CSE574 Introduction to Machine Learning

Machine Learning: Notation and Definitions

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Outline

Notation

Let's breifly revisit the mathematical notation we all learned at school.

Data Structure

A **scalar** is a simple numerical value, like 15 or -3.25 . Variables or constants that take scalar values are denoted by an italic letter, like x or a.

A **vector** is an ordered list of scalar values, called attributes. We denote a vector as a bold character, for example, $\mathbf{x}$ or $\mathbf{w}$.

- Vectors can be visualized as arrows that point to some directions as well as points in a multi-dimensional space.

Illustrations of three two-dimensional vectors,
$\mathbf{a} = [2, 3]$, $\mathbf{b} = [-2, 5]$, and $\mathbf{c} = [1, 0]$ are given in the figure.

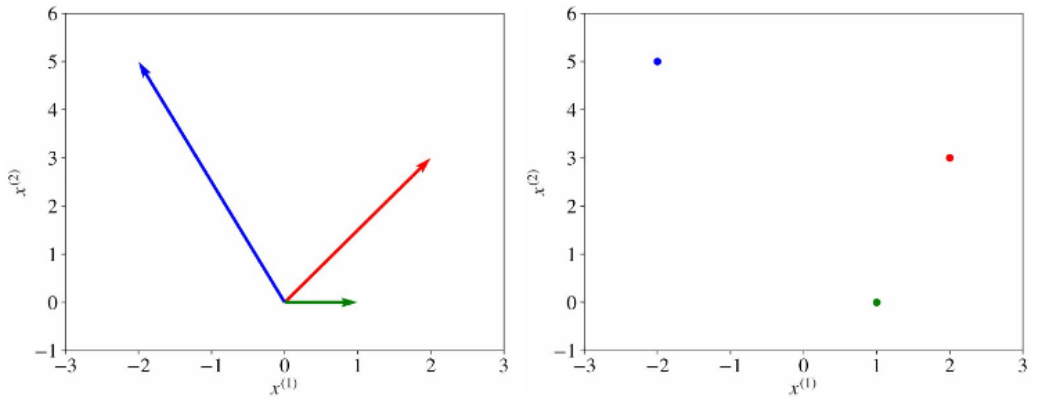

Figure: Three vectors visualized as directions and as points.

We denote an attribute of a vector as an italic value with an index, like this: $w^{(j)}$ or $x^{(j)}$. The index j denotes a specific **dimension** of the vector, the position of an attribute in the list. For instance, in the vector a shown in red in the figure, $a^{(1)} = 2$ and $a^{(2)} = 3$.

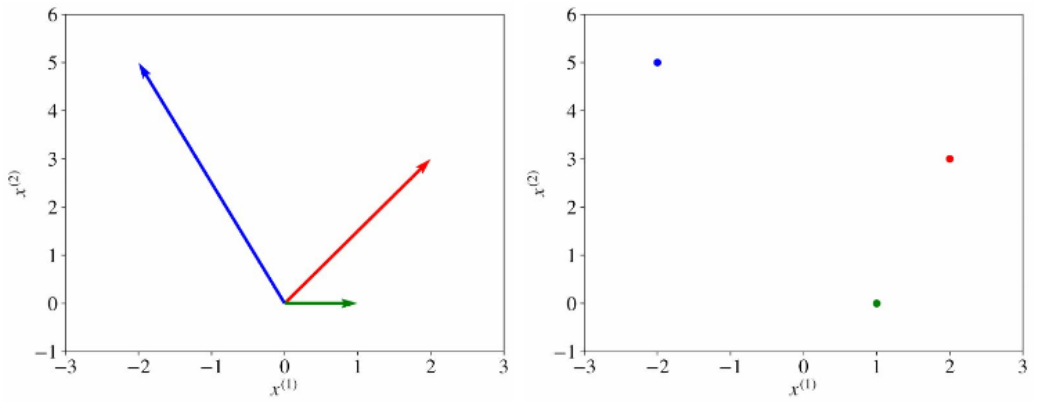

Figure: Three vectors visualized as directions and as points.

The notation $x^{(j)}$ should not be confused with the power operator, such as the 2 in x^2 (squared) or 3 in x^3 (cubed). If we want to apply a power operator, say square, to an indexed attribute of a vector, we write like this: $\left(x^{(j)}\right)^2$.

A variable can have two or more indices, like this: $x_i^{(j)}$ or like this $x_{i,j}^{(k)}$. For example, in neural networks, we denote as $x_{l,u}^{(j)}$ the input feature j of unit u in layer l.

A **matrix** is a rectangular array of numbers arranged in rows and columns. Below is an example of a matrix with two rows and three columns,

$$\begin{bmatrix} 2 & 4 & -3 \\ 21 & -6 & -1 \end{bmatrix}$$

Matrices are denoted with bold capital letters, such as $\mathbf{A}$ or $\mathbf{W}$.
A **set** is an unordered collection of unique elements.

- We denote a set as a calligraphic capital character, for example, $\mathcal{S}$.

A set of numbers can be finite (include a fixed amount of values).

- In this case, it is denoted using accolades, for example, $\{1, 3, 18, 23, 235\}$ or $\{x_1, x_2, x_3, x_4, \ldots, x_n\}$.

A set can be infinite and include all values in some interval.

- If a set includes all values between a and b, including a and b, it is denoted using brackets as $[a, b]$. If the set doesn't include the values a and b, such a set is denoted using parentheses like this: (a, b).
- For example, the set $[0, 1]$ includes such values as $0, 0.0001, 0.25, 0.784, 0.9995$, and 1.0. A special set denoted $\mathbb{R}$ includes all numbers from minus infinity to plus infinity.