Lecture 4: Transformations

Part 1 of several

Last Time

- Canvas pen model
- State stacks
- 2D Primitives
    - polygons

- translate

Today: Transformations

1. Translation - the simplest transformation
    - think in terms of coordinate systems
    - thinking forwards and backwards
2. Undo
3. Composition
4. More types of transformations
    - Scale
    - Rotate
5. Combining types (maybe next time)

Transform vs. Transformation

Transform is a verb or a noun

Transformation is a noun

Thinking about Transformation

with demos...

```
context.fillStyle="blue"
context.fillRect(20,20,20,20);
context.translate(20.0,20.0);
context.fillStyle="red"
context.fillRect(0,0,20,20);
```

(demo 1 - same square)

- both squares in the same place
- translate effects later points
- translate changes the coordinate system
- reading programs backwards

Forwards and Backwards

Transformations move the coordinate system

Global to local

Another way to think about it...

Work backwards from the object

The code is not written this way (backwards)

You cannot run your code **backwards**

(the demo program animates how to think about it)

Caveat to in-class "code"

```
context.fillRect(x,y, w,h);
```

- w,h are not coordinates - how are they transformed?
- demo code actually draws rectangle with lineto
- use a triangle instead today...

An Example Triangle

```
function tri(x,y) {
    context.beginPath();
    context.moveTo(x,y);
    context.lineTo(x+10,y);
    context.drawTo(x,y+20);
    context.closePath();
    context.fill();
    context.stroke();
}
```

- Intentionally not symmetric

We could use a transformation!

```
function tri(x,y) {


    context.beginPath();
    context.moveTo(x,y);
    context.lineTo(x+10,y);
    context.drawTo(x,y+20);
    context.closePath();
    context.fill();
    context.stroke();

}
```

```
function tri(x,y) {
    context.save();
    context.translate(x,y);
    context.beginPath();
    context.moveTo(0,0);
    context.lineTo(10,0);
    context.drawTo(0,20);
    context.closePath();
    context.fill();
    context.stroke();
    context.restore();
}
```

Some key ideas are there...

- save and restore
- transform all points

- Object defined **locally**
- Doesn't know where it goes
- Could put translate outside
- Can re-use the object

```
function tri(x,y) {
    context.save();
    context.translate(x,y);
    context.beginPath();
    context.moveTo(0,0);
    context.lineTo(10,0);
    context.drawTo(0,20);
    context.closePath();
    context.fill();
    context.stroke();
    context.restore();
}
```

Three Triangles, Three Transformations

```
context.fillStyle="red"
triangle(context,0,0);
context.translate(20.0,0.0);
context.fillStyle="blue"
triangle(context,0,0);
context.translate(0.0,20.0);
context.fillStyle="green"
triangle(context,0,0);
```

- apply transformations in the **current coordinate system**

(demo)

Undo!

- Transformation applied in the **current coordinate system**
- Coordinate system is state
- How do we get back?

1. save/restore
2. Remember the transform and apply the opposite

```
context.translate(x,y)
// draw something
context.translate(-x,-y)
```

Instancing

Create the graphics object

Use transformations to put it into the right place

Re-use, Re-use, Re-use

Three trangles, Three Ways

```
tri(0,0);

tri(20,0);

tri(20,20);
```

```
tri();

context.translate(20,0);
tri();

context.translate(0,20);
tri();
```

Triangle assumes 0,0

Its own coordinates

local coordinates

```
tri();
context.save();
context.transtlate(20,0);
tri();
context.restore();
context.save()
context.translate(20,20);
tri();
context.restore()
```

Composition of Transforms

- Transformations "add up"
- Steps combine into one "bigger"

$$T_1 + T_2 = T_{12}$$

- Translations are easy (literally add)
- More generally, transformations **compose**

$$T_1 \circ T_2 = T_{12}$$

- T_{12} is some new transformation (that combines T_1 and T_2)

Why Compose?

- Use simple building blocks
  - translateX, translateY --> translate
- Build things up gradually
  - use intermediate results
  - easier to understand
- More important when combining transform types
  - when we have multiple types...

What Types of Transformations?

Can be anything $(x', y') = \mathbf{f}(x, y)$

We'll start with a few basic types...

What Types of Transformations?

- Basic ones
  - translate, rotate, scale
  - skew
- Why these?
- Linear transformations
  - mathematically nice (next time)
  - really useful!
- More over the course of the semester

Why Compose?

- Use simple building blocks
  - translateX, translateY --> translate
- Build things up gradually
  - use intermediate results
  - easier to understand
- More important when combining transform types
  - when we have multiple types...

What Types of Transformations?

- Why these?
- Linear transformations
  - mathematically nice (next time)
  - really useful!
- Basic ones
  - translate, rotate, scale
  - skew
  - projection (when we get to 3D)
- More over the course of the semester

Scale Transformations

Make it bigger! (factor of 2)

```
function tri() {
    let s = 2;
    context.save();

    context.beginPath();
    context.moveTo(0  *s, 0  *s);
    context.lineTo(10 *s, 0  *s);
    context.drawTo(0  *s, 20 *s);
    context.closePath();
    context.fill();
    context.restore();
}
```

Note:

- multiply all coordinates by s (=2)
- it will be twice as big
- zero stays zero (center)

Scale Transformations

Make it bigger! (factor of 2)

```
function tri() {
    let s = 2;
    context.save();

    context.beginPath();
    context.moveTo(0  *s, 0  *s);
    context.lineTo(10 *s, 0  *s);
    context.drawTo(0  *s, 20 *s);
    context.closePath();
    context.fill();
    context.restore();
}
```

```
function tri() {
    let s = 2;
    context.save();
    context.scale(s,s); // note x,y
    context.beginPath();
    context.moveTo(0    , 0 );
    context.lineTo(10   , 0 );
    context.drawTo(0    , 20);
    context.closePath();
    context.fill();
    context.restore();
}
```

Non uniform scale

```
context.scale(sx,sy);
```

means:

```
context.moveTo(0  *sx, 0  *sy);
context.lineTo(10 *sx, 0  *sy);
context.drawTo(0  *sx, 20 *sy);
```

Use for Stretching or Flipping

```
context.scale(1,-1);
```

Scales compose by multiplication

```
scale(s1x,s1y)
scale(s2x,s2y)

context.moveTo(0  *s1x*s2x, 0  *s1y*s2y);
context.lineTo(10 *s1x*s2x, 0  *s1y*s2y);
context.drawTo(0  *s1x*s2x, 20 *s1y*s2y);
```

Change of coordinates?

Yes, stretch (scale the coordinates)

Notice that **zero** stays in place

(demo)

Scaling

- has a **center**
- everything else moves

- this is inconvenient

Combining Scale and Translate

```
context.scale(sx,sy);
context.lineTo(X,Y);
```

means for all X,Y:

```
context.lineTo(X *sx, Y  *sy);
```

```
context.translate(tx,ty)
context.lineTo(X,Y);
```

means for all X,Y:

```
context.lineTo(X +tx, Y +ty);
```

Scale and Translate

```
context.scale(sx,sy);
context.translate(tx,ty);
context.lineTo(X,Y);
```

means for all X,Y:

```
context.lineTo( sx* (X+tx), sy* (Y+ty) );
```

Isn't this backwards?

Scale and Translate

```
context.scale(sx,sy);

context.translate(tx,ty);

context.lineTo(x,y);
```

set up a **scaled** coordinate system

translate in the **scaled** coordinate system

draw in the **scaled** and **translated** csys

So...

context.lineTo(sx* (X+tx), sy* (Y+ty));

Order Matters!

```
context.scale(sx,sy);
context.translate(tx,ty);
context.lineTo(X,Y);
```

means for all X,Y:

```
context.lineTo( sx* (X+tx), sy* (Y+ty));
```

```
context.translate(tx,ty);
context.scale(sx,sy);
context.lineTo(X,Y);
```

means for all X,Y:

```
context.lineTo( tx+ (X*sx), ty+ (Y*sy) );
```

this needs a demo....

Scale and Translate

```
context.scale(sx,sy);
context.translate(tx1,ty1);
context.lineTo(X,Y);
```

means for all X,Y:

```
context.lineTo( sx* (X+tx1), sy* (Y+ty1));
```

```
context.translate(tx2,ty2);
context.scale(sx2,sy2);
context.lineTo(X,Y);
```

means for all X,Y:

```
context.lineTo( tx2+ (X*sx), ty2+ (Y*sy) );
```

These translate a different amount

We could find `tx2` so it does the same thing as `tx1`

Forwards and Backwards

Transformations move the coordinate system

Global to local

Another way to think about it...

Work backwards from the object

The code is not written this way (backwards)

You cannot run your code **backwards**

(the demo program animates how to think about it)

Center of Scaling

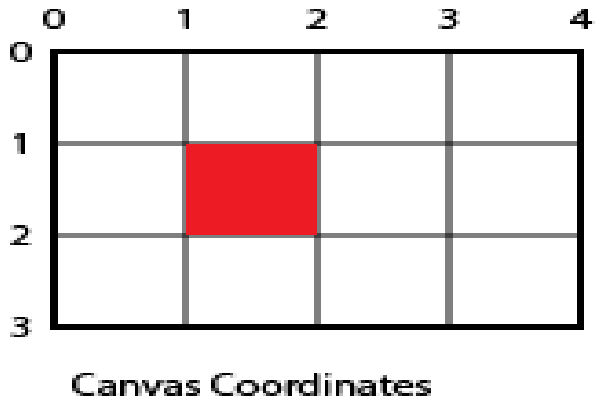
Choose what point doesn't change

Pick **some** point to be the center

(demo)

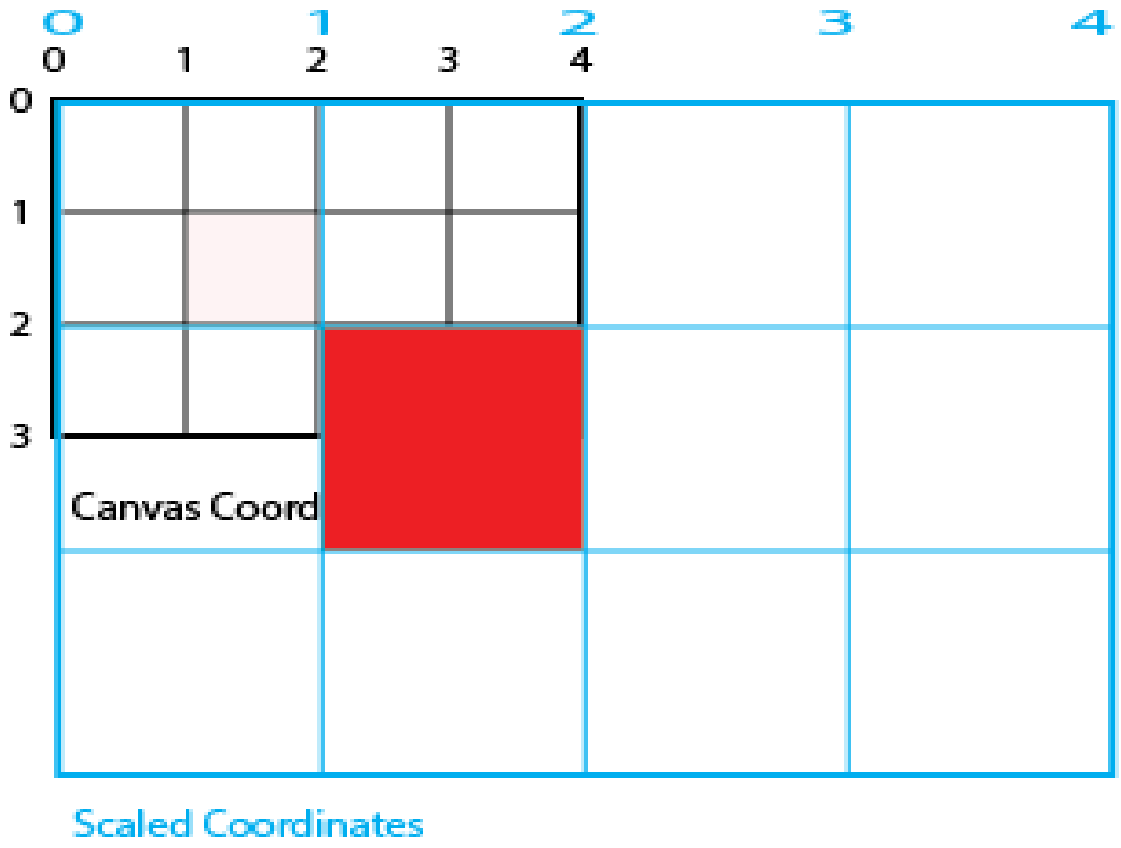
```
context.save();

context.translate(cx,cy);
context.scale(sx,sy);
context.translate(-cx,-cy);

context.restore();
```

Scale about Center

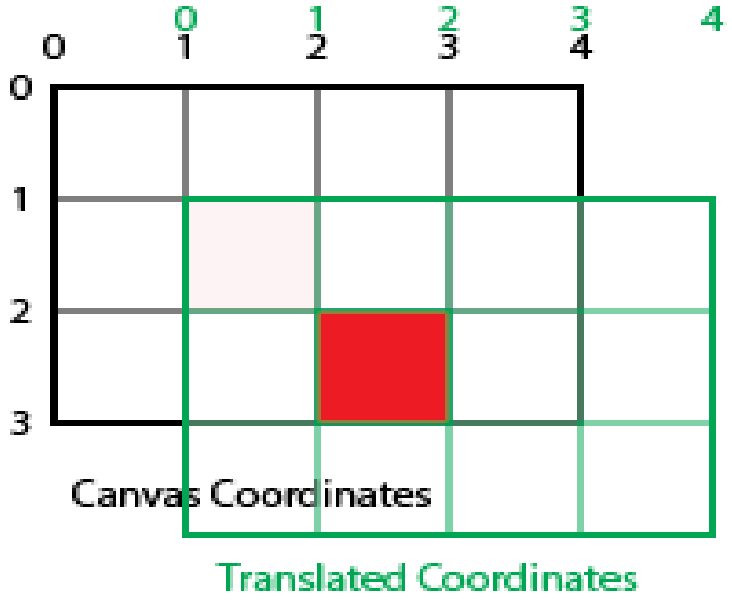


Canvas Coordinates

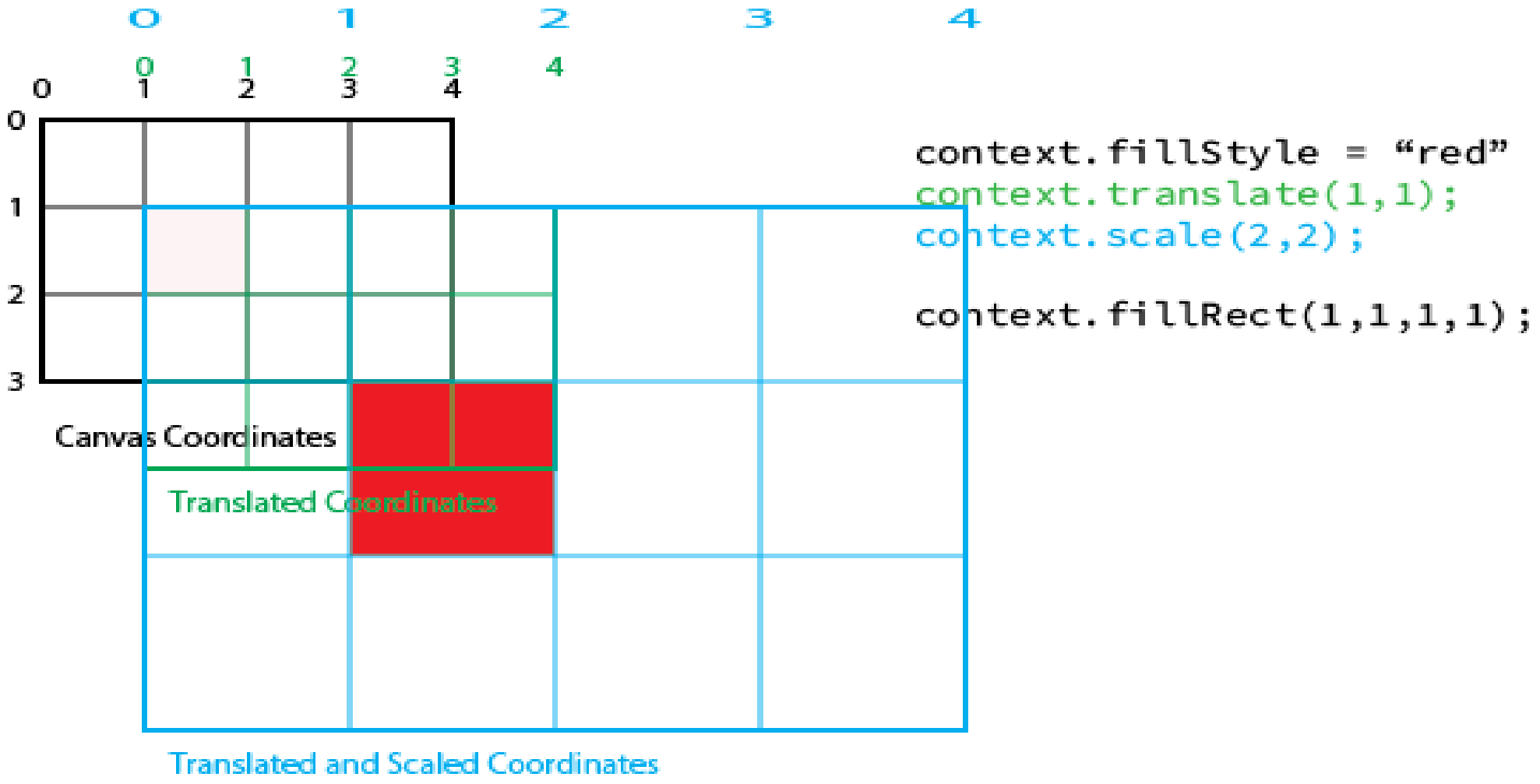
```
context.fillStyle = “red”

context.fillRect(1,1,1,1);
```

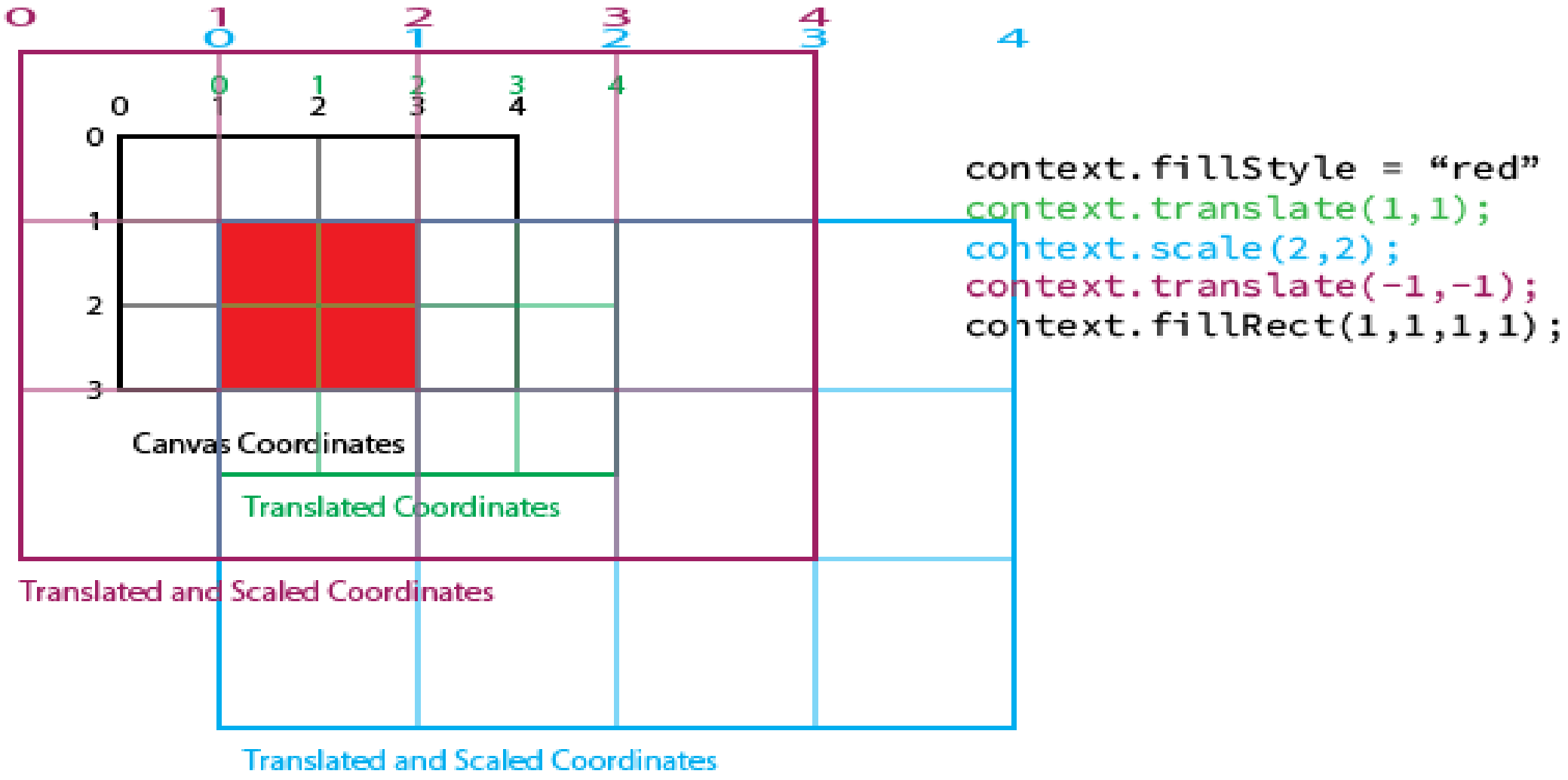


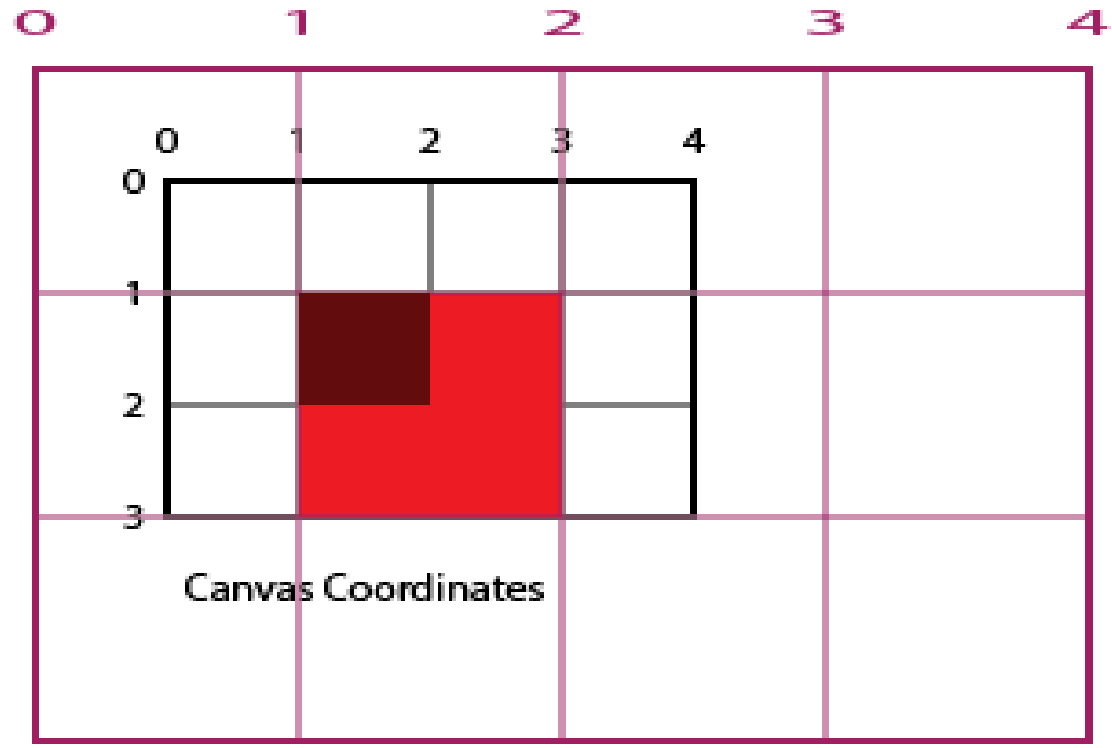

```
context.fillStyle = “red”

context.scale(2,2);

context.fillRect(1,1,1,1);
```




```
context.fillStyle = “red”
context.translate(1,1);

context.fillRect(1,1,1,1);
```


0 1 2 3 4
0 1 2 3 4
0 1 2 3 4
0
1
2
3
Canvas Coordinates
Translated Coordinates
Translated and Scaled Coordinates
context.fillStyle = "red"
context.translate(1,1);
context.scale(2,2);
context.fillRect(1,1,1,1);


0 1 2 3 4
0 1 2 3 4
0 1 2 3 4
0 1 2 3 4
0
1
2
3
Canvas Coordinates
Translated Coordinates
Translated and Scaled Coordinates
Translated and Scaled Coordinates
context.fillStyle = “red”
context.translate(1,1);
context.scale(2,2);
context.translate(-1,-1);
context.fillRect(1,1,1,1);



Translated and Scaled Coordinates

```
context.save();
context.fillStyle = “red”
context.translate(1,1);
context.scale(2,2);
context.translate(-1,-1);
context.fillRect(2,1,1,1);
context.restore();
```

```
context.fillStyle=“darkred”
```

```
context.fillRect(1,1,1,1);
```

Did we have to ...

```
translate(cx,cy)
scale(s,s)
translate(-cx,-cy)
```

couldn't we just

```
scale(s,s)
translate(ty,ty)
```

for some ??,??

Yes, but... this is a general way to figure out those values

And it will work for everything else later too

Non-Uniform Scale

Scale differently in each direction

Scale in X, Scale in Y

Useful for "flipping" (make Y go up)

What about stretch in a different direction?

Rigid Transformations

Distances between points are preserved

Handedness is preserved

Two types of transforms have these properties:

- translation (all points move the same)
- rotation (spin around a center point)

Handedness

Curl your fingers from X to Y

- right-handed vs. left-handed (will make sense in 3D)

Rotates around your thumb

- clockwise (Canvas coordinates, Y down, thumb into screen)
- counter-clockwise (math coordinates, Y up, thumb out of screen)

Handedness - preserves the direction of your thumb

Mirror reflection does not preserve handedness (it is not rigid)

Rotations

(in 3D, this is more complicated)

Spin the coordinate system around a point

Center - the point that doesn't move

Measuring rotations (in 2D)

radians = amount a point moves on the unit circle

(convert to degrees - if that's easier)

Properties of rotations

- There is a **zero** (point that doesn't move)
- Distances are preserved
- Handedness is preserved (not mirror reflection)

Rotated Coordinate Systems

Axes point in new directions

Convention

We rotate around the center of the coordinate system

We change change the center

- just like we did with scale

Demos

Translate then rotate

Rotate then translate

Rotation is about the **center** of rotation

Useful idiom

Rotate around a center point

- translate the origin to **some point**
- rotate about the origin
- translate the origin back from **some point**
- draw the object

rotate around a point

```
context.translate(cx,cy);
context.rotate(angle);
context.translate(-cx,-cy);
drawThing();
```

- make the object
- move the center point to origin
- rotate around the origin
- move the center point back
- **this is reading backwards**

Summary: Transformations

- Think of transformation as changing the **coordinate system**

- translate
- rotate
- scale

- compose transformations to do useful things
- more useful things next time
- math soon too!