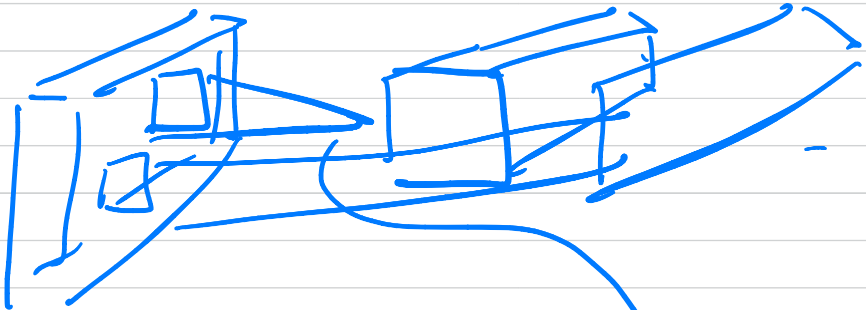
FYS5429/9429, Lecture February 13, 2025

FYS 5429/9429, Feb 13, 2025

CNNs basic ingredients

1) Several convolutional in parallel



... many scaled down copies

$28 \times 28 \times 1$

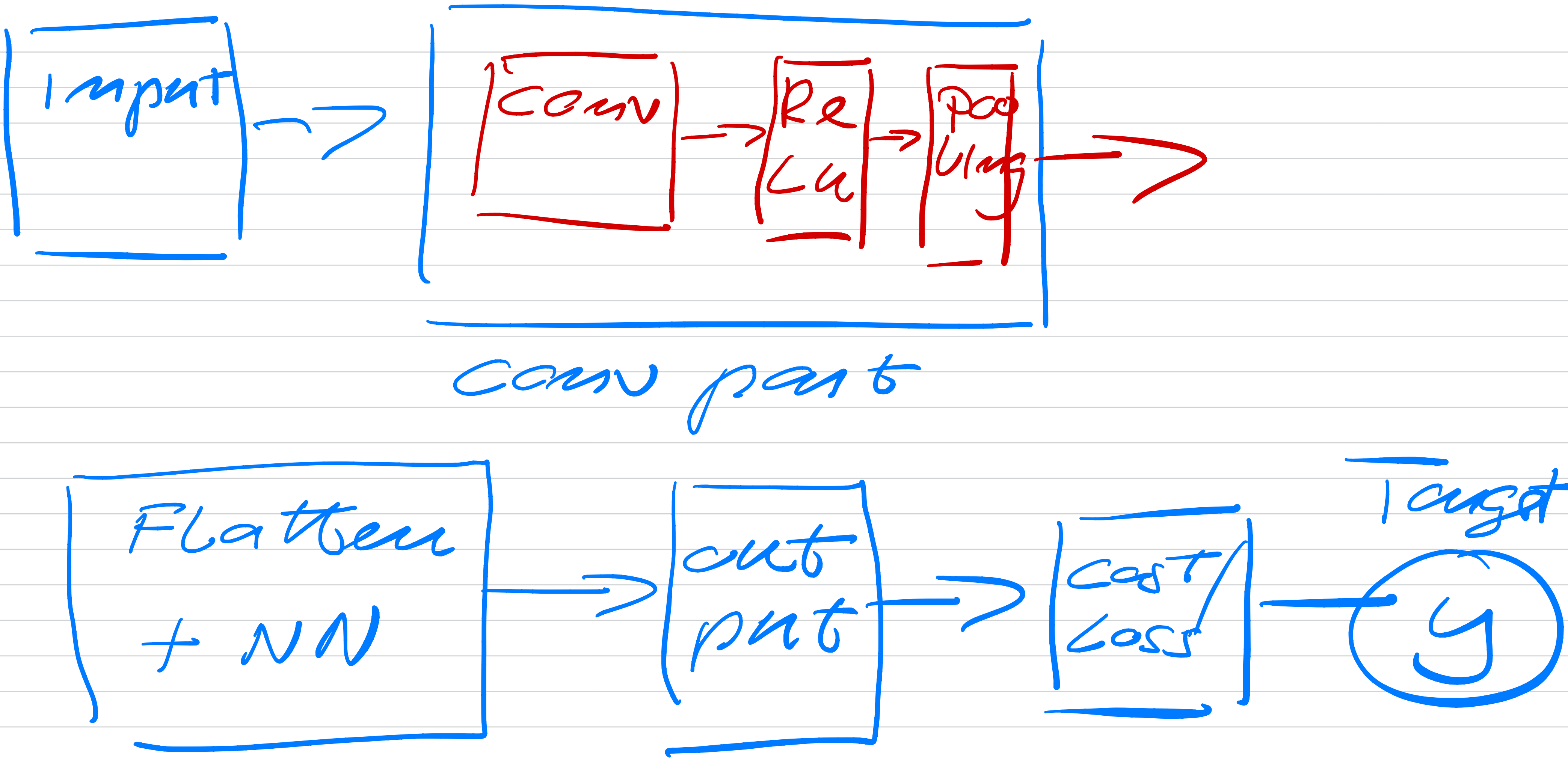
Filter

parameters to be trained by Backpropagation

2) Detector stage or
ReLU activation stage
(only hyperparameters)

3) Pooling stage (hyper parameters)
- Max pooling
- average pooling
scale further down image

4) Flattening stage + NN
Train with Back propagation.


Input
Conv
ReLu
Pooling
conv part
Flatten + NN
output
Cost/Loss
Target
y

Convolution stage

input X Filter W

$s = 1$ s = stride

3x3

x_{00}	x_{01}	x_{02}
x_{10}	x_{11}	x_{12}
x_{20}	x_{21}	x_{22}

w_{00}	w_{01}
w_{10}	w_{11}

2x2

output

$$\begin{bmatrix} x_{00}w_{00} + x_{01}w_{01} + x_{10}w_{10} + x_{11}w_{11} & x_{01}w_{00} + x_{02}w_{01} + x_{11}w_{10} + x_{12}w_{11} \\ x_{10}w_{00} + x_{11}w_{01} + x_{20}w_{10} + x_{21}w_{11} & x_{11}w_{00} + x_{12}w_{01} + x_{21}w_{10} + x_{22}w_{11} \end{bmatrix}$$

with a bias 4 (weights) + 1
parameters to train

N = Dim of x $(N \times N)$

F = Dim of W $(F \times F)$

output dimensionality

$$N_2 = (N_1 - F)/S + 1$$

$N_1 = 3$ $F = 2$ $S = 1$

$N_2 = 2$

with padding P

$$N_2 = (N_1 - F + 2P)/S + 1$$

Example 2

$N_1 = 32$ input volume

$32 \times 32 \times 3$

10 copies through 5×5

Filter, $P = 0$, $S = 1$

$$N_2 = (32-5)/1 + 1 = 28$$

28 x 28 x 10 replicate
with colours for each Filter
5 x 5 x 3 + 1 = 76
parameters
Bias

with 10 images 760
parameters

Require 4 new hyperparameters

- K = number of filter
- F = spatial extent
- S = stride
- P = amount of padding

input volume :

$W_1 \times H_1 \times D_1$

width heigth Depth

produces an output

$$W_2 \times H_2 \times D_2$$

$$W_2 = (W_1 - F + 2P)/S + 1$$

$$H_2 = (H_1 - F + 2P)/S + 1$$

Typical choices

$F = 3 \quad S = 1 \quad P = 1$

$F = 5 \quad S = 1 \quad P = 2$

$F = 5 \quad S = 2 \quad P =$ open

convolution, Example

$$p(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2$$

$$s(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3$$

$$z(t) = p(t) \cdot s(t)$$

$$= \delta_0 + \delta_1 t + \delta_2 t^2 + \delta_3 t^3 + \delta_4 t^4 + \delta_5 t^5$$

$$\delta_0 = \alpha_0 \beta_0 \; ; \; \delta_1 = \alpha_1 \beta_0 + \alpha_0 \beta_1$$

$$\delta_2 = \alpha_0 \beta_2 + \alpha_1 \beta_1 + \alpha_2 \beta_0$$

$$\delta_3 = \alpha_1 \beta_2 + \alpha_2 \beta_1 + \alpha_0 \beta_3$$

$$\delta_4 = \alpha_2 \beta_2 + \alpha_1 \beta_3$$

$$\delta_5 = \alpha_2 \beta_3$$

$\alpha_i = 0$ except for $i = \{0, 1, 2\}$

$\beta_i = 0$ — , — $\{0, 1, 2, 3\}$

$$\delta_j = \sum_{i=-\infty}^{\infty} \alpha_i \beta_{j-i}$$

$$= (\alpha * \beta)_j$$

convolution

$$y(t) = \int_{s \in D} x(s)\, w(t-s)\, ds$$

$$= (x * w)$$

$$\delta = \begin{bmatrix} \alpha_0 & 0 & 0 & 0 \\ \alpha_1 & \alpha_0 & 0 & 0 \\ \alpha_2 & \alpha_1 & \alpha_0 & 0 \\ 0 & \alpha_2 & \alpha_1 & \alpha_0 \\ 0 & 0 & 0 & \alpha_2 \end{bmatrix} \cdot \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$$

$$\delta = \begin{bmatrix} \beta_0 & 0 & 0 \\ \beta_1 & \beta_0 & 0 \\ \beta_2 & \beta_1 & \beta_0 \\ \beta_3 & \beta_2 & \beta_1 \\ 0 & \beta_3 & \beta_2 \\ 0 & 0 & \beta_3 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix}$$

Toeplitz matrix

$$A = \begin{bmatrix} a_0 & 0 & 0 \\ a_1 & a_0 & 0 \\ a_2 & a_1 & a_0 \\ a_3 & a_2 & a_1 \\ 0 & a_3 & a_2 \\ 0 & 0 & a_3 \end{bmatrix}$$

$$a_{ij} = a_{i+j,\, j+1} = a_{i-j}$$

replace β with x (input)

— " — α with w (filter)

— " — δ with y (output)

$$y(i) = (x * w)(i) = \sum_{k=0}^{k=m-1} w(k)\, x(i-k)$$

$m = 3$

$$w = \{ \underset{\alpha_0}{w(0)}, \underset{\alpha_1}{w(1)}, \underset{\alpha_2}{w(2)} \}$$

$$x = \{ x(0), x(1), x(2), x(3) \}$$

For specific value -i-
we get $x(-1)$ and $x(-2)$

increase size of x

$n = 4 \Rightarrow n + 2P$

$P = 2$

$x(0) = 0$; $x(1) = 0$

$x(2) = \beta_0 (= x(0))$; $x(3) = \beta_1$

$x(4) = \beta_2$; $x(5) = \beta_3$

$x(6) = x(7) = 0$

$$y(i) = \sum_{k=0}^{k=m-1} w(k)\, x(i+(m-1)-k)$$

$$y(4) = x(6)\,w(0) + x(5)\,w(1) + x(4)\,w(2)$$

$$= 0 \cdot \alpha_0 + \beta_3 \alpha_1 + \beta_2 \alpha_2$$

$$y(5) = x(7)\,w(0) + x(6) \cdot w(1) + x(5)\,w(2)$$

$$= \alpha_2 \beta_3$$

redefine $w \Rightarrow \tilde{w}$

$$\tilde{w}(0) = w(2) = \alpha_2$$

$$\tilde{w}(1) = w(1) = \alpha_1$$

$$\tilde{w}(2) = w(0) = \alpha_0 \Rightarrow$$

$$y(i) = X(i : i+(m-1))\,\tilde{w}$$