FYS-STK3155/4155, lecture November 3, 2025

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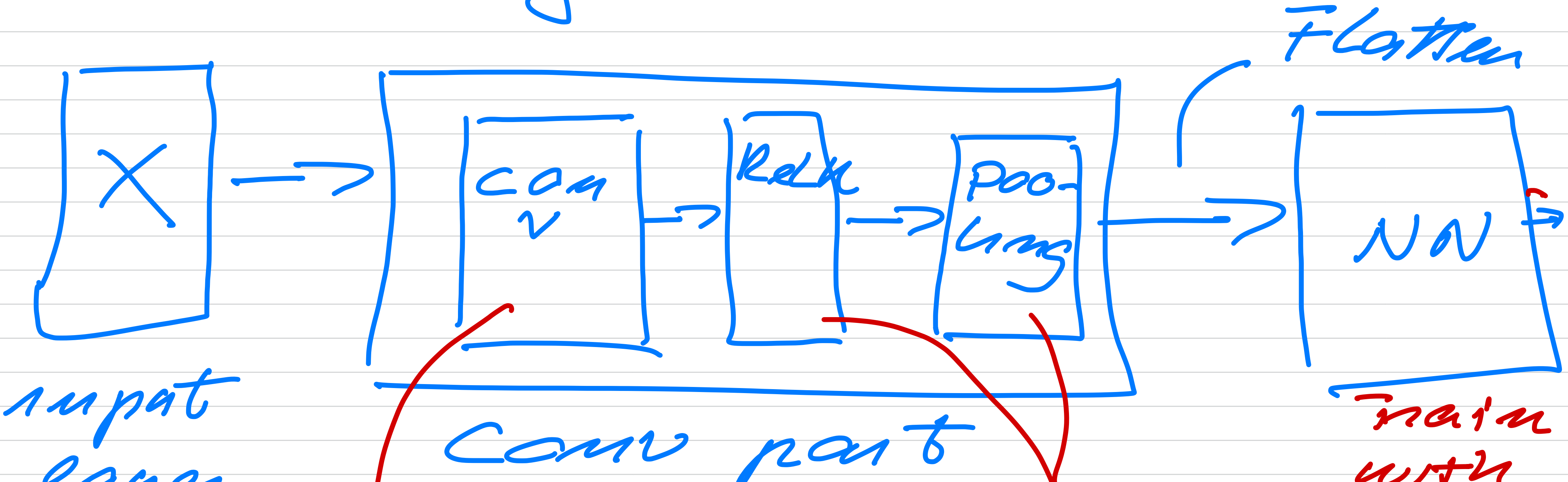
Basics of CNNs



Filter/Kernel parameters to train by backprop.

hyper parameters

output

X 3x3 matrix

W 2x2 filter/kernel

$$\underbrace{\begin{bmatrix} x_{00} & x_{01} & x_{02} \\ x_{10} & x_{11} & x_{12} \\ x_{20} & x_{21} & x_{22} \end{bmatrix}}_{X} * \begin{bmatrix} w_{00} & w_{01} \\ w_{10} & w_{11} \end{bmatrix} \qquad (X * W)$$

$$\begin{bmatrix} x_{00}w_{00} + x_{01}w_{01} + x_{10}w_{10} + x_{11}w_{11} & x_{01}w_{00} + x_{02}w_{01} + x_{11}w_{10} + x_{12}w_{11} \\ x_{10}w_{00} + x_{11}w_{01} + x_{20}w_{10} + x_{22}w_{11} & x_{11}w_{00} + x_{12}w_{01} + x_{21}w_{10} + x_{22}w_{11} \end{bmatrix}$$

STRIDE $= 1 = S$

hyper parameter

Padding $P = 0 \quad (0, 1, 2)$
hyperparameter,

input $N \times N \; (\times 3)$

Filter $F \times F$

output $N_2 = (N - F)/S + 1$

$N = 3 \quad F = 2 \quad S = 1$

$N_2 = 2 \quad (2 \times 2)$

with padding

$N_2 = (N - F + 2P)/S + 1$

Example 2

$N = 32$

input $32 \times 32 \times 3$

10 copies of original

Filter 5×5, $P = 0$, $S = 1$

$$N_2 = (32-5)/1 + 1 = 28$$

with colors for each filter

$$5 \times 5 \times 3 + \underbrace{1}_{\text{Bias}} = 76 \text{ parameters}$$

in total $76 \times 10 = 760$

Require 4 new parameters (hyperparameters)

- K = # of filters/kernels
- F = spatial extent of filter
- S = stride
- P = amount of padding

Typical choice

$F=3$ $\quad S=1$ $\quad P=1$

$F=5$ $\quad S=1$ $\quad P=2$ or 1

$F=5$ $\quad S=2$ $\quad P=$ open

Example

$$y(t) = (x * w) = \int_{s\in D} x(s)\, w(t-s)\, ds$$

$$p(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2$$

$$s(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3$$

$$z(t) = p(t) \cdot s(t)$$

$$= \delta_0 + \delta_1 t + \delta_2 t^2 + \delta_3 t^3 + \delta_4 t^4 + \delta_5 t^5$$

$$\delta_0 = \alpha_0 \beta_0 \; ; \quad \delta_1 = \alpha_1 \beta_0 + \alpha_0 \beta_1$$

$$\delta_2 = \alpha_0 \beta_2 + \alpha_1 \beta_1 + \alpha_2 \beta_0$$

$$\delta_3 = \alpha_1 \beta_2 + \alpha_2 \beta_1 + \alpha_0 \beta_3$$

$$\delta_4 = \alpha_2 \beta_2 + \alpha_1 \beta_3 \; ; \quad \delta_5 = \alpha_2 \beta_3$$

$\alpha_i = 0$ except $i = \{0, 1, 2\}$

$\beta_i = 0 \; - \; 1 \; -$ $i = \{0, 1, 2, 3\}$

$$\delta_j = \sum_{i=-\infty}^{\infty} \alpha_i \beta_{j-i}$$

$$= (\alpha * \beta)_j$$

$$\delta = \underbrace{\begin{bmatrix} \beta_0 & 0 & 0 \\ \beta_1 & \beta_0 & 0 \\ \beta_2 & \beta_1 & \beta_0 \\ \beta_3 & \beta_2 & \beta_1 \\ 0 & \beta_3 & \beta_2 \\ 0 & 0 & \beta_3 \end{bmatrix}}_{\text{Toeplitz matrix}} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix}$$

replace β with x (input)

— " — α with w (weight filter)

— " — δ with y (output)

$$y(i) = (x * w)(i)$$

$$= \sum_{k=0}^{k=m-1} w(k)\, x(i-k).$$

$$m = 3$$

$$\alpha_0 \qquad \alpha_1 \qquad \alpha_2$$

$$w = \{ w(0),\ w(1),\ w(2) \}$$

$$x = \{ x(0), x(1), x(2), x(3) \}$$

For specific -i- values $x(-1)$ and $(x-2)$, not defined

increase size of x

$n = 4 \Rightarrow n + 2P$

padding

$P = 2$

$x(0) = 0$; $x(1) = 0$

$x(2) = \beta_0$; $x(3) = \beta_1$

$x(4) = \beta_2$; $x(5) = \beta_3$

$x(6) = x(7) = 0$

$$y(i) = \sum_{k=0}^{k=m-1} w(k) \times (i+(m-1)-k)$$

redefine

$$w \Rightarrow \tilde{w}$$

$$\tilde{w}(0) = w(2) = \alpha_2$$

$$\tilde{w}(1) = w(1) = \alpha_1$$

$$\tilde{w}(2) = w(0) = \alpha_0$$

$$y(i) = x(i : i+(m-1)) \cdot \tilde{w}$$