

Board questions set 4

Problem 1: Variances

- (a) Prove that if $X \sim \text{Bernoulli}(p)$, then $\text{Var}(X) = p(1 - p)$.
- (b) Prove that if $X \sim \text{Bin}(n, p)$, then $\text{Var}(X) = np(1 - p)$.
- (c) Suppose X_1, X_2, \dots, X_n are independent and all have the same standard deviation $\sigma = 2$. Let \bar{X} be the average of X_1, X_2, \dots, X_n . What is the standard deviation of \bar{X} ?

Problem 2: Covariance

Flip a fair coin 3 times. Let X be the number of heads in the first 2 flips and let Y be the number of heads in the last 2 flips. Compute $\text{Cov}(X, Y)$.

Problem 3: More covariance

Toss a fair coin $2n + t$ times. Let X be number of heads in the first $n + t$ flips and let Y be number of heads in the last $n + t$ flips. Compute $\text{Cov}(X, Y)$ and $\text{Cor}(X, Y)$.