Cellular Automata - An Introduction

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Major contributors: John von Neumann
Stanislaw Ulam
John Conway
Stephen Wolfram

October 3, 2013

Hello

Part I

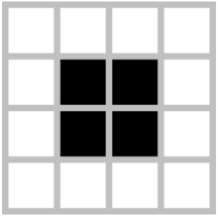
Conway's Game of Life

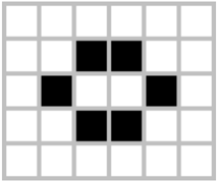
The rules

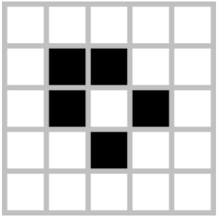
- *Loneliness*[1]*:* A live cell with 1 or fewer neighbors dies in the next iteration
- *Stasis:* A live cell with 2 or 3 neighbors lives on
- *Overpopulation:* A live cell with 4 or more neighbors dies
- *Reproduction:* A dead cell with exactly 3 neighbors becomes alive in the next generation

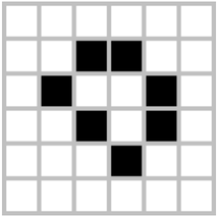
[1] "Loneliness" is just a convenient label. Conway's Game of Life is not intended to represent real life. However these rules can be viewed from a biological standpoint, which is one of the reasons why the automaton is called "Life"

Still lifes

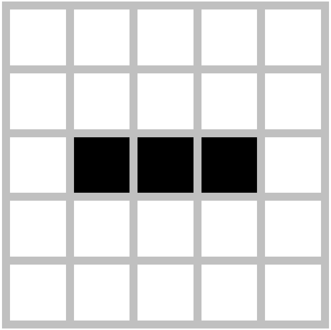




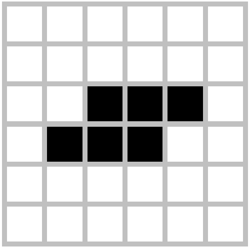




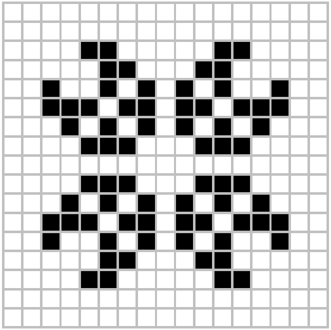
Oscillators



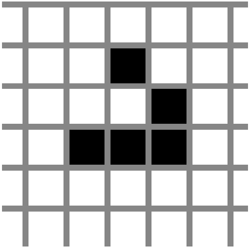
Oscillators



Oscillators



Glider



Motivation behind the rules

While the biological analog seems to be a likely motivation for choosing this particular rule set, Conway's motivations are (Verbatim from the original article[2])

- There should be no initial pattern for which there is a simple proof that the population can grow without limit.
- There should be initial patterns that apparently do grow without limit.
- There should be simple initial patterns that grow and change for a considerable period of time before coming to end in three possible ways: fading away completely (from overcrowding or becoming too sparse), settling into a stable configuration that remains unchanged thereafter, or entering an oscillating phase in which they repeat an endless cycle of two or more periods.

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[2] Gardner, M. (1970). Mathematical Games - The fantastic combinations of John Conway's new solitaire game "life". In *Scientific American* 223 (pp 120-123.)

Part II

Generalized cellular automata

Building blocks for a cellular automaton

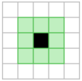
To build a cellular automaton, one needs to make a suitable choice for the following:

- A suitable grid type. For example, we can have a 1D/2D/3D rectangular grid, a hexagonal grid, or even a Penrose tiling[3]
- The number of states a cell can have. This number must be finite for it to be a cellular automaton[4]
- The type of neighborhood being considered. E.g. Moore, von Neumann, Margolus.
- The rules. The rules let one calculate the next iteration of a cell given its current state and the state of its surroundings.

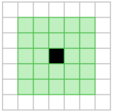
[3] Owens, N., & Stepney, S. (2010). The Game of Life rules on Penrose tilings: still life and oscillators. In *Game of Life Cellular Automata* (pp. 331-378). Springer London.

[4] Otherwise it is a continuous automaton

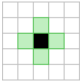
Neighborhood types



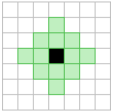
(a) Moore neighborhood, range 1



(b) Moore neighborhood, range 2



(c) von Neumann neighborhood, range 1



(d) von Neumann neighborhood, range 2

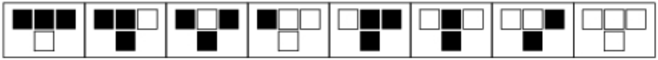
Rules

- If the evolution of a cell depends on only the sum of the states of the cell and those in its neighborhood, the rule is *totalistic*.
- If the evolution of a cell depends on only the sum of the states the cells in its neighborhood and the state of the cell itself, the rule is *outer totalistic*. (E.g. Life)
- Rules need not be totalistic
- Rules need not be deterministic!

Part III

Elementary cellular automata

Rule 110



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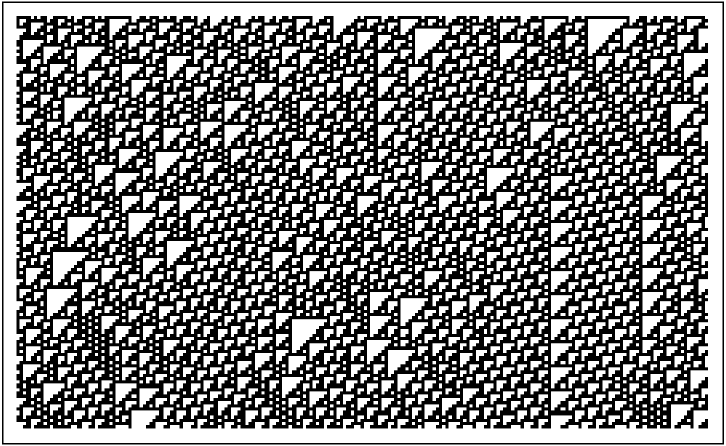
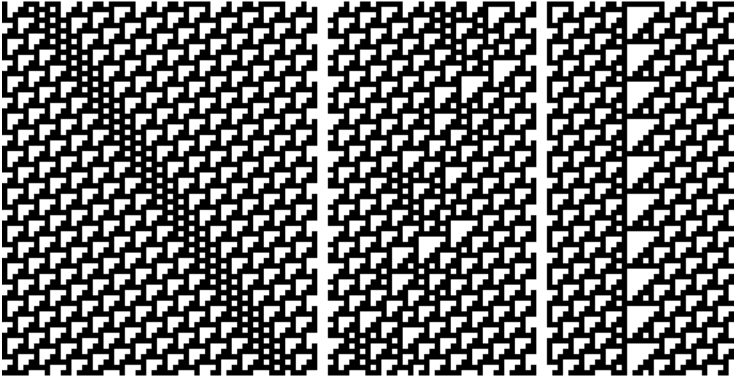
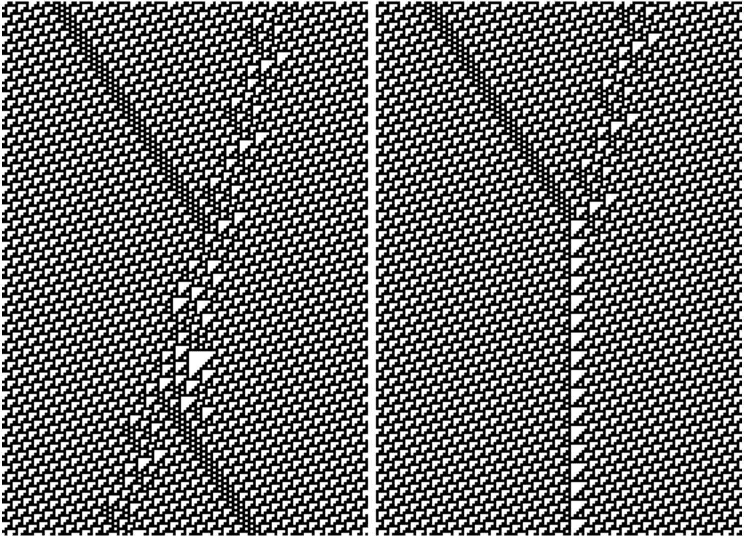


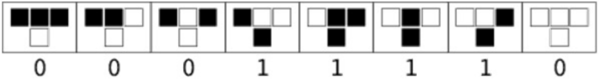
Gliders in 110

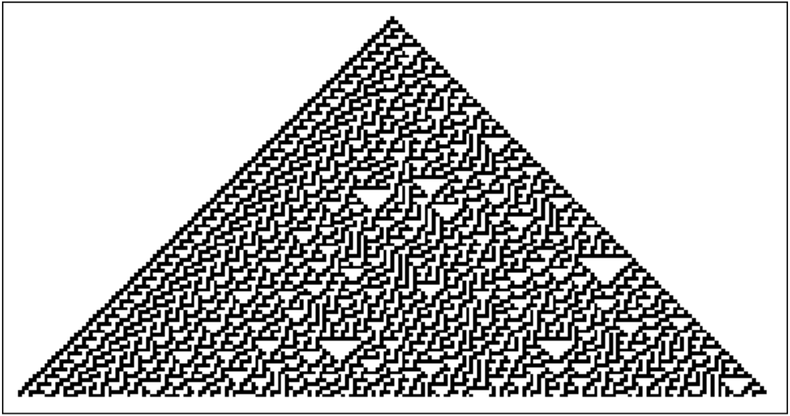


Interaction of gliders



Rule 30



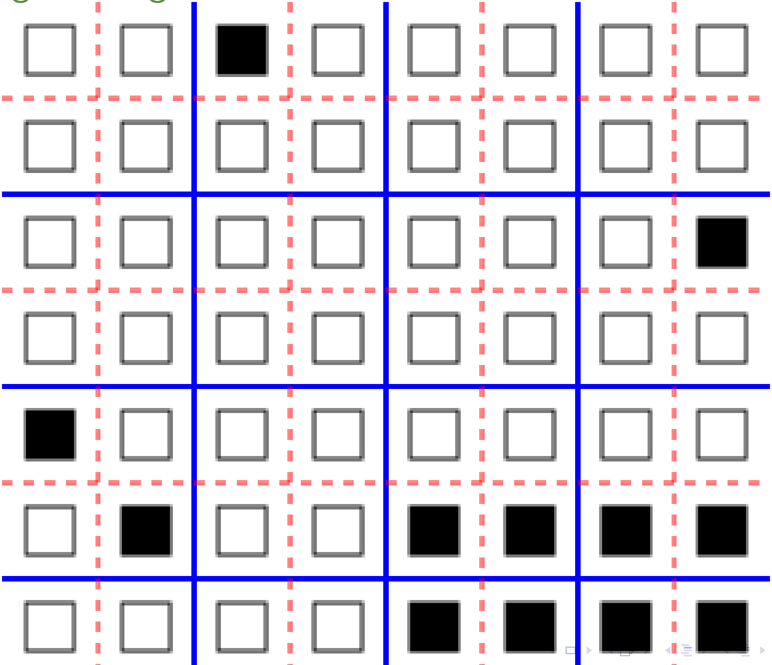


Rule 30 in nature

Part IV

Block Cellular automata

Margolus neighborhood



Part V

Applications

Cryptography

- Pseudo-random number generators
- Hash functions
- Encryptions

Modelling the real world

- Predator-prey interactions (stochastic)
- Oscillatory chemical reactions
- Lattice gases (reversible Margolus)
- Ising model (reversible Margolus)
- Crystal growth, shell patterns, other emergent phenomena

Miscellaneous

- Universal computation in theoretical CS
- Mental exercise; *Life* is one example of a CA which has been extensively researched and analyzed; with the discoveries of many new, interesting patterns.