ME598/494 Homework 4

1. (10 points) Sketch graphically the problem

$$\begin{aligned}
&\min\ f(\mathbf{x}) = (x_1+1)^2 + (x_2-2)^2 \\
&\text{subject to } g_1 = x_1 - 2 \leq 0, \quad g_3 = -x_1 \leq 0, \\
&g_2 = x_2 - 1 \leq 0, \quad g_4 = -x_2 \leq 0.
\end{aligned}$$

   Find the optimum graphically. Determine directions of feasible descent at the corner points of the feasible domain. Show the gradient directions of f and g_is at these points. Verify graphical results analytically using the KKT conditions.

2. (10 points) Graph the problem

$$\begin{aligned}
&\min f = -x_1, \text{subject to} \\
&g_1 = x_2 - (1-x_1)^3 \leq 0 \quad \text{and} \quad x_2 \geq 0.
\end{aligned}$$

   Find the solution graphically. Then apply the optimality conditions. Can you find a solution based on the optimality conditions? Why? (From Kuhn and Tucker, 1951.)

3. (30 points) Find a local solution to the problem

$$\begin{aligned}
&\max\ f = x_1x_2 + x_2x_3 + x_1x_3 \\
&\text{subject to } h = x_1 + x_2 + x_3 - 3 = 0.
\end{aligned}$$

   Use three methods: direct elimination, reduced gradient, and Lagrange multipliers. Compare. Is the solution global?

4. (20 points) Use reduced gradient to find the value(s) of the parameter b for which the point $x_1 = 1$, $x_2 = 2$ is the solution to the problem

$$\begin{aligned}
&\max\ f = 2x_1 + bx_2 \\
&\text{subject to } g_1 = x_1^2 + x_2^2 - 5 \leq 0 \\
&\text{and } g_2 = x_1 - x_2 - 2 \leq 0.
\end{aligned}$$

5. (30 points, MAE 598) Find the solution for

$$\begin{aligned}
&\min\ f = x_1^2 + x_2^2 + x_3^2 \\
&\text{subject to } h_1 = x_1^2/4 + x_2^2/5 + x_3^2/25 - 1 = 0 \\
&\text{and } h_2 = x_1 + x_2 - x_3 = 0,
\end{aligned}$$

   by implementing the generalized reduced gradient method (e.g., using MATLAB). See template code here.