and $Y$ as before. Compute $\text{Cov}(X,Y)$ without first using the joint distribution.

## Problem 3: More covariance

Toss a fair coin $2n+t$ times. Let $X$ be number of heads in the first $n+t$ flips and let $Y$ be number of heads in the last $n+t$ flips. Compute $\text{Cov}(X,Y)$ and $\text{Cor}(X,Y)$.

CREDITS Adapted from MIT course 18-05 by Jeremy Orloff and Jonathan Bloom, see ocw.mit.edu

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