

# Board questions set 5

## Problem 1: Sufficient Statistics

You are given a data set  $x = x_1^n$  of  $n$  independent, geometrically distributed observations. Show that  $\sum_{i=1}^n x_i$  is a sufficient statistic for the geometric distribution.

## Problem 2: Covariance

A coin is taken from a box containing three coins, which give heads with probability  $p = 1/3, 1/2$ , and  $2/3$ . The mysterious coin is tossed 80 times, resulting in 49 heads and 31 tails.

- (a) What is the likelihood of this data for each type of coin and which coin gives the maximum likelihood?
- (b) Now suppose that we have a single coin with unknown probability  $p$  of landing heads. Find the likelihood and log likelihood functions given the same data. What is the maximum likelihood estimate for  $p$ ?

## Problem 3: Dice

There are five fair dice each with a different number of sides: 4,6,8,12,20. Jon picks one of them uniformly at random rolls it and reports a 13.

- (a) Compute the posterior probability for each die to have generated this outcome.
- (b) Compute the posterior probabilities if the result had been a 5 instead.  
*Hint: Drawing a table may help here. And please do use a calculator!*

## Problem 4: Geometric maximum likelihood estimator

Recall that if  $Y$  is geometrically distributed, then  $P(Y = y) = (1 - \theta)^y \theta$ . You are given a set of independent, geometrically distributed observations  $x = x_1^n$ . Find the maximum likelihood estimator of  $\theta$ .