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Board questions set 4

Problem 1: Variances

(a) Prove that if $X \sim \text{Bernoulli}(p)$, then $\text{Var}(X) = p(1-p)$.

(b) Prove that if $X \sim \text{Bin}(n, p)$, then $\text{Var}(X) = np(1-p)$.

(c) Suppose $X_1, X_2, \ldots, X_n$ are independent and all have the same standard deviation σ. Let $\overline{X}$ be the average of $X_1, X_2, \ldots, X_n$. What is the standard deviation of $\overline{X}$? What does this mean?

Problem 2: Covariance

(a) Flip a fair coin 3 times. Let X be the number of heads in the first 2 flips and let Y be the number of heads in the last 2 flips. Give a table describing the joint distribution of X and Y and directly compute $\text{Cov}(X, Y)$.

(b) Let X_1, X_2, X_3 be the results of the three fair coin flips and let X and Y as before. Compute $\text{Cov}(X, Y)$ without first using the joint distribution.

Problem 3: More covariance

Toss a fair coin $2n+t$ times. Let X be number of heads in the first $n+t$ flips and let Y be number of heads in the last $n+t$ flips. Compute $\text{Cov}(X, Y)$ and $\text{Cor}(X, Y)$.

CREDITS Adapted from MIT course 18-05 by Jeremy Orloff and Jonathan Bloom, see ocw.mit.edu