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Practice problem set 1

This week's exercises deal with sets, counting and uniform probabilities. You do not have to hand these exercises in; they are optional and for practicing only. If you have questions about them, please post them to the discussion forum and try to help each other. We will also keep an eye on that.

Problem 1

There are 11 students in a class: 4 boys and 7 girls. We need to form a group of 5 people.

(a) How many different groups can you make?

(b) How many different groups are possible with at least 4 girls?

(c) If you pick the group (uniformly) at random, what is the probability that there are at least 3 boys in the group?

Problem 2: Words

(a) How many 'words' of length 5 can you make using each letter of the alphabet at most once?

(b) And how many if the order of the letters is irrelevant? (I.e., if we treat 'words' and 'sword' as the same word.)

(c) And how many words can you make if you can use every letter as many times as you like?

(d) In how many unique ways can the letters in the word 'error' be arranged?

(e) Consider a word of n letters in which two letters occur more than once: p and q times respectively. How many unique 'words' of the same length can you make of the n letters?

Problem 3: Books

You have 3 books on complexity theory, 2 on probability theory, and 1 novel.

(a) In how many ways can the books be arranged?

(b) And what if the books on complexity theory must be together but the other books can be arranged in any order?

Problem 4: Poker hands

Calculate the probability of drawing each of these poker hands.

(a) **Two-pair** Two cards have one rank, two cards have another rank, and the remaining card has a third rank. Example: two 2's, two 5's and a king.

(b) **Three-of-a-kind** Three cards have one rank and the remaining two cards have two other ranks. Example: three 2's, a five and a king.

CREDITS Some questions are from MIT course 18-05 by Jeremy Orloff and Jonathan Bloom, see ocw.mit.edu.