Optimization of a Golf Driver

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Abstract

In this study, the relationship between a golf ball and driver will be optimized to produce the farthest golf shot. There are various factors that affect this system, but the two that will be focused on are the impact model between the ball and the club head and the aerodynamic model of the ball's flight through the air. The two variables that will be optimized are the loft angle of the club head and the swing elevation angle.

The impact model depends on three things: the momentum of the ball and the club during impact, the impulse of the system during impact, and the swing elevation angle. The momentum during impact depends on the kinetic energy, translational velocity, and rotational velocity of the ball and the club.

The aerodynamic model is dependent on the velocities calculated in the impact model. The translational and rotational velocities of the ball after impact are used to determine the coefficient of lift and drag.

Since the fmincon function on Matlab wouldn't work due to noisy data, metamodeling was used for the optimization. The loft angle was optimized for a constant swing elevation angle, and then the elevation angle was optimized for constant loft. The maximum distance determined was 248 meters by using a loft of 17.68 degrees and a swing elevation angle of 7.3 degrees.

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Subsystem A - Impact Model

1.1 Problem Statement

The goal of the impact model is to determine the twelve velocity components of the club head and the golf ball after impact. All of the velocity components will be solved in the I,J,K directions, which is relative to the ground.

1.2 Nomenclature

u_i =translational velocity components for the club head where i=1,2,3
u_i =rotational velocity components for the club head where i=4,5,6
u_i =translational velocity components for the golf ball where i=7,8,9
u_i =rotational velocity components for the golf ball where i=10,11,12
$V^C=[u_1,u_2,u_3]$
$\omega^H=[u_4,u_5,u_6]$
$V^B=[u_7,u_8,u_9]$
$\omega^G=[u_{10},u_{11},u_{12}]$, Spin rate
I^H=Inertia of the club head
I^G=Inertia of the golf ball
I_H=Mass of the club head
I_G=Mass of the golf ball
p_i = 15x1 matrix for the initial momentum components of the club head and golf ball.
V^P=Velocity components at the impact point on the club head face
$V^{P'}$=Velocity components at the impact point on the golf ball
$r^{P/C}$=Position vector from the club head center of mass to the impact point
$r^{P'/B}$=Position vector from the golf ball center of mass to the impact point
r_1=Position vector in i' direction
r_2=Position vector in j' direction
r_3=Position vector in k' direction
R=Resultant impulse vector
I=15x1 generalized impulse vector
V_s=Relative velocity of the club head and golf ball just after impact
V_a=Relative velocity of the club head and golf ball just before impact
e_N=normal coefficient of restitution
p_f=15x1 matrix representing the final momentum of the club head and the golf ball
K^H=kinetic energy of the club head
K^G=kinetic energy of the golf ball

1.3 Mathematical Model

For this study, the velocity in the X and Z directions were considered, as seen in Figure 2.3.1. The impact of the club head and golf ball was modeled using the initial momentum of the the club head and the golf ball, the impulse created by the contact between the two bodies, and assumptions made about the

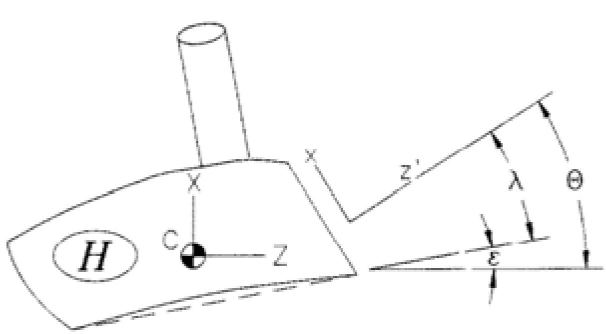

Figure 2.3.1: Side view of the club head

relative velocities between the two bodies. This created 15 variables and gave 15 equations. Using these equations, a system of linear equations was constructed and solved to calculate the velocity vectors of the golf ball just after impact.

The initial velocity of the golf ball was zero, while the club head was given an initial translational swing speed with a magnitude of 44.7 m/s. This created a non-zero initial momentum for the club head in the translational X and Z directions. The momentum was found using the following equation.

$$p_i = \partial K^H/\partial u_i + \partial K^G/u_i$$

To model the impulse, first the velocity vector at the point of impact had to be calculated for each body. This was found using the following equations.

$$V^P = V^C + \omega^H \times r^{P/C}, \;\; V^{P\prime} = V^B + \omega^G \times r^{P/B}$$

The time integral of the contact force between the two bodies was modeled as the following equation.

$$R = R_1\mathrm{I} + R_2\mathrm{J} + R_3\mathrm{K}$$

Next, the impulse was found in similar fashion to the initial momentum, by using the final equation.

$$I = (\partial V^P/\partial u_i) \cdot R - (\partial V^{P'}/\partial u_i) \cdot R$$

The initial momentum was then added to the impulse, and set equal to the final momentum. After solving for the velocity vectors, the first 12 rows of the 15x15 matrix used in the system of linear of equations can be constructed. The relative velocities between the two bodies were then used to compose the final three equations.

$$V_A = V^P(0) - V^{P'}(0)\,, V_S = V^P(t) - V^{P'}(t)$$

The relative velocity just before impact was assumed to be zero in the X and Y directions. The coefficient of restitution was then used to equate the relative velocity in the Z direction just before impact, and immediately after.

$$V_A \cdot \boldsymbol{i'} = 0$$
$$V_A \cdot \boldsymbol{j'} = 0$$
$$-e_N V_A \cdot \boldsymbol{k'} = V_S \cdot \boldsymbol{k'}$$

Using these final three equations, a system of linear equations was constructed to determine the translational and rotational velocities of the golf ball immediately after contact (u_7-u_{12}).

Subsystem B - Aerodynamic Model

2.1 Problem Statement

The goal of the aerodynamic model is to determine the drag and lift forces acting on the golf ball while it is in flight. Drag and lift forces are dependent upon the coefficient of drag, coefficient of lift, and the spin rate of the ball. Each of these values must be updated for each time step.

2.2 Nomenclature

F_d=Drag force
F_l=Lift force
C_d=Drag coefficient
C_l=Lift coefficient
ρ=Density of air
A=Projected cross sectional area of golf ball
$\mathbf{F}^G$=Applied gravitational and aerodynamic forces
$\mathbf{F}^{*G}$=Inertia forces of golf ball

2.3 Mathematical Model

To solve for the velocities at each time step for the ball, a differential equation had to be solved. Using MATLAB's built in ode45 solver, the differential equation of $\mathbf{F}^G + \mathbf{F}^{*G} = 0$ was solved. This equation was solved in each direction, resulting in translational velocities of the golf ball in each of the x, y, and z directions. To get to this differential equation, the drag and lift forces had to be found using the drag and lift coefficients. These drag and lift coefficients were found using an equation that depended on the rotational velocities[2] .Where the coefficient of drag was modeled as,

$$C_D = 0.3 + 2.58 * 10^{-4} * \omega$$

And the coefficient of lift was modeled as,

$$C_L = 3.10 * 10^{-1}[1 - \exp(-2.48 * 10^{-3}\omega)]$$

The model was solved using a while loop that would end when the height of the ball was zero. The height was found using the simple physics equation of velocity multiplied by the time step, which then was added onto the previous height from the time step before. This was repeated until the height was zero. The distance of the golf ball was found using the same simple physics equation and was again added up over each time step. To increase accuracy of the solver, the time step could be minimized.

3. Model Analysis

There were many simplifications made to solve this problem. When thinking about hitting a golf ball, it is apparent that unless you hit the ball directly in the center of the driver head (the sweet spot), there will be more than just a back spin (a rotational velocity in the j-direction for this case). For this model, it was assumed that the ball was hit in the sweet spot. This also meant that the golf ball went directly down the center of the fairway. Also, from simple physics, it would be assumed that the optimal angle of attack (the swing elevation angle plus the angle of loft of the driver) would be 45 degrees for the maximum driving distance. But as will be discussed later, due to the mathematical model used and the boundary conditions for the elevation and loft angles, this was not the case.

Also, the translational velocity of the driver head at impact depended on the swing elevation angle, in the x and z directions. The inertia dyadic of the club head and golf ball were kept constant and were found using a 3D model.

4. Optimization Study

The optimization study was done to determine the maximum distance a golf ball would travel down the center of the fairway depending on the optimum swing elevation angle for a given club head loft angle, as well as, the optimum club head loft angle for a given swing elevation angle. These results were then compared to find the overall optimum. Initially, the fmincon function in MATLAB was attempted to solve for the optimum angles, but it was found that due to the while loop (that was used to find when the ball landed) the data was noisy because the loop did not stop exactly when the ball hit the ground. The loop had the conditions to stop when the height was no longer greater than zero, so this meant that technically the ball could have a negative height.

This meant that the solver was finding a local solution within the noise, but a global solution was needed. The next step was to look into using one of MATLAB's built in genetic algorithms that could find the global solution. It was decided that this would be too computationally expensive and also was quite unnecessary. The team decided to use a simple regression metamodeling technique.

To solve for these optimum angles; a range was set for each angle, given a constant for the other angle. So for example, a range of 0 to 20 degrees was given for the driver head loft, while the swing elevation angle was constant at 0 degrees. This was done for many different constant angles of elevation, and then was done vice versa for the club head loft. Using regression, data was selected within these ranges of angles, respectively. Then using the fmincon on the regression, the optimum angle was found. These resultant optimum angles will be discussed later in the report.

Since this study had been done previously, the results were then compared to this previous study[1]. It was assumed the results would not be exactly the same because some of the parameters used differed between what was done here and what was done in the previous study.

In the end it was found that neither of the constraints was active, and that monotonicity analysis was negligible. The solution was an interior solution for each of the respective constant angles.

5. Parametric Study

A parametric study was essential to this problem. It is quite apparent from the mathematical models that changing the parameters will change the optimum solution. From our study it was found that changing many different parameters will affect the optimum solution, which leads to the desire to perform more optimization analysis where these parameters are too optimized.

For example, when the mass of the ball is either increased or decreased, the optimum angle for both the club head loft and swing elevation angle is affected. This is because the mass of the ball is included in the impact and aerodynamic models.

Another parameter that was not tested but in the future would be very interesting to study, is the club head inertia dyadic. Golf club designers are always trying to optimize the shape of the club head to outperform their competitors. From our study, it is known that the shape of the club head has a large impact on the distance the ball flies, but the results were never collected.

6. Discussion of Results

The results from the optimization study are very interesting. The two parameters that are constrained, as mentioned before, are the swing elevation angle, ε, and the loft angle, λ, of the club. The first optimization study discussed here is when the swing elevation angle was held constant and the loft angle was optimized. The figure below shows the result of the metamodeling of the system with the optimum values using the fmincon solver in MATLAB.

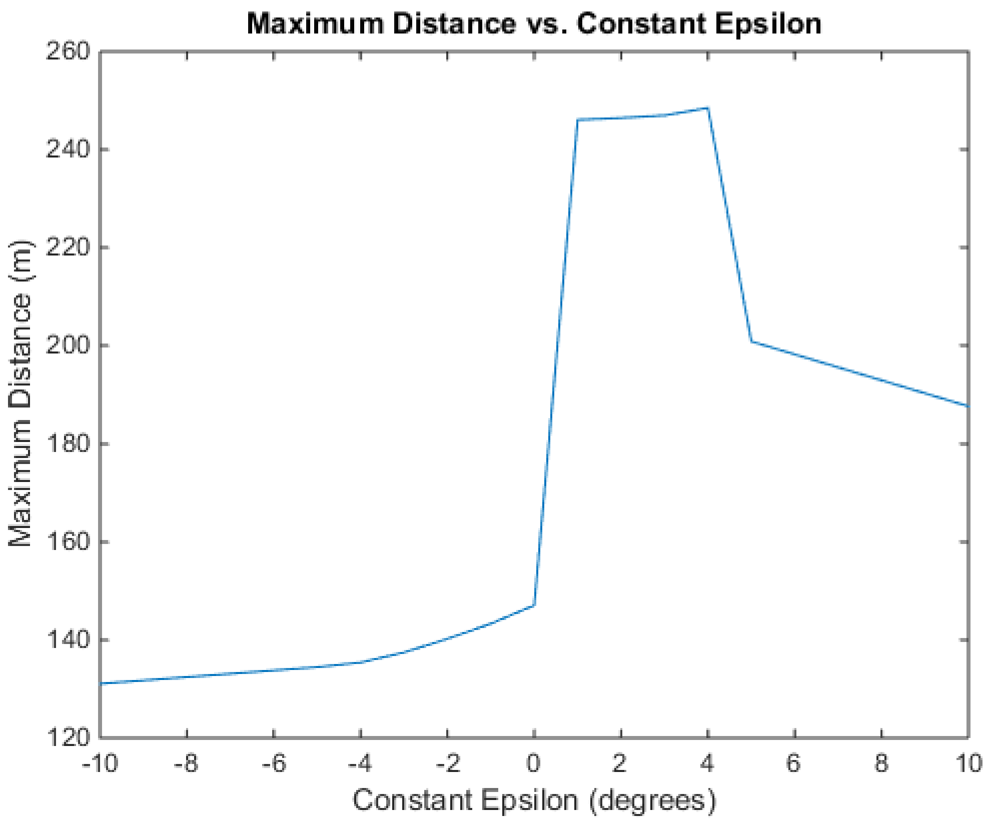

Figure 1: This is a plot of the maximum distance vs. the constant swing elevation angle. It can be observed from the figure that with a swing elevation angle in the range of about 1-4°, the golf ball travels a much farther distance. A table of the data that makes the figure above with the optimum swing elevation angle is listed below.

Table 1: Optimization Results with Constant Swing Elevation Angle

Swing Elevation Angle, ε (degrees)	Optimum Loft Angle λ (degrees)	Maximum Distance (m)
-10	11.5713	131.08
-5	4.3575	134.47
-4	4.4011	135.41
-3	4.4368	137.47
-2	4.4644	140.24
-1	4.4824	143.33
0	4.4946	147.09
1	19.2683	246.05
2	18.605	246.44
3	18.02	246.96
4	17.6815	248.48
5	9.4692	200.83
10	4.4166	187.59

From the figure and table above it can be seen that the optimum loft angle of the club is 17.68°, with a maximum distance of 248.48 m. The next study done was to keep the loft angle constant and optimize the swing elevation angle.

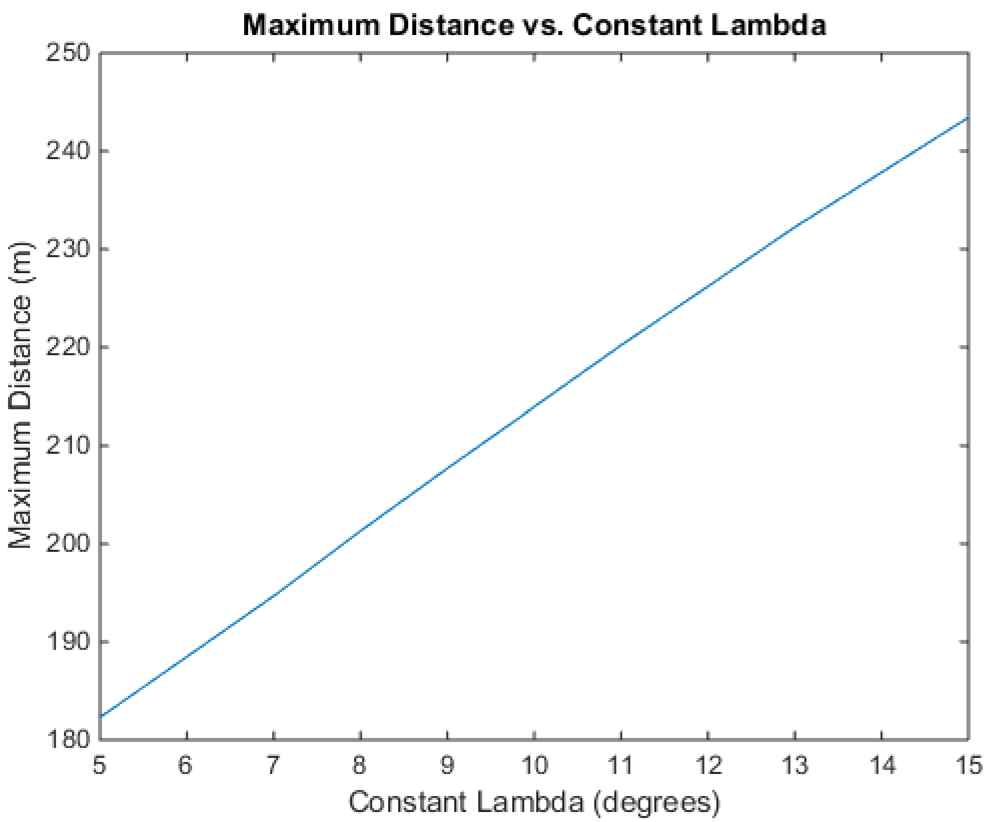

Figure 2: This is a plot of the maximum distance vs. the constant loft of the club. The observation to be made with this study is that the swing elevation angle does not have as much of an impact as the loft of the club. When comparing figure 1 to figure 2, it is clear that there is an optimum loft angle of the club; because in figure one a maximum value is

observed. In figure 2, optimizing the swing elevation angle does not yield a maximum value. The table below shows the data that make up figure 2 with the optimum swing elevation angle.

Table 2: Optimum Swing Elevation Angle with Constant Loft Angle

Loft Angle λ (degrees)	Optimum Swing Elevation Angle ε (degrees)	Maximum Distance (m)
5	7.5236	182.3
7	7.4428	194.68
8	7.4111	201.31
9	7.3827	207.7
10	7.3428	213.93
11	7.3060	220.20
12	7.2875	226.2
13	7.2593	232.24
15	7.2053	243.43

From table 2 it is observed that the optimum swing elevation angle does not change much, but as the loft angle increases the maximum distance also increases. Using the data from table 1, it is assumed that when the loft angle is increased to around 17° a maximum value would be found. Based on the two studies conducted, the optimum conditions for the longest drive would be a loft angle of 17.68° with a swing elevation angle of about 7.3°. A third study was done to check to see if the optimization of our mathematical model was correct. This was done by using a swing elevation angle range of 0-10° and a range of 8-18° for the loft angle of the club. Keeping the loft angle constant, the distance of a drive was found changing the swing elevation angle by one degree every time until 10° was reached. Then, an average distance was found for each constant loft angle. The process was repeated until the loft angle reached 18°. The plot of the data is shown below.

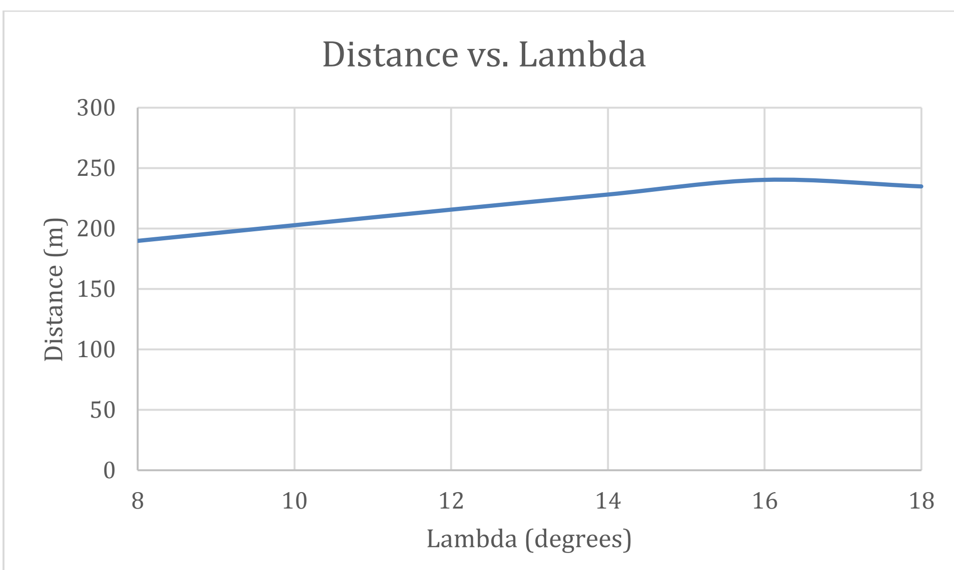

Figure 3: This is the average distance for the swing elevation range vs. the constant loft angle of the club. It is seen that the maximum distance occurs when the loft angle of the

club is 16°. This value is consistent with the optimization study conducted for our mathematical model using metamodeling and the fmincon solver in MATLAB.
After conducting research about average golf driver lofts, it was discovered that most people have drivers with a 9-10° loft, but they may get better results with a 12-16° lofted club[3]. This corresponds well to what was found in the optimization study.
The common variable in this study, between the two subsystems, are the velocities of the ball directly after impact, and are constrained by the swing elevation angle and loft angle of the club. It is observed that the results from the optimization study are consistent with products in the market currently.

7. System Integration Study

The results listed in section seven are the results of a system integration study. The impact model solved for the velocities of the ball and club directly after impact, and those velocities were inputted into the aerodynamic model to determine the distance the ball traveled. It is observed that the combined subsystem optimum yields the overall system optimum for the distance the golf ball after impact.

References

1. Tan, T.E. , & Winfield, D.C. (1993). *Optimization of Clubhead Loft and Swing Elevation Angles for Maximum Distance of a Golf Drive*
2. Robinson, Garry, & Robinson, Ian (2013). *The motion of an arbitrarily rotating spherical projectile and its application to ball gam*es
3. Southern, Mike. Demand Media. *Correct Loft for a Golf Driver*. Retreived from http://golftips.golfsmith.com/

Appendix

Matlab Optimization Code:

```
%MATHEMATICAL MODEL

%Impact Model

    lambda=4.4166*pi/180; %Loft angle (radians)
epsilon=10*pi/180; %Elevation angle (radians)
theta=lambda+epsilon;

vc=[44.7*sin(epsilon) 0 44.7*cos(epsilon)]; %Initial Translational
velocity of club head m/s
wh=[0 0 0]; %Initial Angular velocity of club head rad/s
vb=[0 0 0]; %Initial Translational velocity of ball m/s
wg=[0 0 0]; %Initial Angular velocity of club head rad/s

mh=.230; %Mass of club head
mg=.049; %Mass of golf ball

Ih=[0.0002057,0.000377,0.0002738]; %Inertia dyadic club head kg*m^2
Ig=0.00000585; %Inertia dyadic ball kg*m^2

dc=.037;
db=.021;
rc=[0 0 dc];
rb=[0 0 db];

Area=[1 0 0 0 0 0 0 0 0 0 0 0 -sin(epsilon)/mh 0 0
   0 1 0 0 0 0 0 0 0 0 0 0 0 -1/mh 0
   0 0 1 0 0 0 0 0 0 0 0 0 0 0 -cos(epsilon)/mh
   0 0 0 1 0 0 0 0 0 0 0 0 0 -rc(3)*cos(epsilon)/Ih(2) 0
   0 0 0 0 1 0 0 0 0 0 0 0 (rc(3)*cos(epsilon))/Ih(1) 0 -
(rc(3)*sin(epsilon))/Ih(3)
   0 0 0 0 0 1 0 0 0 0 0 0 0 rc(3)*sin(epsilon)/Ih(2) 0
   0 0 0 0 0 0 1 0 0 0 0 0 sin(theta)/mg 0 0
   0 0 0 0 0 0 0 1 0 0 0 0 0 1/mg 0
   0 0 0 0 0 0 0 0 1 0 0 0 0 0 cos(theta)/mg
   0 0 0 0 0 0 0 0 0 1 0 0 0 -rb(3)*cos(theta)/Ig 0
   0 0 0 0 0 0 0 0 0 0 1 0 rb(3)*cos(theta)/Ig 0 -rb(3)*sin(theta)/Ig
   0 0 0 0 0 0 0 0 0 0 0 1 0 rb(3)*sin(theta)/Ig 0
   0 0 cos(epsilon) 0 -rc(3)*sin(epsilon) 0 0 0 -cos(theta) 0
rb(3)*sin(theta) 0 0 0 0
   -sin(epsilon) 0 0 0 rc(3)*cos(epsilon) 0 -sin(theta) 0 0 0 -
rb(3)*cos(theta) 0 0 0 0
   0 1 0 -rc(3)*cos(epsilon) 0 rc(3)*sin(epsilon) 0 -1 0
rb(3)*cos(theta) 0 -rb(3)*sin(theta) 0 0 0];

b=[vc(1)
   vc(2)
   vc(3)
   wh(1)
   wh(2)
```

```
    wh(3)
    vb(1)
    vb(2)
    vb(3)
    wg(1)
    wg(2)
    wg(3)
    -0.7*vc(3)
    0
    0];
x=inv(Area)*b;

u7=x(7);
u8=x(8);
u9=x(9);
u10=x(10);
u11_original=x(11)*2*pi;
u12=x(12);




%Aerodynamic Model

rho=1.225; %Density of Air kg/m^3
Area=(0.042672/2)^2*pi; %Cross sectional area of golf ball m^2
g=9.81;  %Gravity kg/m^2
m_G=.049 ;   %Mass of Golf Ball kg  NEED TO EDIT



%u7=10.14;
%u8=0;
%u9=94.56;
%u10=0;
%u11_original=-94;
%u12_original=-242.37;  %Rad/s
%u12=0;



h=.01;                                                    % step size
time = 0:h/10:h;                                                % Calculates
tt=length(time);

distance=0;
height=0;

count=1;

time_total=0;

while height>=0
```

```
 u11=u11_original*exp(-.7*h*time_total);
 Cd=.3+2.58e-4*abs(u11);
Cl=(3.19e-1)*(1-exp(-2.48e-3*abs(u11)));




    u_vb_norm=(u7^2+u8^2+u9^2)^0.5;
u_w_norm=(u10^2+u11^2+u12^2)^0.5;

n_vb=[u7;u8;u9]./u_vb_norm;
n_w=[u10;u11;u12]./u_w_norm;

Fd=0.5*Cd*rho*(u7^2+u8^2+u9^2)*Area;
Fl=0.5*Cl*rho*(u7^2+u8^2+u9^2)*Area;



n_l=[n_w(2)*n_vb(3)-n_w(3)*n_vb(2);n_w(3)*n_vb(1)-
n_w(1)*n_vb(3);n_w(1)*n_vb(2)-n_w(2)*n_vb(1)];




  f_u7= @(t,u7) ((Fl*n_l(1)-Fd*n_vb(1))/m_G)-g;

  f_u8= @(t,u8) (Fl*n_l(2)-Fd*n_vb(2))/m_G;

  f_u9= @(t,u9) (Fl*n_l(3)-Fd*n_vb(3))/m_G;



   [ti_u7,u7_new]=ode45(f_u7,time,u7);

   u7_final=u7_new(tt,1);

   [ti_u8, u8_new]=ode45(f_u8,time,u8);

   u8_final=u8_new(tt,1);

   [ti_u9,u9_new]=ode45(f_u9,time,u9);

   u9_final=u9_new(tt,1);


   u7=u7_final;
   u8=u8_final;
   u9=u9_final;
```

```
   u7_check(count,:)=[count,u7];
    u8_check(count,:)=[count,u8];
    u9_check(count,:)=[count,u9]
height=height+(h*u7)
distance=distance+(h*u9);
height_check(count,:)=[count,height];
distance_check(count,:)=[count,distance];
Cd_check(count,:)=[count,Cd];
Cl_check(count,:)=[count,Cl];

count=count+1;

time_for_plot=0:h:h*time_total;
time_total=time_total+1;
end

%plot(time_for_plot,height_check(:,2))
D=max(distance_check,[],1);
dd=-D(1,2)
hh=max(height_check,[],1);
hhh=hh(1,2);

%REGRESSION
function [lambdastar]=metamodel_eps_negative10(X)
d=[9.7  191.2   222.5   249 98.3];
lambda=[0 5*pi/180 10*pi/180 15*pi/180 20*pi/180];
plot(lambda,d)

x=[1 lambda(1)^2 lambda(1)^3 lambda(1)^4 lambda(1)^5;
    1 lambda(2)^2 lambda(2)^3 lambda(2)^4 lambda(2)^5;
    1 lambda(3)^2 lambda(3)^3 lambda(3)^4 lambda(3)^5;
    1 lambda(4)^2 lambda(4)^3 lambda(4)^4 lambda(4)^5;
    1 lambda(5)^2 lambda(5)^3 lambda(5)^4 lambda(5)^5;];

beta=inv((x'*x))*x'*d';
lambdastar=-(beta(1)+beta(2)*X+beta(3)*X^2+beta(4)*X^3+beta(5)*X^4);
end
%OPTMIZATION

A=[]; b=[]; Aeq=[ ]; Beq=[ ]; % matrix/vectors for defining linear
constraints (not used)
lb =[0*pi/180]; % lower bounds on the problem
ub = [20*pi/180]; % upper bounds on the problem (not used)
%i0=0;

l_star_1=(180/pi)*fmincon('metamodel_eps_negative10',10*pi/180,A,b,Aeq,
Beq,lb,ub)
```

www. grabcad.com was used to model the club and ball in SolidWorks where the inertia dyadics were then found.