Problem 1

$$d_P = 17/8 = 2.125 \text{ in}$$

$$d_G = \frac{N_2}{N_3} d_P = \frac{1120}{544} (2.125) = 4.375 \text{ in}$$

$$N_G = Pd_G = 8(4.375) = 35 \text{ teeth} \qquad Ans.$$

$$C = (2.125 + 4.375)/2 = 3.25 \text{ in} \qquad Ans.$$

Problem 2

The smallest pinion that will mesh with a gear ratio of m_G = 2.5, from Eq. (13-11) is

$$\begin{split} N_{P} &\geq \frac{2k}{\left(1 + 2m\right)\sin^{2}\phi} \left(m + \sqrt{m^{2} + \left(1 + 2m\right)\sin^{2}\phi}\right) \\ &\geq \frac{2\left(1\right)}{\left[1 + 2\left(2.5\right)\right]\sin^{2}20^{\circ}} \left\{2.5 + \sqrt{2.5^{2} + \left[1 + 2\left(2.5\right)\right]\sin^{2}20^{\circ}}\right\} \\ &\geq 14.64 \quad \rightarrow \quad 15 \text{ teeth} \quad Ans. \end{split}$$

The largest gear-tooth count possible to mesh with this pinion, from Eq. (13-12) is

$$N_G \le \frac{N_P^2 \sin^2 \phi - 4k^2}{4k - 2N_P \sin^2 \phi}$$

$$\le \frac{15^2 \sin^2 20^\circ - 4(1)^2}{4(1) - 2(15)\sin^2 20^\circ}$$

$$\le 45.49 \rightarrow 45 \text{ teeth} \quad Ans.$$

Problem 3

Applying Eq. (13-30), $e = (N_2 / N_3) (N_4 / N_5) = 45$. For an exact ratio, we will choose to factor the train value into integers, such that

$$N_2 / N_3 = 9$$
 (1)

$$N_4 / N_5 = 5$$
 (2)

Assuming a constant diametral pitch in both stages, the geometry condition to satisfy the in-line requirement of the compound reverted configuration is

$$N_2 + N_3 = N_4 + N_5$$
 (3)

With three equations and four unknowns, one free choice is available. It is necessary that all of the unknowns be integers. We will use a normalized approach to find the minimum free choice to guarantee integers; that is, set the smallest gear of the largest stage to unity, thus $N_3 = 1$. From (1), $N_2 = 9$. From (3),

$$N_2 + N_3 = 9 + 1 = 10 = N_4 + N_5$$

Substituting $N_4 = 5 N_5$ from (2) gives

$$10 = 5 N_5 + N_5 = 6 N_5$$

 $N_5 = 10 / 6 = 5 / 3$

To eliminate this fraction, we need to multiply the original free choice by a multiple of 3. In addition, the smallest gear needs to have sufficient teeth to avoid interference. From Eq. (13-11) with k = 1, $\phi = 20^{\circ}$, and m = 9, the minimum number of teeth on the pinion to avoid interference is 17. Therefore, the smallest multiple of 3 greater than 17 is 18. Setting $N_3 = 18$ and repeating the solution of equations (1), (2), and (3) yields

$$N_2$$
 = 162 teeth
 N_3 = 18 teeth
 N_4 = 150 teeth
 N_5 = 30 teeth Ans.

Problem 4

$$d_P = 16/6 = 2.667 \text{ in,}$$
 $d_G = 48/6 = 8 \text{ in}$
$$V = \frac{\pi(2.667)(300)}{12} = 209.4 \text{ ft/min}$$

$$W^t = \frac{33\ 000(5)}{209.4} = 787.8 \text{ lbf}$$

Assuming uniform loading, $K_o = 1$.

Eq. (14-28):
$$Q_v = 6$$
, $B = 0.25(12 - 6)^{2/3} = 0.8255$ $A = 50 + 56(1 - 0.8255) = 59.77$

Eq. (14-27):
$$K_v = \left(\frac{59.77 + \sqrt{209.4}}{59.77}\right)^{0.8255} = 1.196$$

Table 14-2: $Y_P = 0.296$, $Y_G = 0.4056$ From Eq. (a), Sec. 14-10 with F = 2 in

$$(K_s)_P = 1.192 \left(\frac{2\sqrt{0.296}}{6}\right)^{0.0535} = 1.088$$

 $(K_s)_G = 1.192 \left(\frac{2\sqrt{0.4056}}{6}\right)^{0.0535} = 1.097$

From Eq. (14-30) with $C_{mc} = 1$

$$C_{pf} = \frac{2}{10(2.667)} - 0.0375 + 0.0125(2) = 0.0625$$

 $C_{pm} = 1$, $C_{ma} = 0.093$ (Fig. 14 - 11), $C_e = 1$
 $K_m = 1 + 1[0.0625(1) + 0.093(1)] = 1.156$

Assuming constant thickness of the gears $\rightarrow K_B = 1$

$$m_G = N_G/N_P = 48/16 = 3$$

With N (pinion) = 10^8 cycles and N (gear) = $10^8/3$, Fig. 14-14 provides the relations:

$$(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$$

 $(Y_N)_G = 1.3558(10^8 / 3)^{-0.0178} = 0.996$

Fig. 14-6:
$$J_P = 0.27$$
, $J_G \to 0.38$
Table 14-10: $K_R = 0.85$
 $K_T = C_f = 1$

Eq. (14-23):
$$I = \frac{\cos 20^{\circ} \sin 20^{\circ}}{2(1)} \left(\frac{3}{3+1}\right) = 0.1205$$
 Table 14-8:
$$C_p = 2300 \sqrt{\mathrm{psi}}$$

Strength: Grade 1 steel with $H_{BP} = H_{BG} = 200$

Fig. 14-2:
$$(S_t)_P = (S_t)_G = 77.3(200) + 12800 = 28260$$
 psi

Fig. 14-5:
$$(S_c)_P = (S_c)_G = 322(200) + 29\ 100 = 93\ 500\ psi$$

Fig. 14-15:
$$(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$$

 $(Z_N)_G = 1.4488(10^8/3)^{-0.023} = 0.973$

Sec. 14-12:
$$H_{BP}/H_{BG} = 1$$
 :: $C_H = 1$

Pinion tooth bending

Eq. (14-15):
$$(\sigma)_P = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

= 787.8(1)(1.196)(1.088) $\left(\frac{6}{2}\right) \left[\frac{(1.156)(1)}{0.27}\right]$
= 13 170 psi Ans.

Eq. (14-41):
$$(S_F)_P = \left[\frac{S_t Y_N / (K_T K_R)}{\sigma} \right]$$

= $\frac{28 \ 260(0.977) / [(1)(0.85)]}{13 \ 170} = 2.47$ Ans.

Gear tooth bending

Eq. (14-15):
$$(\sigma)_G = 787.8(1)(1.196)(1.097) \left(\frac{6}{2}\right) \left[\frac{(1.156)(1)}{0.38}\right] = 9433 \text{ psi}$$
 Ans.
Eq. (14-41): $(S_F)_G = \frac{28 \ 260(0.996) \ / \ [(1)(0.85)]}{9433} = 3.51 \text{ Ans.}$

Pinion tooth wear

Eq. (14-16):
$$(\sigma_c)_P = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_p F} \frac{C_f}{I} \right)_P^{1/2}$$

=
$$2300 \left[787.8(1)(1.196)(1.088) \left(\frac{1.156}{2.667(2)} \right) \left(\frac{1}{0.1205} \right) \right]^{1/2}$$

= 98760 psi Ans.

Eq. (14-42):

$$(S_H)_P = \left[\frac{S_c Z_N / (K_T K_R)}{\sigma_c}\right]_P = \left\{\frac{93\ 500(0.948) / [(1)(0.85)]}{98\ 760}\right\} = 1.06$$
 Ans.

Gear tooth wear

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2} (\sigma_c)_P = \left(\frac{1.097}{1.088}\right)^{1/2} (98760) = 99170 \text{ psi}$$
 Ans.
 $(S_H)_G = \frac{93500(0.973)(1)/[(1)(0.85)]}{99170} = 1.08 \text{ Ans.}$

The hardness of the pinion and the gear should be increased.

Fig. 14-5:
$$(S_c)_P = (S_c)_G = 322(200) + 29\ 100 = 93\ 500\ psi$$

Note that in general the pinion and the gear will have different life as the pinion will experience more number of revolutions than the gear.

Fig. 14-15:
$$(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$$

 $(Z_N)_G = 1.4488(10^8/3)^{-0.023} = 0.973$

Could also use Z_N =0.948 for both the pinion and gear.

Sec. 14-12:
$$H_{BP}/H_{BG} = 1$$
 \therefore $C_H = 1$ for gear

Pinion tooth bending

Eq. (14-15):
$$(\sigma)_P = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{I} = 787.8(1.6)(1.196)(6)/2/0.27 = 16750$$
psi

Eq. (14-41):
$$(S_F)_P = \left\lceil \frac{S_t Y_N / (K_T K_R)}{\sigma} \right\rceil = (28260)(0.977)/.85/16750 = 1.9392$$

Gear tooth bending

Eq. (14-15):
$$(\sigma)_G = 787.8(1.6)(1.196)(6)/2/0.38 = 11902$$
psi

Eq. (14-41):
$$(S_F)_G = (28260)(0.977)/.85/11902 = 2.7292$$

If
$$(Y_N)_G = 0.996$$
 is used instead, $(S_F)_G = (28260)(0.996)/.85/11902 = 2.7822$

Pinion tooth wear

Eq. (14-16):
$$(\sigma_c)_P = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_p F} \frac{C_f}{I} \right)_P^{1/2}$$

=2300(787.8(1.196)(1.6)/2.667/2/0.1205)^(1/2)=111390psi

Eq. (14-42):

$$(S_H)_P = \frac{\frac{(93500)(0.948)}{0.85}}{111390} = 0.9362$$

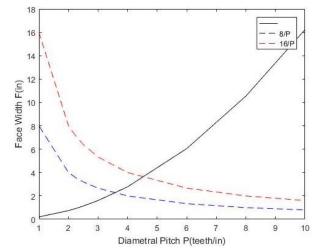
Gear tooth wear

$$(S_H)_G = \frac{\frac{(93500)(0.973)}{0.85}}{111390} = 0.9609$$

If
$$(Z_N)_G = 0.948$$
 is used instead, $(S_H)_G = 0.9362$

Phion:
$$S_{t} = 40 \text{ ksi}$$
, $S_{F} = 2$, $d_{f} = \frac{N\rho}{D} = \frac{2\beta}{P}$ $V_{t} = \frac{Tt n_{f} d\rho}{12} = \frac{T(1000) \frac{13}{P}}{12} = V_{t}(f)$ $V_{t} = \frac{33000 \text{ H}}{V_{t}} = \frac{33000 \text{ H}}{V_{t}} = W_{t}(f)$ $V_{t} = W_{t}(f$

and plot F(P) with \frac{8}{E} and \frac{16}{E}.



From plot: select P=4+eety/in.

use l=4, calculate F= a. 7811 in.

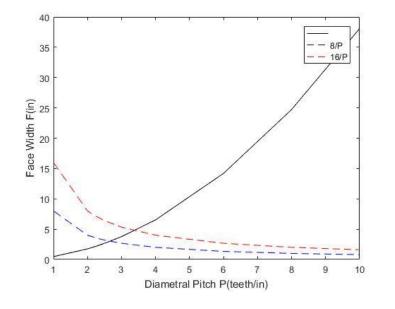
me F= 2.8 M.

update Km=1.62 and iterate.

P=4 teeth/in. F= 3 in. updated km=1.625.

Gear. Different parameters:

St = 13 ksi, plug all variables to egn (x) based on plot F(R), & and I



select P = 3 + eeth/in. $F = 3.767 \pm in$. we F = 3.8 in. update $K_m = 1.645$ and iterate P = 3 + eeth/in. F = 4.0 in. updated $K_m = 1.65$

For the gearset, use
$$P=3+exty/in$$
. $F=4.0in$. $Q_{p}=\frac{NP}{P}=7.67in$. $Q_{q}=\frac{NQ}{P}=19in$.

realized F.S.s,

pininion:

= 4.8302 >2
(SF)G =
$$\frac{(S_{4})_{4}(N)_{6}/(k_{1}N)_{7}}{W^{4}K_{0}K_{1}K_{2}} = \frac{K_{1}K_{1}}{K_{2}}$$

= 2.0593 >2 satisfactory

refer to solutions from problem 2 for most of the parameters,

$$\frac{\overline{pinion}}{Gp=2100 \sqrt{psi}}$$

$$S_{c}=165 \text{ ksi},$$

$$G_{H}=1$$

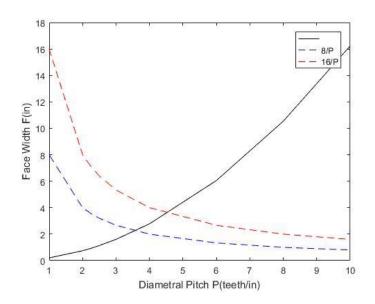
$$G_{f}=1$$

$$I = \frac{\sin 20^{\circ} \cos 20^{\circ}}{2} = \frac{57}{23} + 1 = 0.1145$$

$$(Z_N)_{\ell} = (Z_N)_{\ell} = 2.466 (10^5)^{-0.056} = 1.2942$$

pluf all variables to egn (**) to obtain $F(P)$

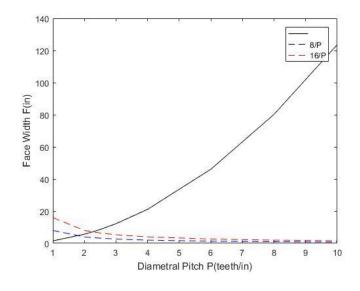
Plot F(R) and & . . 16.



from plot , select P=3 teeth/in. F=2.8749 in. we F=3.0 in. update $K_m=1.675$ and iterate P=3 teeth/in. F=3.0 in. updated $K_m=1.675$

Gene Sc= 80 ksi,

GH= | Since A'=0 for $\frac{H_{BP}}{H_{BG}} = \frac{230}{200} = 1.15 < 1.2$ repeat the process and plot F(R), $\frac{8}{R}$, $\frac{16}{R}$



from plot, select
$$l=2$$
 teeth/ in ., $F=5.6648$ in .

where $F=5.8$ in .

update $K_{m}=1.695$ and iterate

Select $l=2$ teeth/ in . $F=6.0012$ in .

where $F=7$ in . $K_{m}=1.725$

georset = $l=2$ teeth/ in . $F=7$ in . $k_{m}=1.725$

$$l=2$$
 teeth/ in . $l=7$ in .

$$l=1.725$$

$$l=2$$
 $l=1.5$ in .

$$l=2$$
 $l=1.5$ in .

$$l=2$$
 $l=1.5$ in .

$$l=3$$
 $l=4$ $l=3$ $l=4$ l

Additional Problems with Solutions:

1. A spur gearset has 17 teeth on the pinion and 51 teeth on the gear. The pressure angle is 20° and the overload factor $K_{o} = 1$. The diametral pitch is 6 teeth/in and the face width is 2in. The pinion speed is 1120rev/min and its cycle life is to be 10° revolutions at a reliability R=0.99. The quality number is 5. The material is a through-hardened steel, grade 1, with Brinell hardnesses of 232 core and case of both gears. For a design factor of 2, rate the gearset for these conditions using the AGMA method.

Solutions:

Given: R = 0.99 at 10^8 cycles, $H_B = 232$ through-hardening Grade 1, core and case, both gears. $N_P = 17T$, $N_G = 51T$,

Table 14-2: $Y_P = 0.303, Y_G = 0.4103$

Fig. 14-6: $J_P = 0.292, J_G = 0.396$

 $d_P = N_P / P = 17 / 6 = 2.833$ in, $d_G = 51 / 6 = 8.500$ in.

Pinion bending

From Fig. 14-2:

$$_{0.99}(S_t)_{10^7} = 77.3H_B + 12800$$

= 77.3(232) + 12800 = 30734 psi

Fig. 14-14: $Y_N = 1.6831(10^8)^{-0.0323} = 0.928$

$$V = \pi d_p n / 12 = \pi (2.833)(1120 / 12) = 830.7 \text{ ft/min}$$

$$K_T = K_R = 1, \quad S_F = 2, \quad S_H = \sqrt{2}$$

$$\sigma_{\text{all}} = \frac{30.734(0.928)}{2(1)(1)} = 14.261 \text{ psi}$$

$$Q_v = 5, \quad B = 0.25(12 - 5)^{2/3} = 0.9148$$

$$A = 50 + 56(1 - 0.9148) = 54.77$$

$$K_v = \left(\frac{54.77 + \sqrt{830.7}}{54.77}\right)^{0.0535} = 1.089 \Rightarrow \text{use } 1$$

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{md}C_e)$$

$$C_{mc} = 1$$

$$C_{pf} = \frac{F}{10d} - 0.0375 + 0.0125F$$

$$= \frac{2}{10(2.833)} - 0.0375 + 0.0125(2) = 0.0581$$

$$C_{pm} = 1$$

$$C_{ma} = 0.127 + 0.0158(2) - 0.093(10^{-4})(2^2) = 0.1586$$

$$C_e = 1$$

$$K_m = 1 + 1[0.0581(1) + 0.1586(1)] = 1.217$$

$$K_B = 1$$

$$Eq. (14-15): \quad W' = \frac{FJ_p\sigma_{\text{all}}}{K_oK_vK_sP_dK_mK_B}$$

$$= \frac{2(0.292)(14.261)}{1(1.472)(1)(6)(1.217)(1)} = 775 \text{ lbf}$$

$$H = \frac{W'V}{33.000} = \frac{775(830.7)}{33.000} = 19.5 \text{ hp}$$

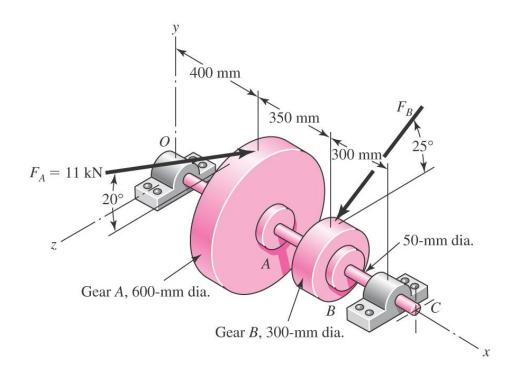
Pinion wear

Fig. 14-15:
$$Z_N = 2.466N^{-0.056} = 2.466(10^8)^{-0.056} = 0.879$$

$$\begin{split} m_G &= 51 \, / \, 17 = 3 \\ \text{Eq. (14-23):} \qquad I = \frac{\cos 20^\circ \sin 20^\circ}{2} \bigg(\frac{3}{3+1} \bigg) = 1.205, \quad C_H = 1 \\ \text{Fig. 14-5:} \qquad _{0.99} (S_c)_{10^7} &= 322 H_B + 29\,100 \\ &= 322(232) + 29\,100 = 103\,804\,\text{psi} \\ \sigma_{c,\text{all}} &= \frac{103\,804(0.879)}{\sqrt{2}(1)(1)} = 64\,519\,\text{psi} \\ \text{Eq. (14-16):} \qquad W^t = \bigg(\frac{\sigma_{c,\text{all}}}{C_p} \bigg)^2 \frac{F d_p I}{K_o K_v K_s K_m C_f} \\ &= \bigg(\frac{64\,519}{2300} \bigg)^2 \bigg[\frac{2(2.833)(0.1205)}{1(1.472)(1)(1.2167)(1)} \bigg] \\ &= 300\,\text{lbf} \\ H &= \frac{W^t V}{33\,000} = \frac{300(830.7)}{33\,000} = 7.55\,\text{hp} \end{split}$$

The pinion controls, therefore $H_{\text{rated}} = 7.55 \text{ hp}$ Ans.

2. The countershaft in Problem 3-73, is part of a speed reducing compound gear train using 20° spur gears. A gear on the input shaft drives gear A with a 2 to 1 speed reduction. Gear B drives a gear on the output shaft with a 5 to 1 speed reduction. The input shaft runs at 1800rev/min. All gears are to be of the same material. Since gear B is the smallest gear, transmitting the largest load, it will likely be critical, so a preliminary analysis is to be performed on it. Use a module of 18.75 mm/tooth, a facewidth of 4 times the circular pitch, a Grade 2 steel through-hardened to a Brinell hardness of 300, and a desired life of 12kh with a 98 percent reliability. Determine factors of safety for bending and wear.



Solutions:

m = 18.75 mm/tooth, d = 300 mm

$$N = d/m = 300 / 18.75 = 16$$
teeth

$$F = b = 4p = 4(\pi m) = 4\pi (18.75) = 236 \text{ mm}$$

$$\sum M_x = 0 = 300(11)\cos 20^\circ - 150F_B \cos 25^\circ$$

$$F_B = 22.81 \text{ kN}$$

$$W^{t} = F_{B} \cos 25^{\circ} = 22.81 \cos 25^{\circ} = 20.67 \text{ kN}$$

$$n = 1800 / 2 = 900 \text{ rev/min}$$

$$V = \frac{\pi dn}{60} = \frac{\pi (0.300)(900)}{60} = 14.14 \text{ m/s}$$

We will obtain all of the needed factors, roughly in the order presented in the textbook.

Fig. 14-2:
$$S_t = 0.703(300) + 113 = 324 \text{ MPa}$$

Fig. 14-5:
$$S_c = 2.41(300) + 237 = 960 \text{ MPa}$$

Fig. 14-6:
$$J = Y_J = 0.27$$

Eq. (14-23):
$$I = Z_I = \frac{\cos 20^{\circ} \sin 20^{\circ}}{2(1)} \left(\frac{5}{5+1}\right) = 0.134$$

Table 14-8:
$$Z_E = 191\sqrt{\text{MPa}}$$

Assume a typical quality number of 6.

Eq. (14-28):
$$B = 0.25(12 - Q_v)^{2/3} = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.8255) = 59.77$$

Eq. (14-27):
$$K_v = \left(\frac{A + \sqrt{200V}}{A}\right)^B = \left(\frac{59.77 + \sqrt{200(14.14)}}{59.77}\right)^{0.8255} = 1.69$$

To estimate a size factor, get the Lewis Form Factor from Table 14-2, Y = 0.296.

Similar to Eq. (a) of Sec. 14-10 but for SI units:

$$K_s = \frac{1}{k_h} = 0.8433 \left(mF \sqrt{Y} \right)^{0.0535}$$

$$K_s = 0.8433 \Big[18.75(236) \sqrt{0.296} \Big]^{0.0535} = 1.28$$

Convert the diameter and facewidth to inches for use in the load-distribution factor equations. d = 300/25.4 = 11.81 in, F = 236/25.4 = 9.29 in

Eq. (14-31): $C_{mc} = 1$ (uncrowned teeth)

Eq. (14-32):
$$C_{pf} = \frac{9.29}{10(11.81)} - 0.0375 + 0.0125(9.29) = 0.1573$$

Eq. (14-33): $C_{pm} = 1.1$

Fig. 14-11: $C_{ma} = 0.27$ (commercial enclosed gear unit)

Eq. (14-35): $C_e = 1$

Eq. (14-30):
$$K_m = K_H = 1 + 1[0.1573(1.1) + 0.27(1)] = 1.44$$

For the stress-cycle factors, we need the desired number of load cycles.

 $N = 12\,000\,h\,(900\,\text{rev/min})(60\,\text{min/h}) = 6.48\,(10^8)\,\text{rev}$

Fig. 14-14: $Y_N = 0.9$

Fig. 14-15: $Z_N = 0.85$

Eq. 14-38:
$$K_R = 0.658 - 0.0759 \ln(1-R) = 0.658 - 0.0759 \ln(1-0.98) = 0.955$$

With no specific information given to indicate otherwise, assume $K_o = K_B = K_T = Z_R = 1$.

Tooth bending

Eq. (14-15):
$$\sigma = W^{t} K_{o} K_{v} K_{s} \frac{1}{bm_{t}} \frac{K_{H} K_{B}}{Y_{J}}$$

$$= 20 670(1)(1.69)(1.28) \left[\frac{1}{236(18.75)} \right] \left[\frac{(1.44)(1)}{0.27} \right] = 53.9 \text{ MPa}$$

$$\text{Eq. (14-41):} \qquad S_{F} = \left[\frac{S_{t} Y_{N} / (K_{T} K_{R})}{\sigma} \right]$$

$$= \frac{324(0.9) / [(1)(0.955)]}{53.9} = 5.66 \quad \text{Ans.}$$

Tooth wear

Eq. (14-16):
$$\sigma_c = Z_E \left(W^t K_o K_v K_s \frac{K_H}{d_{wl} b} \frac{Z_R}{Z_I} \right)^{1/2}$$

$$= 191 \left[20 670(1)(1.69)(1.28) \left(\frac{1.44}{300(236)} \right) \left(\frac{1}{0.134} \right) \right]^{1/2}$$

$$= 498 \text{ MPa}$$

Since gear B is a pinion, C_H is not used in Eq. (14-42) (see p. 757), where

$$S_{H} = \frac{S_{c}Z_{N} / (K_{T}K_{R})}{\sigma_{c}}$$

$$= \frac{960(0.85) / [(1)(0.955)]}{498} = 1.72 \quad Ans$$