ME598/494 Homework 1 Solution

1. Solve the following problem using two of the three solvers: Excel Solver, Matlab's *fmincon*, and Scipy optimization. Use initial point: $\mathbf{x}_0 = (1, 1, 1, 1)$. (15 points each, 30 in total)

$$
\begin{aligned}
\text{minimize:} \quad & 24.55x_1 + 26.75x_2 + 39.00x_3 + 40.50x_4 \\
\text{subject to:} \quad & 2.3x_1 + 5.6x_2 + 11.1x_3 + 1.3x_4 - 5 \geq 0 \\
& 12x_1 + 11.9x_2 + 41.8x_3 + 52.1x_4 - 21 \\
& - 1.645(0.28x_1^2 + 0.19x_2^2 + 20.5x_3^2 + 0.62x_4^2)^{1/2} \geq 0 \\
& x_1 + x_2 + x_3 + x_4 - 1 = 0 \\
& 0 \leq x_i, \ i = 1, \ldots, 4
\end{aligned}
$$

Note: Here are some common programming issues:

(a) First and foremost, please learn how to use **break points**. I will not address your programming questions if you have not learned how to debug your code.

(b) Please note that your solutions from all solvers should be **the same**.

(c) For some versions of Matlab, **space** is interpreted as comma when there is a bracket around. For example, [2 + 3] could be considered an array with two elements [2, 3] rather than a single number 5.

(d) For some versions of fmincon, treating linear constraints as non-linear ones will lead to different local solutions. Regardless, it is always recommended that linear constraints are treated as linear, as they directly provide gradient information (no finite difference is needed).

Solution $x^* = [0.6355, 0, 0.3127, 0.0518]$

Objective function:

```
function f = fun(x);
f = [24.55, 26.75, 39, 40.5]*x;
```

Nonlinear constraint function:

```
function [c,ceq] = funcon(x);
    c = [-12, -11.9, -41.8, -52.1]*x + 21 + ...
        1.645*sqrt( x   *diag([0.28, 0.19, ...
        20.5, 0.62])*x);
    ceq = [];
```

Main file:

```
x0 = ones(4,1);
A = [-2.3 -5.6 -11.1 -1.3];
b = [-5];
Aeq = ones(4,1);
beq = 1;
lb = zeros(4,1);
ub = [];
[x,fval] = fmincon(@fun, x0, A, b, Aeq, beq, ...
    lb, ub, @funcon);
```

2. Design a **cylindrical** cola can of volume V to minimize material usage. Formulate an optimization problem by the following steps: (1) Define design variables and the objective (10 points), (2) state constraints (5 points), (3) discuss assumptions made during modeling (5 points).

   Solve your optimization problem with a realistic cola can volume. Is your optimal solution close to the reality? If not, briefly discuss why (not restricted to engineering considerations). (20 points)

   Solution Let radius and height be r and h, respectively. The problem of minimizing material usage becomes:

$$\begin{aligned} \min_{r,h} \quad & 2\pi r^2 + 2\pi r h \\ \text{subject to:} \quad & \pi r^2 h = V \end{aligned} \tag{1}$$

   Assumptions include (1) can is in perfect cylindrical shape; (2) thickness of the material used in production is fixed and same everywhere on the can; (3) thickness of the material is good enough to withstand the internal pressure; (4) thickness of the material will satisfy other manufacturing or packaging constraints.

Use $V = 330ml$, the optimal solution is $r^* = 37.4mm$ and $h^* = 74.9mm$, while real dimensions are $r = 32mm$ and $h = 122mm$. Possible reasons for finding different results than the real dimensions: (1) top, bottom, and side thicknesses are not the same. Usually $t_{bottom} > t_{top} > t_{sides}$; (2) a can is not in a perfect cylindrical shape due to some packaging and (pressure related) engineering constraints; (3) a can must also be easy to hold. There needs to be an upper bound on r.

3. Consider the problem of grocery shopping with a budget of 30 dollars. You will need 10 units of protein and 20 units of vitamin. You can choose from Products A, B, and C. The (protein, vitamin) values for them are (5,0), (0,6), and (3,2) per unit, and their unit prices are 5, 6, 7 dollars, respectively. What is the best way to spend your money? Define "best". (30 points)

   Solution We can formulate the problem as to minimize the cost while constraining the nutrition requirements:

$$\begin{aligned} \min_{x} \quad & 5x_1 + 6x_2 + 7x_3 \\ \text{subject to} \quad & -5x_1 - 3x_3 \leq -10 \\ & -6x_2 - 2x_3 \leq -20 \end{aligned} \tag{2}$$

   The solution to this problem is $x_1^* = 2$, $x_2^* = 3.33$, and $x_3^* = 0$. This is not a integer solution and may not be feasible when products are sold in units. A feasible integer solution with cost lower than 30 does not exist. But without cost constraint, the best solution is $x_1^* = 2$, $x_2^* = 4$, and $x_3^* = 0$. One can also solve the problem of minimizing the nutrition gap while keeping the cost lower than 30.