ME 555 Exam 1 - February 13, 2013
Closed Book Closed Notes

Honor Code

I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code.

Problem 1 (20 points)

Check if the following statements are true or not. Explain.

(a) If x_* is a stationary point of a continuous and differentiable function f, then x_* is a local minimum of f.

(b) The intersection of two convex sets is also convex.

(c) A well-constrained linear problem (both the objective and constraints are linear functions of x) can have an interior optimal solution, i.e., with no active constraints.

(d) When using Newton's method, the step size in the line search will be 1 when x_k is close to a local solution.

Problem 2 (20 points)

Solve the problem below using monotonicity analysis. Identify active constraints and prove the global optimum.

$$\begin{aligned} \min_{x_1,x_2} \quad & x_1^2 + x_2^2 - 4x_1 + 4 \\ \text{subject to} \quad & g_1 = -x_1 + 1 \le 0, \\ & g_2 = -x_2 \le 0, \\ & g_3 = x_2 - (1 - x_1)^3 \le 0 \end{aligned}$$

Problem 3 (25 Points)

Consider the problem of finding the minimum of the function

$$f = x_1^2 + x_2^2 - 3x_1x_2$$

(a) Find the stationary point. (5 Points)

(b) Determine what is the nature of the point found above. (5 Points)

(c) Now add the constraints $x_1 \geq 0$ and $x_2 \geq 0$. Can you determine the minimum for this revised problem? Explain. (Hint: Check if there are directions of downslopes away from the stationary point. Also notice that the problem is *symmetric*. If there exists a local solution (x_1^*, x_2^*), what will be the relationship between x_1^* and x_2^*?) (10 Points)

(d) Now add the constraint $g_1 = x_1^2 + x_2^2 - 6 \leq 0$. Can you determine the minimum for this further revised problem? Explain. (Hint: Check if g_1 will be active or not.) (5 Points)

Problem 4 (35 Points)

(a) State and prove the second order sufficiency condition for a minimum of an unconstrained function f in $\mathbb{R}$ that is continuous and differentiable. (5 Points)

(b) *Derive* the iteration formula for solving n-dimensional unconstrained problems with the gradient method without line search. (5 Points)

(c) *Derive* the iteration formula for solving n-dimensional unconstrained problems with Newton's method without line search. (10 Points)

(d) Explain briefly the meaning of the line search and why it is used to modify (b) and (c) above. (5 Points)

(e) Discuss briefly advantages and disadvantages of using Newton's method in solving optimization problems. (10 Points)