# Midterm solutions

**Problem 1**

$S_{ut} = 1200\ MPa, \sigma_{rev} = 800MPa$

$S_{ut} = 1200\ MPa = 174\ kpsi$. From graph (fig 6.18) $f = 0.785$

$S_e = 0.5 * S_{ut} = 0.5 * 1200 = 600.$

$a = \frac{(fS_{ut})^2}{S_e} = \frac{(0.785*1200)^2}{600} = 1479\ MPa.$

$b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.785*1200}{600}\right) = -0.0653.$

$N = \left(\frac{\sigma_{rev}}{a}\right)^{\frac{1}{b}} = \left(\frac{800}{1479}\right)^{1/-0.0653} = 12216\ cycles.$

**Problem 2**

MSS theory:

From Table A-20 for 1020 CD steel, Sy = 66 kpsi. From Eq. (3-42), p.116,

$$T = \frac{63025H}{n} \qquad (1)$$

where n is the shaft speed in rev/min. From Eq. (5-3), for the MSS theory,

$$\tau_{max} = \frac{S_y}{2n_d} = \frac{16T}{\pi d^3} \qquad (2)$$

where nd is the design factor. Substituting Eq. (1) into Eq. (2) and solving for d gives

$$d = \left[\frac{32(63025)Hn_d}{n\pi S_y}\right]^{\frac{1}{3}}$$

$$d = \left[\frac{32(63025)15(2)}{1000\pi(66)10^3}\right]^{\frac{1}{3}} = 0.663\ in$$

DE theory:

$$\sigma' = \frac{S_y}{n_d} \qquad (3)$$

$$\sigma' = (\sigma_x^2 - \sigma_y\sigma_x + \sigma_y^2 + 3\tau_{xy})^{\ 1/2} \quad (4)$$

In this case, only got shear stress

So the equation will be

$$\sqrt{3}\tau_{max} = \frac{S_y}{n_d}$$

Solve d,

$$d = \left[\frac{\sqrt{3}(16)(63025)15(2)}{1000\pi(66)10^3}\right]^{\frac{1}{3}} = 0.632\ in$$

**Problem 3**

Table A-20: $S_{ut} = 68\ kpsi\ \ S_y = 57$

N = (1300 rev/min)(10 hr)(60 min/hr) = 780 000 cycles

One approach is to guess a diameter and solve the problem as an iterative analysis problem. Alternatively, we can estimate the few modifying parameters that are dependent on the diameter and solve the stress equation for the diameter, then iterate to check the estimates. We'll use the second approach since it should require only one iteration, since the estimates on the modifying parameters should be pretty close.

First, we will evaluate the stress. From a free-body diagram analysis, the reaction forces at the bearings are $\boldsymbol{R_1}$ **= 2 kips** and $\boldsymbol{R_2}$ **= 6 kips**. The critical stress location is in the middle of the span at the shoulder, where the bending moment is high, the shaft diameter is smaller, and a stress concentration factor exists. If the critical location is not obvious, prepare a complete bending moment diagram and evaluate at any potentially critical locations. Evaluating at the critical shoulder,

$$M = 2\ kip(10\ in) = 20\ kip.in$$

$$\sigma_{rev} = \frac{Mc}{I} = \frac{M\left(\frac{d}{2}\right)}{\frac{\pi d^4}{64}} = \frac{32M}{\pi d^3} = \frac{32(20)}{\pi d^3} = \frac{203.7}{d^3}kpsi$$

Now we will get the notch sensitivity and stress concentration factor. The notch sensitivity depends on the fillet radius, which depends on the unknown diameter. For now, let us estimate a value of q = 0.85 from observation of Fig. 6-20, and check it later.

Fig. A-15- 9: $\frac{D}{d} = \frac{1.4d}{d} = 1.4, \frac{r}{d} = \frac{0.1d}{d} = 0.1, K_t = 1.65$

Eq (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.85(1.65 - 1) = 1.55$

Now, evaluate the fatigue strength.

$$S_e' = 0.5 * 68 = 34\ kpsi$$

$$k_a = 2.7(68)^{-0.265} = 0.882$$

Since the diameter is not yet known, assume a typical value of $k_b$ = 0.85 and check later. All other modifiers are equal to one.

$$Se = (0.882)(0.85)(34) = 25.5\ kpsi$$

Determine the desired fatigue strength from the S-N diagram

Fig 6-18 f= 0.9

$a = \frac{(fS_{ut})^2}{S_e} = \frac{(0.9*68)^2}{25.5} = 146.88.$

$b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.9*68}{25.5}\right) = -0.1267.$

$S_f = aN^b = 146.88(780000)^{-0.1267} = 26.328.$

Compare strength to stress and solve for the necessary d.

$$n_f = \frac{S_f}{K_f\sigma_{rev}} = \frac{26.328}{(1.55)\left(\frac{203.7}{d^3}\right)} = 1.6$$

$$d = 2.67\ in$$

From here, we can update the notch sensitivity and the size modification factor, and iterate the calculation again to update $d$. Note, the iteration is not necessary to get full credit.

**Problem 4**

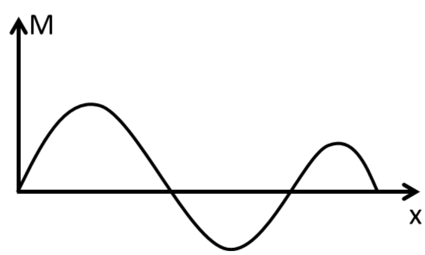
4.1

The sketch should have the following qualitative properties:

(1) Bending moments are zero at the pin supports

(2) Bending moments goes above zero from the left support

(3) The curve is piece-wise quadratic



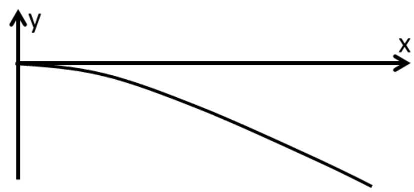

4.2

The sketch should have the following qualitative properties:

(1) The deflect is negative

(2) The slope increases

(3) The rate of increase is decreasing




4.3. True: Both yielding and fracture can cause failure

4.4. This question is a little ambiguous. If you consider the 3D element, the max stress is non-zero if the normal stresses are non-zero. If you only consider 2D, then the max stress within the plane is zero (or tau_xy if you consider it as nonzero). You will get full credit either way, as long as you can explain clearly.

# Common Mistake

Problem 2

(1) Forget to change kpsi to psi. Cause the unit inconsistence.

(2) Half of the students didn't do distortion energy method.

Problem 3

(1) Most of the students didn't use this formula to calculate d

$$n_f = \frac{S_f}{K_f \sigma_{rev}}$$

(2) Students usually use following equation

$$N = (\frac{\sigma_{rev}}{a})^{1/b}$$

But this equation only applicable for completely reversed loading.

(3) Some students didn't calculate reversed stress $\sigma_{rev}$