Problem 1.

Ma = 50Nm, Ta = 35Nm, Mm = 55Nm and Tm = 25Nm

Goodman Formula is

$$d = \sqrt[3]{\left[\frac{16n}{\pi}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)\right]}$$

First calculate A & B

$$A = \sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2} = \sqrt{4(2.4 \times 50)^2 + 3(2 \times 35)^2} = 268.8\ Nm$$

$$B = \sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2} = \sqrt{4(2.4 \times 55)^2 + 3(2 \times 25)^2} = 277.8\ Nm$$

Bring back to Goodman's formula

S_e = 220 Mpa and S_{ut} = 800 Mpa

$$d = \sqrt[3]{\left[\frac{16n}{\pi}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)\right]} = \sqrt[3]{\left[\frac{16 \times 1.5}{\pi}\left(\frac{268.8}{200(10^6)} + \frac{277.8}{800(10^6)}\right)\right]} = 23.4\ mm$$

Problem 2

$F \cos 20°(d / 2) = T_A$, $F = 2\,T_A / (d \cos 20°) = 2(340) / (0.150 \cos 20°) = 4824$ N.
The maximum bending moment will be at point C, with $M_C = 4824(0.06) = 289.4$N·m.
Due to the rotation, the bending is completely reversed, while the torsion is constant.
Thus, $M_a = 289.4$ N·m, $T_m = 340$ N·m, $M_m = T_a = 0$.

For sharp fillet radii at the shoulders, from Table 7-1, $K_t = 2.7$, and $K_{ts} = 2.2$.
Examining Figs. 6-20 and 6-21 (pp. 303 and 304 respectively) with $S_{ut} = 560$ MPa, conservatively estimate $q = 0.8$ and $q_s = 0.9$. These estimates can be checked once a specific fillet radius is determined.

Eq. (6-32): $$K_f = 1 + 0.8(2.7 - 1) = 2.4$$

$$K_{fs} = 1 + 0.9(2.2 - 1)$$

(a) We will choose to include fatigue stress concentration factors even for the static analysis to avoid localized yielding.

Eq. (7-15): $$\sigma'_{\max} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}T_m}{\pi d^3}\right)^2\right]^{1/2}$$

Eq. (7-16): $$n = \frac{S_y}{\sigma'_{\max}} = \frac{\pi d^3 S_y}{16}\left[4\left(K_f M_a\right)^2 + 3\left(K_{fs}T_m\right)^2\right]^{-1/2}$$

Solving for d,

$$d = \left\{\frac{16n}{\pi S_y}\left[4\left(K_f M_a\right)^2 + 3\left(K_{fs}T_m\right)^2\right]^{1/2}\right\}^{1/3}$$

$$= \left\{\frac{16(2.5)}{\pi(420)(10^6)}\left[4(2.4 \times 289.4)^2 + 3(2.1 \times 340)^2\right]^{1/2}\right\}^{1/3}$$

$d = 0.0383$ m = 38.3 mm *Ans.*

(b)

The shaft is machined from steel
From table 6-2, a = 4.51and b = -0.265

$$\mathrm{K}_a = \mathrm{a}S_{ut}{}^b = 0.84$$

Assume $k_b = 0.85$ for now. Check later once a diameter is known.

$$S_e = 0.84(0.85)(0.5)(560) = 200 \text{ MPa}$$

Selecting the DE-ASME Elliptic criteria, use Eq. (7-12) with $M_m = T_a = 0$.

$$\mathrm{d} = \left\{\frac{16(2.5)}{\pi}\left[4\left(\frac{2.4(289.4)}{200(10^6)}\right)^2 + 3\left(\frac{2.1(340)}{420(10^6)}\right)^2\right]^{1/2}\right\}^{1/3} = 0.0457\,m = 45.7mm$$

With this diameter, we can refine our estimates for k_b and q.

Eq. (6-20):

$$\mathrm{K}_b = 1.24d^{-0.107} = 1.24(45.7)^{-0.107} = 0.82$$

Assuming a sharp fillet radius, from Table 7-1, $r = 0.02d = 0.02\,(45.7) = 0.914$ mm.

Fig. (6-20): $q = 0.72$
Fig. (6-21): $q_s = 0.77$

Iterating with these new estimates,

Eq. (6-32): $K_f = 1 + 0.72\,(2.7 - 1) = 2.2$
$K_{fs} = 1 + 0.77\,(2.2 - 1) = 1.9$
Eq. (6-18): $S_e = 0.84(0.82)(0.5)(560) = 191$ MPa
Eq. (7-12):

$$\mathrm{d} = \left\{\frac{16(2.5)}{\pi}\left[4\left(\frac{2.2(289.4)}{191(10^6)}\right)^2 + 3\left(\frac{1.9(340)}{420(10^6)}\right)^2\right]^{1/2}\right\}^{1/3} = 0.045\,m = 45\,mm \quad \textit{Ans.}$$

Further iteration does not change the results.

Problem 3

The yield strength of 1020 CD steel is S_y=390 Mpa. The diameter of shaft at part B is d = 50 mm and Factor of safety n = 1.1.

Since we got a torque T = 3101 Nm.

We can calculate the force F at the surface of the shaft.

$$\mathrm{F} = \frac{T}{r} = \frac{3101}{50(10^{-3})/2} = 124040\ N$$

Using distortion energy theory to get shear strength

$$S_{sy} = 0.577 \times S_y = 0.577 \times 390 = 225\ Mpa$$

The shear stress is

$$\tau = \frac{F}{t \times l}$$

In this problem t is width or height because it is a square key. And *l* is the length of key.

From table 7-6, we assume our t = 12.7mm. (Change mm into inch and check table)

After all,

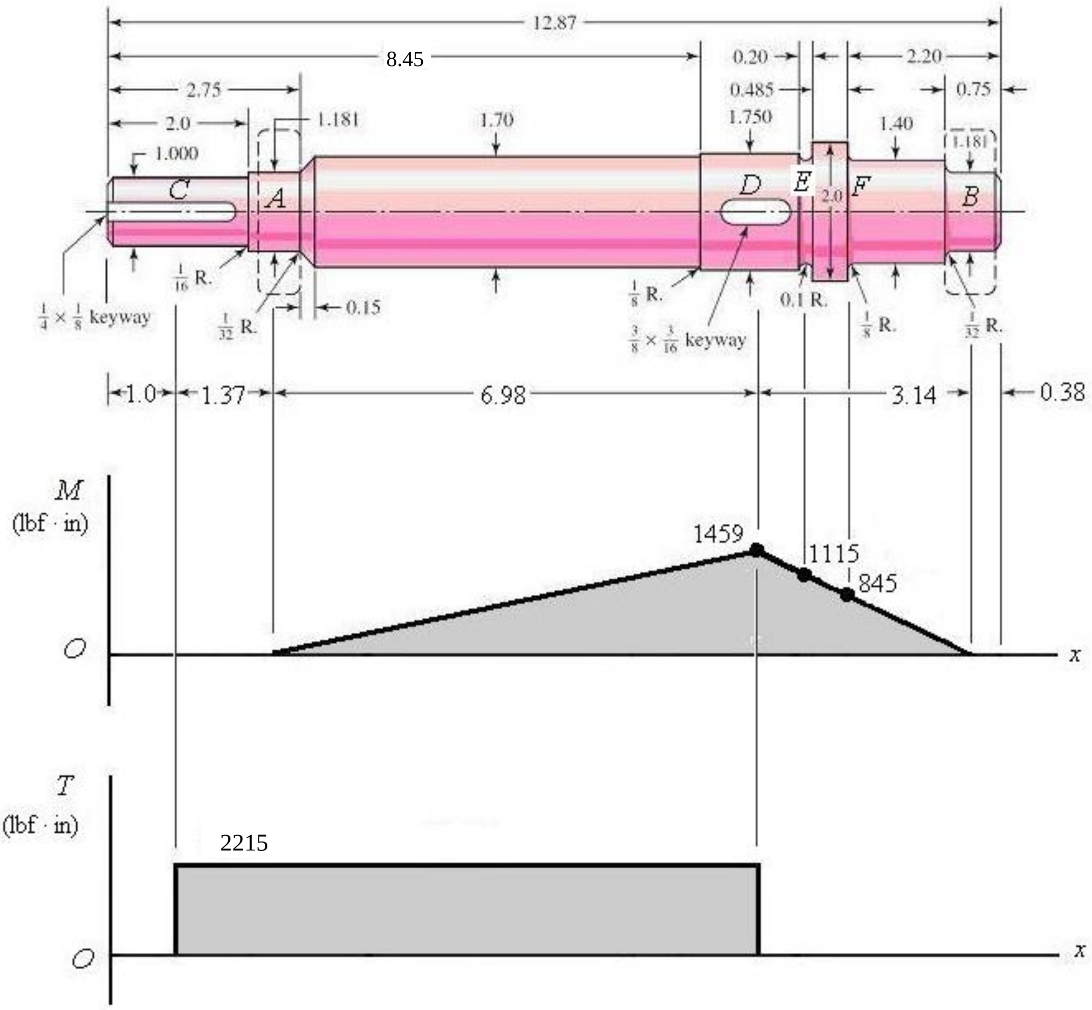
$$\frac{S_{sy}}{n} = \frac{F}{t \times l} = \frac{225(10^6)}{1.1} = \frac{124040}{12.7(10^{-3}) \times l}$$

$$l = 0.047\ m = 47\ mm$$

$$w = h = 12.7\ mm$$

Problem 4

Label the approximate locations of the effective centers of the bearings as *A* and *B*, the fan as *C*, and the gear as *D*, with axial dimensions as shown. Since there is only one gear, we can combine the radial and tangential gear forces into a single resultant force with an accompanying torque and handle the statics problem in a single plane. From statics, the resultant reactions at the bearings can be found to be R_A = 209.9 lbf and R_B = 464.5 lbf. The bending moment and torque diagrams are shown, with the maximum bending moment at *D* of M_D = 209.9(6.98) = 1459 lbf·in and a torque transmitted from *D* to *C* of *T* = 633 (7/2) = 2215.5 lbf·in. Due to the shaft rotation, the bending stress on any stress element will be completely reversed, while the torsional stress will be steady. Since we do not have any information about the fan, we will ignore any axial load that it would introduce. It would not likely contribute much compared to the bending anyway.



Potentially critical locations are identified as follows:

- Keyway at *C*, where the torque is high, the diameter is small, and the keyway creates a stress concentration.
- Keyway at *D*, where the bending moment is maximum, the torque is high, and the keyway creates a stress concentration.
- Groove at *E*, where the diameter is smaller than at *D*, the bending moment is still high, and the groove creates a stress concentration. There is no torque here, though.
- Shoulder at *F*, where the diameter is smaller than at *D* or *E*, the bending moment is still moderate, and the shoulder creates a stress concentration. There is no torque here, though.
- The shoulder to the left of *D* can be eliminated since the change in diameter is very slight, so that the stress concentration will undoubtedly be much less than at *D*.

Table A-20: S_{ut} = 68 kpsi, S_y = 57 kpsi

Eq. (6-8): $S'_e = 0.5(68) = 34.0$ kpsi

Eq. (6-19): $k_a = 2.70(68)^{-0.265} = 0.883$

Keyway at C

Since there is only steady torsion here, only a static check needs to be performed. We'll use the maximum shear stress theory.

$$\tau = \frac{Tr}{J} = \frac{2215(1/2)}{\pi(1^4)/32} = 11280 = 11.28\ kpsi$$

Eq. (5-3):

$$n_y = \frac{S_y/2}{\tau} = \frac{57/2}{11.28} = 2.52$$

Keyway at D

Assuming *r / d* = 0.02 for typical end-milled keyway cutter (p. 365), with *d* = 1.75 in,

r = 0.02*d* = 0.035 in.

Table 7-1: $K_t = 2.14$, $K_{ts} = 3.0$

Fig. 6-20: $q = 0.66$

Fig. 6-21: $q_s = 0.72$

Eq. (6-32): $K_f = 1 + q\left(K_t - 1\right) = 1 + 0.66(2.14 - 1) = 1.8$

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.72(3.0 - 1) = 2.4$$

Eq. (6-20): $k_b = \left(\dfrac{1.75}{0.30}\right)^{-0.107} = 0.828$

Eq. (6-18): $S_e = 0.883(0.828)(34.0) = 24.9$ kpsi

We will choose the DE-Gerber criteria since this is an analysis problem in which we would like to evaluate typical expectations.

Using Eq. (7-9) with $M_m = T_a = 0$,

$$\mathrm{A} = \sqrt{4(K_f M_a)^2} = \sqrt{4(1.8 \times 1459)^2} = 5252\ lbs \cdot in = 5.252 lbs \cdot in$$

$$\mathrm{B} = \sqrt{43(K_{fs} T_m)^2} = \sqrt{3(2.4 \times 2215)^2} = 9207\ lbs \cdot in = 9.207 lbs \cdot in$$

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$= \frac{8(5.252)}{\pi(1.75)^3(24.9)}\left\{1 + \left[1 + \left(\frac{2(9.207)(24.9)}{(5.252)(68)}\right)^2\right]^{1/2}\right\}$$

$n = 3.797$ *Ans.*

Groove at E

We will assume Figs. A-15-14 is applicable since the 2 in diameter to the right of the groove is relatively narrow and will likely not allow the stress flow to fully develop. (See Fig.7-9 for the stress flow concept.)

$r / d = 0.1 / 1.55 = 0.065$, $D / d = 1.75 / 1.55 = 1.13$

Fig. A-15-14: $K_t = 2.1$

Fig. 6-20: $q = 0.76$

Eq. (6-32): $K_f = 1 + q\left(K_t - 1\right) = 1 + 0.76(2.1 - 1) = 1.8$

Eq. (6-20): $k_b = \left(\dfrac{1.55}{0.30}\right)^{-0.107} = 0.839$

Eq. (6-18): $S_e = 0.883(0.839)(34) = 25.2$ kpsi

Using Eq. (7-9) with $M_m = T_a = T_m = 0$,

$$A = \sqrt{4\left(K_f M_a\right)^2} = \sqrt{4\left[(1.8)(1115)\right]^2} = 4122 \text{ lbf}\cdot\text{in} = 4.122 \text{ kip}\cdot\text{in}$$

$$B = 0$$

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1+\left[1+\left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$= \frac{8(4.122)}{\pi\left(1.55^3\right)(25.2)}\left\{1+\left[1+(0)^2\right]^{1/2}\right\}$$

$n = 4.47$ *Ans.*

Shoulder at F

$r/d = 0.125/1.40 = 0.089$, $D/d = 2.0/1.40 = 1.43$

Fig. A-15-9: $K_t = 1.7$

Fig. 6-20: $q = 0.78$

Eq. (6-32): $K_f = 1 + q\left(K_t - 1\right) = 1 + 0.78(1.7 - 1) = 1.5$

Eq. (6-20): $k_b = \left(\dfrac{1.40}{0.30}\right)^{-0.107} = 0.848$

Eq. (6-18): $S_e = 0.883(0.848)(34) = 25.5$ kpsi

Using Eq. (7-9) with $M_m = T_a = T_m = 0$,

$$A = \sqrt{4\left(K_f M_a\right)^2} = \sqrt{4\left[(1.5)(845)\right]^2} = 2535 \text{ lbf}\cdot\text{in} = 2.535 \text{ kip}\cdot\text{in}$$

$B = 0$

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1+\left[1+\left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$= \frac{8(2.535)}{\pi(1.40^3)(25.5)}\left\{1+\left[1+(0)^2\right]^{1/2}\right\}$$

$n = 5.42$ *Ans.*

(b) The deflection will not be much affected by the details of fillet radii, grooves, and keyways, so these can be ignored. Also, the slight diameter changes, as well as the narrow 2.0 in diameter section, can be neglected. We will model the shaft with the following three sections:

Section	Diameter (in)	Length (in)
1	1.00	2.00
2	1.70	7.70
3	1.40	2.20

The deflection problem can readily (though tediously) be solved with singularity functions. For examples, see Ex. 4-7, p. 173, or the solution to Prob. 7-24. Alternatively, shaft analysis software or finite element software may be used. Using any of the methods, the results should be as follows:

Location	Slope (rad)	Deflection (in)
Left bearing *A*	0.000290	0.000000
Right bearing *B*	0.000400	0.000000
Fan *C*	0.000290	0.000404
Gear *D*	0.000146	0.000928

Comparing these values to the recommended limits in Table 7-2, we find that they are all within the recommended range.