BASIC PROBABILITY: THEORY
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TEACHERS Alexandre Cremers and Jakub Dotlačil TA Bas Cornelissen

Board questions set 5

Problem 1: Sufficient Statistics

You are given a data set $x = x_1^n$ of n independent, geometrically distributed observations, so $p(x_i) = \theta(1-\theta)^{x_i}$. Show that $\sum_{i=1}^n x_i$ is a sufficient statistic for the geometric distribution.

Problem 2: Covariance

A coin is taken from a box containing three coins, which give heads with probability $p = 1/3, 1/2$, and $2/3$. The mysterious coin is tossed 80 times, resulting in 49 heads and 31 tails.

(a) What is the likelihood of this data for each type of coin and which coin gives the maximum likelihood?

(b) Now suppose that we have a single coin with unknown probability p of landing heads. Find the likelihood and log likelihood functions given the same data. What is the maximum likelihood estimate for p?

Problem 3: Dice

There are five fair dice each with a different number of sides: 4,6,8,12,20. Jon picks one of them uniformly at random rolls it and reports a 13.

(a) Compute the posterior probability for each die to have generated this outcome.

(b) Compute the posterior probabilities if the result had been a 5 instead. *Hint: Drawing a table may help here. And please do use a calculator!*

Problem 4: Geometric maximum likelihood estimator

Recall that if Y is geometrically distributed, then $P(Y = y) = (1-\theta)^y\theta$. You are given a set of independent, geometrically distributed observations $x = x_1^n$. Find the maximum likelihood estimator of θ.

CREDITS Adapted from MIT course 18-05 by Jeremy Orloff and Jonathan Bloom, see ocw.mit.edu

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