ME 598/494 Exam 2 - Nov. 9, 2017

Honor Code

I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code.

Problem 1 (30 Points)

Check if the following statements are true or not. Explain concisely.

(a) The Lagrangian multipliers for inequality constraints can be negative at the optimal solution.

(b) Line search in GRG is the same as that in the gradient descent or Newton's method.

(c) A KKT point is a solution to a minimization problem, **and** a minimal solution must satisfy the KKT conditions.

(d) GRG can be used when every step of the search is required to be constrained in the feasible domain.

(e) Fitting a nonlinear model to data can always be done through Newton-Ralphson.

(f) Scaling the variables will affect the Lagrangian multipliers at the optimal solution.

Problem 2 (30 Points)

Consider a product with price x, market demand $s(x) = 1 - x^2$, and total cost $c(x) = 0.5s(x)$ linearly increasing with the demand.

(a) Formulate an optimization problem for maximizing the profit with respect to the price, with a cost limit of $c_{max} = 9/50$. (5 Points)

(b) Derive the KKT conditions, and show that $x = 4/5$, $\mu = 3/20$ is an optimal solution. (15 Points)

(c) How would increasing the cost limit (from 9/50) affect the optimal profit? (5 Points)

(d) Let c^* be a value such that any cost limit $c_{max} > c^*$ will not change the optimal solution. Discuss how the smallest c^* can be calculated. (5 Points, Optional for MAE494)

Problem 3 (25 Points)

For the linearly constrained problem (where $\mathbf{A}$ and $\mathbf{b}$ are a matrix and a column vector of parameters, respectively):

$$\min \quad f(x), \text{ subject to } \mathbf{A}\mathbf{x} = \mathbf{b}$$

(a) Derive the reduced gradient. (10 Points)

(b) **Concisely** state all GRG steps for solving **this** problem. (10 Points)

(c) Explain what simplifications occur in GRG due to the linearity of constraints. (5 Points, Optional for MAE494)

Problem 4 (15 Points)

Consider the following problem, with constant m-by-n matrix $\mathbf{A}$ ($m < n$), constant m-by-1 vector $\mathbf{b}$:

$$\min_{\mathbf{x}} \quad \mathbf{x}^T\mathbf{x}, \text{ subject to } \mathbf{A}\mathbf{x} = \mathbf{b}$$

(a) Is this problem convex? Why? (5 Points)

(b) Derive the KKT conditions and find the optimal solution $\mathbf{x}^*$. (10 Points, Optional for MAE494)

Problem 5 (extra 10 points)

(Principal Component Analysis) Consider the following problem where $\mathbf{A}$ is a symmetric and positive semidefinite matrix:

$$\begin{aligned} \max_{\mathbf{x}} \quad & f = \mathbf{x}^T\mathbf{A}\mathbf{x} \\ \text{subject to} \quad & h = \mathbf{x}^T\mathbf{x} = 1 \end{aligned}$$

Derive the optimal solution. What can you tell about the solution $\mathbf{x}^*$ and λ^*?