Challenges in stochastic time series prediction

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Abstract

Predicting the future development of time series is of interest in different areas of computational biology. The time series in biological models exhibit challenging characteristics such as chaotic or stochastic behavior. In this report, time series prediction is done on different kind of biological time series using deep neural networks. In our evaluation, we identify challenges and limitations of this approach, and compare different architectures of deep neural networks with regard to their performance.

1 Introduction

In various kind of diseases, it is important to choose the correct treatment at the current time point. Since the human body is too complex to be described directly using mathematics, special features are modeled with mathematical systems such as the Mackey-Glass equations for respiratory and hematopoietic diseases [5]. These models make use of available data about previous information in order to reduce the uncertainty under which a decision is done. This is especially important when the consequences of a decision are severe, for example the decision of breast cancer treatment after radiation. This particular problem can be addressed by using a stochastic differential equation, as done by Oroji *et al* [7].

In different kinds of biological models, chaotic behavior is observed. Even the Mackey-Glass equations obtain chaotic behavior for certain choices of their parameters, as shown by Fischer [2]. That means that even small perturbations of the time series prediction increase exponentially, which means that the time series prediction should be as robust as possible against noise.

This gets imporant in particular because another characterstic of dealing with biological data is the presence of noise. In this report, we show that different kinds of noise assumptions can impact the results of time series prediction. We evaluate all our investigated problems with regard to the popular *i.i.d.* random noise as well as memorizing random noise. Since mathematical models of biological systems can not describe their characteristics perfect and error-free, it is assumed that the latter type of noise is more realistic. But unfortunately this kind of noise increases the difficulty of time series prediction, even for simple mathematical functions, as seen in section 3.1.

The remaining report is structured as follows. First, we perform proper time series prediction on the *sine* function and show how different noise models impact the prediction ability of our network models. We conduct an evaluation how these results can be extended to the prediction of ordinary differential equations (**ODE**s) at the example of the differential equations of the harmonic oscillator. After that, we show how the stochasticity of noise impactss the possibility to predict deterministic chaotic time series at the example of the *Mackey Glass* time series. Last but not least, we report how the poor results of numerical approximations on stocastic differential equations (**SDE**s) can be explained based on our previous results.

2 Methods

We perform

3 Evaluation

3.1 Time series prediction of continuous functions

3.2 Time series prediction of ODEs

3.3 Mackey Glass time series prediction

In order to model diseases related to dynamic respiratory and hematopoietic diseases, Mackey *et al.* proposed the mackey-glass equations, a kind of first-order nonlinear differential delay equations [5]. If the delayed time ($x_\tau = x(t-\tau)$) exceeds the delay $\tau > 16.8$, then equation 1 behaves chaotic [2].

$$\frac{dx}{dt} = \beta \cdot \frac{x_\tau}{1 + x_\tau^n} \tag{1}$$

The first approach to predict the short-time behavior of chaotic time series was done by Farmer *et al.* who proposed a `local approximation` technique [3]. After improval of predictions using support vector machines by Mller *et al.* [6], the focus in research shifted towards artifical neural networks which enable even better predictions. Two of the latest developments are the usage of Wavelet Networks [1] and particle swarm optimization [4].

3.4 SDE time series prediction

4 Conclusion

References

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