BASIC PROBABILITY: THEORY

Master of Logic, University of Amsterdam, 2018 — LICENSE CC BY-NC-SA 4.0 TEACHERS Alexandre Cremers and Jakub Dotlačil TA Bas Cornelissen

Board questions set 5

Problem 1: Sufficient Statistics

You are given a data set $x = x_1^n$ of n independent, geometrically distributed observations. Show that $\sum_{i=1}^n x_i$ is a sufficient statistic for the geometric distribution.

Problem 2: Covariance

A coin is taken from a box containing three coins, which give heads with probability p=1/3,1/2, and 2/3. The mysterious coin is tossed 80 times, resulting in 49 heads and 31 tails.

- (a) What is the likelihood of this data for each type of coin and which coin gives the maximum likelihood?
- **(b)** Now suppose that we have a single coin with unknown probability p of landing heads. Find the likelihood and log likelihood functions given the same data. What is the maximum likelihood estimate for p?

Problem 3: Dice

There are five fair dice each with a different number of sides: 4,6,8,12,20. Jon picks one of them uniformly at random rolls it and reports a 13.

- (a) Compute the posterior probability for each die to have generated this outcome.
- **(b)** Compute the posterior probabilities if the result had been a 5 instead. *Hint: Drawing a table may help here. And please do use a calculator!*

Problem 4: Geometric maximum likelihood estimator

Recall that if Y is geometrically distributed, then $P(Y = y) = (1 - \theta)^y \theta$. You are given a set of independent, geometrically distributed observations $x = x_1^n$. Find the maximum likelihood estimator of θ .

CREDITS Adapted from MIT course 18-05 by Jeremy Orloff and Jonathan Bloom, see ocw.mit.edu