

```

sum: THEORY
  BEGIN

    n: VAR nat

    sum(n): RECURSIVE nat =
      (IF n = 0 THEN 0 ELSE n + sum(n - 1) ENDIF)
      MEASURE (LAMBDA n: n)

    closed_form: THEOREM sum(n) = n * (n + 1)/2

  END sum

```

```

--:-- sum.pvs      14:06 0.02      (PVS :ready)--L10--All-----

```

```

closed_form.2 :

  |-----
{1}   FORALL (j: nat):
      sum(j) = j * (j + 1) / 2 IMPLIES sum(j + 1) = (j + 1) * (j + 1 + 1) / 2

```

Rule? (postpone)  
Postponing closed\_form.2.

```

closed_form.1 :

  |-----
{1}   sum(0) = 0 * (0 + 1) / 2

```

Rule? (expand "sum")  
Expanding the definition of sum,  
this simplifies to:  
closed\_form.1 :

```

  |-----
{1}   0 = 0 / 2

```

Rule? (assert)  
Simplifying, rewriting, and recording with decision procedures,  
This completes the proof of closed\_form.1.

```

closed_form.2 :

  |-----
{1}   FORALL (j: nat):
      sum(j) = j * (j + 1) / 2 IMPLIES sum(j + 1) = (j + 1) * (j + 1 + 1) / 2

```

Rule?

```

--:** *pvs*      14:06 0.02      (ILISP :ready)--L??--Bot-----

```