MixedNilPotent

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$$P(\Omega, A, B, l) = A + \sum_{k=0}^{\infty} \sum_{m=0}^{k+1} \frac{\Omega^{k-m+1} A B^m}{(k+2)!}$$

We can split this series into the terms with omega and those without.

$$P(\Omega, A, B, l) = A + \sum_{k=0}^{\infty} \frac{AB^{k+1}}{(k+2)!} + \sum_{k=0}^{\infty} \sum_{m=0}^{k} \frac{\Omega^{k-m+1} A B^m}{(k+2)!}$$

$$P_0(\Omega, A, B, l) = A + \sum_{k=0}^{\infty} \frac{AB^{k+1}}{(k+2)!}$$

$$P_1(\Omega, A, B, l) = \sum_{k=0}^{\infty} \sum_{m=0}^{k} \frac{\Omega^{k-m+1} A B^m}{(k+2)!}$$

$$P(\Omega, A, B, l) = P_0(\Omega, A, B, l) + P_1(\Omega, A, B, l)$$

Since

$$B^l = 0$$

$$P_0(\Omega, A, B, l) = A + \sum_{k=0}^{l-2} \frac{AB^{k+1}}{(k+2)!}$$

$$P_0(\Omega, A, B, l) = A \sum_{m=0}^{l-1} \frac{B^m}{(m+1)!}$$

We can reoder the indices of the last term so that the powers of B are in the outer summation:

$$k' = k = m$$

$$P_1(\Omega, A, B, l) = \sum_{m=0}^{\infty} \left[\sum_{k'=0}^{\infty} \frac{\Omega^{k'+1}}{(k'+m+2)!} \right] AB^m$$

Since

$$B^{l+1} = 0$$

$$P_1(\Omega, A, B, l) = \sum_{m=0}^{l-1} \left[\sum_{k=0}^{\infty} \frac{\Omega^{k+1}}{(k+m+2)!} \right] AB^m$$

$k = 2k'$

$k = 2k''$

$$P_1(\Omega, A, B, l) = \sum_{m=0}^{l-1} \left[\sum_{k'=0}^{\infty} \frac{\Omega^{2k'+1}}{(2k'+m+2)!} + \sum_{k'=0}^{\infty} \frac{\Omega^{2k''+2}}{(2k''+m+3)!} \right] AB^m$$

$$P_1(\Omega, A, B, l) = \sum_{m=0}^{l-1} \left[\sum_{k'=0}^{\infty} \frac{(-1)^k \theta^{2k} \Omega}{(2k'+m+2)!} + \sum_{k'=0}^{\infty} \frac{(-1)^k \theta^{2k} \Omega^2}{(2k''+m+3)!} \right] AB^m$$

$$c_m(\theta) = \sum_{k=0}^{\infty} \frac{(-1)^k \theta^{2k}}{(2k+m+2)!}$$

$$P_1(\Omega, A, B, l) = \Omega A \sum_{m=0}^{l-1} c_m(\theta) B^m + \Omega^2 A \sum_{m=0}^{l-1} c_{m+1}(\theta) B^m$$

$$P(\Omega, A, B, l) = A \sum_{m=0}^{l-1} \frac{B^m}{(m+1)!} + \Omega A \sum_{m=0}^{l-1} c_m(\theta) B^m + \Omega^2 A \sum_{m=0}^{l-1} c_{m+1}(\theta) B^m$$

$$c_m(\theta) = \sum_{k=0}^{\infty} \frac{(-1)^k \theta^{2k}}{(2k+m+2)!}$$

$$B^l = 0$$