FYS-STK3155/4155, lecture
October 27, 2025

FYS-STK3155/4155 October 27

Solving differential eqs with Deep learning

$$\frac{dg}{dt} = -\gamma g(t)$$

$$\gamma > 0 \qquad t \in [0, \infty)$$

$$g_0 = g(\cancel{t}=0)$$

$$g(t) = g_0 \exp(-\gamma t)$$

$$f(t, g(t), g'(t))$$

$$f(t, g, g') = \frac{dg}{dt} - \gamma g(t) = 0$$

in general

$$f(t, g, g', g^{(2)}, \dots g^{(m)})$$

$$g \Rightarrow g_T(t) = h_1(t) + h_2(t, \underline{N(t, \theta)})$$

trial

Boundary + initial conditions

neural network with parameters $\underline{\theta}$

$$g(t) = g_0 \exp(-\gamma t)$$

$$g_T(t) = \underbrace{g_0}_{\text{initial conditions}} + t N(t, \theta)$$

$t = 0$

$g_T(0) = g_0$

$$C(\theta) = \frac{1}{n} \sum_{i=0}^{n-1} \left[f(t, g_T, g_T') \right]^2$$

$$\hat{\theta} = \arg\min_{\theta} C(\theta)$$

Taylor expansion

$$g(t \pm \Delta t) = g(t) \pm \Delta t \frac{dg}{dt}\Big|_t + \frac{(\Delta t)^2}{2!} \frac{d^2 g}{dt^2}\Big|_t + O(\Delta t^3)$$

Discretize

$$t \to t_i = t_0 + i\Delta t$$

$$i = 0, 1, 2, \ldots n$$

$$\Delta t = \frac{t_n - t_0}{n}$$

$$g(t + \Delta t) \Rightarrow g(t_i + \Delta t) = g_{i+1}$$

$$g_{i+1} = g_i + \Delta t\, g_i' + O(\Delta t^2)$$

$$g_{i+1}' \simeq g_i' + \Delta t\, g_i'' \Rightarrow$$

$$g_i'' \cong \frac{g_{i+1}' - g_i'}{\Delta t}$$

$$\left.\begin{aligned} g_1 &= g_0 - \gamma g_0 \\ g_2 &= g_1 - \gamma g_1 \\ &\vdots \\ g_n &= g_{n-1} - \gamma g_{n-1} \end{aligned}\right\} \text{Euler's method}$$

$$-g''(x) = f(x)$$

known function

$$x \in [0, 1]$$

$$g(x=0) = g(x=1) = 0$$

$$f(x) = (3x + x^2)\exp(x)$$

$$g(x) = x(1-x)\exp(x)$$

$$g_T(x) = x(1-x)\,N(x, \epsilon)$$

$$g''(x) \simeq \frac{g(x+\Delta x)+g(x-\Delta x)-2g(x)}{(\Delta x)^2}$$

$$g(x) \Rightarrow g(x_i) = g_i$$

$$-g_i'' = -\left(\frac{g_{i+1}+g_{i-1}-2g_i}{(\Delta x)^2}\right)$$

$$= f(x_i) = f_i$$

$$g_0 = g_n = 0$$

$$g = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_{n-1} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 & & 0 \\ -1 & 2 & -1 & \\ & -1 & \ddots & 1 \\ 0 & & -1 & 2 \end{bmatrix}$$

$$A \cdot g = \Delta x^2 \cdot f$$

$$g = (A^{-1} \cdot f)\, \Delta x^2$$

CNNs overarching view