1)假设log N为ln N，方便证明，既证明存在常数C使得 (ln N)^k<= CN
(ln N)^k < CN
==> (ln N)^k < ((CN)^(1/k)^k
==> (ln N) < (CN)^(1/k)
==> limit((ln N)/((CN)^(1/k))) < 1
洛必达法则
==> limit((1/n)/(C*(1/k)*(CN)^(1/k -1))) < 1
==> limit(1/C^(1/k)*N^(1/k)*(1/k)) < 1
因为 limit(1/C^(1/k)*N^(1/k)*(1/k)) = 0 < 1 故必存在常数C使得 (ln N)^k<= CN

2)证明:
(N!)^2 = (N*(N-1)*(N-2)*...1)*(1*2*3...N) >= N*N*N...N = N^N
log(N!) >= (1/2)Nlog N
N! <= N^N
log(N!) <= Nlog N
(1/2)Nlog N <= log(N!) <= Nlog N
所以 log(N!) 与 Nlog N 同阶

3)
T(1) = 1
T(2) = 2T(1) + 2log 2 = 2 + 2log 2
T(4) = 2T(2) + 4log 4 = 4 + 4log 2 + 4log 4
T(8) = 2T(4) + 8log 8 = 8 + 8log 2 + 8log 4 + 8log 8
..
T(N) = N + Nlog 2 + N*2*log 2 + N*3*log 2… + N*log N*log 2
= N + N*(1+2+3..log N)*log 2
= N*(1+log N)*log N/2*log 2

故 T(N) = 0(N*log^2 N)

4)

1| T(n) = 0(3^n) 理由 T(n) = 5*3^n

2| T(n) = 0(n!)
理由T(n) = 估算为 C*N!
即证明 C1*N! <= T(n) <= C2*N!
易得C1 = 1时成立
又 T(0) = 1 ,T(1) = 2 <= 3*1!,T(2) = 5 <= 3*2!,T(3) = 16 <= 3*3!,T(4) = 65 <= 3*4!..
易得C2 = 3时成立
有 T(n) = 0(n!)

3| T(n) = 0(3^(n + 1))
理由 T(n) = 2^n + 3*2^(n-1) + 3^2*2^(n-2) + .. +3^n*2^0 + 3^(n+1)
n*2^n + 3^(n + 1) <= T(n) <= n*3^n + 3^(n + 1)
n*2^n + 3^(n + 1) = 0(3^(n + 1))
n*3^n + 3^(n + 1) = 0(3^(n + 1))
易得 T(n) = 0(3^(n + 1))

4| T(n) = 0(2^n)
理由 T(n) = n^2 + 2*(n-1)^2 + 2^2*(n-2)^2 + 2^3*(n-3)^2 .. + 2^(n-1)*1^2 + 2^n
= 2^n*∑x^2/2^x
设S(n) = ∑x^2/2^x,利用错位相减2S-S易得S=6-1/2^(n-1)-1/2^(n-3)-n^2/2^n-(n-1)/2^(n-2) limit(s(n)) = 6,故T(n) = 0(2^n)

5| T(n) = 0(n^log3 5)
理由 master定理(一条)

6| T(n) = 0(n^2)
理由   master 定理（一条）

7| T(n) = 0(n)
理由   master 定理（一条）

8| T(n) = 0(n*((5/4)^log n))
理由   T(n) = T(1/2n) + T(3/4n) + n
        = n + n*([1/2] + [3/4]) + n*([1/2] + [3/4])^2 + n*([1/2] + [3/4])^3 + n*([1/2] + [3/4])^4 … + n*([1/2] + [3/4])^log n
        = n*((5/4)^log n)

9| T(n) = 0(n^2)
理由   master 定理（3 条）

10| T(n) = 0(n^(1/2))
理由   master 定理（3 条