

## BASIC PROBABILITY: THEORY

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# Practice problem set 7

This week's exercises deal with the basics of information theory. You only have to hand in the homework problems; these exercises are optional and for practicing only. If you have questions about them, please post them to the [discussion forum](#) and try to help each other. We will also keep an eye on that.

## Problem 1: Entropies

Consider (i) a flip of a fair coin, (ii) a toss of a fair four-sided die, and (iii) a toss of a fair six-sided die. Let a RV  $X$  encode (i), (ii) and (iii) and compute  $H(X)$  in each case.

## Problem 2: Binary entropies

A biased coin comes up heads with a probability of  $\frac{2}{3}$ . Compute the entropy of the outcome of six coin flips.

## Problem 3: More entropy

Calculate the entropy of the following:

1. pixel values whose possible values are all integers in  $[0, 255]$  with uniform probability,
2. dogs sorted by whether or not they are mammals,
3. dogs sorted by whether they are older or not than the population's [median](#),
4. RV  $X$  with  $P(X = 0) = \frac{1}{3}$ ,  $P(X = 1) = \frac{1}{4}$ ,  $P(X = 2) = \frac{1}{6}$ ,  $P(X = 3) = \frac{1}{6}$ ,  $P(X = 4) = \frac{1}{12}$ .

## Problem 4: Joint and conditional entropy

Suppose that

$$\begin{aligned} P_X(x) &= 1/6, & x = 1, 2, \dots, 6; \\ P_{Y|X}(y|x) &= 1/x, & y = 1, 2, \dots, x. \end{aligned}$$

Compute  $H(X)$ ,  $H(Y | X)$ , and  $H(X, Y)$ .