Field correlations and effective two-level atom-cavity systems

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We analyze the properties of the second-order correlation functions of the electromagnetic field in atom-cavity systems that approximate two-level systems. It is shown that a recently developed polariton formalism can be used to account for all the properties of the correlations, if the analysis is extended to include two manifolds—corresponding to the ground state and the states excited by a single photon—rather than just two levels.

The fundamental challenge for nonlinear quantum optics is the realization of dissipation-free photon-photon interactions at the level of a few photons. In conventional nonlinear optical systems, Kerr nonlinearity gives rise to an effective photon-photon interaction that becomes important typically on the level of 10^{10} photons. Enhancement of the atom-field coupling using the techniques of cavity quantum electrodynamics (cavity QED) increases the Kerr nonlinearity. Two spectacular experiments have demonstrated that it is indeed possible to obtain large *conditional phase shifts* that arise from strong photon-photon interactions at the few photon level [1]. The basic cavity QED scheme utilized in these experiments is based on the *Jaynes–Cummings model* (JC) [2] of a two-level atom strongly coupled to a single cavity mode. Despite its success in demonstrating large single-photon conditional phase shifts, this scheme appears to be fundamentally limited by the atomic and cavity dissipation. We note, though, the results of Hofmann *et al.* [3], suggesting that the use of one-sided cavity can significantly improve the phase shifts reported in Ref. [1]. Furthermore, Kojima *et al.* [4] analyzed the nonlinear interaction of two photons and a two-level atom and explained bunching and antibunching effects in the output state of photons in terms of quantum interferences between different absorption and propagation processes.

One possible way to overcome the limitation due to dissipation is to study an *effective two-level system*, rather than a two-level atom. Schmidt and Imamoğlu [5] have predicted that a four-level atomic scheme based on electromagnetically induced transparency (EIT) [6] (called EIT-Kerr scheme) can give rise to several orders of magnitude enhancement in Kerr nonlinearity as compared to conventional two- and three-level schemes. In this scheme, atomic spontaneous emission is avoided through EIT. The prediction has been verified in a recent experiment by Kang and Zhu [7]. It has also been predicted that the presence of such large Kerr nonlinearities in a high-finesse cavity could result in *photon blockade* and effective two-level behavior of the cavity mode [8,9]. Recent progress in cavity QED demonstrates the experimental fea-

sibility of the observation of photon blockade using state-of-the-art cavity QED techniques [10].

Another system predicted to exhibit photon blockade, proposed by Tian and Carmichael [11], is based on the JC model, but involves a single two-level atom strongly coupled to the cavity mode. If the atomic and cavity resonances coincide, and the external driving field is tuned to the lower (or upper) vacuum Rabi resonance, the system shows characteristic two-state behavior.

In this Brief Report, we analyze the effective two-level behavior as exhibited by EIT-Kerr and the extended JC schemes. By the extended JC model we mean a single two-level atom interacting with a single mode of a quantized cavity field, where the interaction of the atom with the field mode can be described by the JC Hamiltonian [2], extended by the driving and dissipation terms. The second-order correlation function $g^{(2)}(\tau)$ [12] has been established as a good measure of photon blockade [8], so our analysis concentrates on the properties of second-order correlations. We show that the recently developed polariton formalism [9] can be used to account for the properties of these correlations, provided that the model includes the entire first excitation manifold, rather than just two levels.

The Hamiltonian of an extended JC model in the absence of dissipation [Fig. 1(a)] is given by $H_0^{JC} = H_{int}^{JC} + H_{pump}$, where

$$H_{int}^{JC} = \hbar\,\theta(\sigma_z/2 + a^\dagger a) + i\hbar(ga^\dagger\sigma_- - g^* a\sigma_+), \qquad (1a)$$

$$H_{pump} = i\hbar\mathcal{E}_p(a - a^\dagger). \qquad (1b)$$

Here, H_{int}^{JC} is the Hamiltonian of an atom interacting with a field mode, with σ's being the atomic pseudospin operators, a and $a^\dagger$ are field annihilation and creation operators. Driving of the cavity by a classical field of amplitude $\mathcal{E}_p$ is described by the Hamiltonian H_{pump}. The atom and cavity mode (coupled with strength g) are assumed to be resonant, while we assume the driving field to be detuned by $\theta = \omega_L - \omega_{cav}$ from both atomic and cavity resonances. Including dissipation leads to the non-Hermitian Hamiltonian

$$H^{JC} = H_0^{JC} - \frac{i\hbar}{2}\kappa a^\dagger a - \frac{i\hbar}{2}\gamma\sigma_+\sigma_-, \qquad (2)$$

where κ and γ are the cavity and spontaneous emission dissipation rates. Naturally, H^{JC} has to be combined with a

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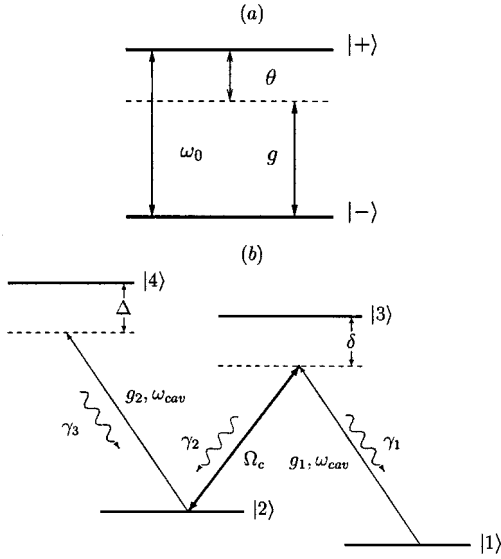
FIG. 1. Energy levels in (*a*) Jaynes-Cummings and (*b*) EIT-Kerr schemes. δ and Δ are the detunings of the field modes from the atomic resonance, and γ_j's are spontaneous emission decay rates for the given decay channel.

gedanken measurement process in order to obtain the complete dynamics of the system. This approach is usually referred to as a quantum trajectory approach [13].

The EIT-Kerr scheme involves a four-level atom in a cavity [see Fig. 1(b)]. The Hamiltonian of this model is $H_0^{EIT} = H_{int}^{EIT} + H_{pump}$, with

$$H_{int}^{EIT} = \hbar(\delta\sigma_{33} + \Delta\sigma_{44}) + i\hbar[g_1(a^\dagger\sigma_{13} - \sigma_{31}a)$$
$$+ (\Omega_c^*\sigma_{23} - \sigma_{32}\Omega_c) + g_2(a^\dagger\sigma_{24} - \sigma_{42}a)], \quad (3)$$

where $\sigma_{\mu\nu}$ are the atomic pseudospin operators, and Ω_c is the Rabi frequency of a (classical) coupling field. Again, dissipation can be included in the same manner as above to get the non-Hermitian Hamiltonian

$$H^{EIT} = H_0^{EIT} - \frac{i\hbar}{2}[\kappa a^\dagger a + (\gamma_1 + \gamma_2)\sigma_{33} + \gamma_3\sigma_{44}]. \quad (4)$$

In both cases we assume a strong atom-field coupling, leading to the natural description in terms of dressed states (polaritons) [9].

The system described by the extended JC model behaves as a two-state system when excited near one of the vacuum Rabi resonances $|e_\pm\rangle = (|0,+\rangle \pm |1,-\rangle)/\sqrt{2}$, where $-$ and $+$ denote ground and excited atomic states, and numbers 0 and 1 denote the number of photons in the cavity mode. The splitting of the dressed states of the *n*th excited manifold is found from Eq. (1a), to be $\epsilon_\pm = (\hbar\theta/2)n \pm \hbar g\sqrt{n}$. If the laser field is tuned to the lower vacuum Rabi resonance ($\theta = g$), the system effectively behaves as a resonantly driven two-level system. Photon blockade occurs, since after the first photon excites the system, the second photon is detuned by $\tilde{\epsilon} = (2-\sqrt{2})\hbar g$ from the resonance of the second excitation.

It is possible to obtain the effective Hamiltonian describing the photon blockade dynamics in terms of the two Rabi-split states $|e_\pm\rangle$. We define the polariton operators $q_\pm$ with $|e_\pm\rangle = q_\pm^\dagger|0,-\rangle$ and find

$$q_\pm\exp(\pm i\hbar\epsilon_\pm) = \frac{1}{\sqrt{2}}(\sigma_- \pm a). \quad (5)$$

Substituting the operators (5) in the Hamiltonian (2), transforming the Hamiltonian to a frame rotating at a laser frequency $\omega_L = \omega_{cav} - \theta$, and performing a rotating wave approximation, we arrive at the effective Hamiltonian

$$\mathcal{H}_{eff}^{JC} = 2\hbar\theta q_+^\dagger q_+ + i\frac{\hbar\mathcal{E}_p}{\sqrt{2}}(q_- - q_-^\dagger + q_+ - q_+^\dagger)$$
$$- i\hbar\frac{\gamma + \kappa}{2}(q_-^\dagger q_- + q_+^\dagger q_+). \quad (6)$$

In the following discussion, we assume $\theta = g$. Hamiltonian (6) contains the effective two-level Hamiltonian of Tian and Carmichael [11], with two additional terms proportional to $q_+^\dagger$ and q_+. We note three key features: (i) This model is valid for large coupling g and weak driving $\mathcal{E}_p$, where the truncated (higher) manifolds do not influence the dynamics. The applicability of the effective model can be determined by the value of $g^{(2)}(0)$, which should ideally be zero. (ii) Large amplitude oscillations in $g^{(2)}(\tau)$, of frequency $\sqrt{2}\mathcal{E}_p$, are predicted by the effective Hamiltonian, to occur for $\mathcal{E}_p > (\kappa + \gamma)/2$. (iii) Small amplitude modulations in $g^{(2)}(\tau)$, of frequency $2\theta = 2g$ occur as a signature of the upper Rabi resonance. The last feature points at the shortcoming of the effective two-level model (also noted by Tian and Carmichael). Hamiltonian (6) represents an *effective two-manifold model*, reducing the dynamics to transitions between the ground state and the entire first excited manifold.

Dressed states analysis for a single atom in the EIT-Kerr configuration has been carried out in Refs. [8,9]. There are three states in the $n=1$ manifold, one of which is resonant with the cavity mode. The second manifold contains four states. The outer two states are detuned far from the resonance and therefore their contribution to the system dynamics is negligible. The inner two states are also detuned, but lie closer to the resonance, with the size of the detuning determined by a coupling strength g_2.

The three states in the first excited manifold are

$$|\phi_0\rangle = \frac{|1,1\rangle + (g_1/\Omega_c)|0,2\rangle}{\sqrt{1 + (g_1/\Omega_c)^2}}, \quad (7a)$$

$$|\phi_\pm\rangle = -\frac{(g_1/\Omega_c)|1,1\rangle + i(\varepsilon_\pm/\Omega_c)|0,3\rangle - |0,2\rangle}{\sqrt{1 + (\varepsilon_\pm/\Omega_c)^2 + (g_1/\Omega_c)^2}}. \quad (7b)$$

Again, the effective two-manifold model can be obtained by following the same method. Defining the polariton operators p_j with $|\phi_j\rangle = p_j^\dagger|0,1\rangle$, the following effective Hamiltonian emerges
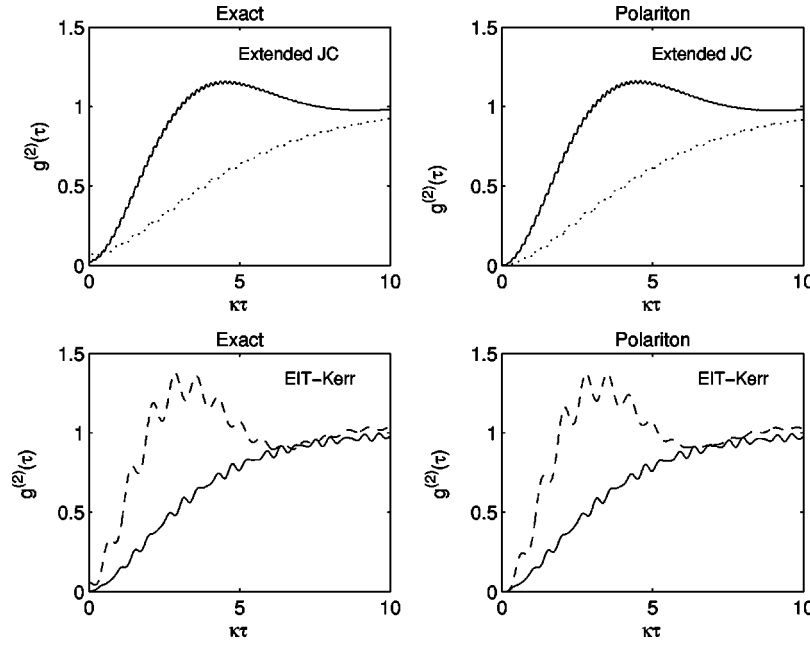
FIG. 2. Second-order correlation functions in Jaynes-Cummings and EIT-Kerr schemes. Left column ("Exact") is obtained from the exact models, while right column ("Polariton") from the effective two-manifold models. In the Exact case, parameters for the extended JC are $\gamma=0.1\kappa$, $\theta=g$ and $g=6\kappa$, $\mathcal{E}_p=0.1\kappa$ for the dashed curve; $g=20\kappa$, $\mathcal{E}_p=0.5\kappa$ for the solid curve. For the EIT-Kerr scheme, parameters are $\gamma_j=0.1\kappa$, $g_j=6\kappa$, $\Delta=0$ and $\delta=0.2\kappa$, $\mathcal{E}_p=0.7\kappa$, $\Omega_c=6\kappa$ (dashed); $\delta=4\kappa$, $\mathcal{E}_p=0.1\kappa$, $\Omega_c=12\kappa$ (solid). The same parameters are used in the Polariton case. In the EIT-Kerr case, this amounts to $\varepsilon_+=8.59\kappa$, $\varepsilon_-=-8.39\kappa$, $\Omega_+=-0.696\kappa$, $\Omega_-=-0.704\kappa$, $\Omega_R=0.99\kappa$, $\Gamma_+=0.348\kappa$, $\Gamma_-=0.352\kappa$ and $\Gamma_0=0.5\kappa$ (dashed); $\varepsilon_+=15.56\kappa$, $\varepsilon_-=-11.56\kappa$, $\Omega_+=-0.06\kappa$, $\Omega_-=-0.06\kappa$, $\Omega_R=0.18\kappa$, $\Gamma_+=0.2\kappa$, $\Gamma_-=0.2\kappa$, and $\Gamma_0=0.8\kappa$ (solid).

$$\mathcal{H}_{eff}^{EIT} = \hbar\varepsilon_- p_-^\dagger p_- + \hbar\varepsilon_+ p_+^\dagger p_+ + i\frac{\hbar\Omega_-}{2}(p_- - p_-^\dagger) + i\frac{\hbar\Omega_+}{2}(p_+ - p_+^\dagger)$$

$$+ i\frac{\hbar\Omega_R}{2}(p_0 - p_0^\dagger) + i\hbar\Gamma_0 p_0^\dagger p_0 + i\hbar\Gamma_-^{(1)} p_-^\dagger p_-$$

$$+ i\hbar\Gamma_+^{(1)} p_+^\dagger p_+, \tag{8}$$

with the effective Rabi frequencies of driving [9],

$$\Omega_\pm = -2\mathcal{E}_p \frac{g_1/\Omega_c}{\sqrt{1 + (\varepsilon_\pm/\hbar\Omega_c)^2 + (g_1/\Omega_c)^2}}, \tag{9a}$$

$$\Omega_R = \frac{2\mathcal{E}_p}{\sqrt{1 + (g_1/\Omega_c)^2}}, \tag{9b}$$

decay rates,

$$\Gamma_0 = \frac{\kappa}{1 + (g_1/\Omega_c)^2}, \tag{9c}$$

$$\Gamma_\pm^{(1)} = \frac{\kappa g_1^2 + (\gamma_1 + \gamma_2)(\varepsilon_\pm)^2}{g_1^2 + \Omega_c^2 + (\varepsilon_\pm)^2}, \tag{9d}$$

and energies $\varepsilon_\pm = \delta/2 \pm \sqrt{(\delta/2)^2 + \Omega_c^2 + g_1^2}$. Key features of this effective two-manifold model can be identified in correspondence to those of the extended JC model. Large oscillations of frequency Ω_R in $g^{(2)}(\tau)$ are predicted to occur for $\Omega_R > \Gamma_0/2$ or $\mathcal{E}_p > (\kappa/4)/\sqrt{1 + g_1^2/\Omega_c^2}$. In addition, small amplitude modulations consisting of *two* frequencies $\varepsilon_\pm$ will be

observed. If $|\varepsilon_+| \approx |\varepsilon_-|$, only one frequency will be visible, whereas if $|\varepsilon_+| \neq |\varepsilon_-|$, oscillations with both frequencies should be apparent.

How well do these effective Hamiltonians describe the dynamics of the full system? We calculate the second-order correlation function using a wave-function simulations [13] of the original Hamiltonians (2) and (4). The photon space was restricted to four photons, resulting in a 10 (20) dimensional Hilbert spaces for the extended JC (EIT-Kerr) model. Results are depicted in Fig. 2. Then, using the same technique, we calculate the second-order correlation function of the effective polariton Hamiltonians (6) and (8), requiring Hilbert spaces of only three and four-dimensions respectively, and also plot the results in Fig. 2.

Identical values of couplings and decay rates are chosen in both schemes to enable better comparison. In the extended JC scheme, a significant antibunching [as measured by $g^{(2)}(\tau=0)$] is found. The particular value of $g^{(2)}(0)$ measures the validity of the truncation of dressed basis after the first manifold. To achieve even better agreement, a stronger coupling is needed ($g/\kappa \sim 20$ or larger) which is experimentally unavailable as of yet. We note the modulation of frequency $2g$, as predicted by two-manifold model.

The other two curves show the simulation results for the EIT-Kerr system. They exhibit essentially the same values at the origin, which are now very close to zero. This means that the effective Hamiltonian (8) captures the significant dynamics well. The modulation of frequency $\varepsilon_+ \approx |\varepsilon_-| \approx \sqrt{g_1^2 + \Omega_c^2}$, as predicted by the effective Hamiltonian is seen on the upper curve. The driving chosen to produce Fig. 2 is such that $\Omega_R > \Gamma_0/2$, implying the presence of large oscillations in the

correlation function. The opposite is true for the lower curve. Note also the existence of two modulation frequencies since the choice $\delta = 4\kappa$ implies $\varepsilon_+ \neq |\varepsilon_-|$.

The two curves in the EIT-Kerr case show vastly different coherence times—a difference attributable to the lifetime of the effective excited state (7a). It follows from the decay rate (9c) that, given the fixed atom-field coupling g_1, the lifetime can be adjusted by the coupling laser (i.e., Ω_c). As a consequence, the coherence time of the effective two-level system can be adjusted to virtually any prescribed value by varying the coupling Ω_c. Indeed, for larger values of g_1/Ω_c, such as the one depicted in the lower EIT-Kerr curve in Fig. 2, there

is a comparatively slow recovery of the correlation function from the origin.

In conclusion, we have presented a polariton description of effective two-level atom-cavity systems in the strong coupling regime of cavity QED. It was shown how a reduction of a more sophisticated polariton structure to a lowest excited manifold can account for the properties of a second-order correlation function of light leaking out of the cavity.

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