ME598/494 Homework 2

1. (20 points) Show that the stationary point of the function

$$f = 2x_1^2 - 4x_1x_2 + 1.5x_2^2 + x_2$$

is a saddle. Find the directions of downslopes away from the saddle. Hint: Use Taylor's expansion at the saddle point. Find directions that reduce f.

2. (a) (10 points) Find the point in the plane $x_1 + 2x_2 + 3x_3 = 1$ in $\mathbb{R}^3$ that is nearest to the point $(-1, 0, 1)^T$. Is this a convex problem? Hint: Convert the problem into an unconstrained problem using $x_1 + 2x_2 + 3x_3 = 1$.

   (b) (40 points) Implement the gradient descent and Newton's algorithm for solving the problem. Attach your codes in the report, along with a short summary of your findings. The summary should include: (1) The initial points tested; (2) corresponding solutions; (3) A log-linear convergence plot. Based on your results, which algorithm do you think is better? Why? Hint: A template can be found **here**.

3. (5 points) Prove that a hyperplane is a convex set. Hint: A hyperplane in $\mathbb{R}^n$ can be expressed as: $\mathbf{a}^T\mathbf{x} = c$ for $\mathbf{x} \in \mathbb{R}^n$, where $\mathbf{a}$ is the normal direction of the hyperplane and c is some constant.

4. Let $f(x)$ and $g(x)$ be two convex functions defined on the convex set $\mathcal{X}$.

   (a) (5 points) Prove that $af(x) + bg(x)$ is convex for $a > 0$ and $b > 0$.

   (b) (5 points) In what conditions will $f(g(x))$ be convex?

5. (15 points, optional for MAE494) Show that $f(\mathbf{x}_1) \geq f(\mathbf{x}_0) + \mathbf{g}_{\mathbf{x}_0}^T(\mathbf{x}_1 - \mathbf{x}_0)$ for a convex function $f(\mathbf{x}) : \mathcal{X} \to \mathbb{R}$ and for $\mathbf{x}_0$, $\mathbf{x}_1 \in \mathcal{X}$.