1.1 Definitions and Notations

Basic Definitions

- **Set**: A collection of objects, called **elements**.
  - Notation: Sets are represented by **uppercase letters**; elements by **lowercase letters**.
  - Example: If a is in set A, we write $a \in A$. If not, $a \notin A$.

Subset

- **Subset**: A set B is a subset of A if every element in B is also in A.
  - Notation: $B \subseteq A$ if every element of B is in A.
  - **Extensionality Principle**: Two sets are equal if they contain the same elements.
    * **Equality**: $A = B$ if $A \subseteq B$ and $B \subseteq A$.

Set Representation

- **Listing Elements**: If a set has finite elements, list them as $\{a_1, a_2, \ldots, a_n\}$.
- **Describing by Properties**: If a set contains elements satisfying a property P, represent it as $\{x \mid x \textbf{ satisfies } P\}$.

Set Operations

1. **Union** ($A \cup B$): Elements in A or B.
2. **Intersection** ($A \cap B$): Elements in both A and B.
3. **Difference** ($A \setminus B$): Elements in A but not in B.
4. **Symmetric Difference** ($A \Delta B$): Elements in either A or B, but not in both.

Properties of Set Operations

- **Associative**: $(A \cup B) \cup C = A \cup (B \cup C)$
- **Commutative**: $A \cup B = B \cup A$
- **Distributive**: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

1.2 Important Set Types

- **Empty Set** ($\emptyset$): The unique set with no elements.
- **Singleton**: A set with only one element.
- **Universal Set** (V): The fixed larger set within which all sets are considered.
- **Complement**: For a set A in the universal set V, the complement $\overline{A} = V \setminus A$.

Complement Properties

- **Identity**: $A \cup \emptyset = A$, $A \cap V = A$
- **Double Complement**: $(\overline{A}) = A$
- **De Morgan's Laws**:
  - $\overline{A \cup B} = \overline{A} \cap \overline{B}$
  - $\overline{A \cap B} = \overline{A} \cup \overline{B}$

1.3 Families of Sets

- **Definition**: A collection of sets, often noted as $\mathcal{A}, \mathcal{B}$, etc.
- **Union of Families**: $\bigcup \mathcal{A} = \{x \mid \exists A \in \mathcal{A} : x \in A\}$
- **Intersection of Families**: $\bigcap \mathcal{A} = \{x \mid \forall A \in \mathcal{A} : x \in A\}$

1.4 Power Set

- **Definition**: The set of all subsets of a set A, including $\emptyset$ and A itself.
  - Notation: 2^A or $\mathcal{P}(A)$
  - **Example**: For $A = \{0, 1\}$, $\mathcal{P}(A) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$

1.5 Cartesian Product

- **Definition**: An ordered pair where order matters, denoted as (a, b).
  - For sets $A_1, \ldots, A_n$, the **Cartesian product** is $A_1 \times \cdots \times A_n = \{(a_1, \ldots, a_n) \mid a_i \in A_i\}$.
  - **Example**: If $A = \{1, 2\}$ and $B = \{x, y\}$, $A \times B = \{(1, x), (1, y), (2, x), (2, y)\}$

Key Points to Remember

- **Set**: A collection of elements denoted with uppercase letters.
- **Subset**: $B \subseteq A$ means all elements in B are in A.
- **Empty Set** ($\emptyset$): Unique set with no elements.
- **Union** ($\cup$) and **Intersection** ($\cap$): Basic operations to combine sets.
- **Complement**: For set A, $\overline{A} = V \setminus A$.
- **Power Set**: Set of all subsets of A.
- **Cartesian Product**: Ordered pairs from multiple sets.