

BASIC PROBABILITY: THEORY

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Practice problem set 3

This week's exercises deal with discrete random variables. You do not have to hand these exercises in; they are optional and for practicing only. If you have questions about them, please post them to the [discussion forum](#) and try to help each other. We will also keep an eye on that.

Problem 1

Let X be a random variable taking any of the values $0, 5, -5$ with probabilities $P(X = -5) = P(X = 0) = 0.3$ and $P(X = 5) = 0.4$. What is $E[X^2]$?

Problem 2

Compute the pmf of the number of tails for 7 subsequent coin tosses.

Problem 3

Calculate the variance of a loaded 6-sided die that has a probability of $\frac{1}{6}$ for all odd numbered sides and $P(\{2\}) = P(\{4\}) = 0.1$ and $P(\{6\}) = 0.3$.

Problem 4: Independence

Three events A, B , and C are pairwise independent if each pair is independent. They are mutually independent if they are pairwise independent and in addition

$$P(A \cap B \cap C) = P(A)P(B)P(C). \quad (1)$$

- (a) Suppose we roll two 6-sided dice. Consider the events:

$$D = \text{'odd on die 1'} \quad E = \text{'odd on die 2'} \quad F = \text{'odd sum'}$$

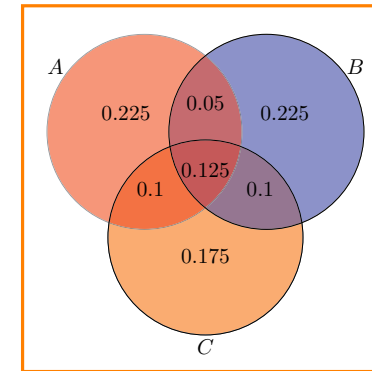


Figure 1: Venn diagram for problem 4 (b)

Are D, E , and F pairwise independent? Are they mutually independent?

- (b) Consider the Venn diagram in figure 1. A, B and C are the overlapping circles and the probabilities of each region are as marked. Does equation 1 hold? Are the events A, B, C mutually independent?
- (c) For families with n children, the events “the family has children of both sexes” and “there is at most one girl” are independent. What is n ?

Problem 5: Trees of cards

There are 8 cards in a hat:

$$\{1\heartsuit, 1\spadesuit, 1\diamondsuit, 1\clubsuit, 2\heartsuit, 2\spadesuit, 2\diamondsuit, 2\clubsuit\}.$$

You draw one card at random. If its rank is 1 you draw one more card; if its rank is two you draw two more cards. Let X be the sum of the ranks on all the cards you have drawn. Find $E(X)$.

Problem 6: Seating arrangements

A total of n people take their seats around a circular table with n chairs. No two people have the same height. What is the expected number of

people who are shorter than both of their immediate neighbours?

Problem 7: Negative binomial distribution

Assume that a number of independent trials, each with a probability of success of p , $0 < p < 1$, are performed until q successes are registered. Let X be equal to the number of trials required, then

$$P(X = n) = \binom{n-1}{q-1} p^q (1-p)^{n-q} \quad n = q, q+1, \dots$$

Any RV X whose probability distribution is given by the above is said to be a *negative binomial RV* with parameters (q, p) . Compute the expectation and variance of this RV with parameters (q, p) .