Points & Surfaces

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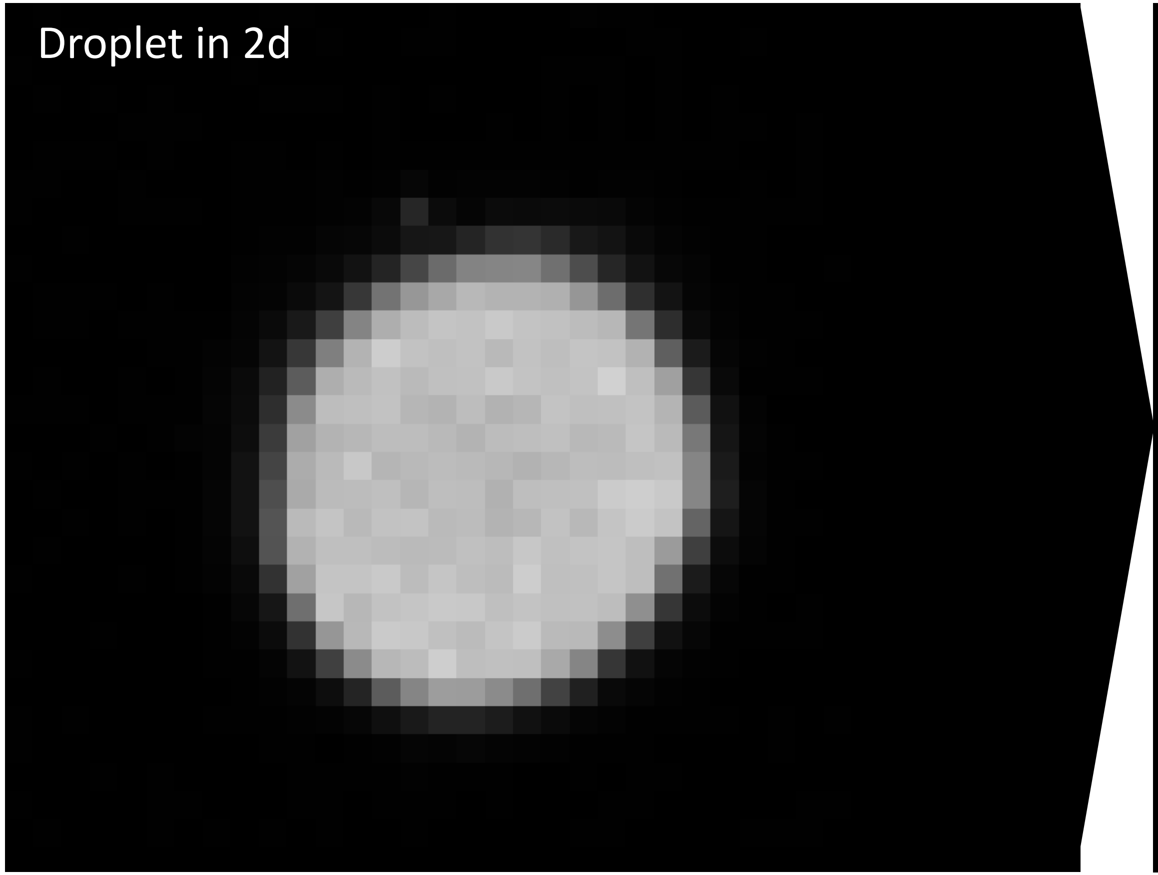
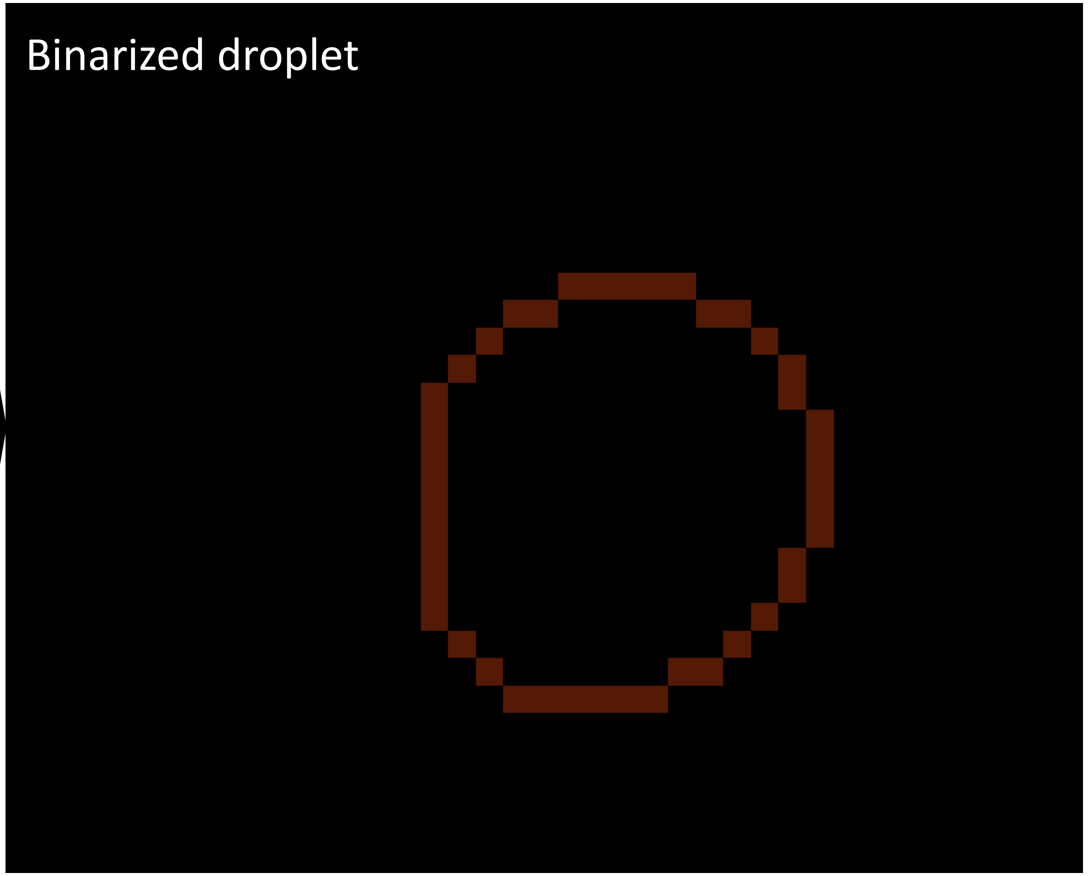
Common image data





Inherent limitation of image data as arrays:

→ Biology does not have edges at pixels

→ Measurements get distorted by pixel artifacts – especially for smaller objects

Source: https://github.com/campaslab/STRESS (https://github.com/campaslab/STRESS/blob/main/LICENSE.md)

Common image data

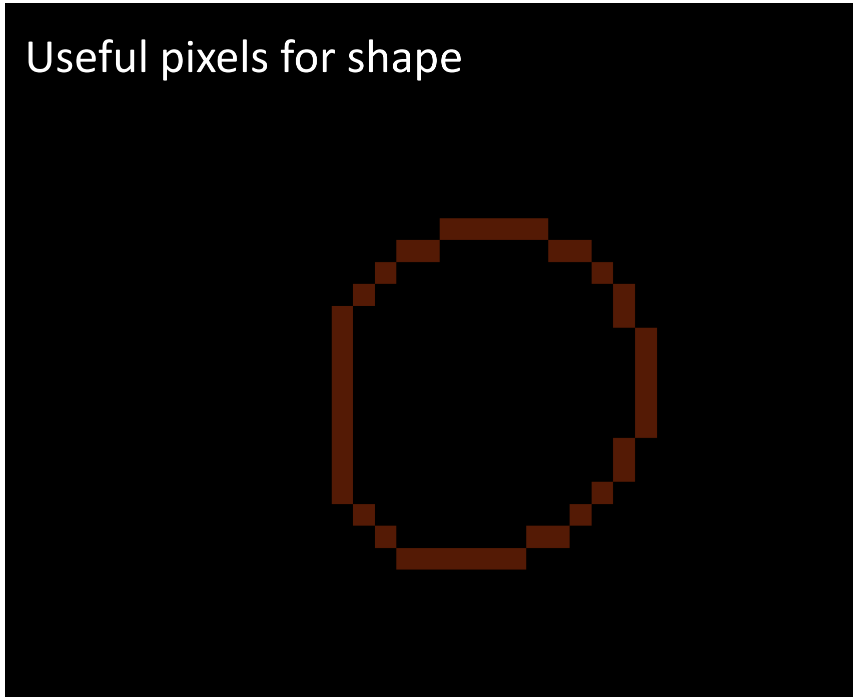
Limitations of image data as arrays:

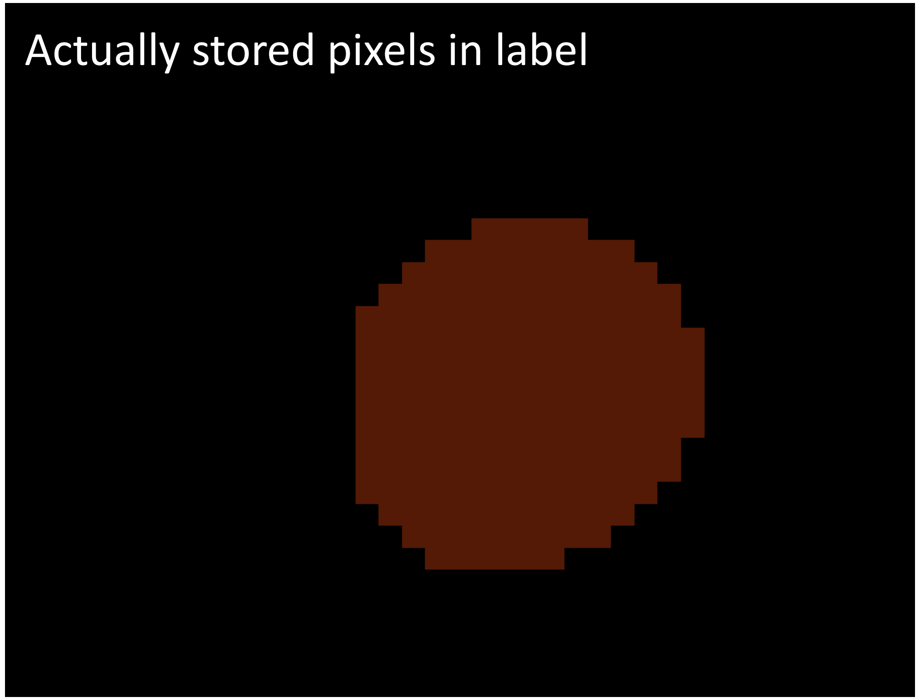
→ **Small objects**: Biology does not have edges at pixels

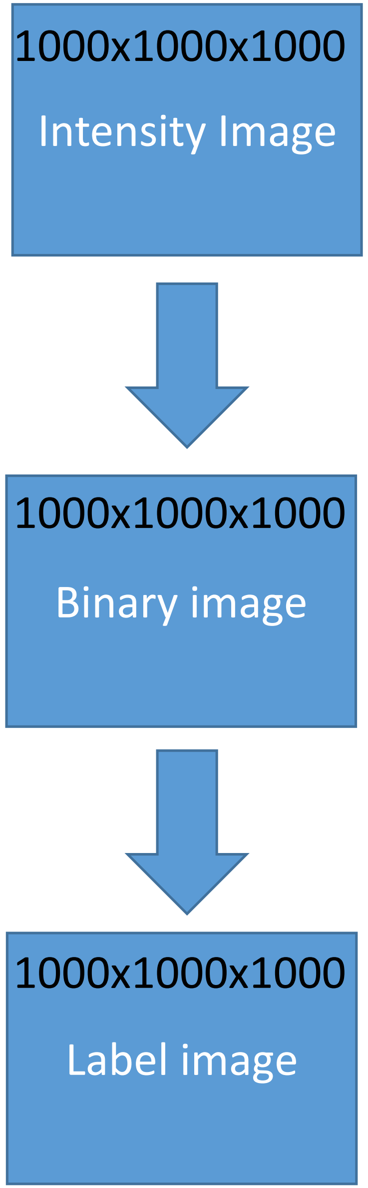
→ **Small objects**: Measurements get distorted by pixel artifacts

→ **Large objects**: Storing raw and processed images requires huge storage!

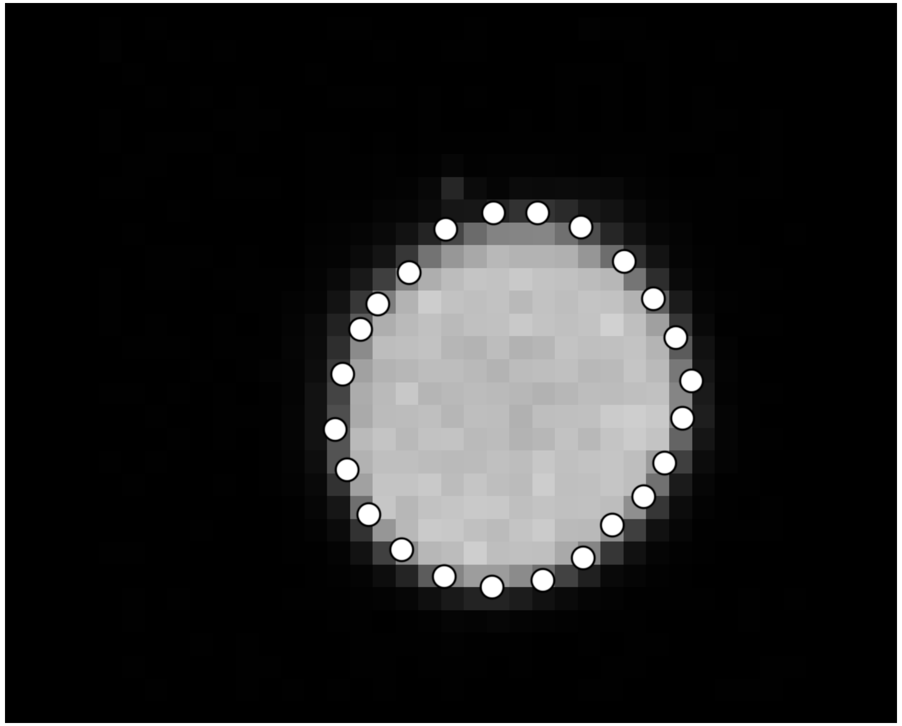
→ **Large objects after segmentation**: Most of the pixels/voxels carry no useful information







Surfaces



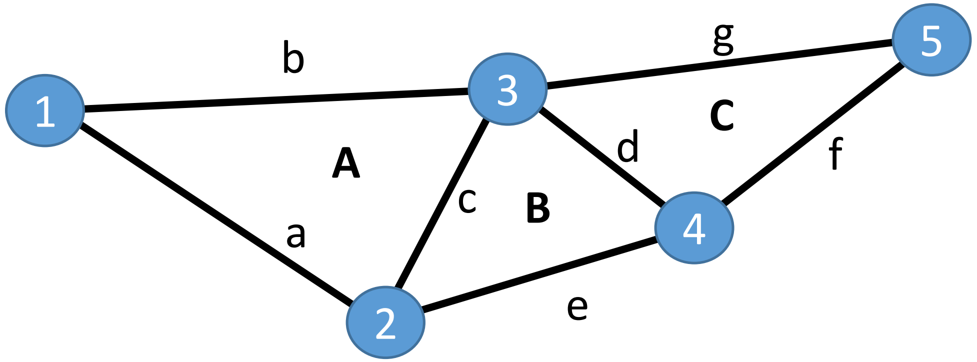
Points:

→ Represent shape of objects by points on surface
→ Points can be between pixels!
→ Coordinates have **physical units**

```
array([[13.1896702 , 10.97073578],
       [15.15516485, 10.18453792],
       [17.57927492,  9.85695548],
       [19.34822011, 10.38108738],
       [21.31371476, 11.33107646],
       [22.85335224, 12.77243921],
       [24.03264903, 14.63965913],
       [24.49126445, 16.73618675],
       [24.19644025, 18.96374736],
       [23.21369292, 20.73269255],
       [21.77233018, 22.01026407],
       [20.5275169 , 23.38611032],
       [19.05339591, 24.30334116],
       [17.08790126, 25.08953902],
       [15.44998905, 25.48263795],
       [13.55001089, 24.79471482],
```

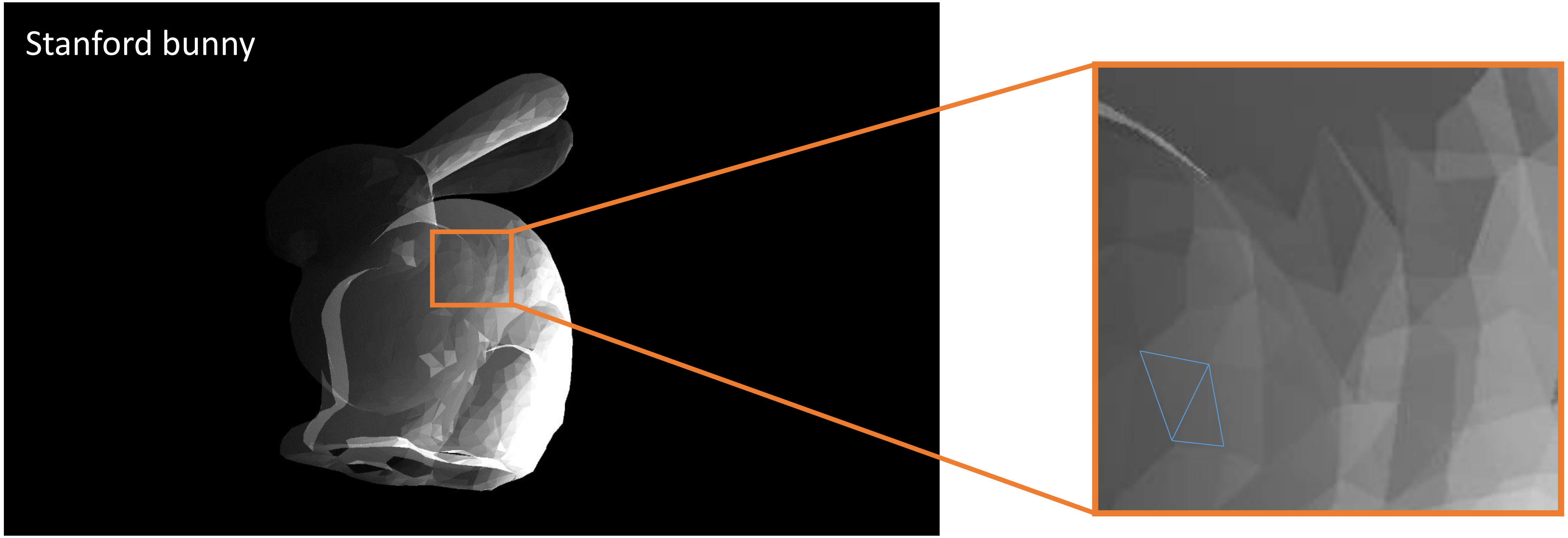
Surfaces/Meshs:

→ Connect neighboring points with graph structure

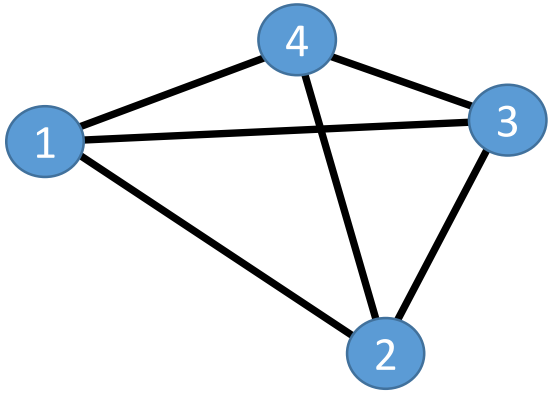


Meshes consist of vertices, edges and faces:
→ Vertices (1, 2, 3, 4, 5)
→ Edges (a, b, c, d, e, f, g)
→ Faces (**A**, **B**, **C**)

Mesh structure



Mesh data structure:



Point x	Point y	Point z
x_1	y_1	z_1
X_2	Y_2	Z_2
X_3	Y_3	Z_3
X_4	Y_4	Z_4
…	…	…

\+

Point 1	Point 2	Point 3
1	2	3
1	2	4
2	3	4
1	3	4

For simplicity: Only triangles

Sources: Stanford Computer Graphics Laboratory
https://graphics.stanford.edu/data/3Dscanrep/

How many vertices would a pyramid have as a mesh object?

3	4	5	6

How many edges would a pyramid have as a mesh object?

5	6	7	8

How many faces would a pyramid have as a mesh object?

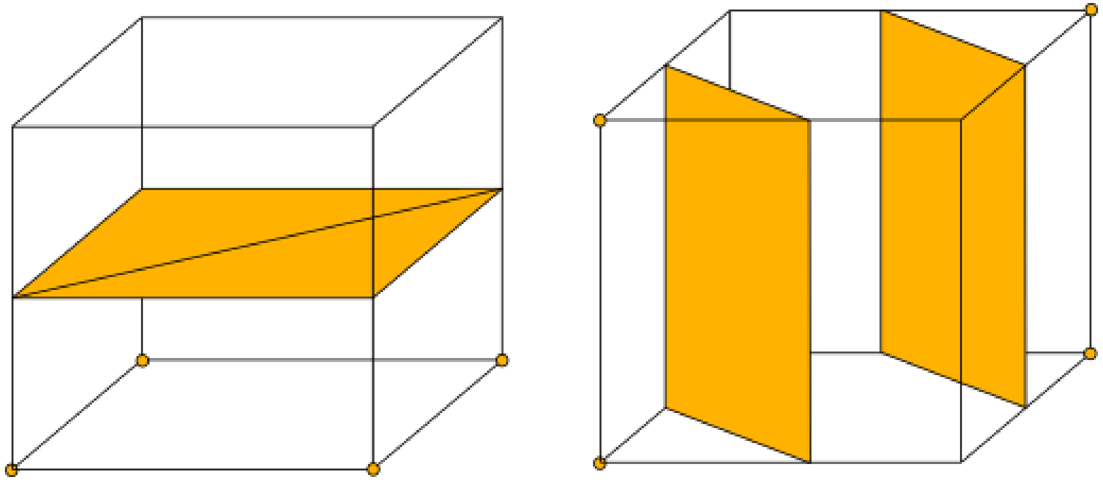
5	6	7	8

Generating meshes from label images

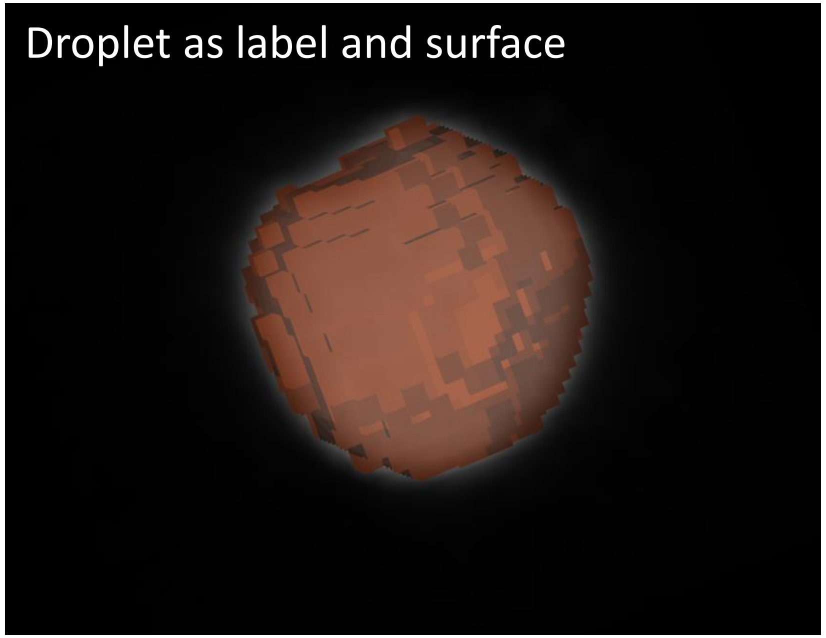
Marching cubes: Converts label image into surface (Cline & Lorensen, 1985)

→Slide cube over image (similar to filter concept)

→Are cube corners inside or outside of the label object?

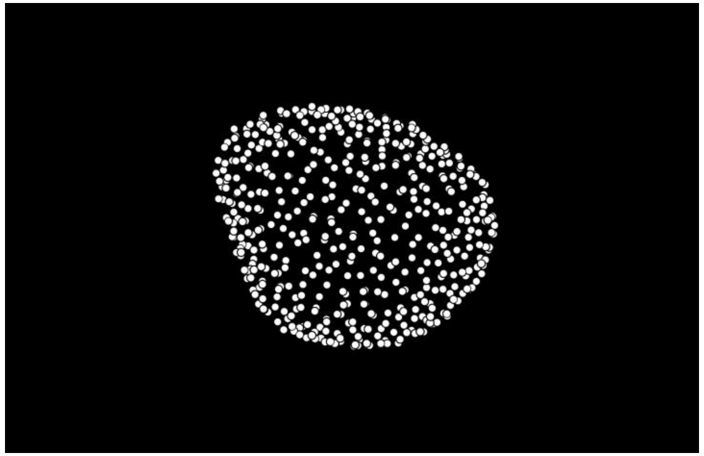


Symmetries: Currently 33 unique topologies of arranging polygons inside cube (Chernyaev et al., 1995)



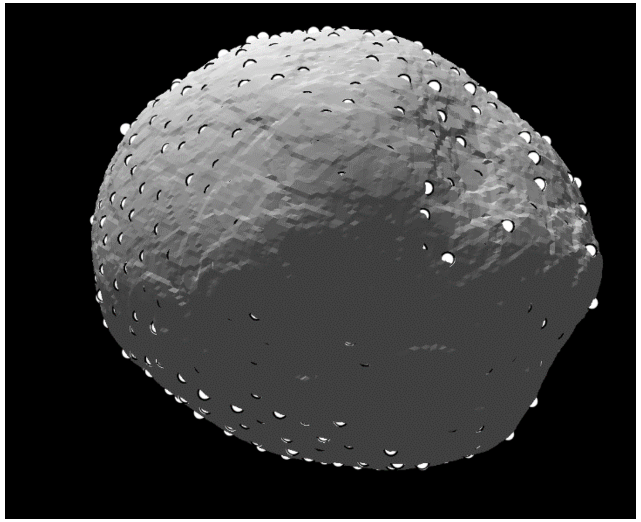
→Finite possibilities of how surface planes can be oriented with respect to the label surface

Generating meshes from points

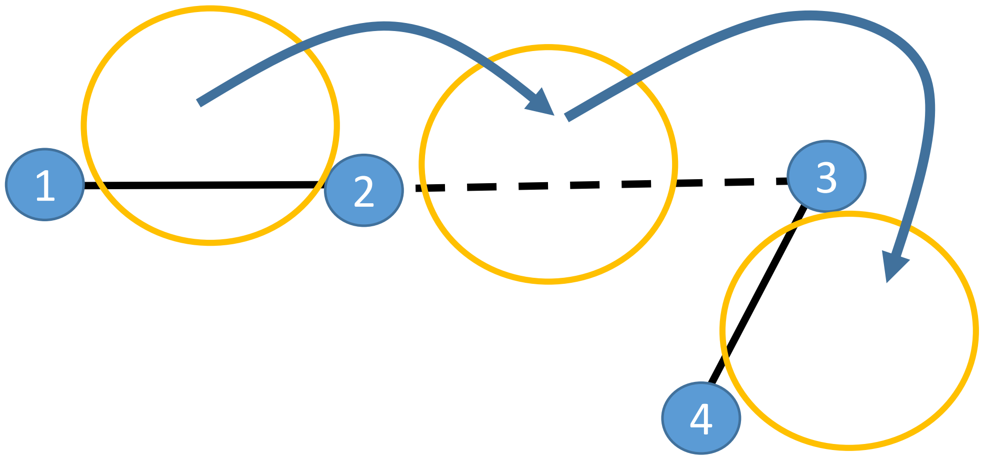


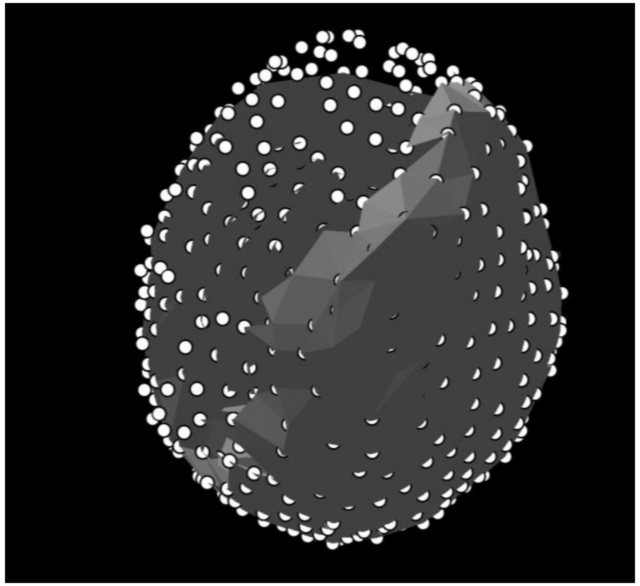
Surface reconstruction algorithms:

- Signed distance + flying edges
  → Calculate distance to nearest point at sample location
  → Fit curve to zero-crossings
  → Surface vertices approximated

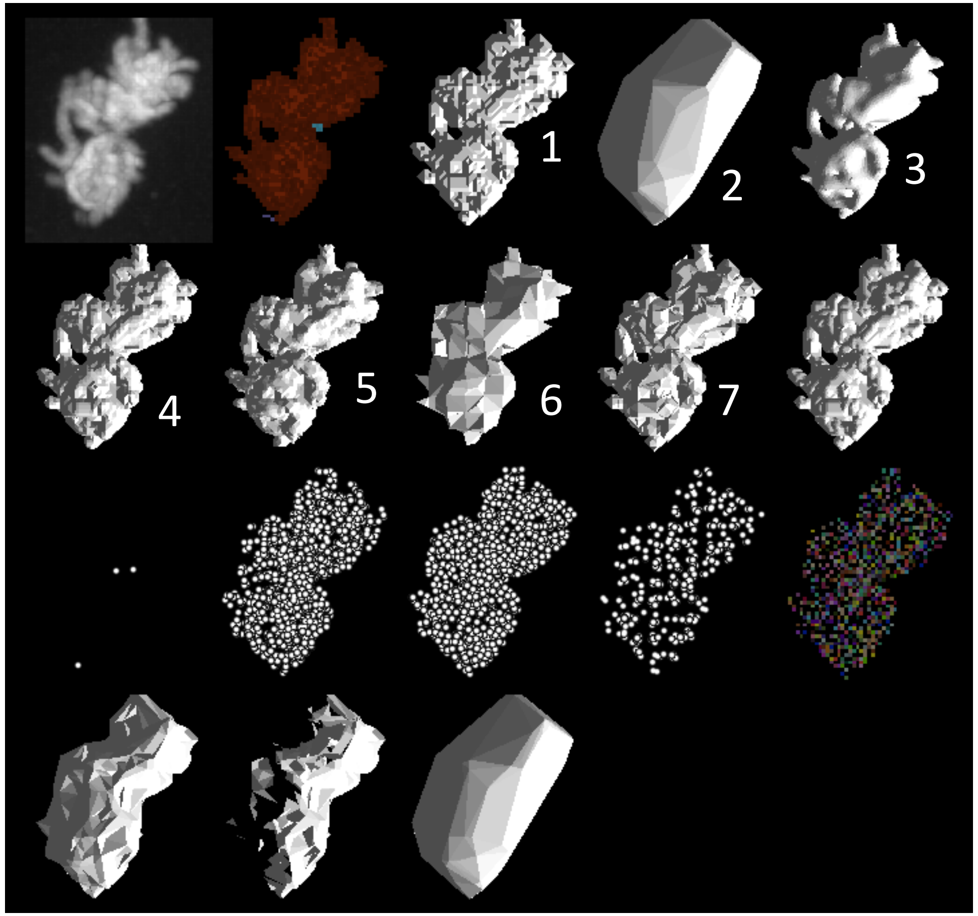


- Ball pivoting
  → Rolling ball over points
  → Preserves original points
  → Can lead to holes





Processing meshes



Source: https://github.com/haesleinhuepf/napari-process-points-and-surfaces

Typical mesh processing algorithms

- Reconstruction (1)
- Convex hull (2)
- Mesh smoothing (simple) (3)
- Mesh smoothing (Laplacian) (4)
- Mesh smoothing (taubin) (5)
- Mesh simplification I (6)
- Mesh simplification II (7)

Further operations:

- Connected components
- Hole filing

Measurable features: basic

Standard shape features

- Volume, surface area (perimeter), elongation

```
print('Surface area:', mesh.area())
print('Volume:', mesh.volume())
```

```
Surface area: 5.646863245482599
Volume: 0.7620278501410354
```

- Asphericity: $\frac{\pi^{\frac{1}{3}}(6V)^{\frac{2}{3}}}{A}$

  (V: Volume, A: Surface area)

  → measure of compactness

  → This is actually just

Circularity in 3D | Roundness in 3D | Solidity in 3D

- Face-wise features: Local surface shape descriptors

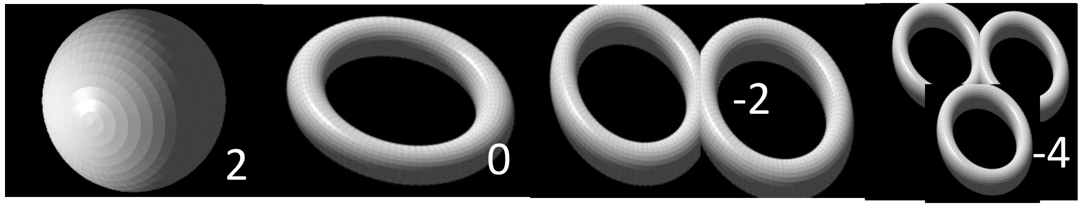
```
- EDGE RATIO, 0
- ASPECT RATIO, 1
- RADIUS RATIO, 2
- ASPECT FROBENIUS, 3
- MED ASPECT FROBENIUS, 4
- MAX ASPECT FROBENIUS, 5
- MIN_ANGLE, 6
- COLLAPSE RATIO, 7
- MAX ANGLE, 8
- CONDITION, 9
- SCALED JACOBIAN, 10
- SHEAR, 11
- RELATIVE SIZE SQUARED, 12
- SHAPE, 13
- SHAPE AND SIZE, 14
- DISTORTION, 15
- MAX EDGE RATIO, 16
- SKEW, 17
- TAPER, 18
- VOLUME, 19
- STRETCH, 20
- DIAGONAL, 21
- DIMENSION, 22
- ODDY, 23
- SHEAR AND SIZE, 24
- JACOBIAN, 25
- WARPAGE, 26
- ASPECT GAMMA, 27
- AREA, 28
- ASPECT BETA, 29
```

Measurable features: fancy

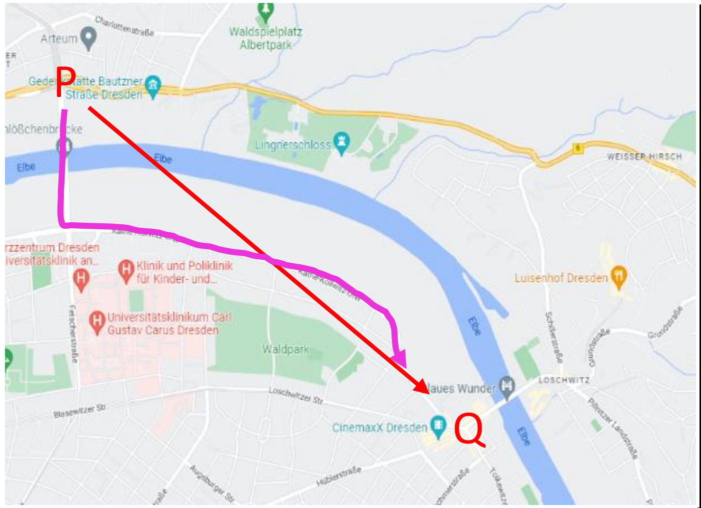
Geodesic distances:
→ Shortest distance on surface between two points
→ Can be found with Djikstra algorithm

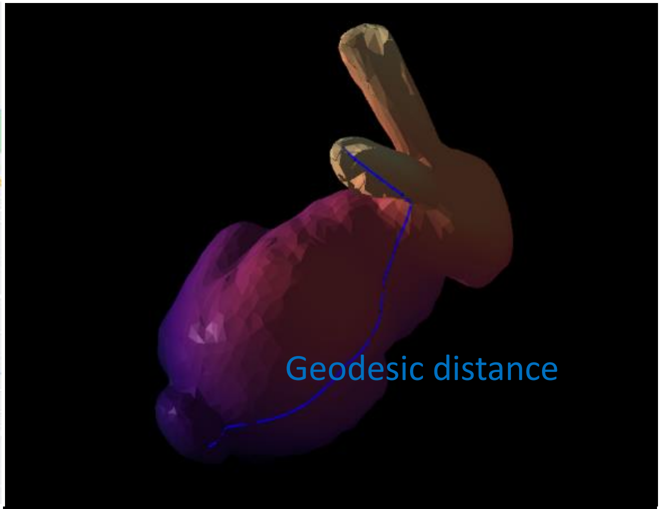
Surface curvature
→ Mean curvature: [0,…]
→ Gaussian curvature: [-… ,…] (identifies ridge regions)

Euler characteristics:
→ Distinguish topologies of objects

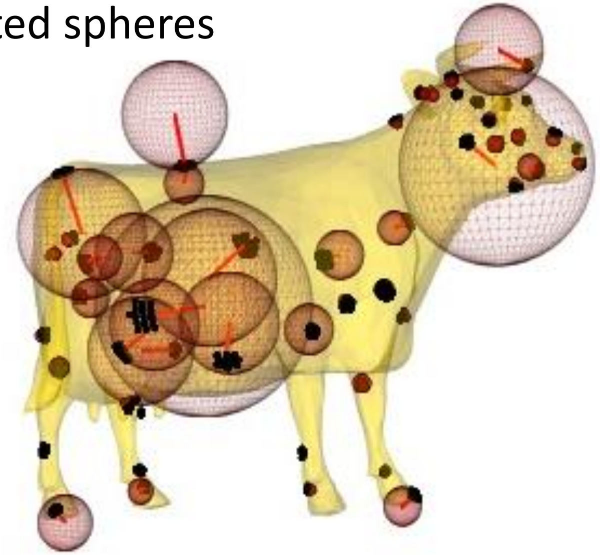


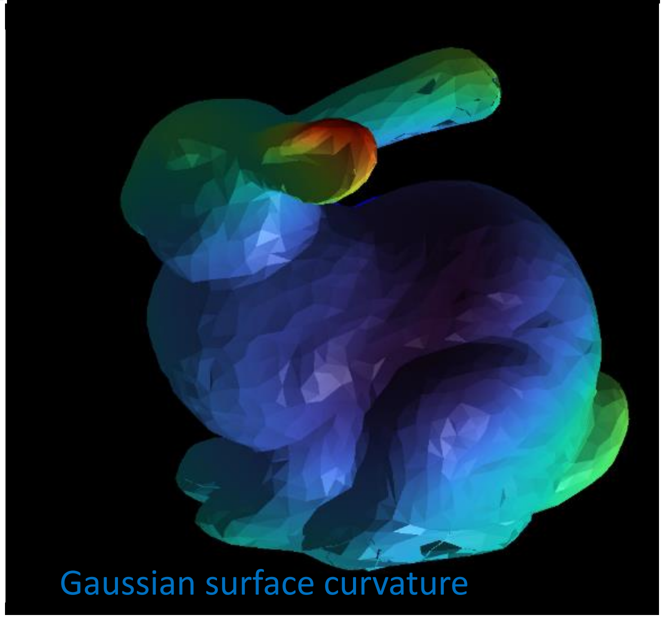
→ Euler characteristic $\chi = n_{points} - n_{edges} + n_{faces}$





Fitted spheres

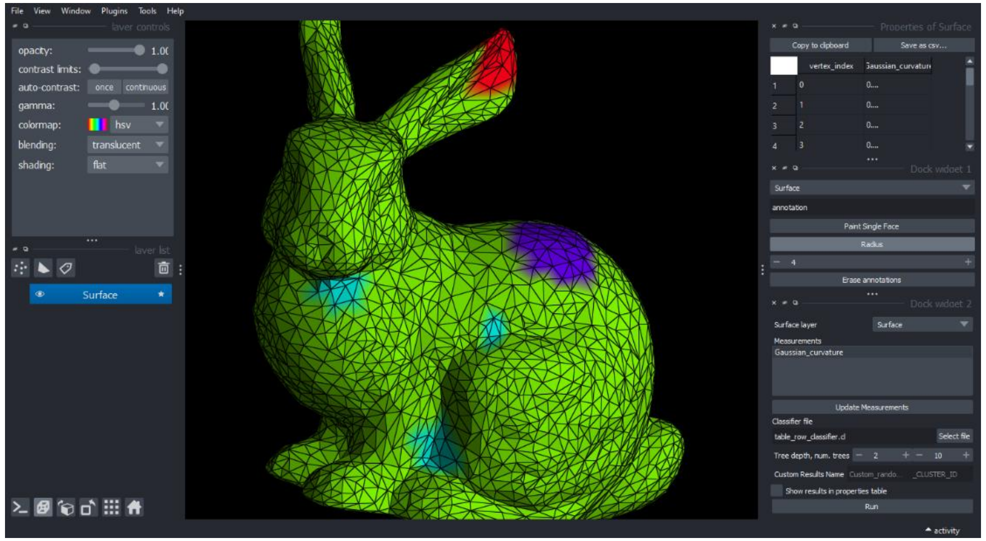




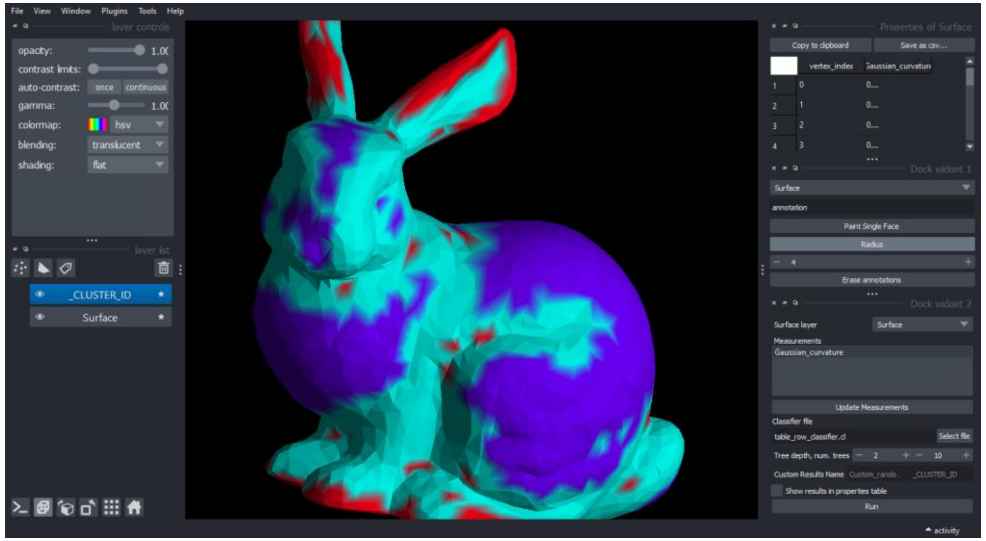
Sources: Stanford Computer Graphics Laboratory https://graphics.stanford.edu/data/3Dscanrep/
https://vedo.embl.es, License: https://vedo.embl.es/examples/data/LICENSE.txt

Where there are features…

There is data science!



Annotated surface regions



Prediction on surface

Recent feature: SVeC (surface vertex classification)
→Interactively annotate surface vertices
→Predict class membership of all other vertices

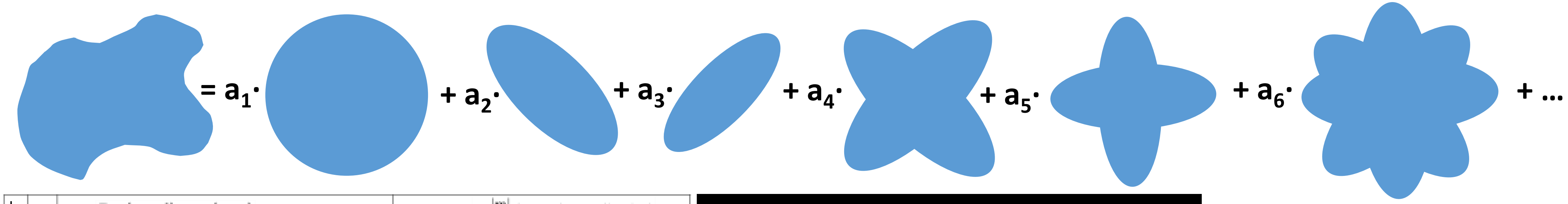
Coming soon:
→ Dimensionality reduction on meshes
→ Clustering on meshes

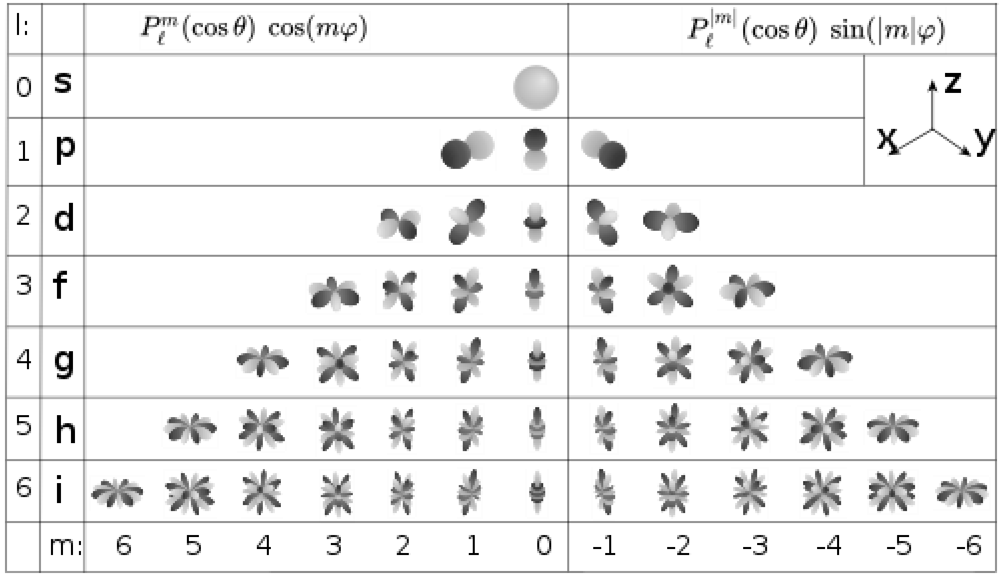
Source: Stanford Computer Graphics Laboratory https://graphics.stanford.edu/data/3Dscanrep/
https://github.com/haesleinhuepf/napari-accelerated-pixel-and-object-classification

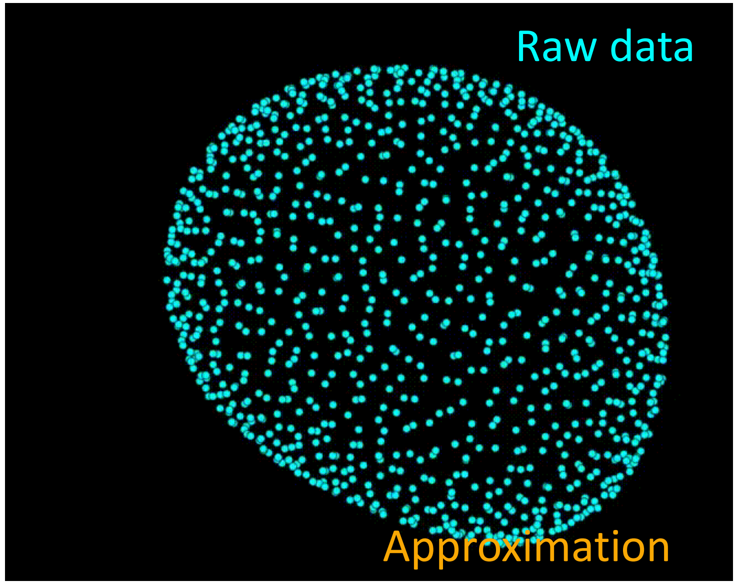
Spherical harmonics

Basic idea:

→ Express object shape as superposition of multiple elemental shapes







→ Final object shape described by spherical harmonics coeffcients

Historical background:

→ Describe probability distributions of electron location in nuclear shell

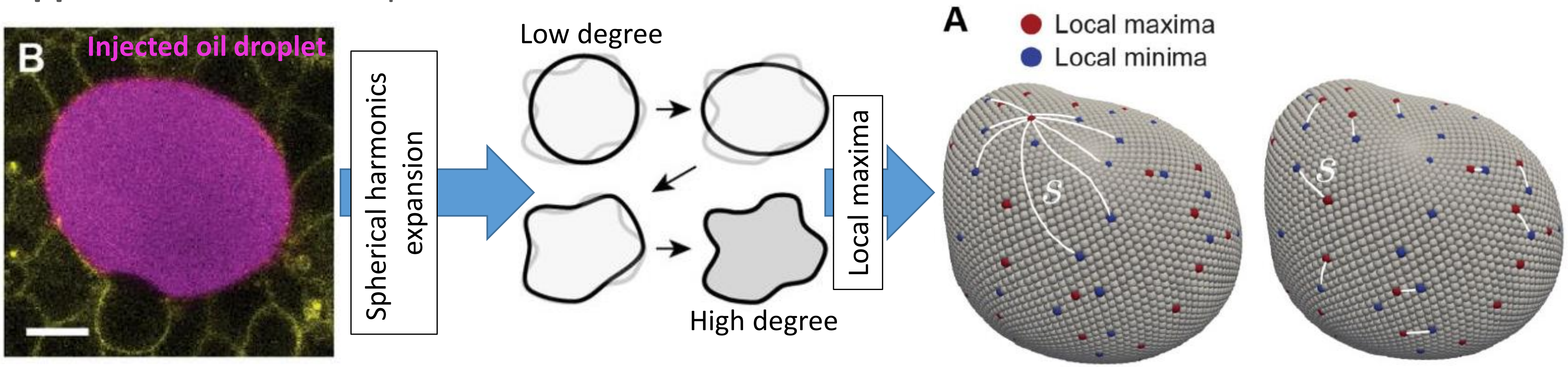
→ First described in 1782

→ Leads to the concept of **shape-spectra** of objects

https://commons.wikimedia.org/wiki/File:Sphericalfunctions.svg

Napari-stress

Application: Measure quantitative tissue stresses



→ Inject droplet of known interfacial tension into tissue
→ Reconstruct surface
→ Spherical harmonics expansion
→ Calculate surface curvature

Local maxima

↕

tight junctions of neighboring cells

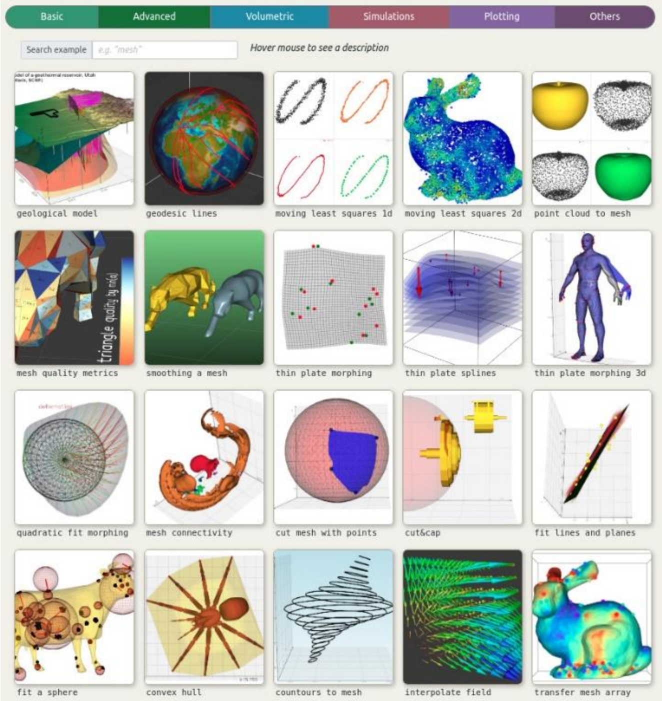
Source: Gross et al. (biorXiv, 2021)

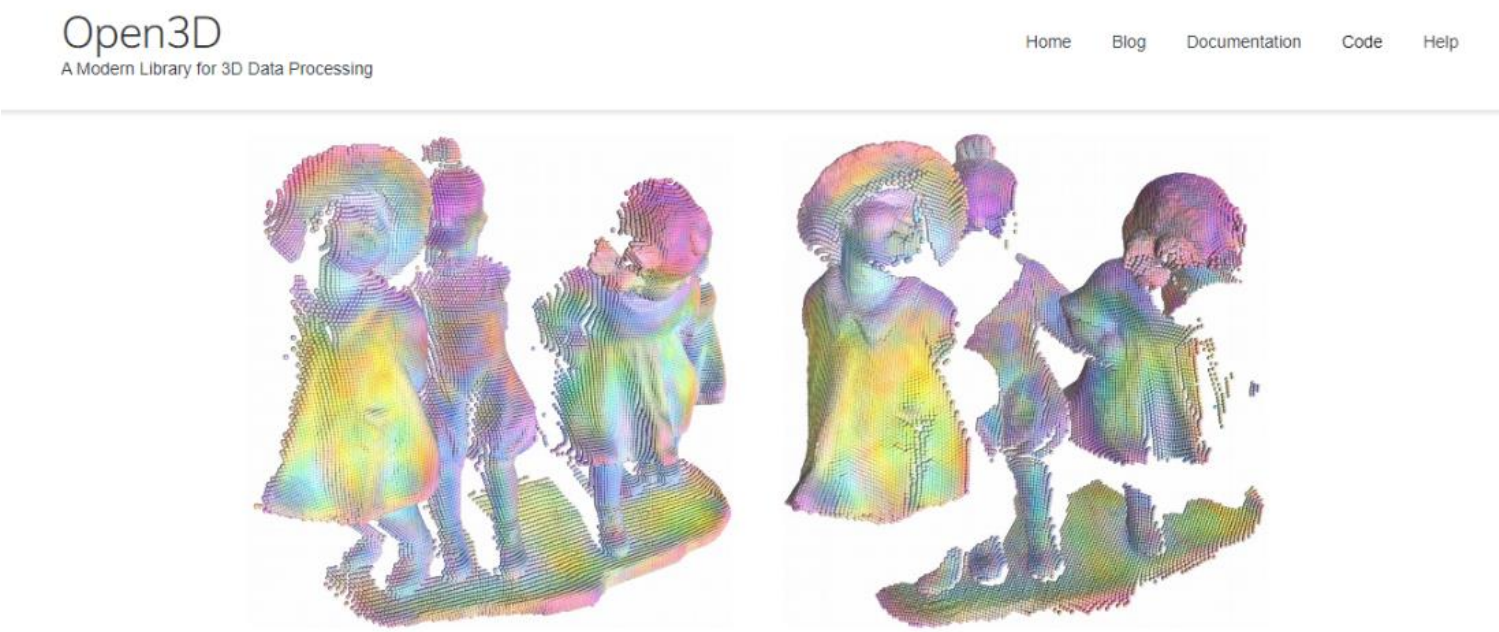
https://www.biorxiv.org/content/10.1101/2021.03.26.437148v1

Why meshes?

- **Lightweight:** Label image of 0.5GB = Surface data of ~7MB
- **Speed:** Mesh structures are fast to render and interact visually
- **Quantitative measurements:** Using Mesh structures capture the shape of objects on a more abstract levels
- **3D-printable:** Meshes are the native format for 3D printers!
- **Relevant Python packages:** Vedo, open3d

3D printed cell nucleus from cryo-EM





Sources: https://github.com/marcomusy/vedo (Shared under MIT license)
http://www.open3d.org/ (Shared under MIT license)

In Napari

```
viewer.add_surface(bunny)
```

```
viewer.add_points(points_array, size=0.01, face_color='red')
```