BASIC PROBABILITY: THEORY

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Board questions set 4

Problem 1: Variances

- (a) Prove that if $X \sim \text{Bernoulli}(p)$, then Var(X) = p(1-p).
- **(b)** Prove that if $X \sim \text{Bin}(n, p)$, then Var(X) = np(1 p).
- (c) Suppose X_1, X_2, \ldots, X_n are independent and all have the same standard deviation $\sigma = 2$. Let \overline{X} be the average of X_1, X_2, \ldots, X_n . What is the standard deviation of \overline{X} ?

Problem 2: Covariance

Flip a fair coin 3 times. Let X be the number of heads in the first 2 flips and let Y be the number of heads in the last 2 flips. Compute Cov(X, Y).

Problem 3: More covariance

Toss a fair coin 2n+t times. Let X be number of heads in the first n+t flips and let Y be number of heads in the last n+t flips. Compute Cov(X,Y) and Cor(X,Y).