

Assignment 1: Part 1 - Linear Regression

Task 1

1.1:

According to

$$h_{\theta}(x) = \theta_0 x_0 + \theta_1 x_1$$

Modify the [calculate_hypothesis.m](#):

```
theta_0 = theta(1);  
theta_1 = theta(2);  
hypothesis = X(training_example, 1) * theta_0 + theta_1 * X(training_example, 2);
```

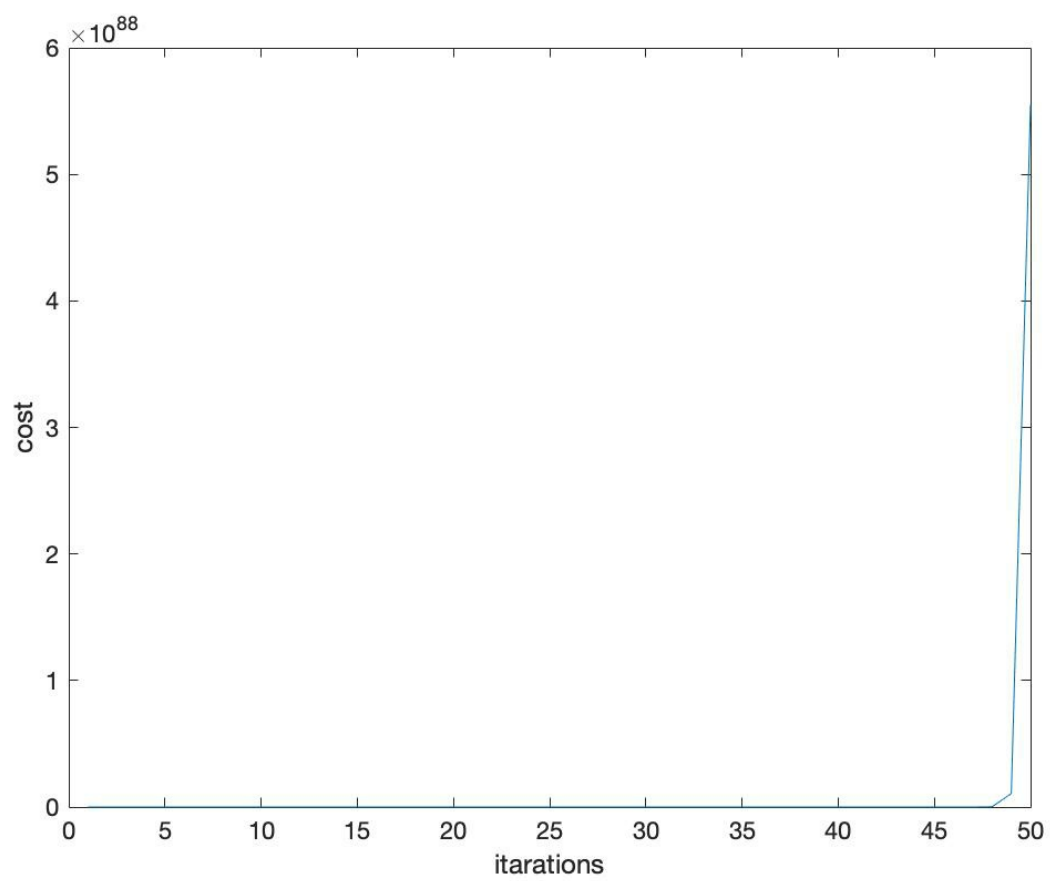
1.2:

Modify the [gradient_descent.m](#)

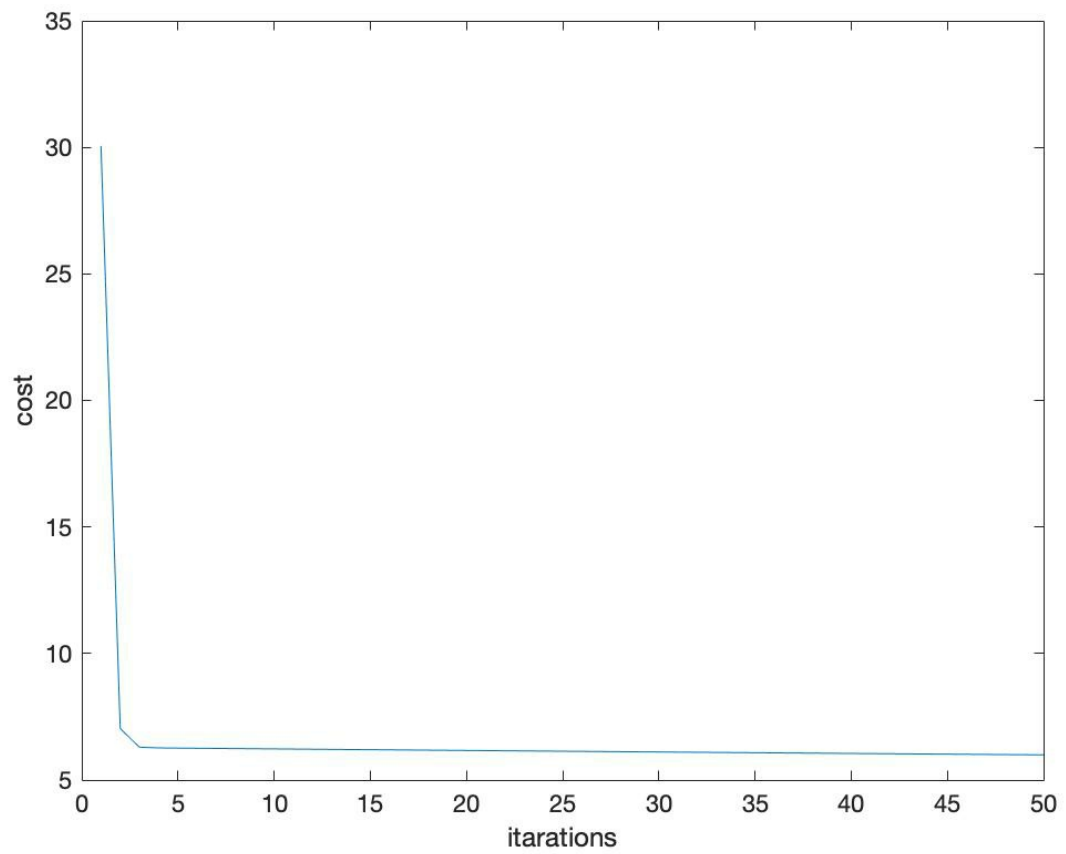
```
% hypothesis = X(i, 1) * theta(1) + X(i, 2) * theta(2);  
hypothesis = calculate_hypothesis(X, theta, i);
```

1.3:

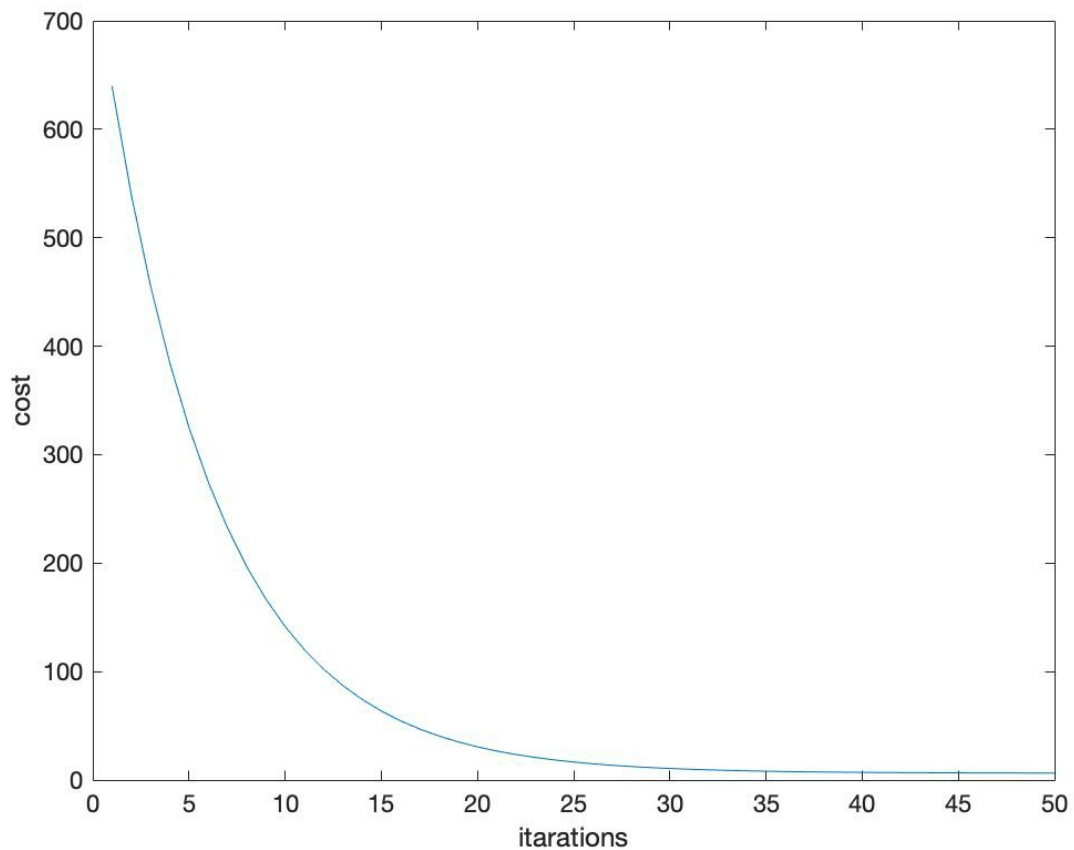
When the learning rate equals to 0.1:



When the learning rate equals to 0.01:



When the learning rate equals to 0.001:



Observations: Low learning rates caused slow convergence and needed more iterations. However, the cost graph shows that it is more likely to get the local optimum than high learning rates.

Task 2:

2.1:

According to

$$h_{\theta}(x) = \theta_0 x_0 + \theta_1 x_1 + \theta_2 x_2$$

Modify the [calculate_hypothesis.m](#):

```
function hypothesis = calculate_hypothesis(X, theta, training_example)
    %CALCULATE_HYPOTHESIS This calculates the hypothesis for a given X,
    %theta and specified training example

    theta_0 = theta(1);
    theta_1 = theta(2);
    theta_2 = theta(3);

    hypothesis = 1 * theta_0 + ...
    theta_1 * X(training_example, 2) + ...
    theta_2 * X(training_example, 3);

    %hypothesis = 0.0;
end
```

Modify the [gradient_descent.m](#), Add:

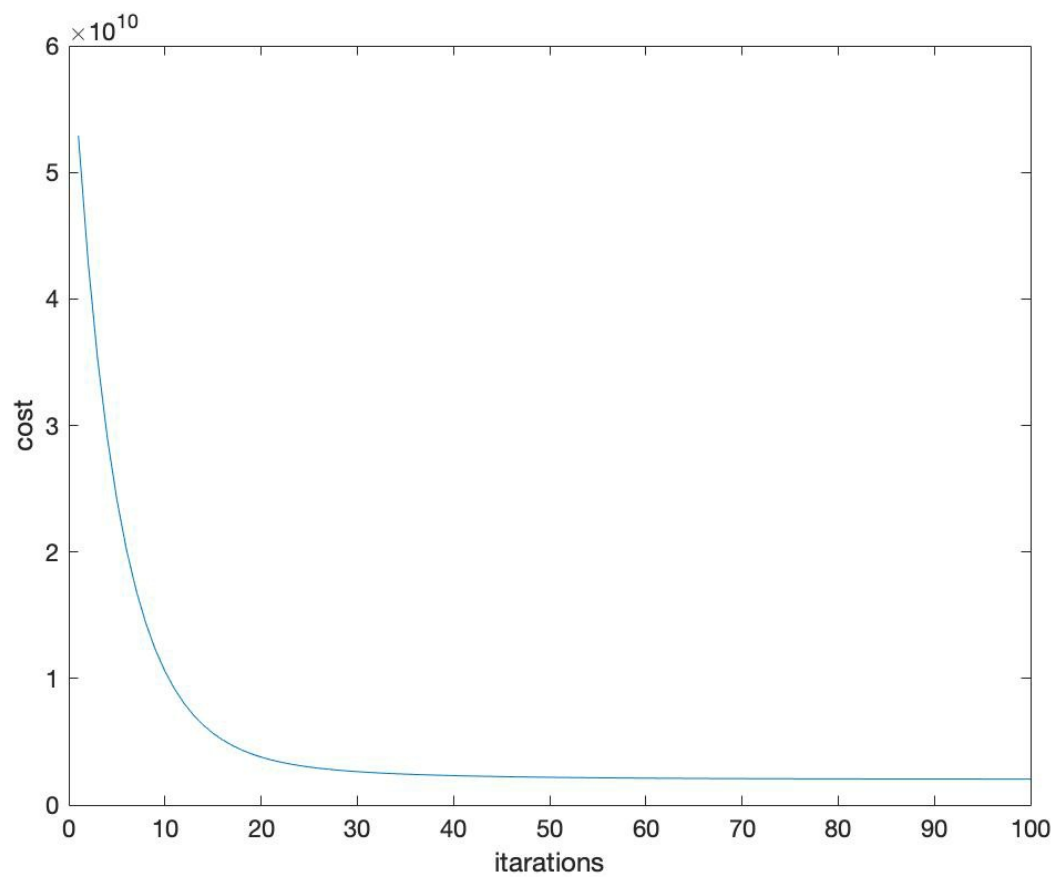
```
for i = 1:m
    %hypothesis = X(i, 1) * theta(1) + X(i, 2) * theta(2);
    %hypothesis = 1 * theta(1) + X(i, 2) * theta(2) + X(i, 3) * theta(3);
    hypothesis = calculate_hypothesis(X, theta, i);
    output = y(i);
    sigma = sigma + (hypothesis - output) * X(i, 3);
end

theta_2 = theta_2 - ((alpha * 1.0) / m) * sigma;

%update theta
theta = [theta_0, theta_1, theta_2];
```

2.2:

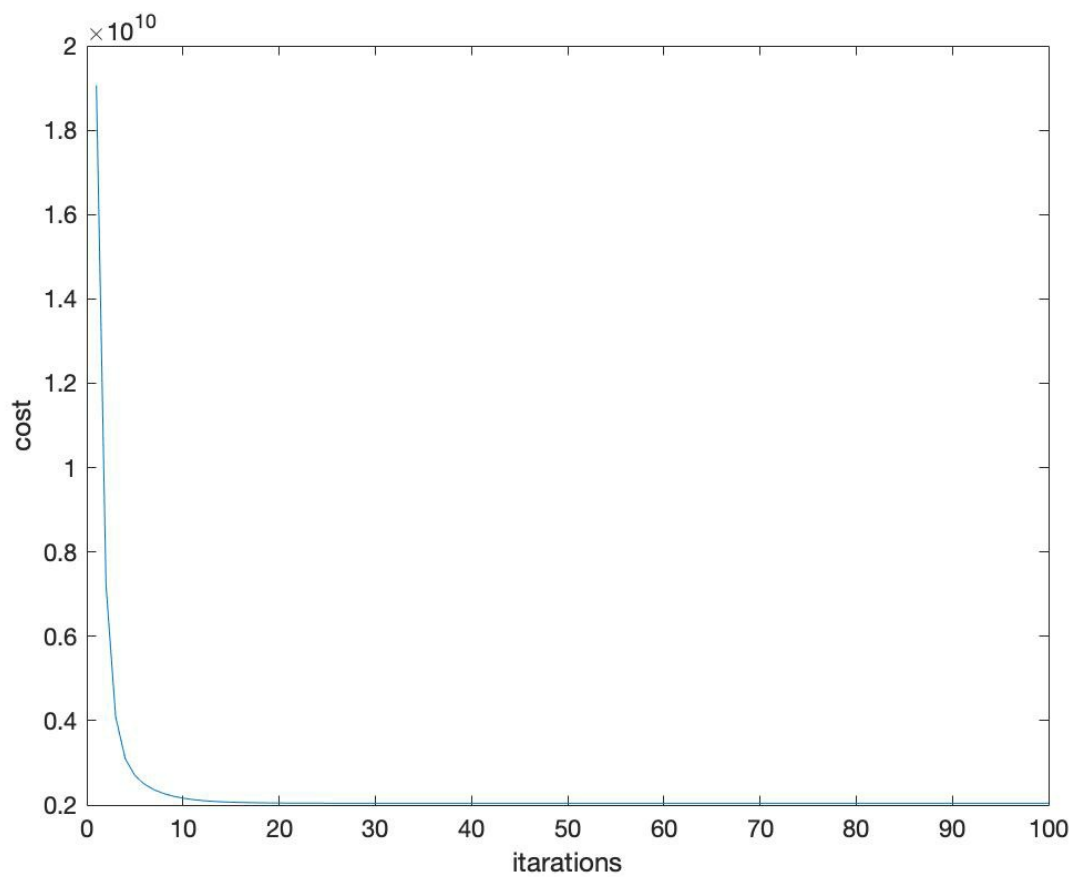
When alpha equals 0.1:



The theta

- $1.0\text{e}+05 * 3.4040$
- $1.0\text{e}+05 * 1.0708$
- $1.0\text{e}+05 * -0.0223$

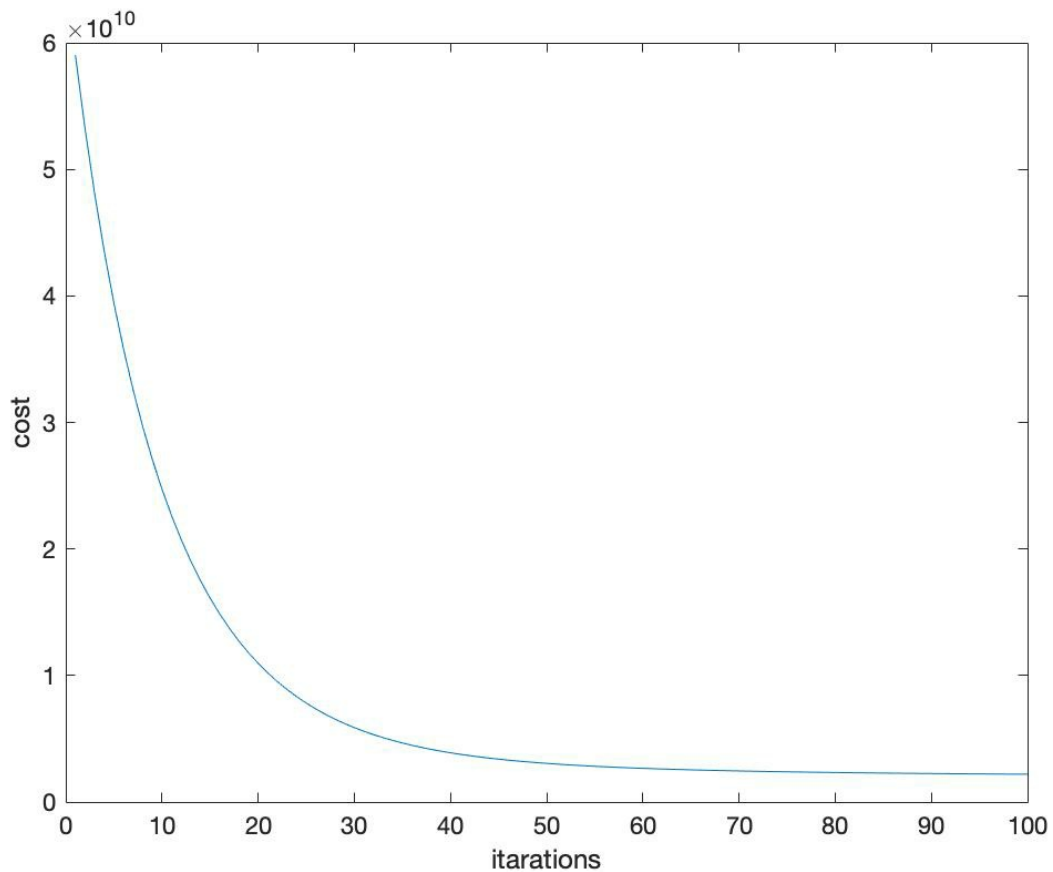
When alpha equals 0.5:



The theta

- $1.0\text{e}+05 * 3.4041$
- $1.0\text{e}+05 * 1.1063$
- $1.0\text{e}+05 * -0.0665$

When alpha equals 0.05:



The theta

- $1.0e+05 * 3.3840$
- $1.0e+05 * 0.9433$
- $1.0e+05 * 0.1360$

Observations: The alpha have strong effect on the theta value finding. At the end of the optimization, When the alpha equals to 0.5, the best theta value is :

- $1.0e+05 * 3.4041$
- $1.0e+05 * 1.1063$
- $1.0e+05 * -0.0665$

2.3:

According to :

$$X_{norm} = \frac{X - \text{mean}}{\text{standard deviation}} \quad (2.2)$$

the code modified to:

```
Xpredict = [3000,4];
Xnorm = (Xpredict - mean_vec)./std_vec;
Xnorm = [1,Xnorm];
Ypredict = sum(Xnorm .* theta);
disp(Ypredict);
```

- When 1650 sq.ft. 3 bedrooms: The predicts price is

2.9308e+05

- When 3000 sq.ft. 4 bedrooms: The predicts price is

4.7228e+05

Task 3:

3.1

Modify the [gradient_descent.m](#),

Add lamda :

```
function theta = gradient_descent(X, y, theta, alpha, iterations,l, do_plot)

...
cost_vector = [cost_vector; compute_cost_regularised(X, y, theta,l)];
...

end
```

Update theta 0 :

```
sigma = 0.0;

for i = 1:m
    %hypothesis = 1 * theta(1) + X(i, 1) * theta(2) + X(i, 2) * theta(3);
    hypothesis = calculate_hypothesis(X,theta,i);
    output = y(i);
    sigma = sigma + (hypothesis - output)*X(i, 1);
end

theta(1) = theta(1) - ((alpha * 1.0) / m) * sigma;
```

Update theta 1 and other theta :

```
sigma = 0.0;

for i = 1:m
    %hypothesis = X(i, 1) * theta(1) + X(i, 2) * theta(2);
    %hypothesis = 1 * theta(1) + X(i, 1) * theta(2) + X(i, 2) * theta(3);
    hypothesis = calculate_hypothesis(X,theta,i);
    output = y(i);
    sigma = sigma + (hypothesis - output) * X(i, 2);
end

theta(2) = theta(2)*(1 - alpha*l/m) - ((alpha * 1.0) / m) * sigma;
```

Modify the [calculate_hypothesis.m](#):

```
function hypothesis = calculate_hypothesis(X, theta, training_example)
    %CALCULATE_HYPOTHESIS This calculates the hypothesis for a given X,

    hypothesis =(X(training_example, 1)) * theta(1) +...
    theta(2) * (X(training_example, 2))^1 + ...
    theta(3) * (X(training_example, 3))^2+...
    theta(4) * (X(training_example, 4))^3+...
    theta(5) * (X(training_example, 5))^4+...
    theta(6) * (X(training_example, 6))^5;

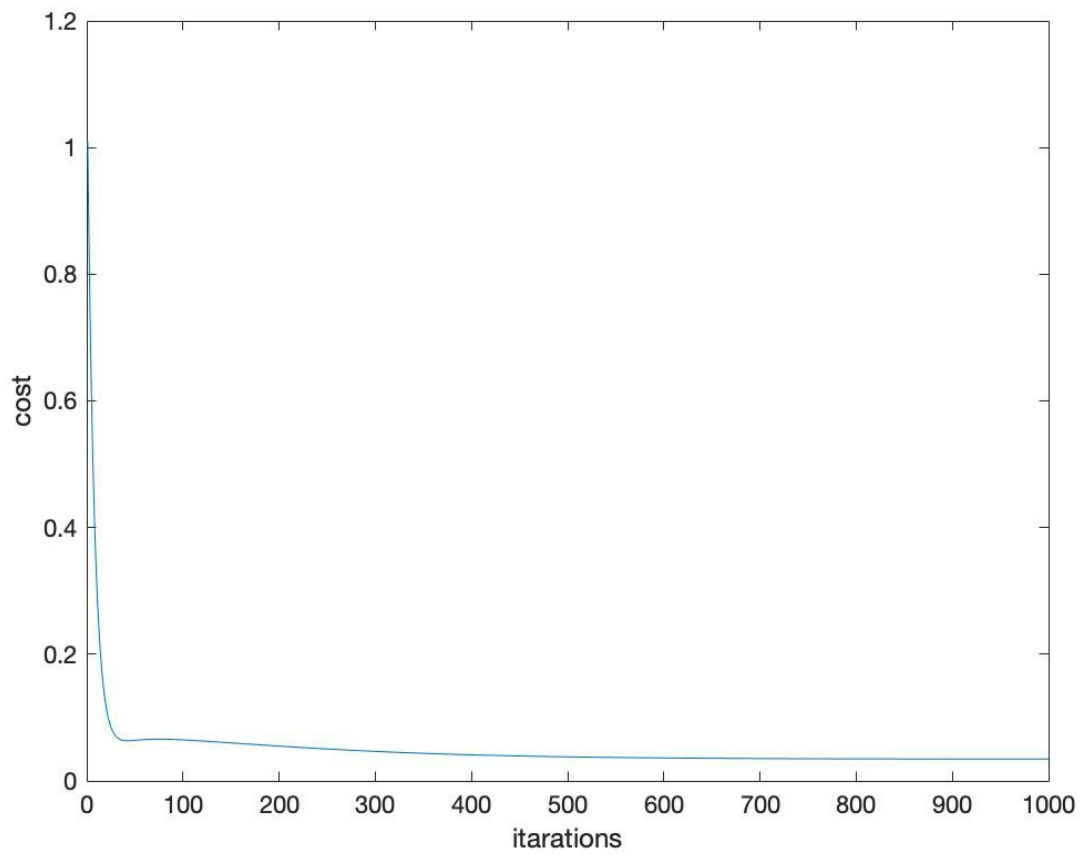
end
```

3.2

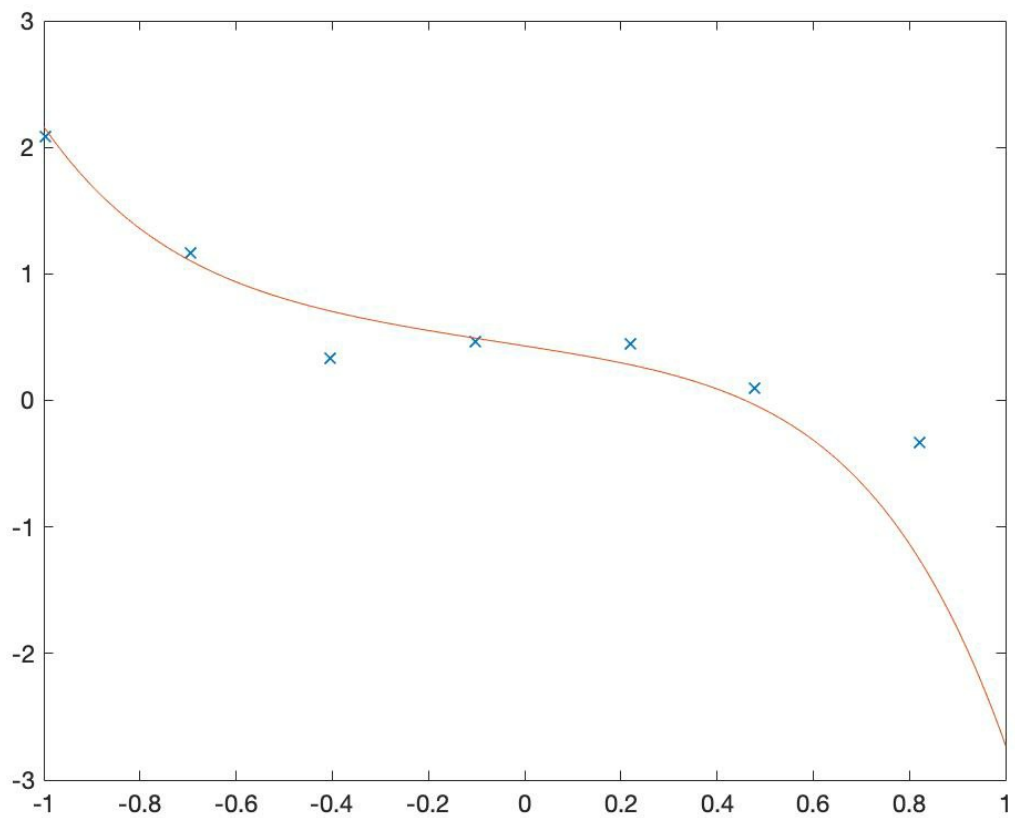
Changing alpha

When alpha equals 0.1, lamda 0.1

The cost graph:

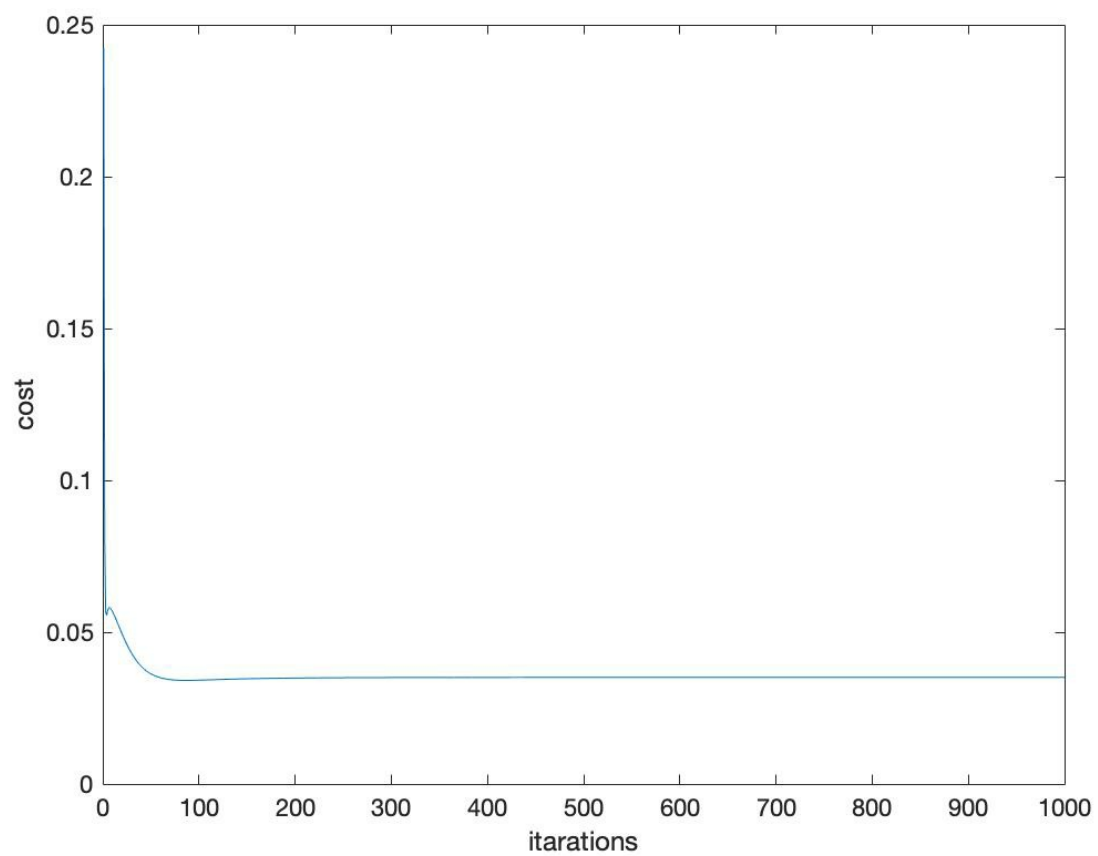


Curve:

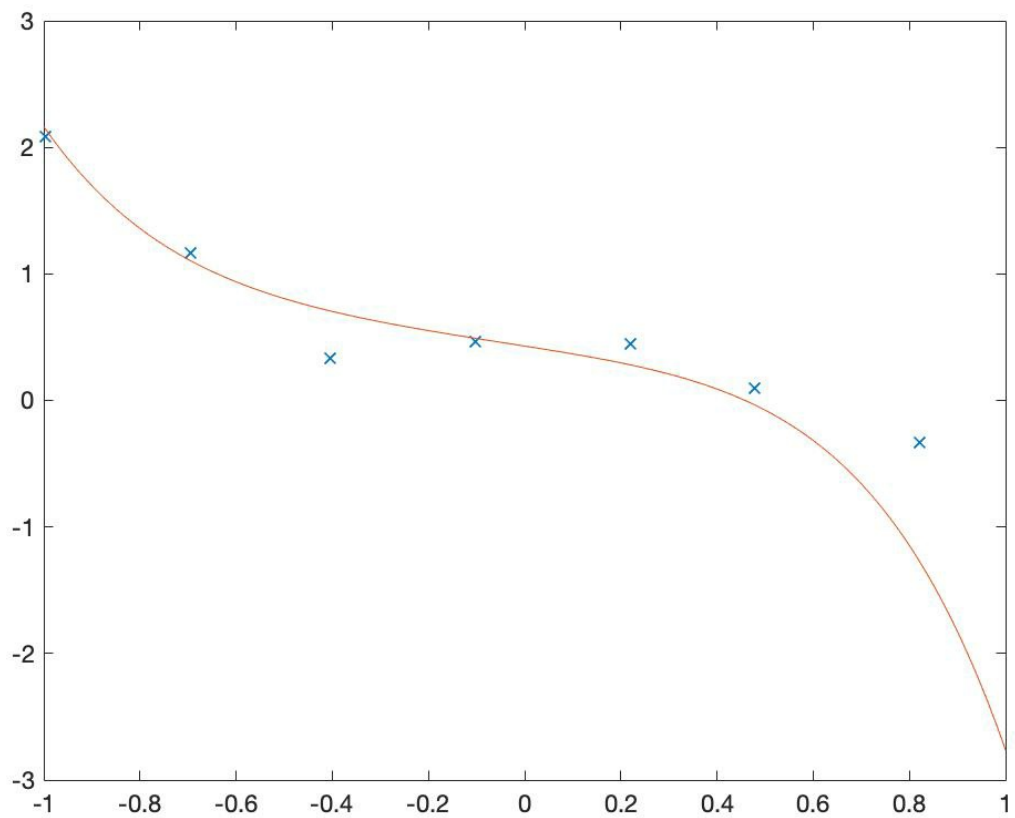


When alpha equals 0.2, lamda 0.1

The cost graph:

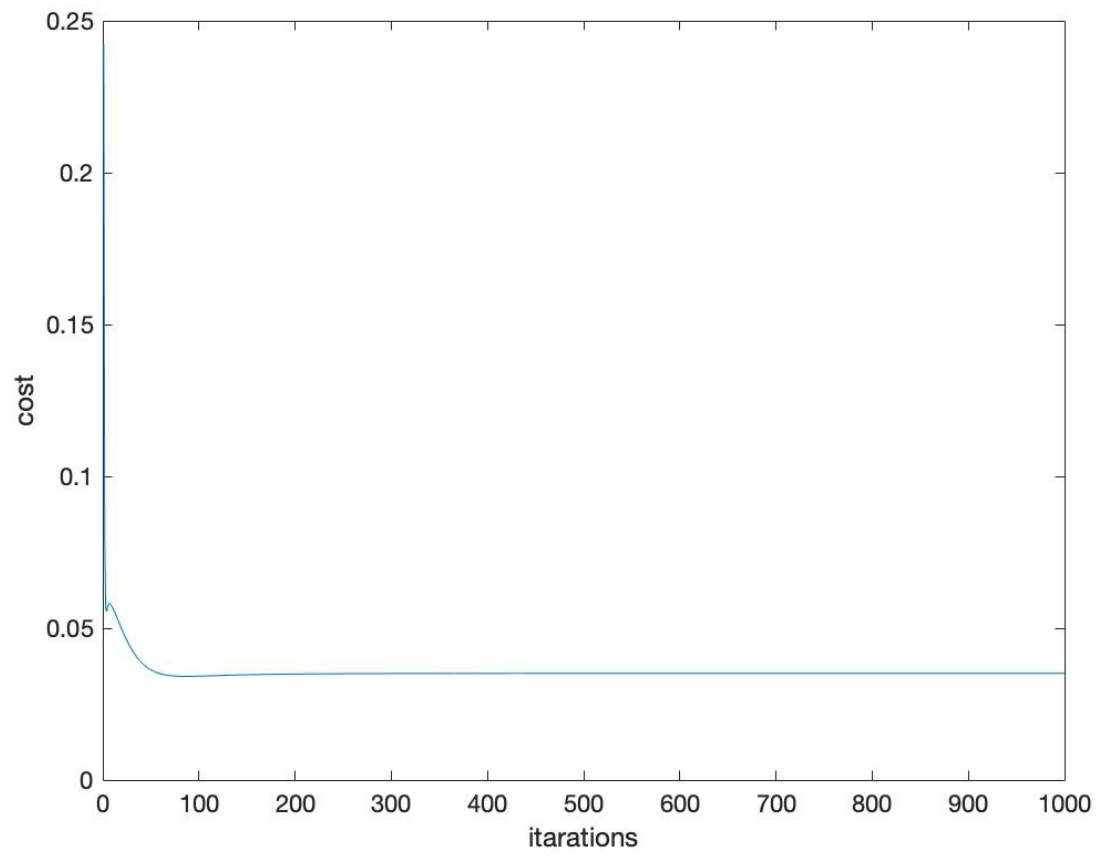


Curve:

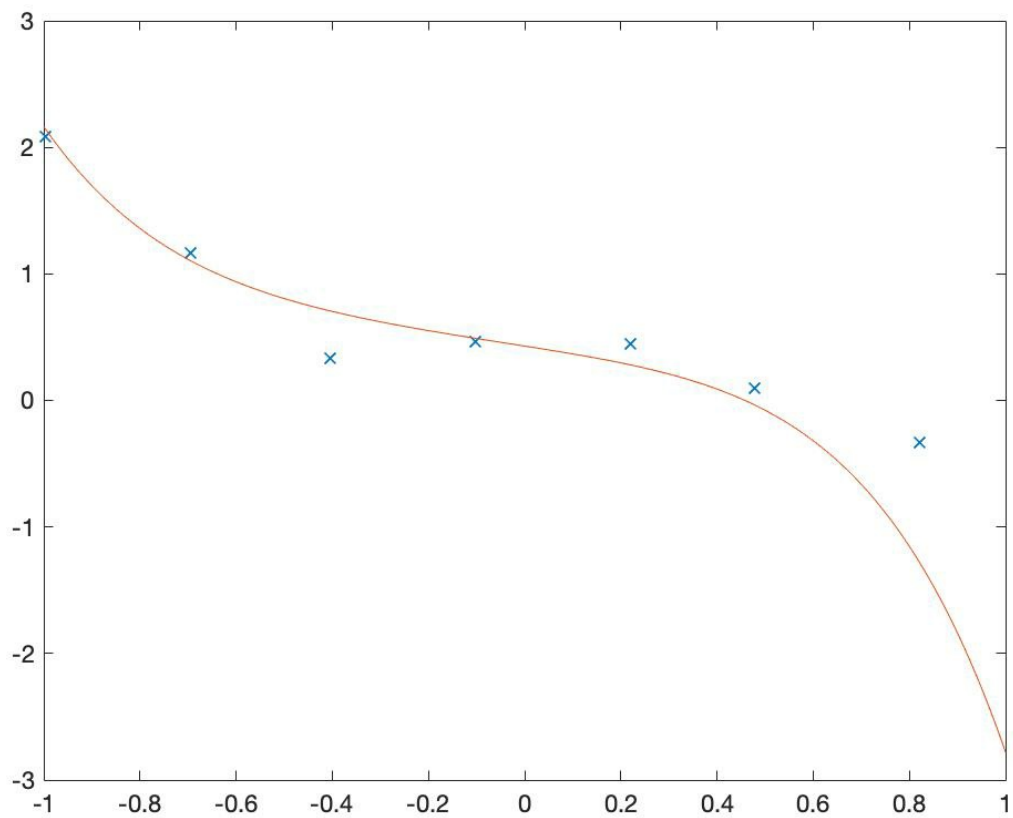


When alpha equals 0.9, lamda 0.1

The cost graph:

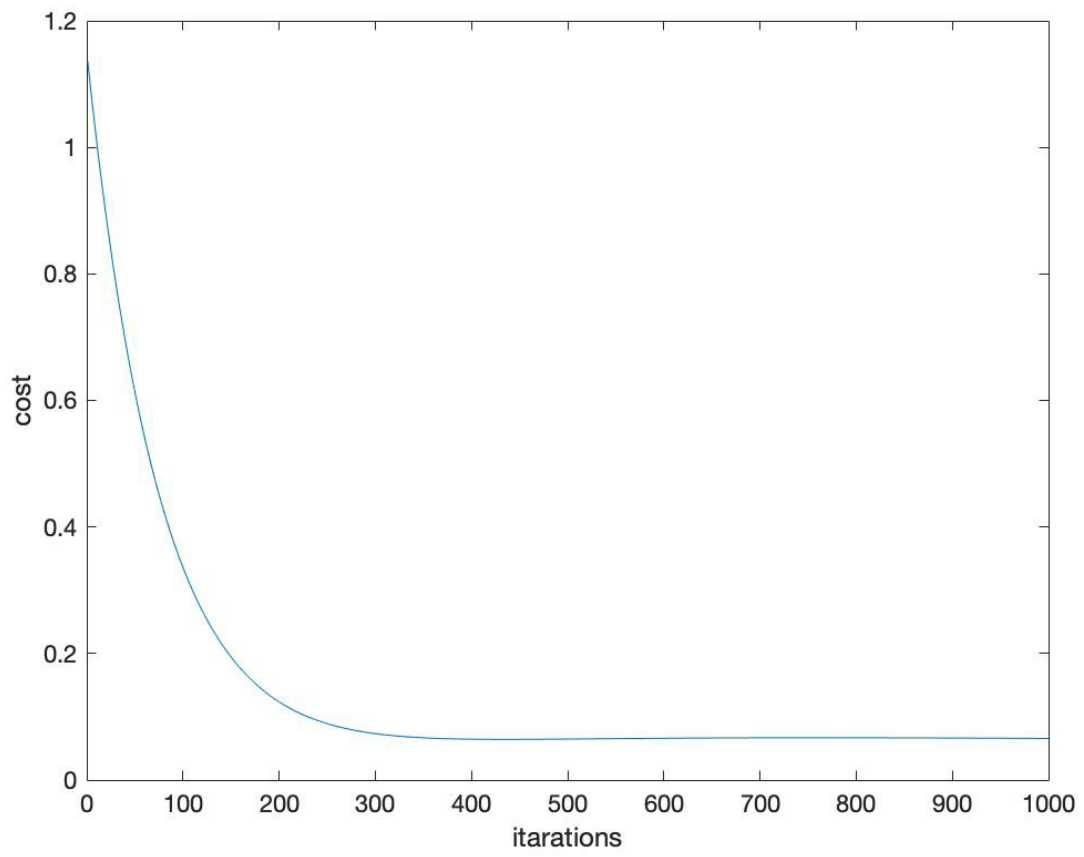


Curve:

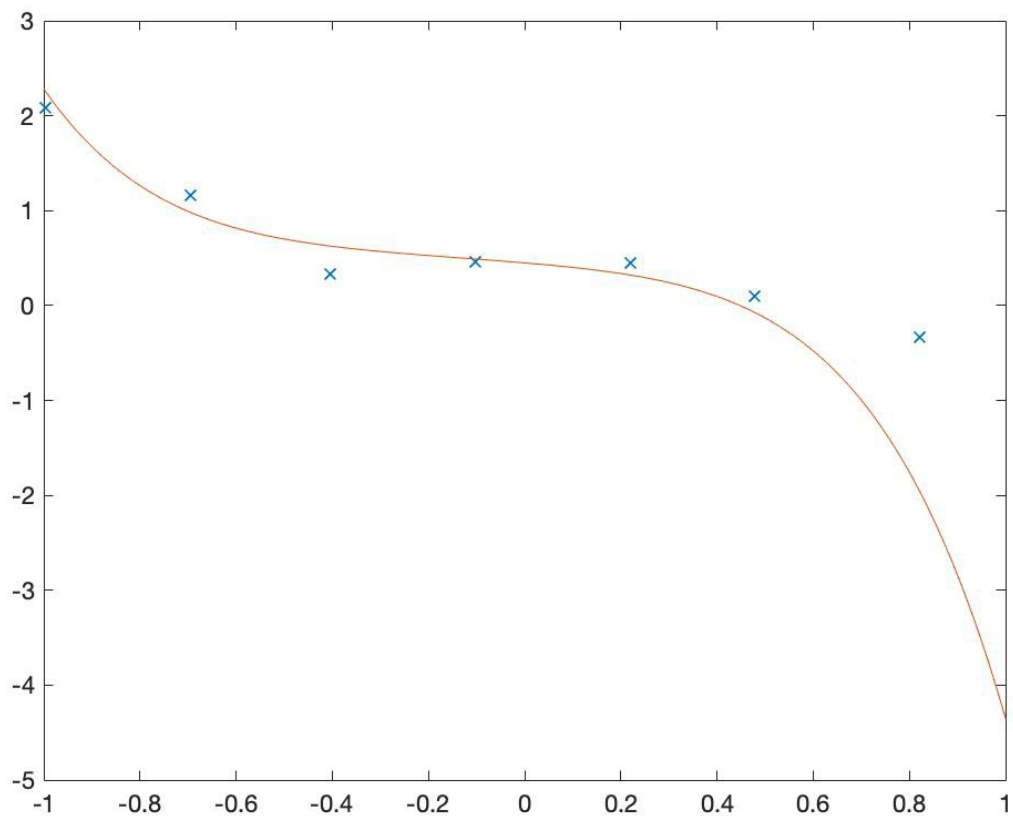


When alpha equals 0.001, lamda 0.1

The cost graph:



Curve:



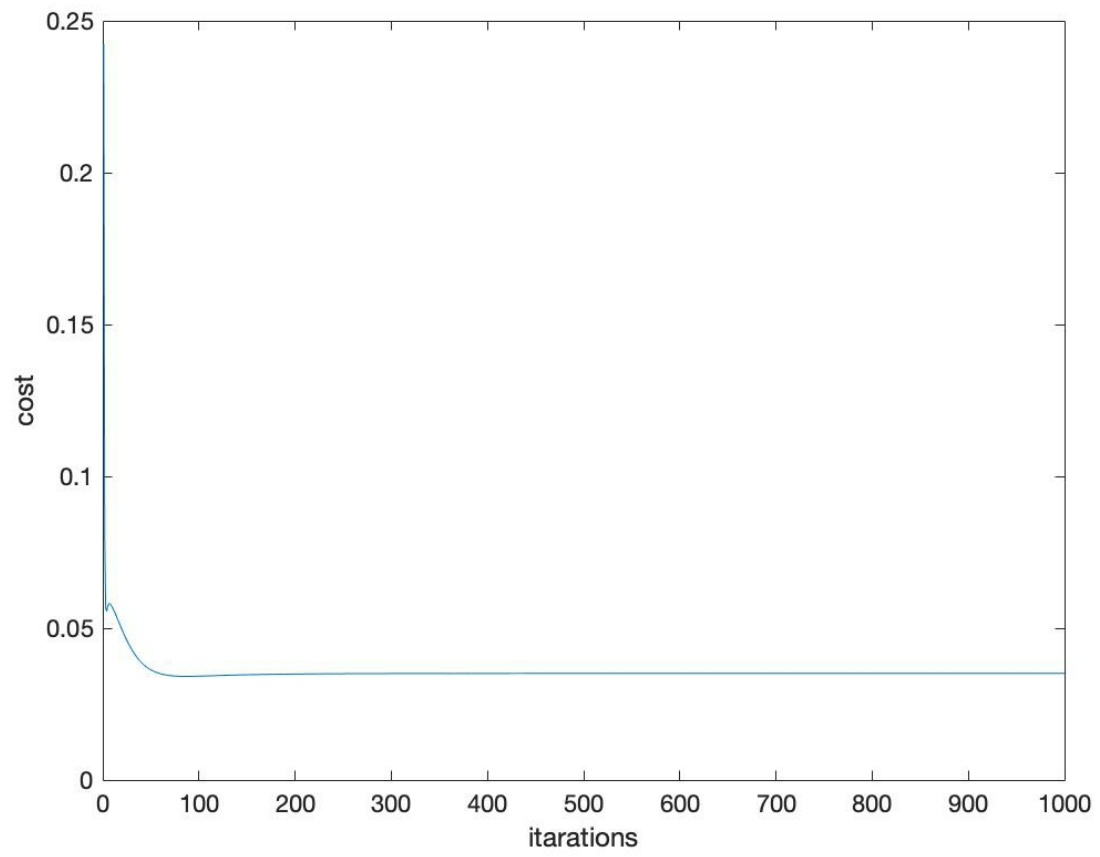
When alpha equals 0.9, is the best value of alpha.

3.3

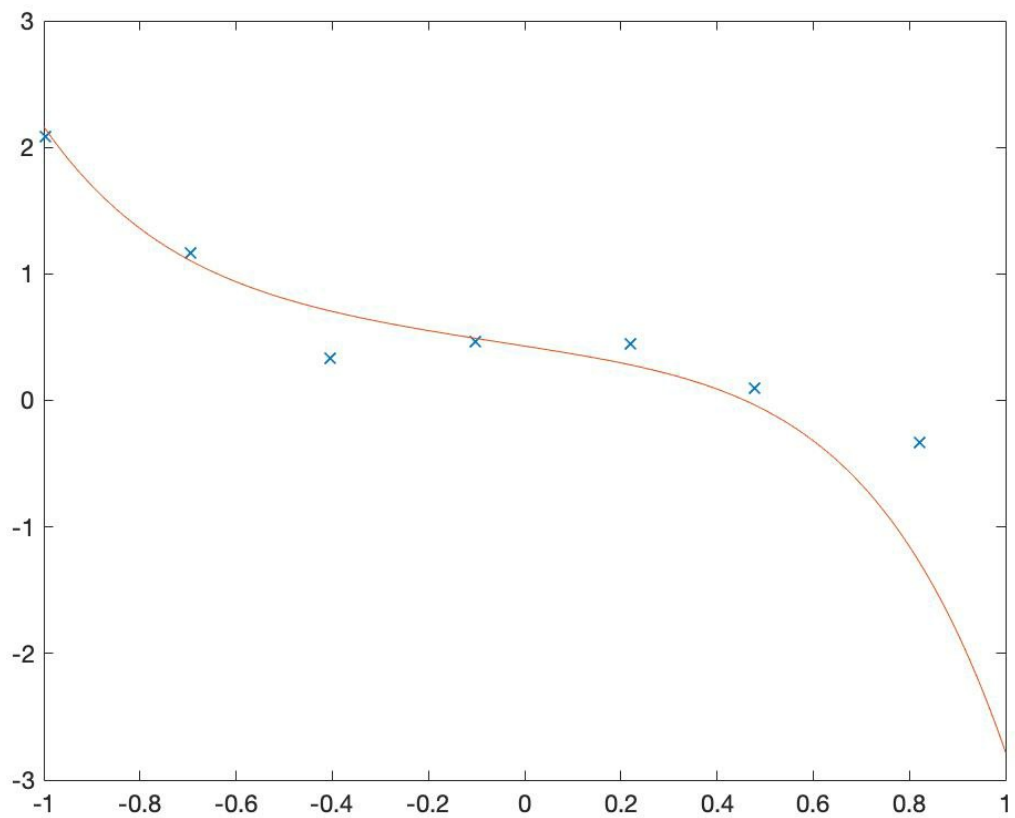
Changing lamda

lamda equals 0.1, alpha 0.9

The cost graph:

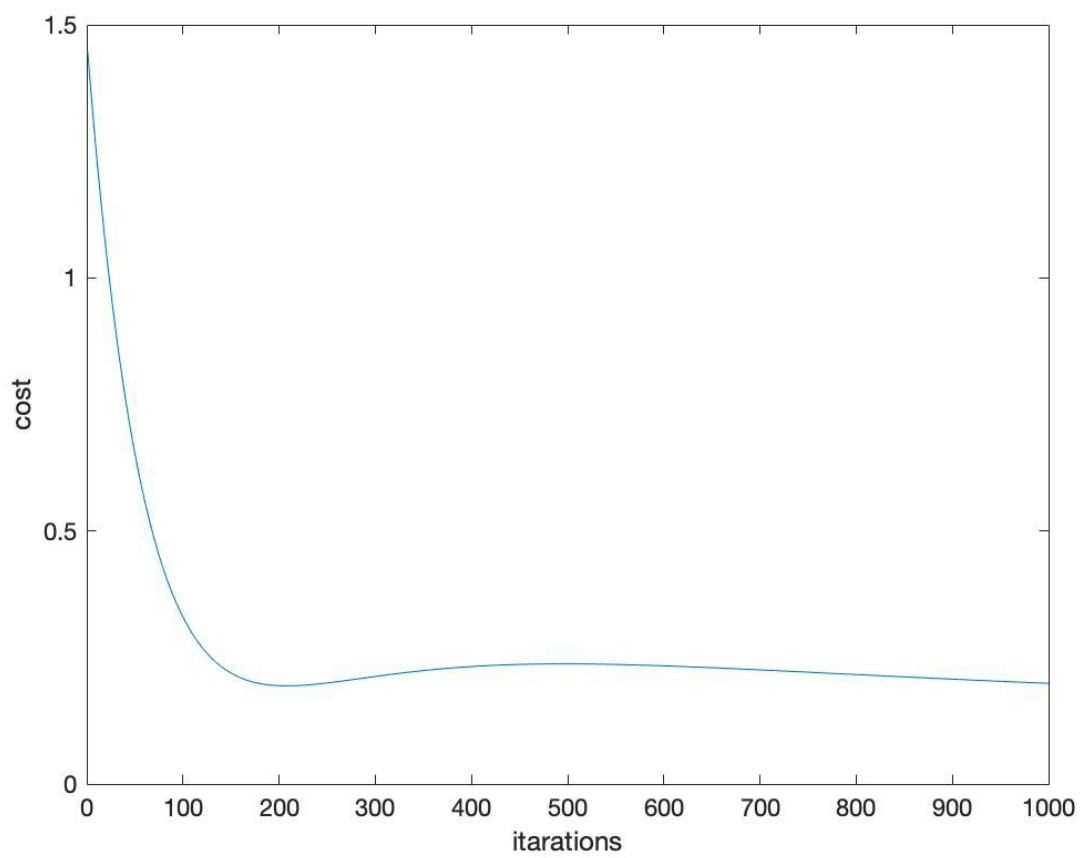


Curve:

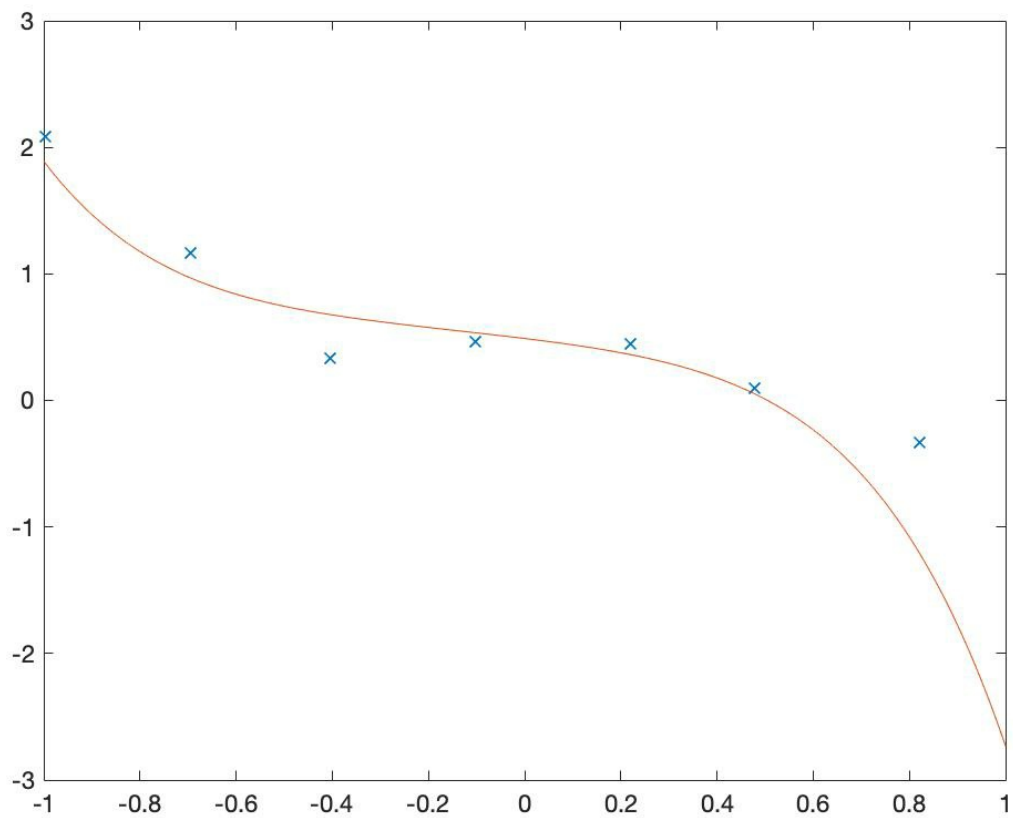


lamda equals 1, alpha 0.9

The cost graph:

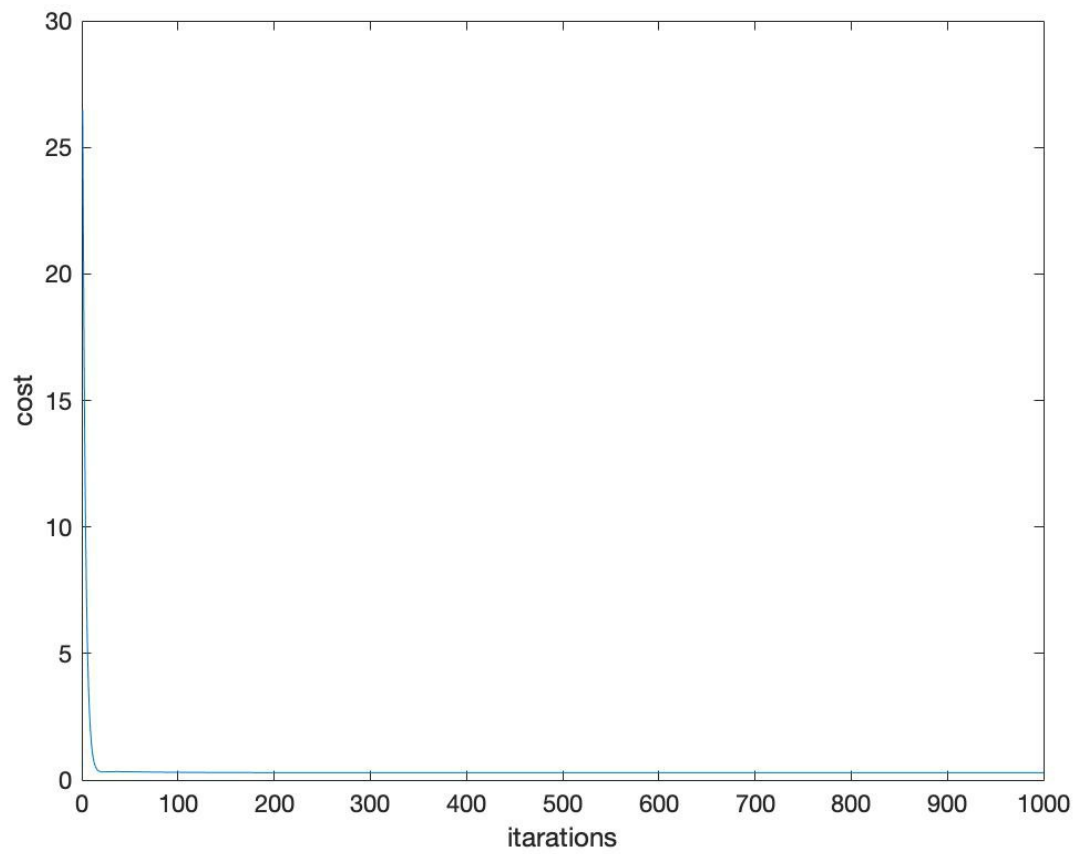


Curve:

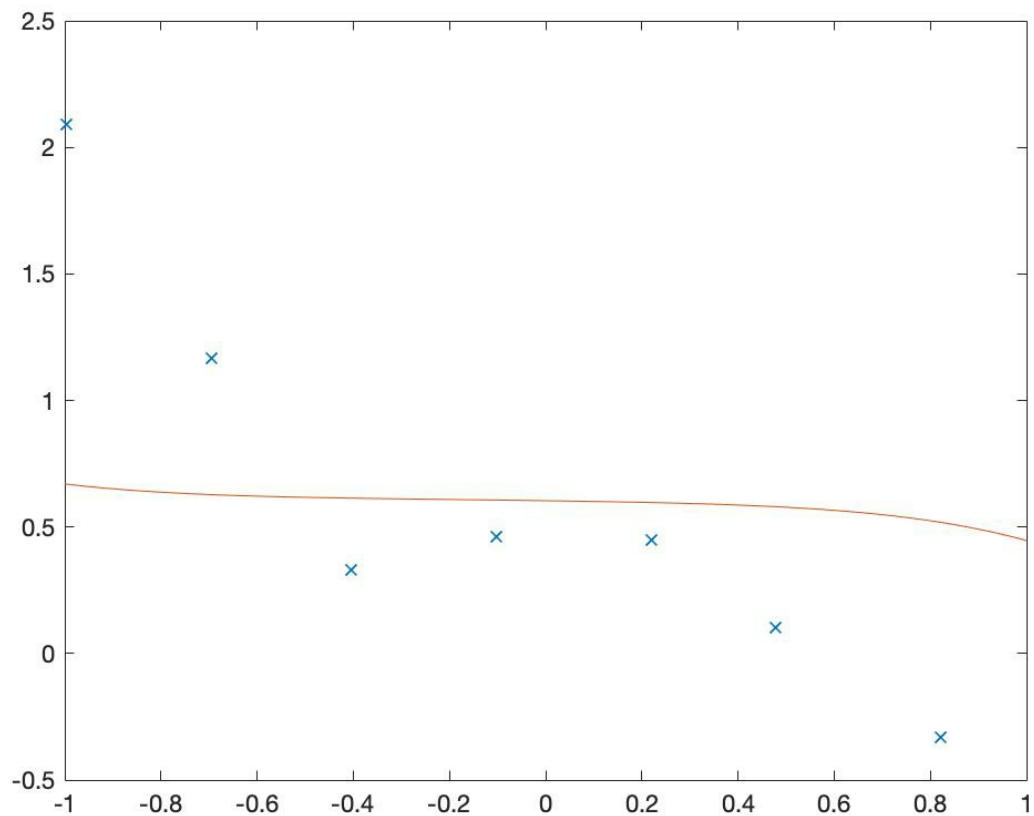


lamda equals 100, alpha 0.9

The cost graph:



Curve:



The lamda is the a kind of penalty function, to prevent the overfitting. The large number lamda will highly affect the curve to fit the dataset.

Assignment 1 – Part 2: Logistic Regression and Neural Networks

Logistic Regression

Task 1:

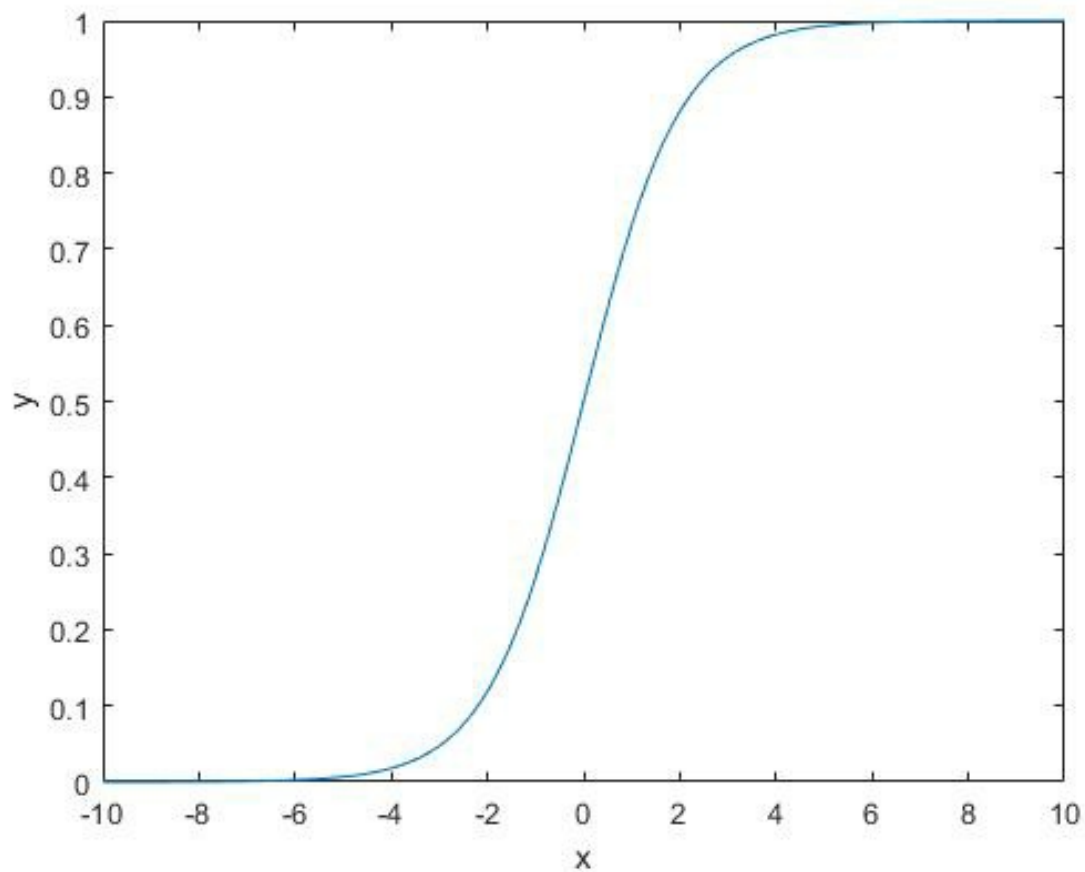
According to :

$$h_{\theta}(x) = g(\theta^T x) = \frac{1}{1+e^{-\theta^T x}}$$

the code modified to

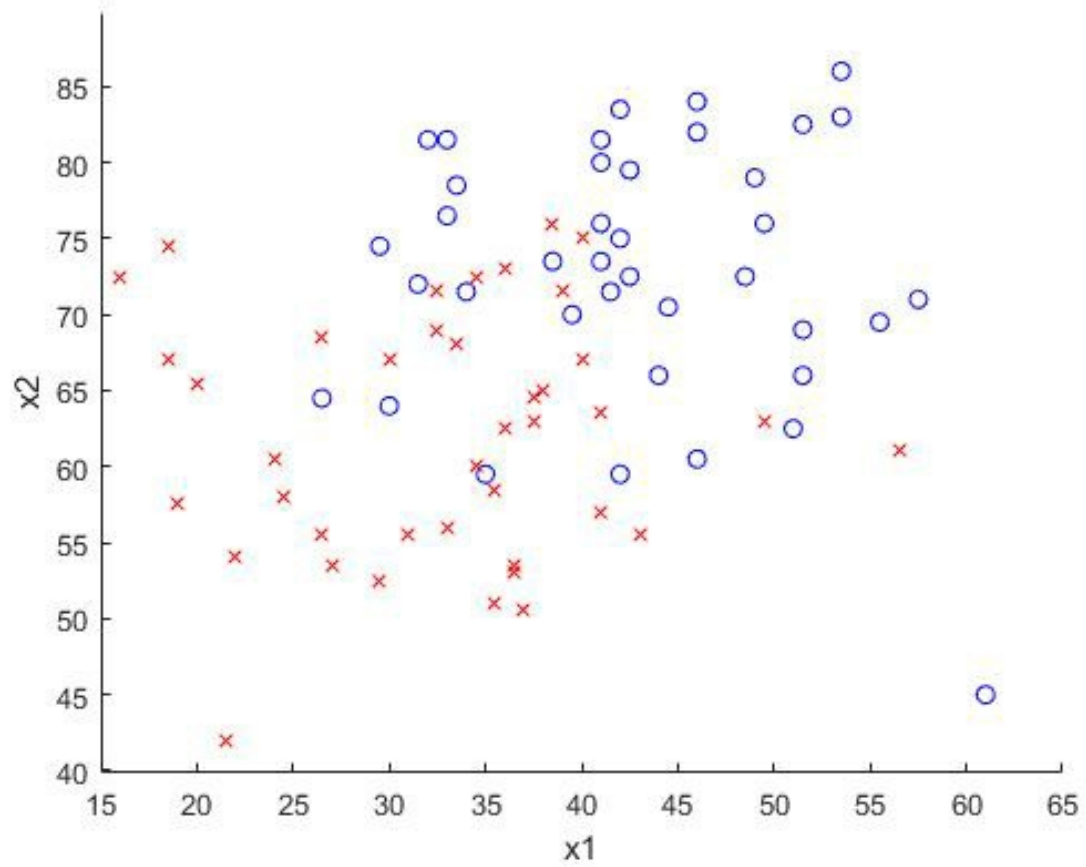
```
output = 1./(1+exp(-z));
```

The figure shows:

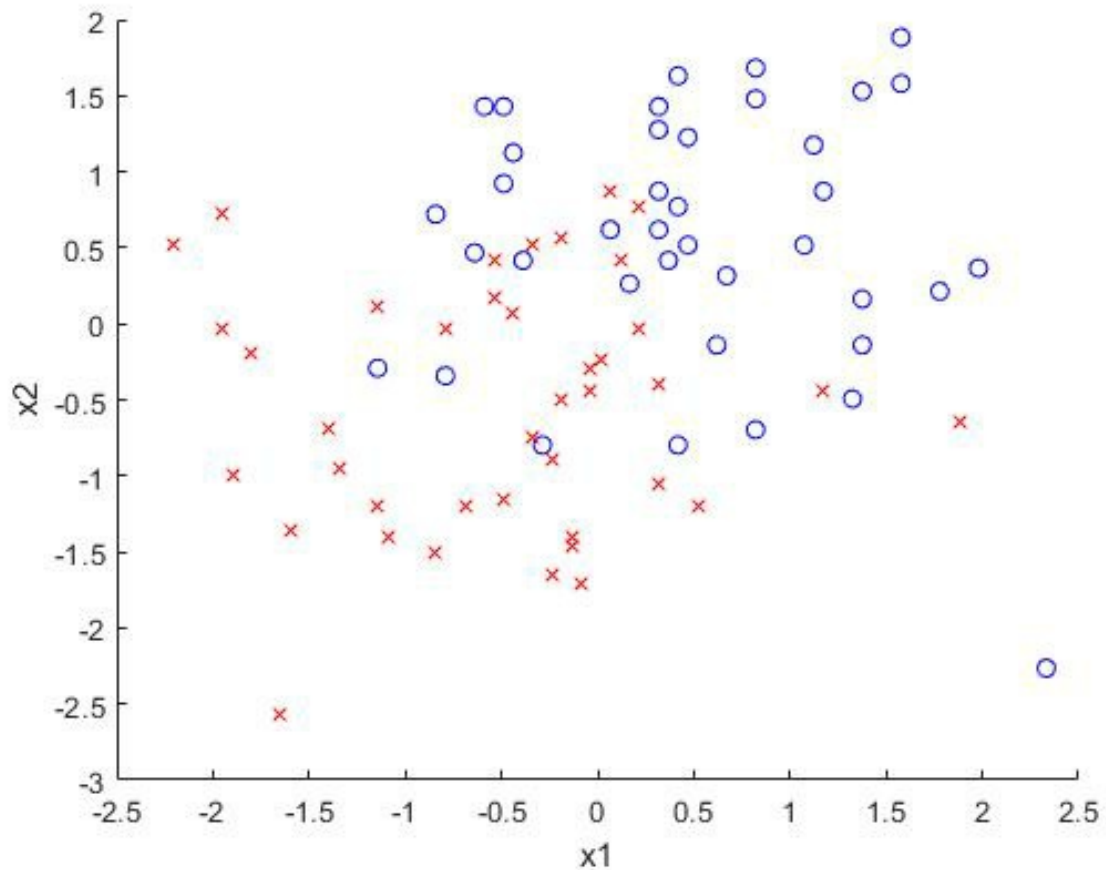


Task 2:

Plot the data:



Normalized the data :



Task 3:

```
function result=calculate_hypothesis(X,theta,training_example)
    hypothesis = 0.0;
    [y , x] = size(theta);
    for i =1 : x
        hypothesis = hypothesis +...
            X(training_example, i)* theta(i);
    end
    result=sigmoid(hypothesis);
end
```

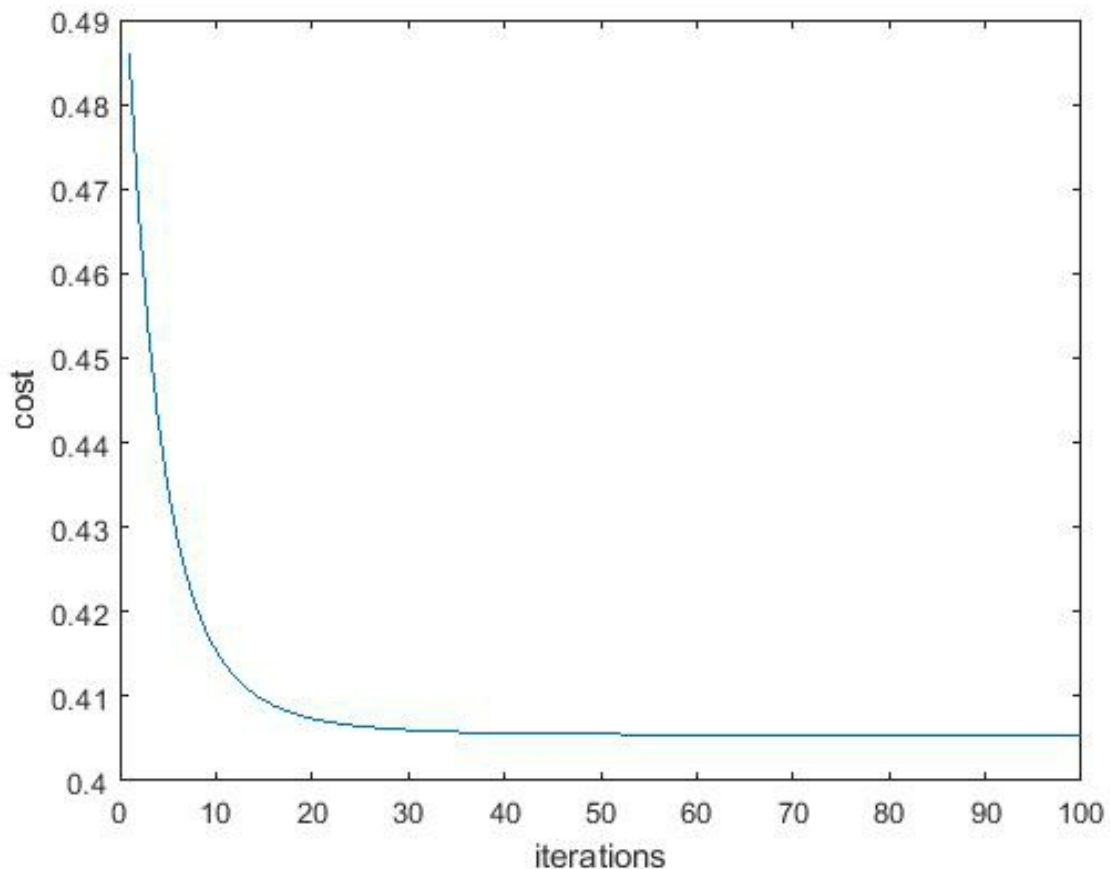
Task 4:

Modify the line:

```
cost = -output*log(hypothesis)-...  
(1-output)*log(1-hypothesis);
```

The Final cost found is 0.40545

The cost graph:

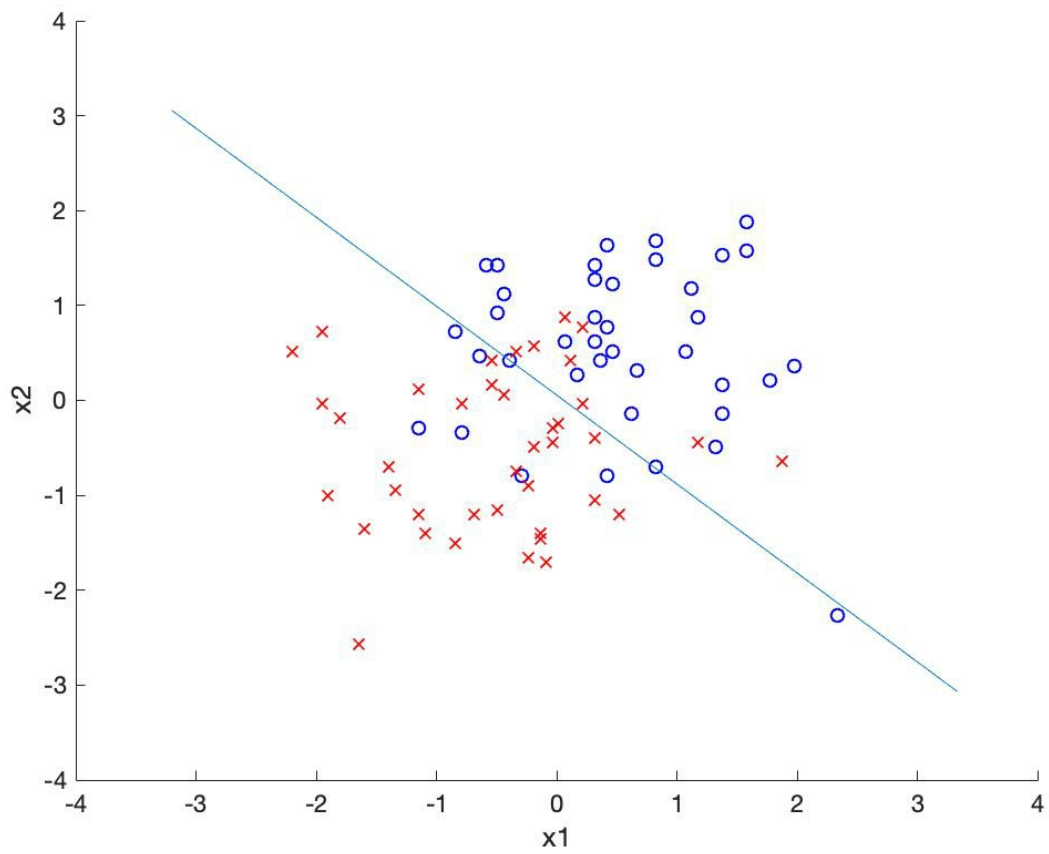


Task 5:

Modify the code

```
y1 = (-1)*(theta(1)+theta(2)*min_x1/theta(3));  
y2 = (-1)*(theta(1)+theta(2)*max_x1/theta(3));  
plot([min_x1,max_x1],[y1,y2], '-')
```

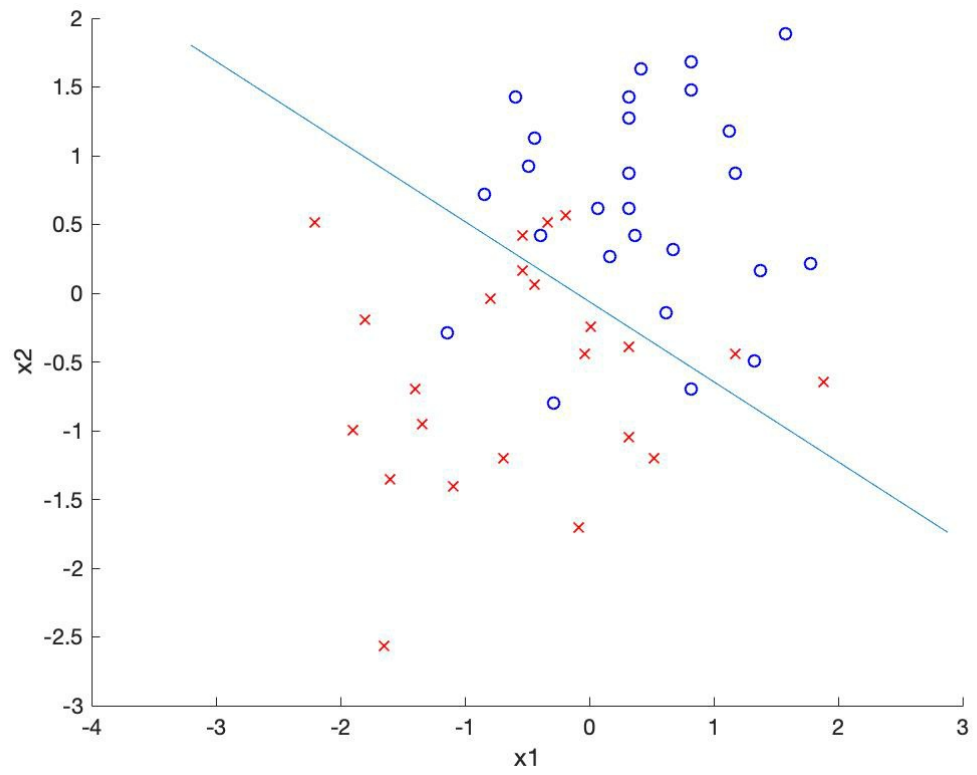
Graph:



Task 6:

1. First time

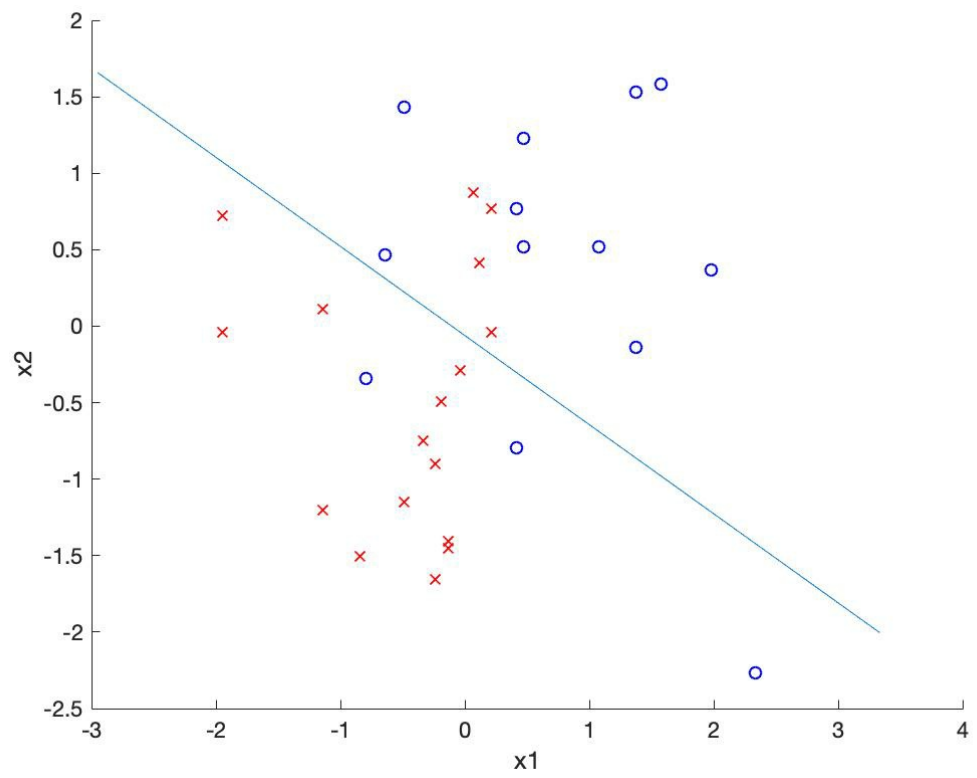
- Training error:0.38123



◦

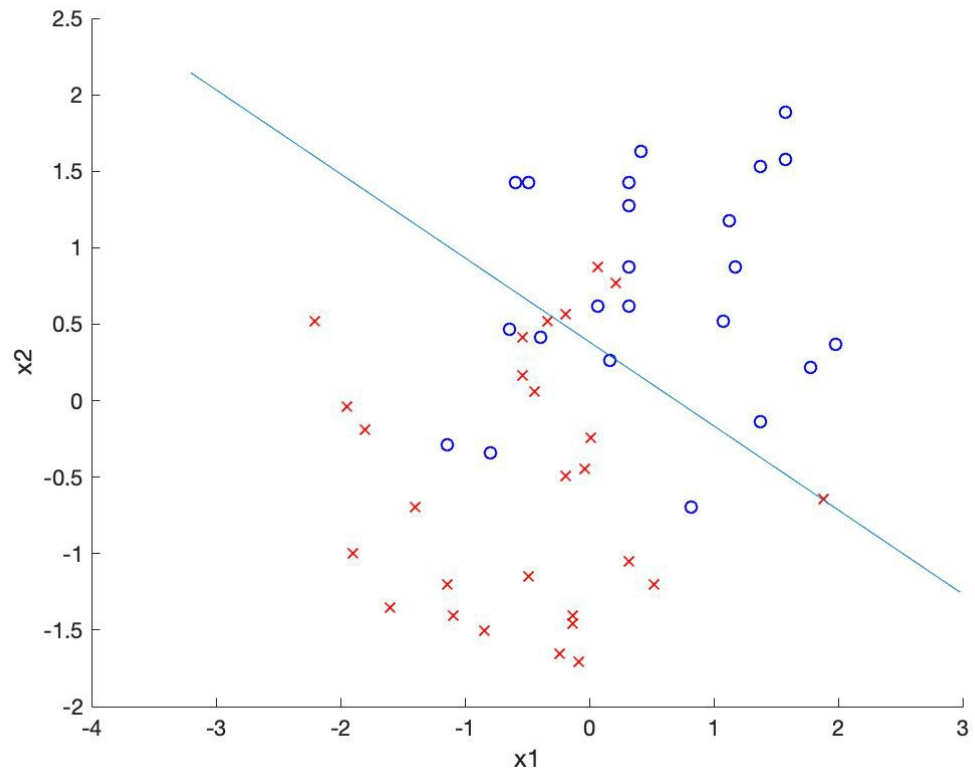
◦ Test error:0.49629

◦ #



2. Second time

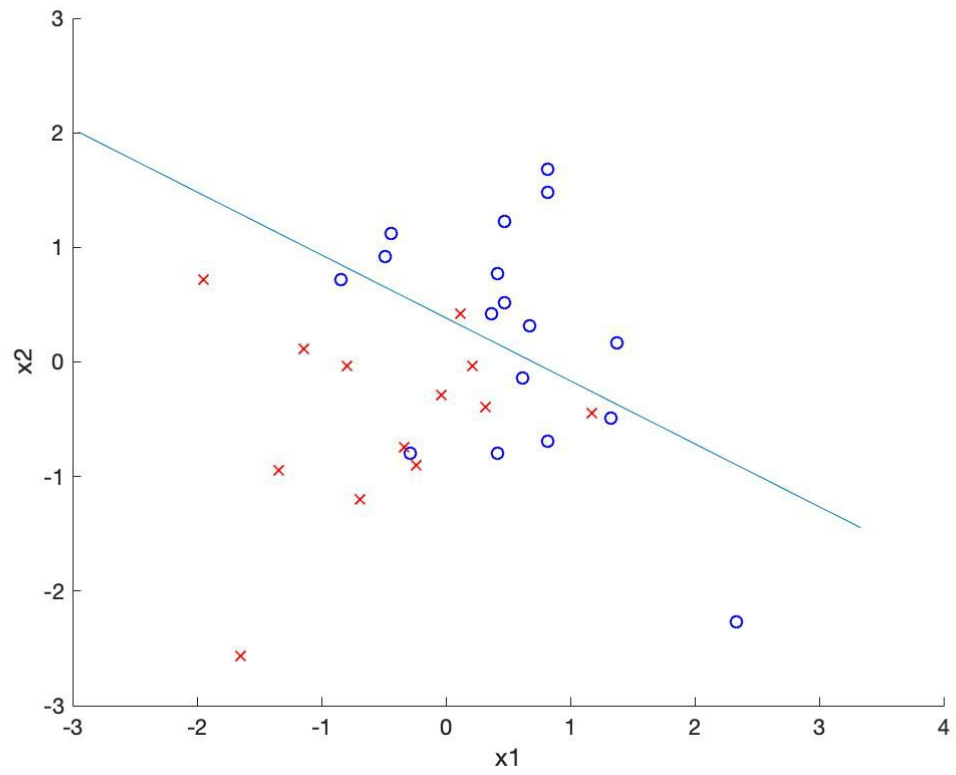
- Training error:0.37281



-
-

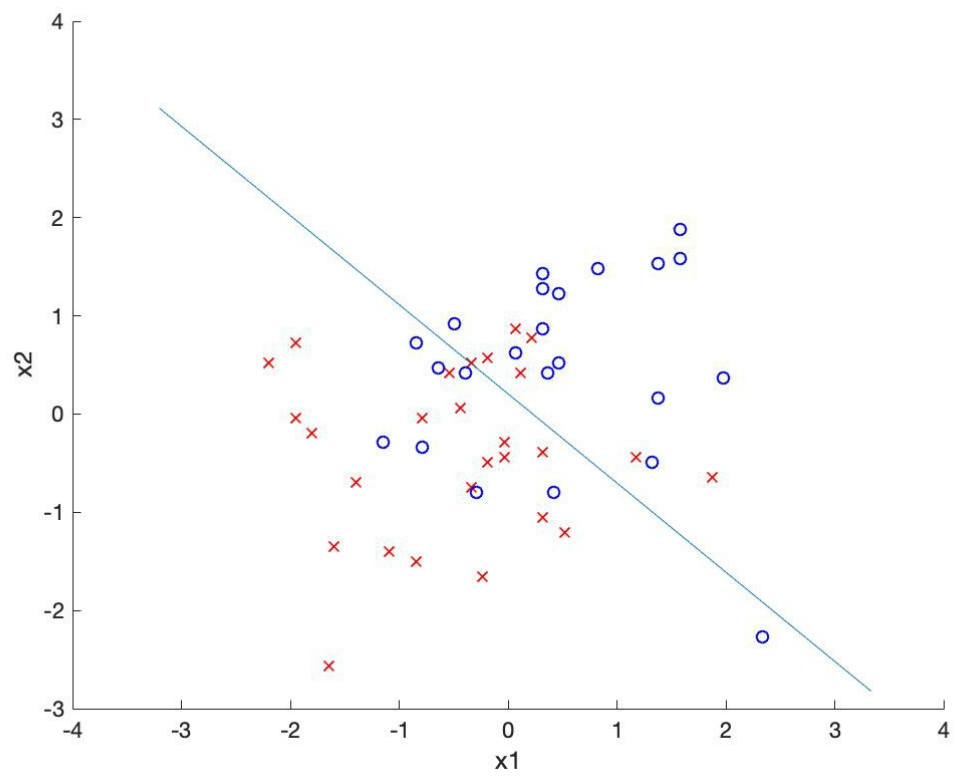
- Test error:0.29797

- #



3. Third Time

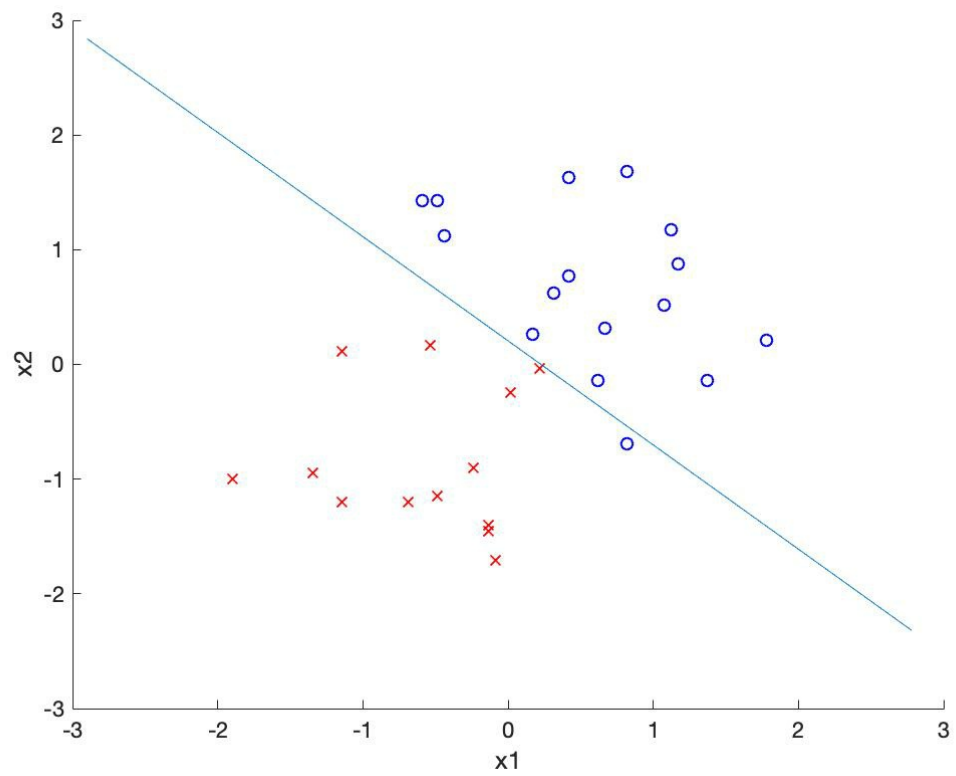
- Training error:0.50007



◦

◦ Test error:0.29389

◦ #



Good Generalisation: Second time(Test error:0.29797)

Bad Generalisation: First time(Test error:0.49629)

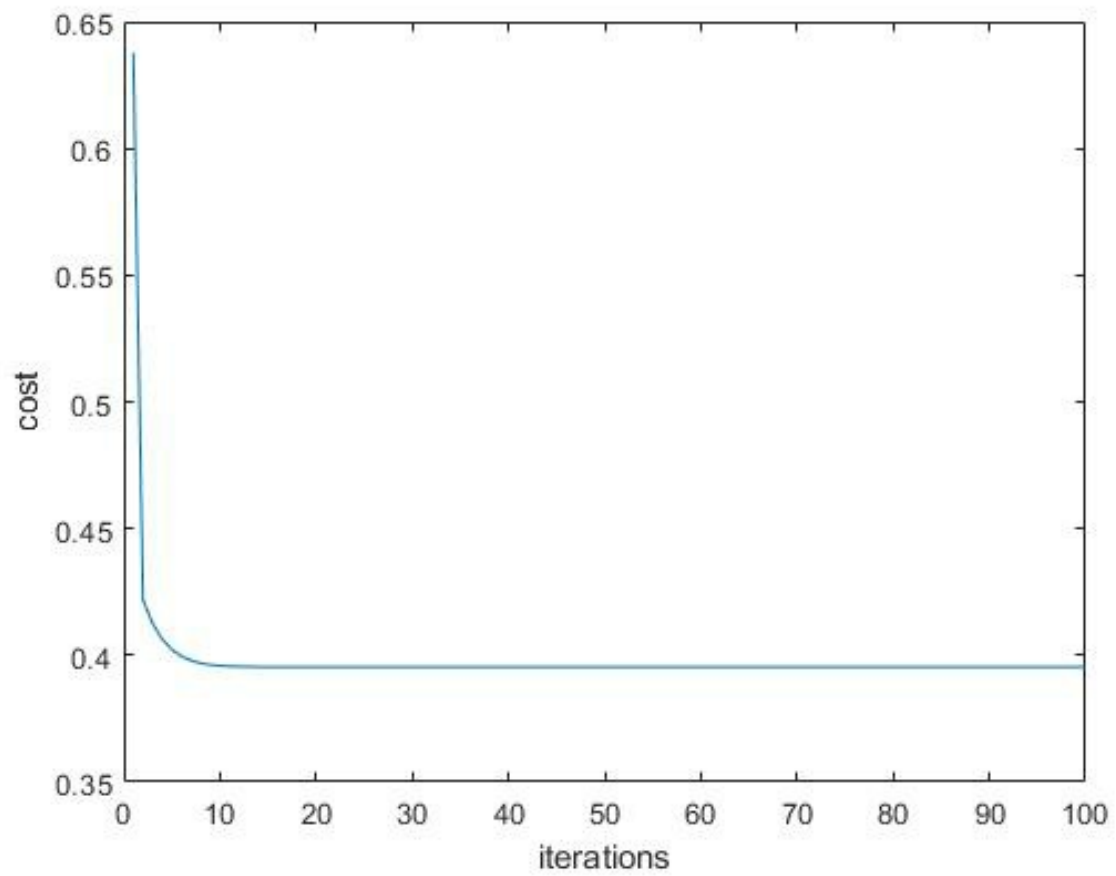
Task 7:

Modify the code

```
% for question 7, modify the dataset X to have more features (in each row)
% append to X(i), the following features:
% here append x_2 * x_3 (remember that x_1 is the bias)
X_2byX_3 = X(:, 2) .* X(:, 3);
% here append x_2 * x_2 (remember that x_1 is the bias)
X_2byX_2 = X(:, 2) .* X(:, 2);
% here append x_3 * x_3 (remember that x_1 is the bias)
X_3byX_3 = X(:, 3) .* X(:, 3);

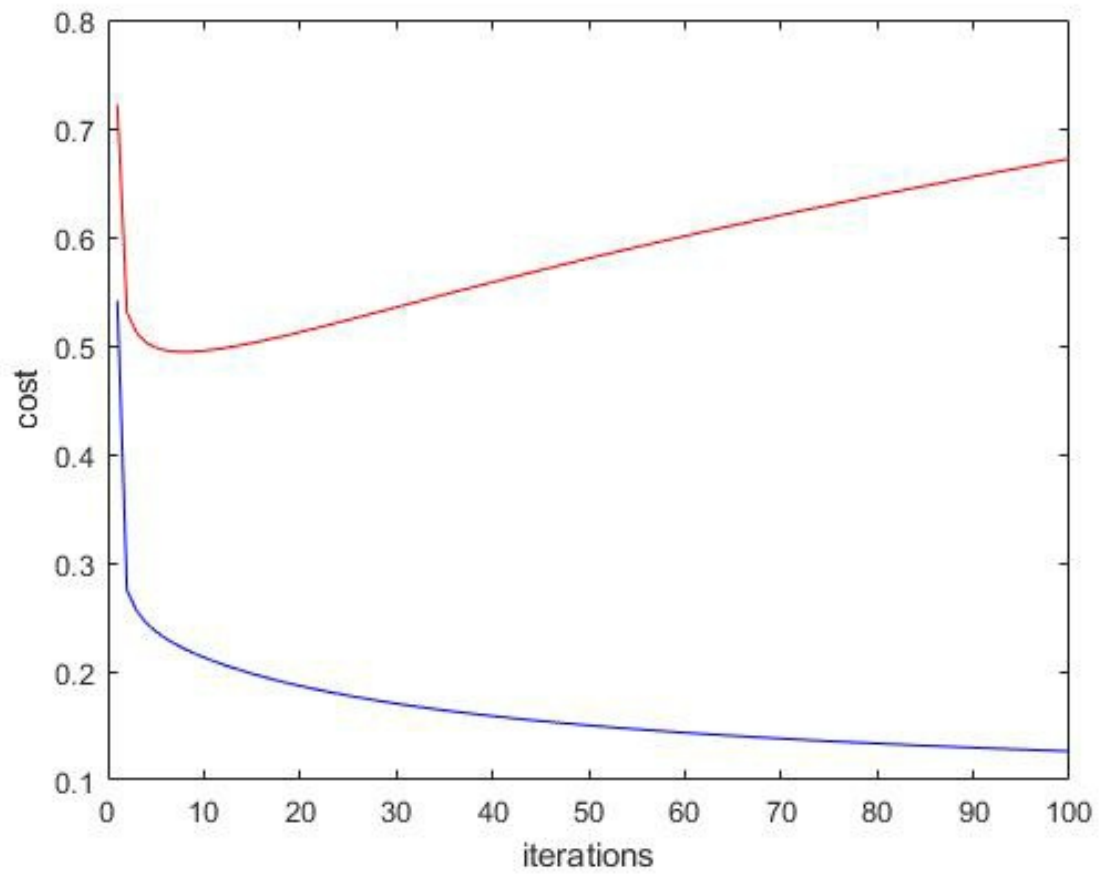
X = [X, X_2byX_3, X_2byX_2, X_3byX_3];
% initialise theta
alpha = 0.05;
iterations = 100;
```

Error is 0.39537, the second order polynomial increase the dimensions, help the curve fitting the data set better.



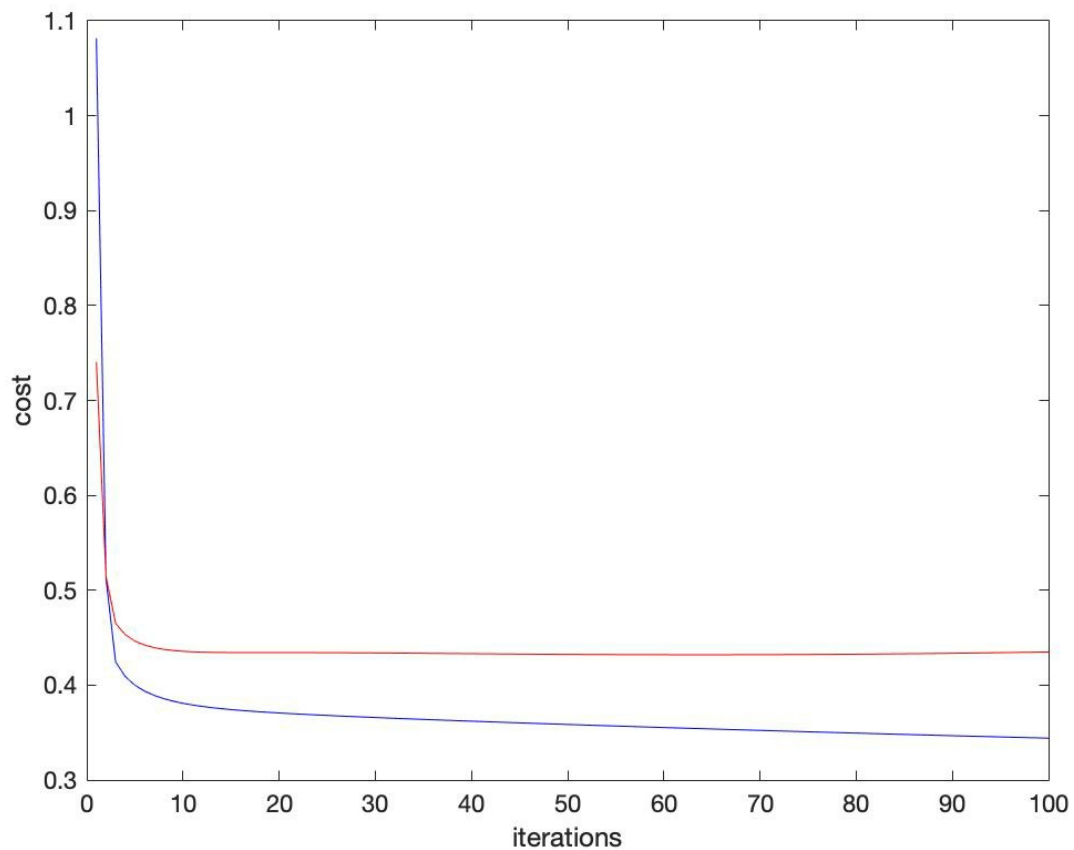
Task 8:

Train set is set to 70 (overfitting):



-
- Training cost :0.12649
 - Test cost :0.67195

Train set is set to 10:



- Training cost :0.34411
- Test cost :0.43497

When the training set is small, the curve can better fit the testing set. The training set is large, the error of the cost of the test curve will increase. This causes overfitting.

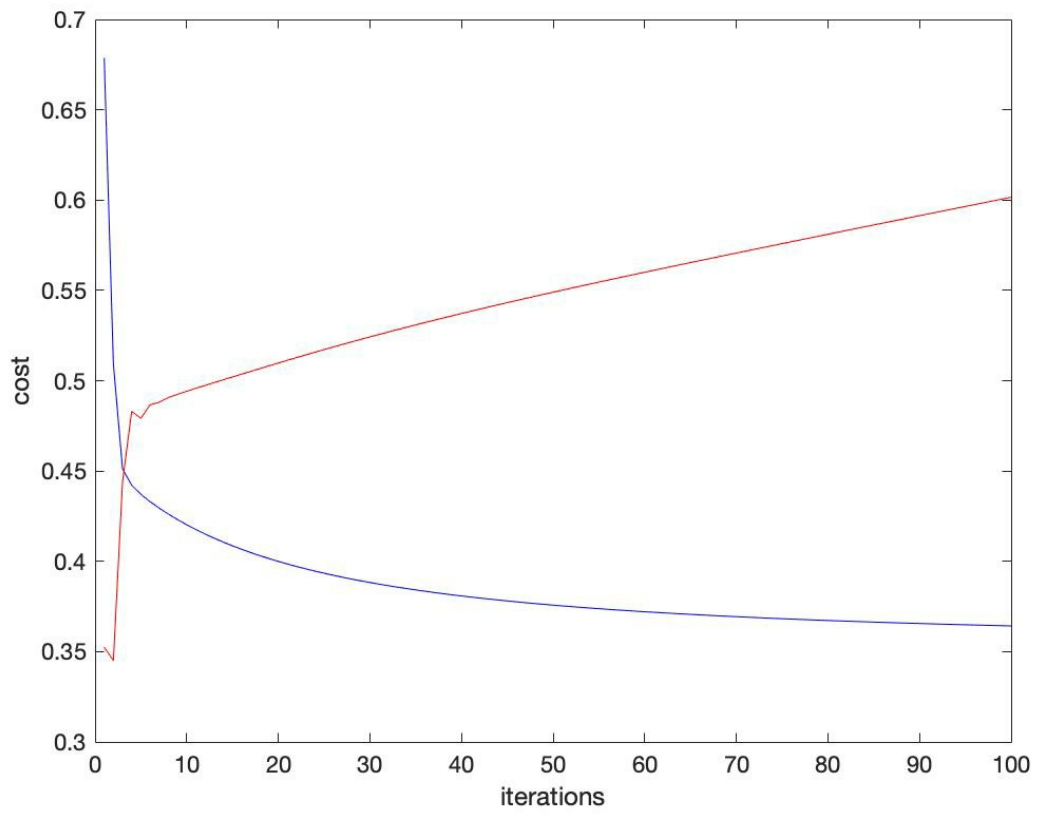
After added a third order $x_1x_1x_1$, and the weight is 1.0

```
%third order
X_2byX_2byX_2 = X(:, 2) .* X(:, 2).*X(:, 2);

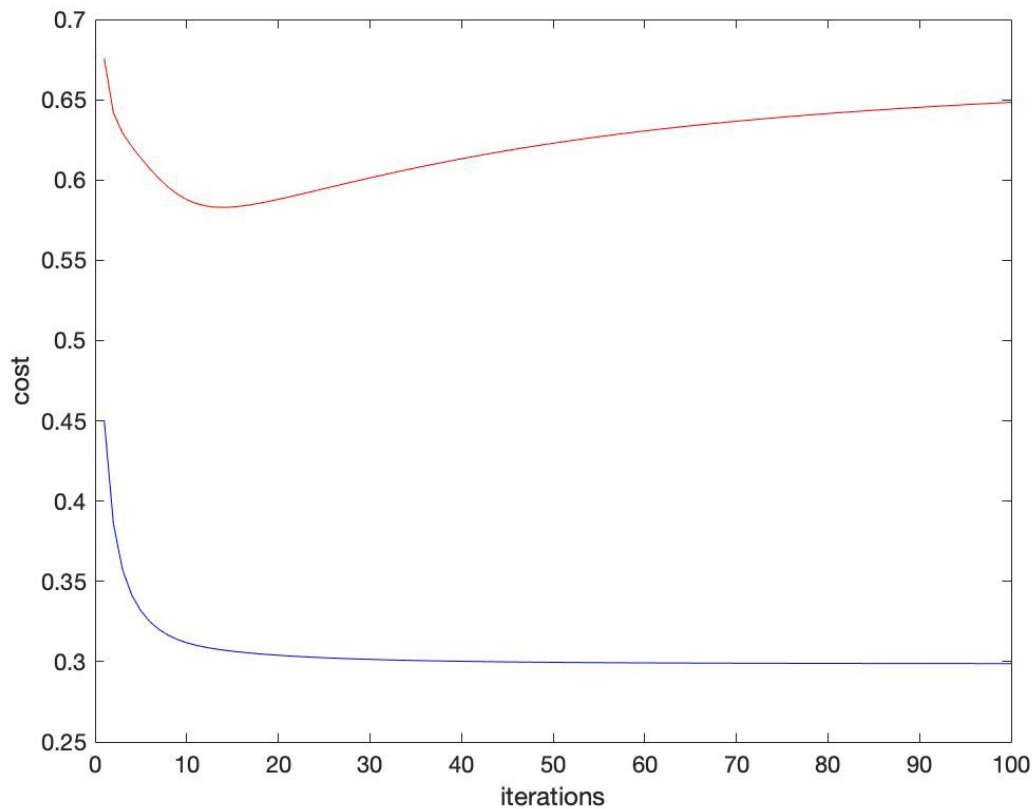
X = [X, X_2byX_3, X_2byX_2, X_3byX_3,X_2byX_2byX_2];
% initialise theta. Remember that theta needs to be
% the same size as one row of X
theta=[1.0,1.0,1.0,1.0,1.0,1.0,1.0];
```

The train set is set to 40:

- Graph 1:



-
- Graph 2:

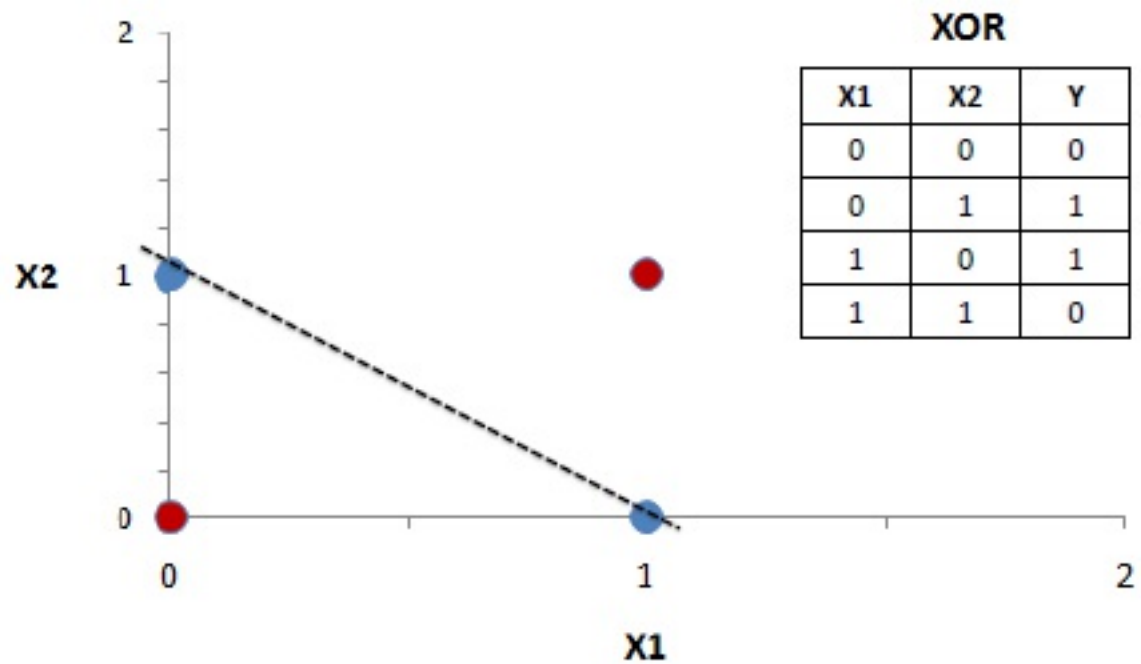


- Training:0.1405
- Test:0.65299

The second graph shows that the red line (test set) goes up. However, the blue line (training set) goes down. The third order polynomial caused the function overfitting.

Task 9:

The logistic regression is a linear model. However, the data points for an xor function are not linearly separable. The logistic regression cannot correctly classify the xor points.



Neural Network

Task 10:

Implement backpropagation:

Step 1:

```
% Step 1\, Output deltas (used to change weights from hidden --> output)
output_deltas = zeros(1,length(nn.output_neurons));
outputs=nn.output_neurons;
for i=1:length(outputs)
    output_deltas(i) = (outputs(i)-...
        targets(i))*sigmoid_derivative(outputs(i));
end
```

Step 2:

```
% Step 2\, Hidden deltas (used to change weights from input --> output).
hidden_deltas = zeros(1,length(nn.hidden_neurons));

for j=1:length(nn.hidden_neurons)
```



```

        accumulate = 0;
        for i=1:length(outputs)
            accumulate = accumulate +...
                output_deltas(i) * nn.output_weights(j,i);
        end
        hidden_deltas(j) = accumulate * sigmoid_derivative(nn.hidden_neurons(1,j));
    end
end

```

Step 3:

```

% Step 3\ . update weights output --> hidden
for i=1:length(nn.hidden_neurons)
    for j=1:length(output_deltas)
        nn.output_weights(i,j) =nn.output_weights(i,j) -...
            (output_deltas(j) * nn.hidden_neurons(i) * learning_rate);
    end
end
end

```

Step 4:

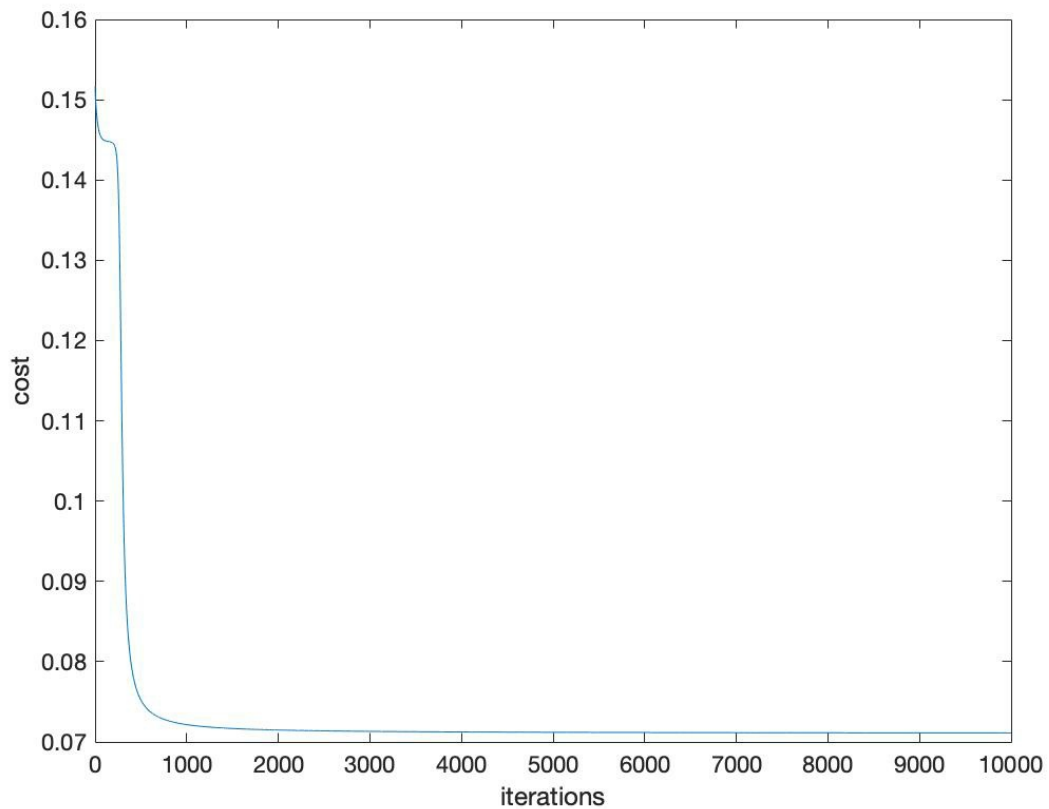
```

% Step 4\ . update weights input --> hidden.
for i=1:length(nn.hidden_neurons)
    for j=1:length(hidden_deltas)
        nn.hidden_weights(i,j) =nn.hidden_weights(i,j) -...
            (hidden_deltas(j) * nn.hidden_neurons(i) * learning_rate);
    end
end
end

```

When learning rate equals to 2 The output :

- target output:0, actual output0.013118
- target output:1, actual output0.98976
- target output:1, actual output0.46662
- target output:0, actual output0.46685



Task 11:

```
training_set_input = [1,0,0,0]
```

Output:

- target output:0, actual output0.0042224
- target output:1, actual output0.68067
- target output:1, actual output0.68067
- target output:1, actual output0.68067

Training outputs is changed to NOR function [1,0,0,0], The output shows the function fits the training set well.

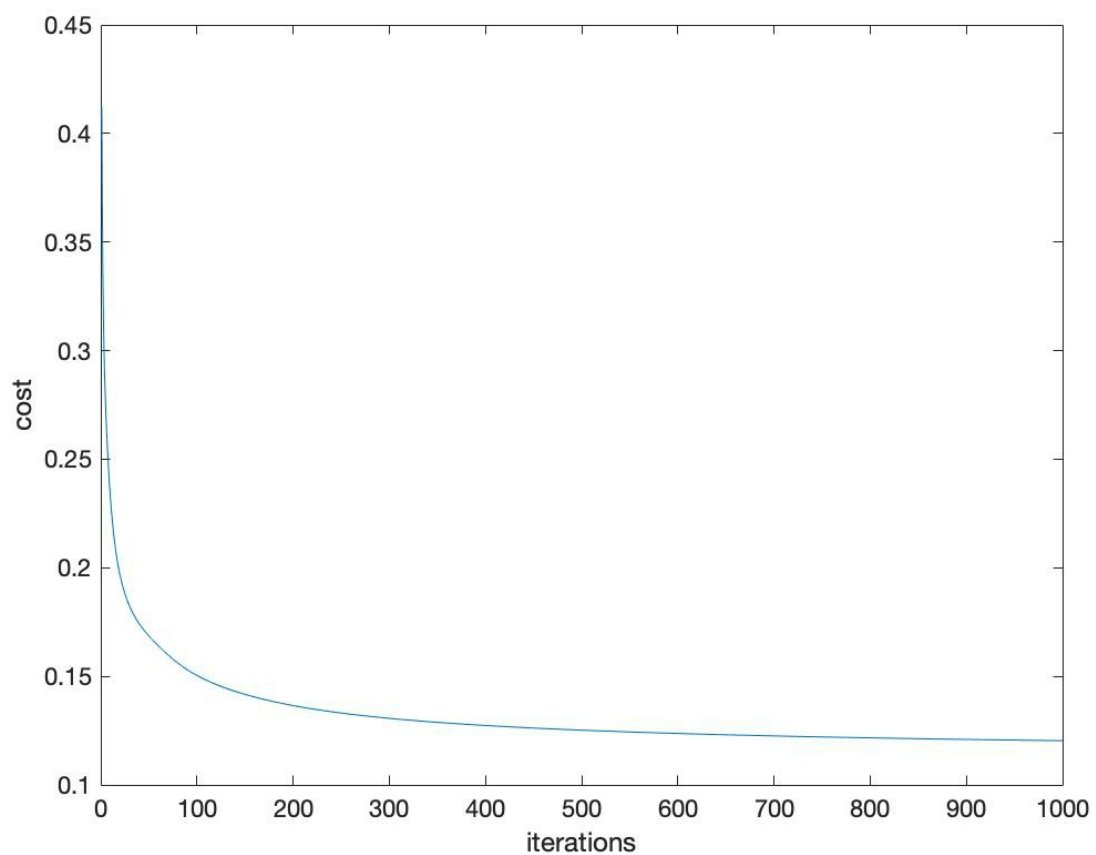
Task 12:

Neurons Network will classify the 3 classes directly. However, Logistic regression will classify one class from the total dataset once.

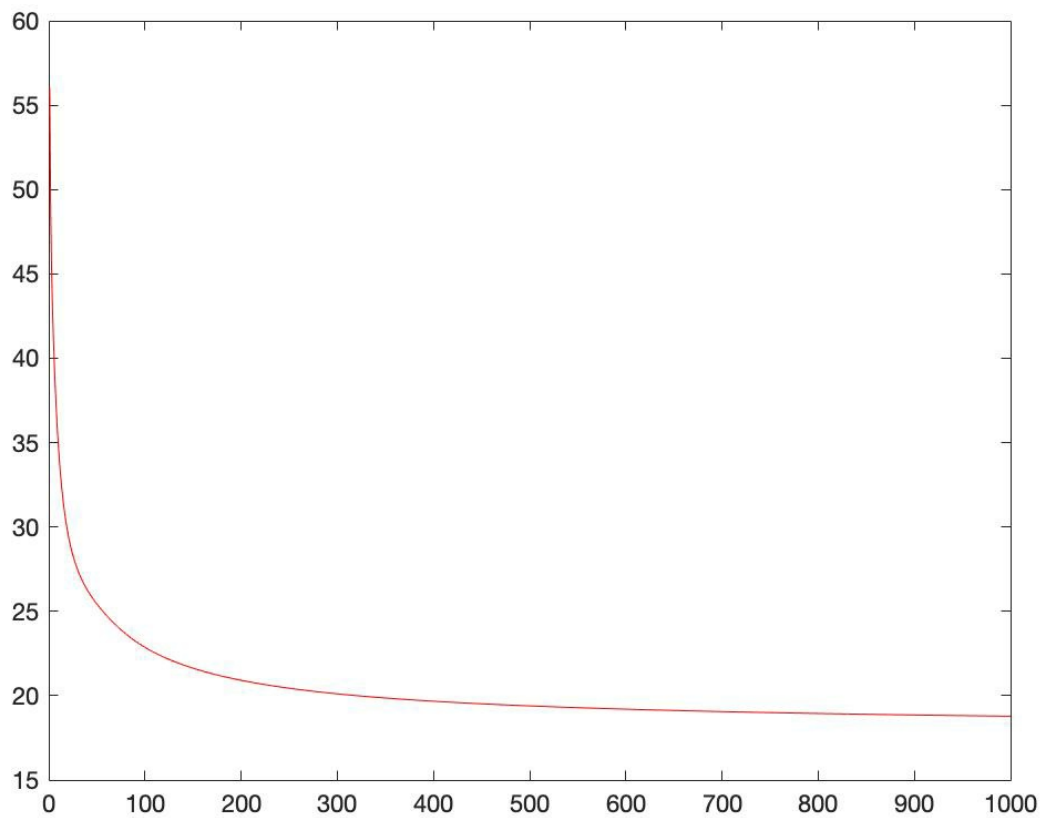
Task 13:

When hidden neurons equals to 1:

Costs of the training set:



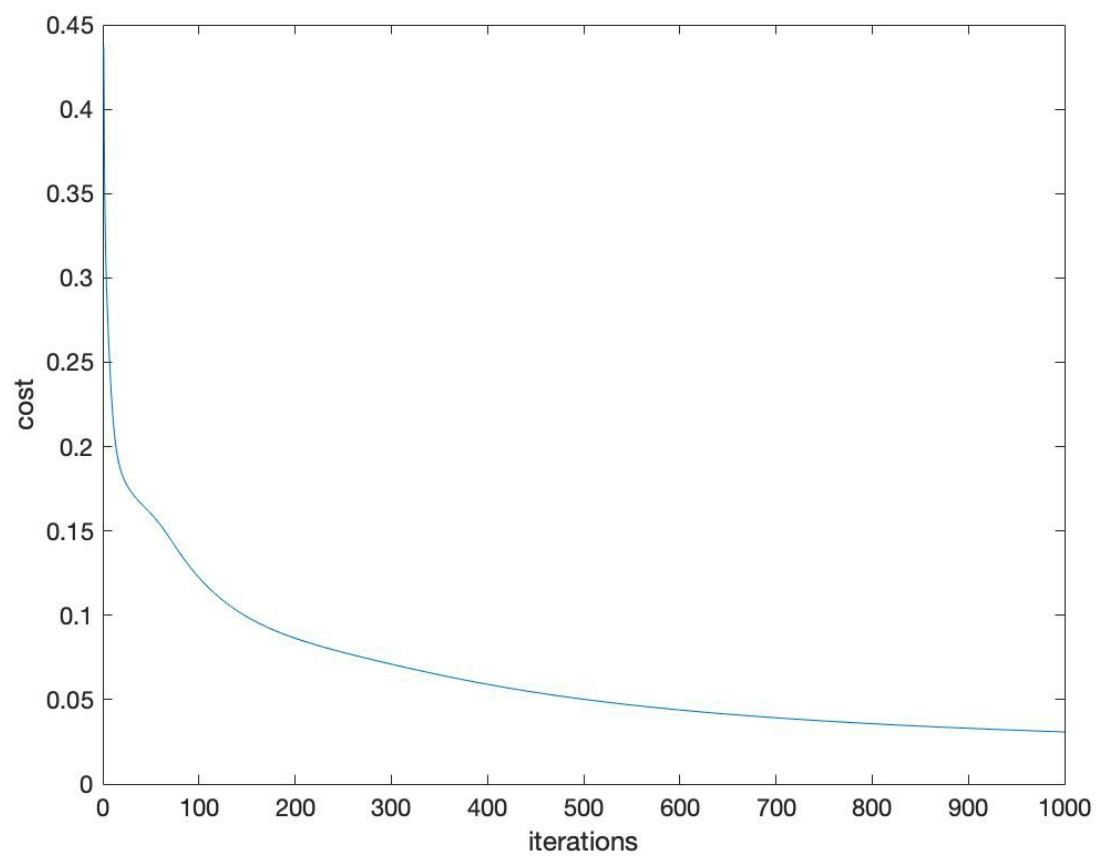
Costs of the test set:



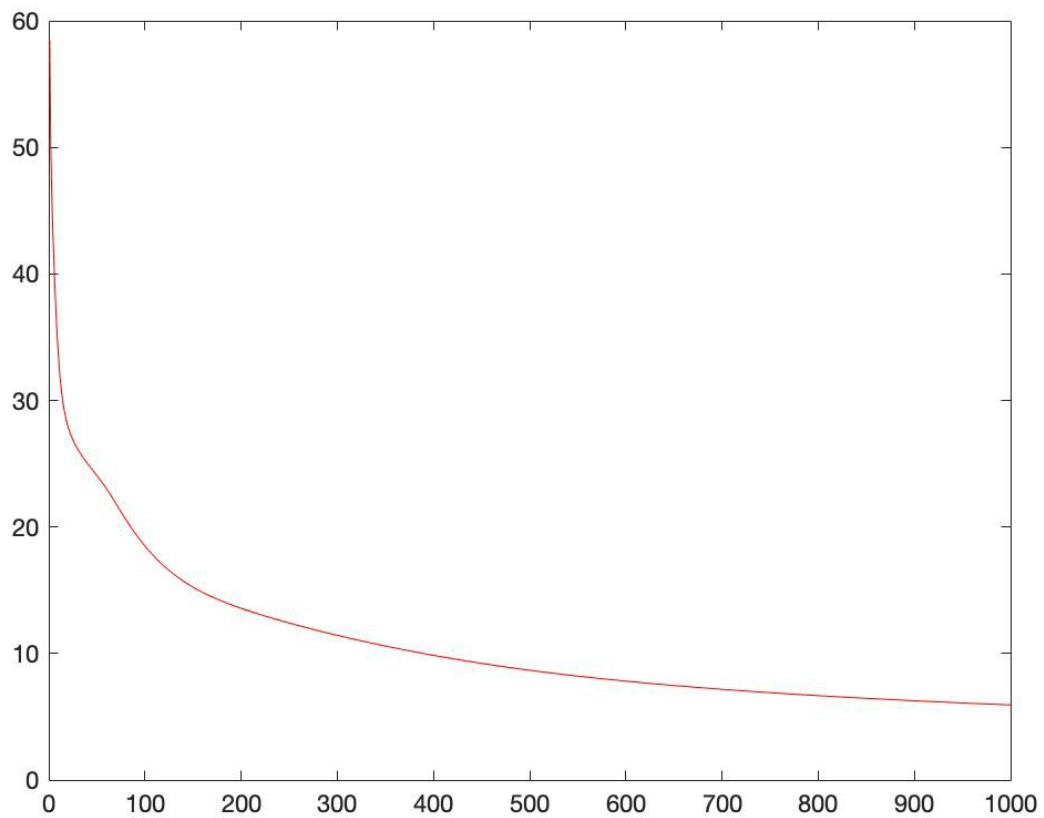
-
- Error training:17.9567
 - Error testing:18.7816

When hidden neurons equals to 2:

Costs of the training set:



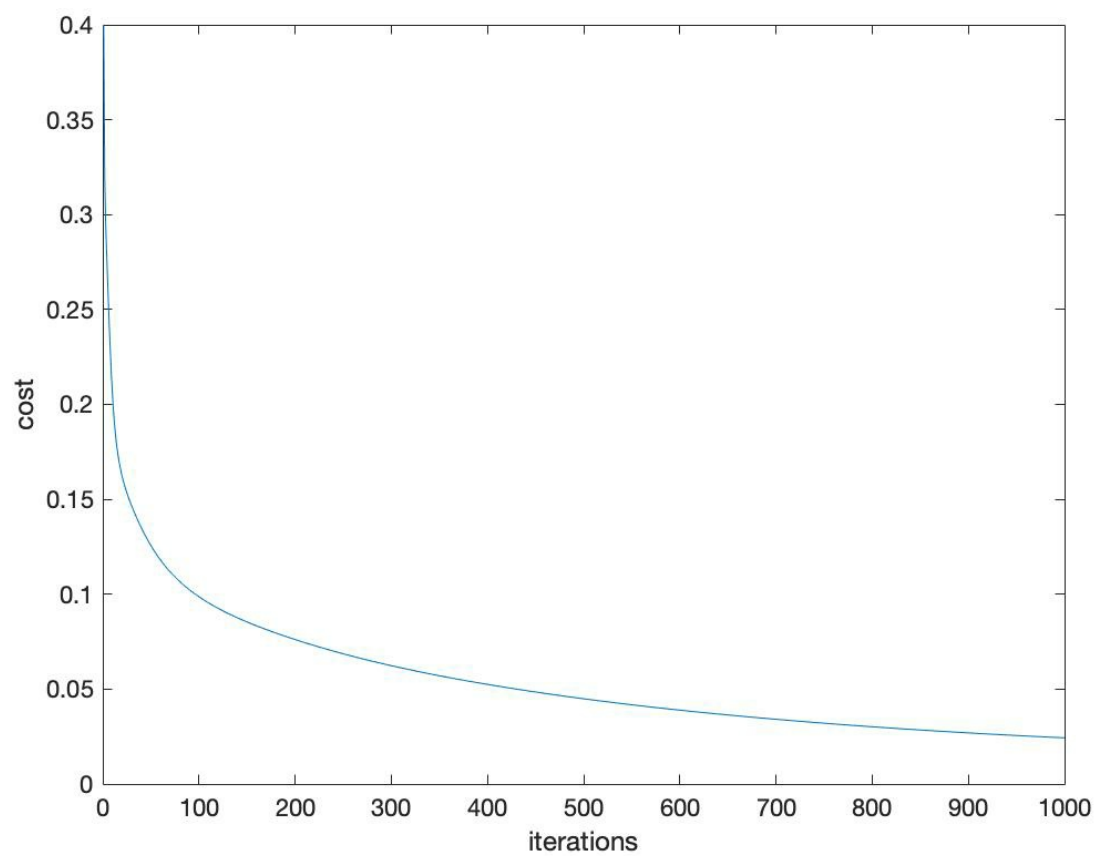
Costs of the test set:



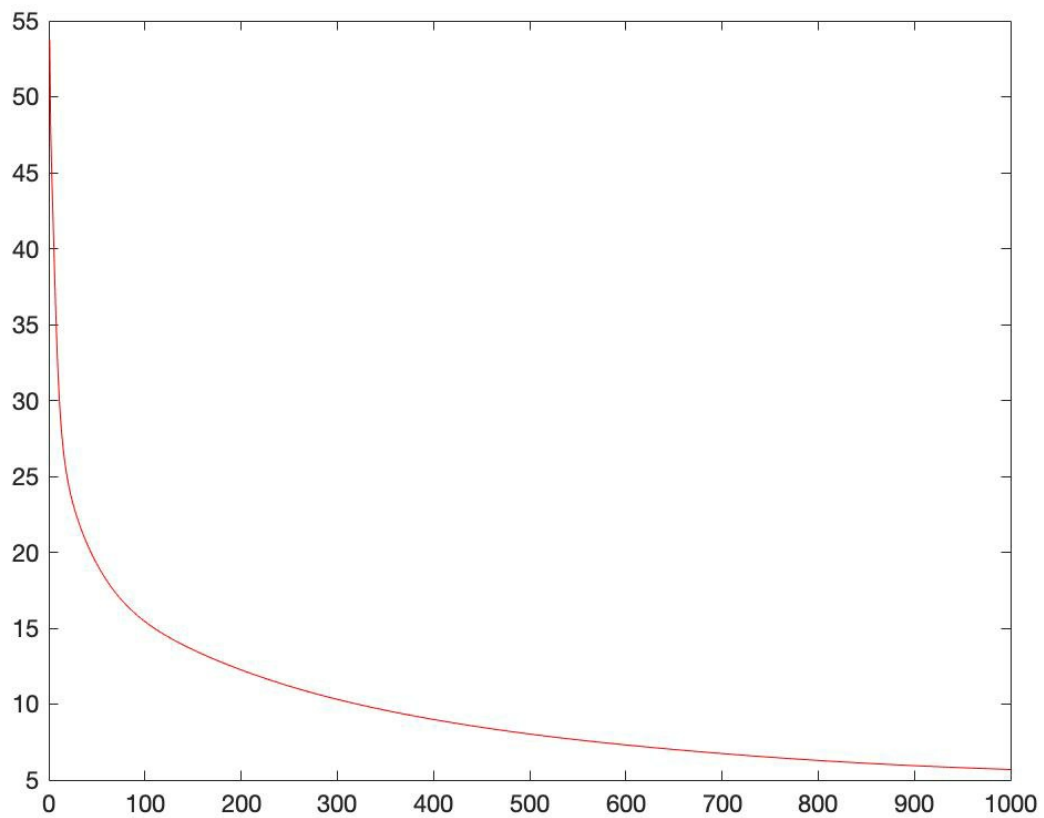
-
- Error training:4.909
 - Error testing:5.9335

When hidden neurons equals to 3:

Costs of the training set:



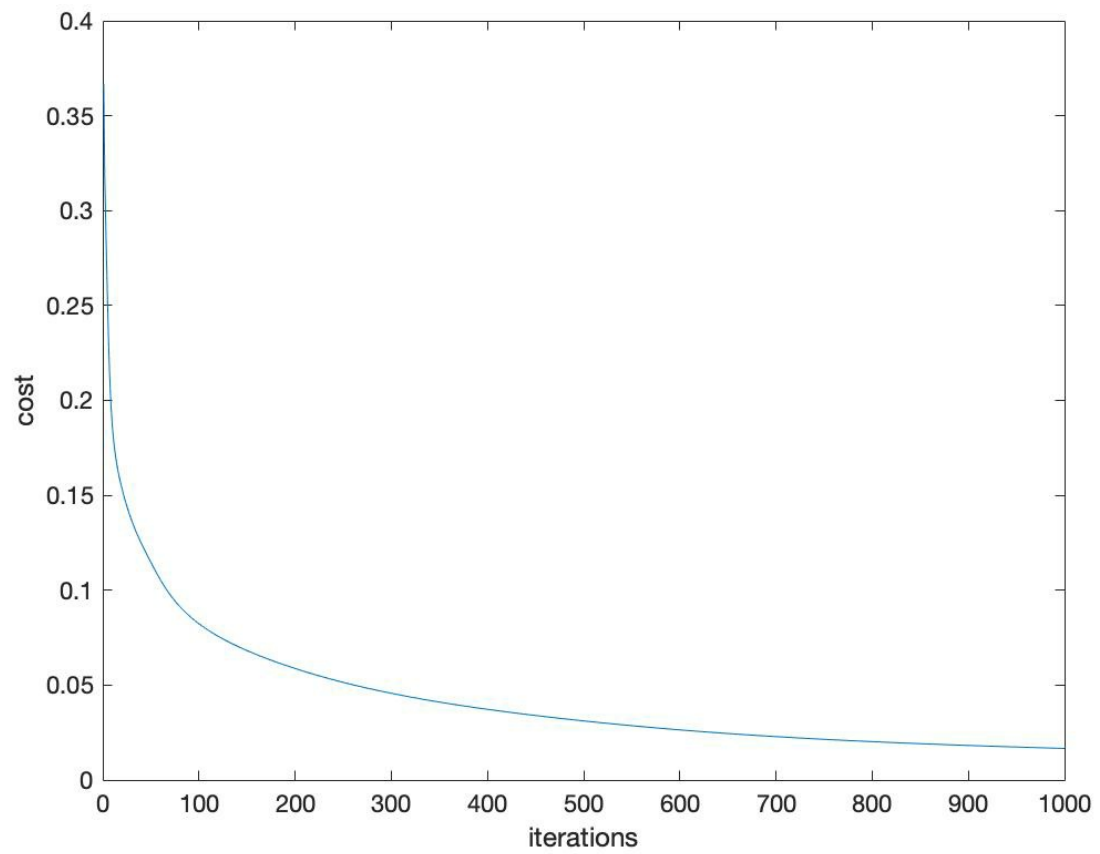
Costs of the test set:



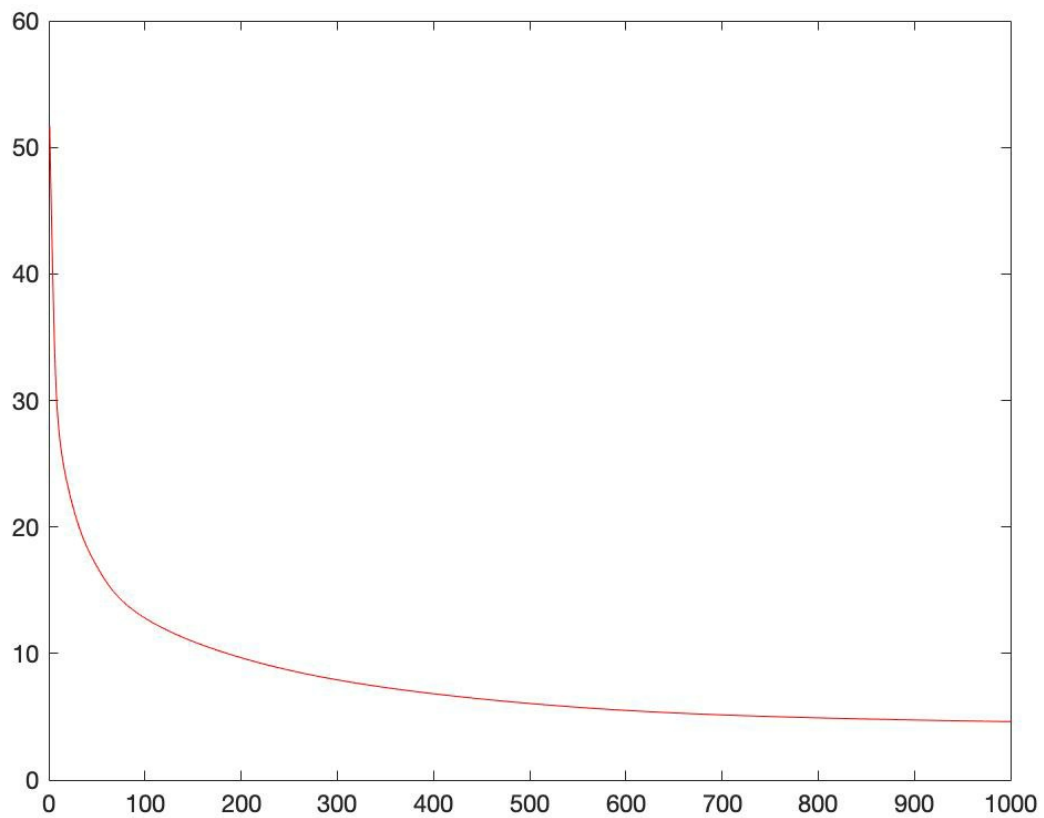
-
- Error training:3.9066
 - Error testing:5.6948

When hidden neurons equals to 5:

Costs of the training set:



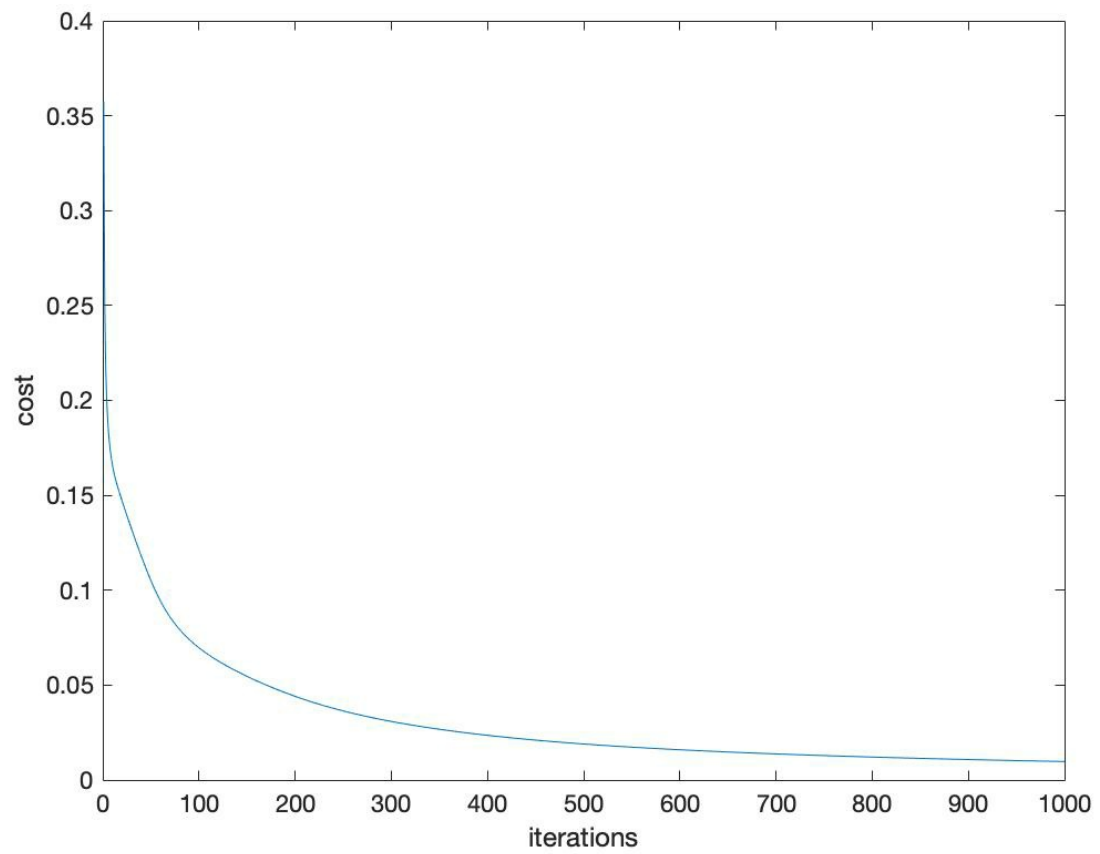
Costs of the test set:



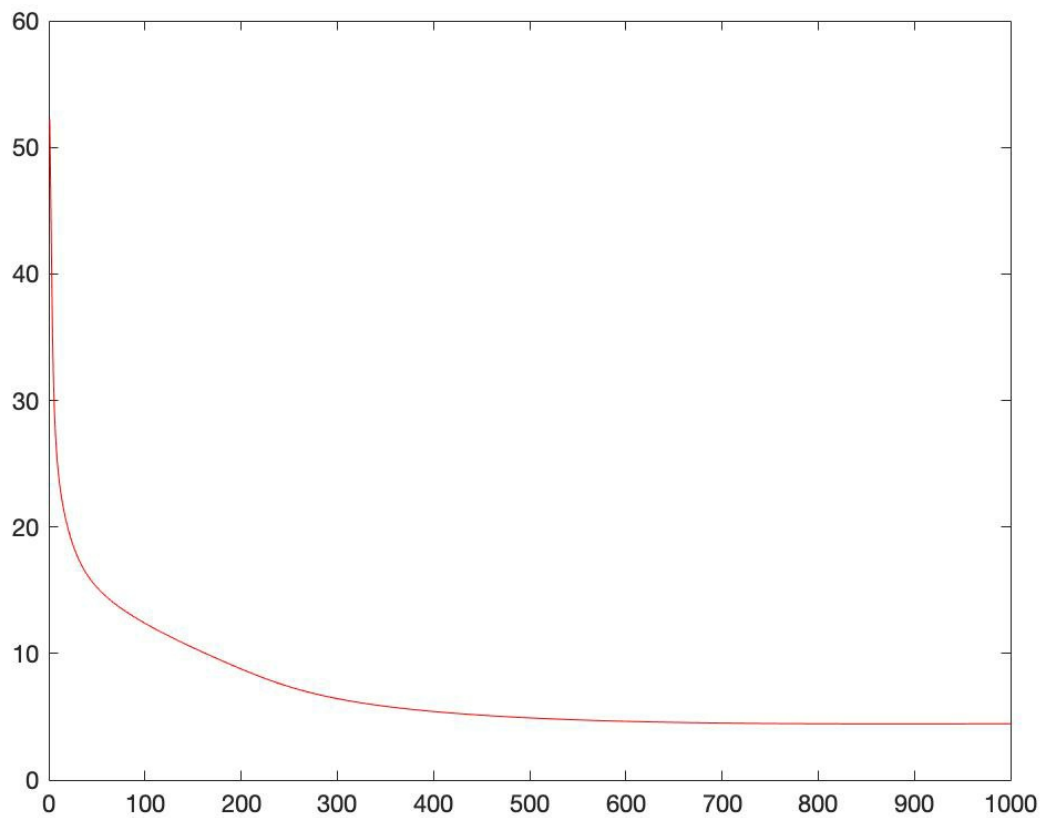
-
- Error training:2.4614
 - Error testing:4.6142

When hidden neurons equals to 7:

Costs of the training set:



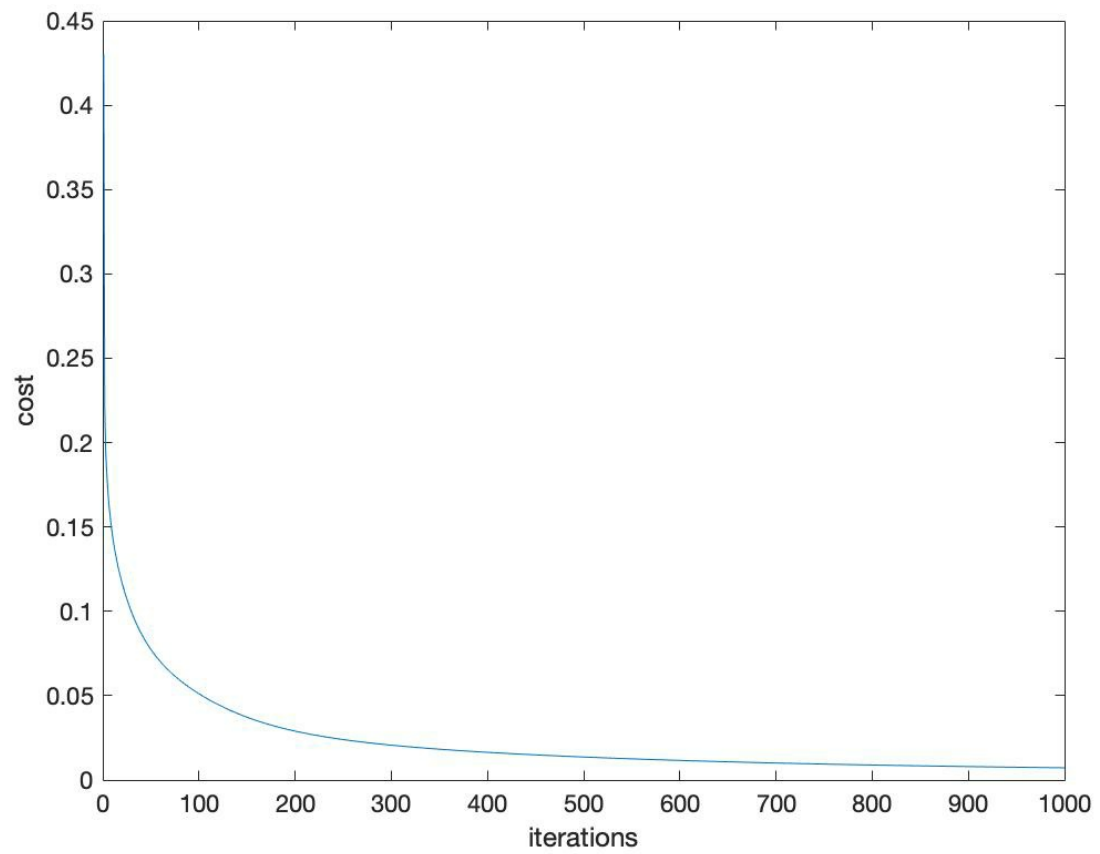
Costs of the test set:



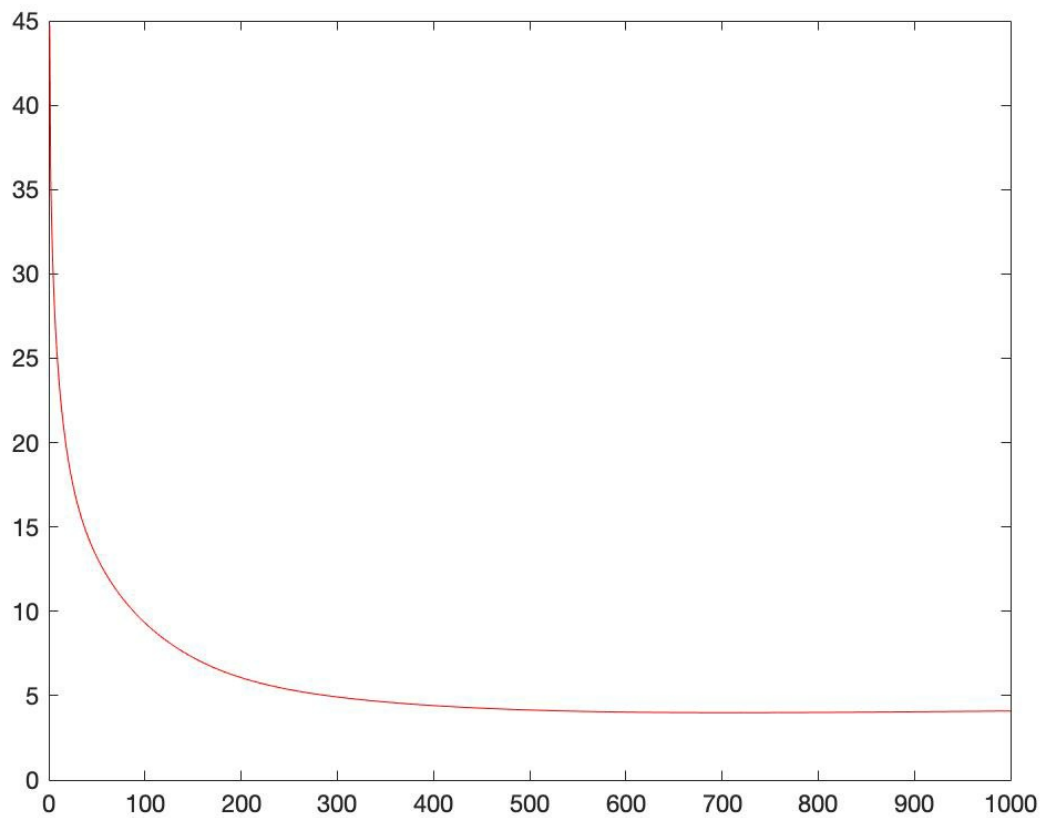
-
- Error training:1.0397
 - Error testing:4.4482

When hidden neurons equals to 10:

Costs of the training set:



Costs of the test set:



-
- Error training:1.0285
 - Error testing:4.0931

As the number of the hidden neurons goes up, the training error and testing error decrease.