University of California Riverside

ME170B Experimental Techniques: Lab 5

Damping of Vibrations

Group A5

Elijah Perez | Soham Saha | Alex Pham

Fall 2024 - Mon-Wed 8AM Session

October 9th, 2024

Abstract

The purpose of this laboratory experiment is to determine the viscosity of a fluid by observing its effects on a viscous damper spring-mass system. This was achieved by placing a weighted hook onto a spring, and measuring the frequencies for various damping setups. Measurements were taken via an accelerometer which was connected to a LabView VI on a computer. This was also used to measure the spring constant. Different experiments featured various damping mediums (air, water and glycerin), as well as various disk shaped dampers submerged in the damping medium. Utilizing the Stokes equation for viscosity and damping force, along with the equation of motion for a spring mass damper, a relationship between damping coefficients and viscosity was developed. Upon the completion of the experiment, the spring constant was calculated to be 140 Nm. The damping coefficients for the 1.5”, 2.0”, and 2.5” diameter samples were c = 0.053, c = 0.053, and c = 0.053 in air. For water, we calculated the damping to be c = 0.22, c = 0.35, and c = 0.7. We also calculated the damping in the glycerin to be c = 0.65, c = 1, c = 1.25. The damping coefficient has units of Ns/m. The fluid viscosity of air was measured to be 0.244662 Pa*s, the fluid viscosity of water was measured to be 1.8322 Pa*s, and the fluid viscosity of glycerine was measured to be 2.6 Pa*s. The air calculation had a relative error of 3332.3333%, the water calculation had an error of 6387.67342%, and the glycerin measurement had an error of 83.8755304%. The major error in the experiment can be attributed to the several assumptions that were made to relate damping coefficient to viscosity, as well as the discrepancies in measurement of the spring constant. Additionally damping effects were hardly noticeable for the water medium and not at all noticeable for the air medium.

Introduction

Mass spring damper systems appear in a variety of applications. In vehicles, the primary role of coils and suspension systems is to absorb energy from road irregularities, providing a smoother and more controlled driving experience. Springs store and release substantial energy due to their structure, and their behavior varies depending on the medium through which energy transfers. In different media, energy may be dissipated, impeded, or passed through with minimal effect, a response largely influenced by the viscosity of the damping fluid. When weight is applied to a spring, the release of stored energy causes oscillations, which experience varying damping effects depending on the surrounding medium. By examining these dynamics, we can calculate the damping force and viscosity between the fluid and the spring system, considering factors such as geometry and weight. This laboratory experiment will analytically explore spring-mass systems, beginning with the determination of the system’s spring constant. Using Hooke’s Law, we’ll apply weights to the spring and measure its deflection. In many applications, reducing vibrations is crucial, and dampers are employed to minimize these. Here, water will act as a viscous damper to study its impact on spring vibrations. Through the use of an accelerometer and LabVIEW to measure frequency, we’ll calculate the damping constants for viscous damping and, ultimately, determine the fluid’s viscosity.

Theory

A spring is a type of mechanical link, which in most applications is assumed to have negligible mass and damping. A spring is said to be linear if the elongation or reduction in length x is related to the applied force F as shown in Equation 1.

$$F = kx \tag{1}$$

In this experiment, we studied how the damping of a system affects its vibrations. We studied the case where the system was under free vibration, which means that there was no forcing function acting on the system. The free-body diagram of the system can be seen

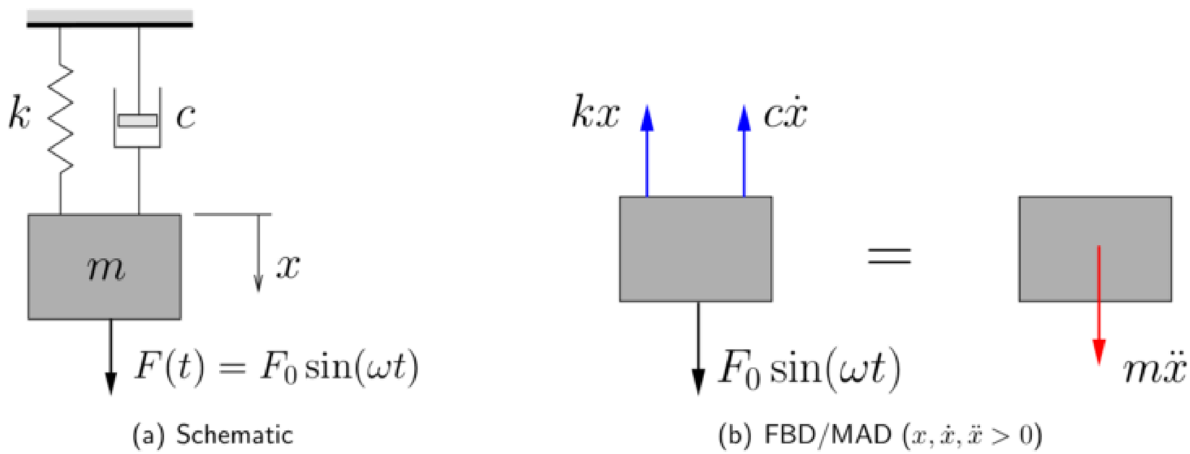

Figure 1: Single-degree-of-freedom system with viscous damper

By doing an energy balance with Newton's Second Law, Equation (2) is derived.

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{2}$$

Equation 2 is an Ordinary Differential Equation (ODE) and its solution gives the location of the mass as a function of time. The solution for the ODE is presented:

$$x(t) = C_1 e^{\frac{-c}{m}t + \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}\,t} + C_2 e^{\frac{c}{m}t - \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}\,t} \tag{3}$$

If the spring-mass system is in a vertical position, the natural frequency can be expressed as,

$$\sqrt{\frac{m}{k}} \tag{4}$$

From Equation 3, the critical damping constant can be determined; This is the quickest time to approach a zero amplitude. The critical damping constant is:

$$c_c = 2\sqrt{km} = 2m\omega_n \tag{5}$$

The frequency of damped vibration ω_d is always less than the undamped natural frequency ω_n. The quantity referred to as the frequency of damped vibration is

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \tag{6}$$

Equation 5 enables us to determine the damping ratio, ζ, which represents the ratio of the damping constant to the critical damping constant. The damping ratio provides insight into the system's behavior: a ratio of $\zeta=1$ indicates critical damping, $\zeta<1$ indicates underdamping, and $\zeta>1$ indicates overdamping. Figure 2 illustrates these behaviors: in a critically damped system, the mass quickly returns to its equilibrium position; in an underdamped system, the mass oscillates before settling; and in an overdamped system, the mass gradually returns to equilibrium without oscillation.

$$\zeta = \frac{c}{c_c} \tag{7}$$

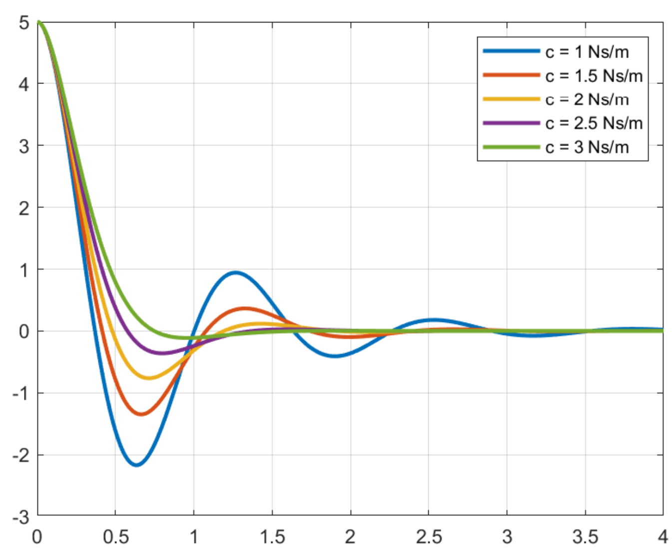

Figure 2: Damping effects of a single degree of freedom (SMD) system

The damping constant can be found in the equation that gives the damped frequency in which the system oscillates. Solving that equation for the damping constant provides Equation 8:

$$c = \frac{2\sqrt{km}}{\sqrt{1-\left(\frac{\omega_d}{\omega_n}\right)}} \tag{8}$$

The damping was estimated using experimental data and theoretical comparisons. We obtained the parameters of the system to simulate the theoretical response of the system and compared the two. Modeling the single degree of freedom in state space representation is shown in Equation (9) and solving the system of equations using the runge kutta method is shown in Equation (10). We successfully estimated the damping coefficients.

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{9}$$

$$y_{n+1} = y_n + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right) \tag{10}$$

Where:

$$\begin{aligned} k_1 &= f\left(t_n, y_n\right), \\ k_2 &= f\left(t_n + \tfrac{h}{2}, y_n + \tfrac{h}{2}k_1\right), \\ k_3 &= f\left(t_n + \tfrac{h}{2}, y_n + \tfrac{h}{2}k_2\right), \\ k_4 &= f\left(t_n + h, y_n + hk_3\right). \end{aligned}$$

Solving the theoretical solution for acceleration and matching it to our experimental data we were able to obtain the damping coefficient for our experiment.

Once the damping coefficient for a given system is obtained, the result can be used to then calculate the dynamic viscosity of the fluid used in the viscous damper. This is given by Equation (11), the Stokes equation for a sphere in a viscous medium of dynamic viscosity μ. V is the velocity of the system as it oscillates. For our purposes, the thin disk can be approximated to be a sphere with the radius R (radius of the disk), as the velocities are low. F represents the force exerted on the system by the damper, given by Equation (12) which describes the relationship between damping coefficient, velocity and damping force.

$$F = -6\pi\mu Rv \tag{11}$$

$$F = cv \tag{12}$$

Setting *F* from Equation (11) and Equation (12) equal to each other allows us to cancel velocity out of both sides of the equation, giving us Equation (13). Rearranging to isolate μ gives us Equation (14) which displays a linear relationship between viscosity and damping coefficient.

$$cv = -6\pi\mu Rv \tag{13}$$
$$c = -6\pi\mu R$$
$$\mu = \frac{-c}{6\pi R} \tag{14}$$

This experiment will seek to prove or disprove Eq (14), which suggests a linear relationship between the damping coefficient of a spring mass damper system and the dynamic viscosity of the fluid used in the damper.

Methods

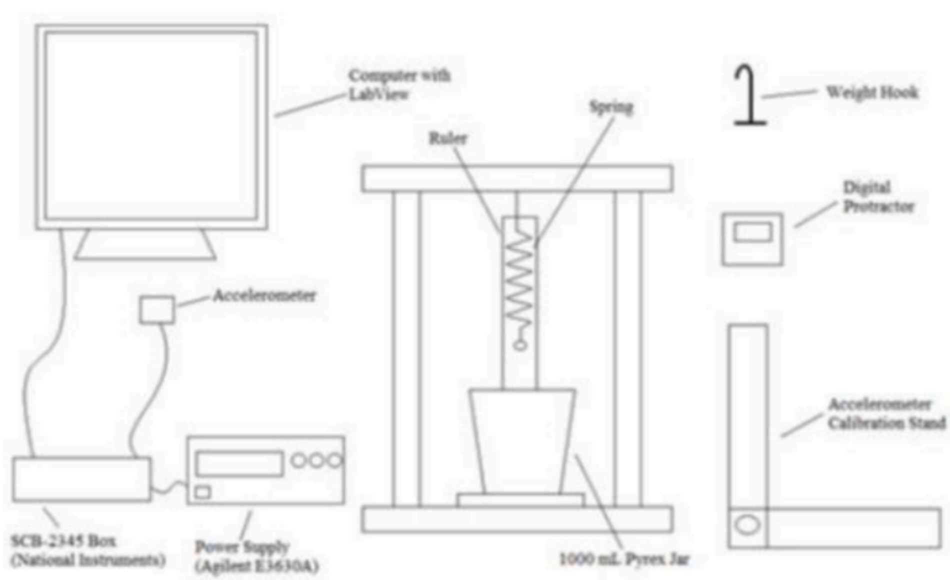

Figure 3: Damping of Vibrations Apparatus with main components listed

The experiment will begin by calibrating the accelerometer. Secure the accelerometer calibration stand to the table, then attach the digital protractor and accelerometer to the stand. Open the LabVIEW program file titled "Accelerometer," turn on the SCB-2345 Box, and set the power supply to 15 volts. Adjust the angle on the digital protractor and input this data into the LabVIEW file, recording the transducer output voltage. Repeat this process three more times to complete the calibration.

After calibration, remove the accelerometer from the stand and attach it to the weight hook. Mount both the hook and accelerometer onto the spring assembly. Add 100-gram weights (one at a time) to the system and record the initial and final lengths. These measurements are used to calculate the spring deflection and, subsequently, determine the spring constant using Hooke's Law. Once complete, remove the 100-gram weights.

Undamped System: Attach one of the three diameter test objects to the bottom of the weighted hook. Open the "Undamped" file in LabVIEW and input the required parameters: attached mass,

spring mass, and spring constant. Run the program and simultaneously pull and release the spring-mass system to allow for oscillations. Stop the program, record the corresponding frequencies, and save the displayed graph. Repeat this procedure, adding 100-gram weights one at a time. Perform the steps above with each different diameter disc to observe how geometry affects oscillation.

Damped System: Fill a Pyrex jar with 800 mL of water and repeat the procedure used for the undamped system. This time, the water will act as a damping fluid to reduce oscillations. Once all measurements have been taken, pour out the water and turn off the SCB-2345 Box, power supply, and computer.

Experimental Results:

Based on these results, we are confident in the accuracy of our damping coefficients. The theoretical damping coefficient shows the same rate of decay as observed in our experimental data. Additionally, the results for different experiments display the different coefficients of damping, with air having the least effect (hardly any damping) and glycerin having the most damping effect.

Although the graphs may not align perfectly due to experimental limitations, such as the challenge of reproducing the exact initial conditions for acceleration without machine-level precision, the primary focus is on the accuracy of the decay rate. This rate is the most critical factor affected by damping in this single-degree-of-freedom experiment. Any phase shift between the experimental and theoretical graphs is expected, but it does not impact the validity of our findings on damping behavior.

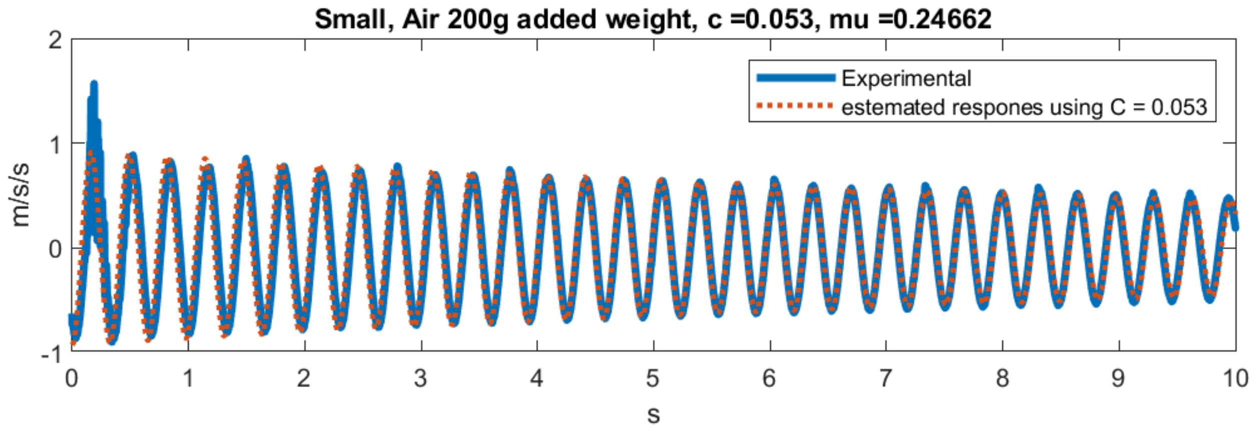

Figure 4: Small diameter plate in air

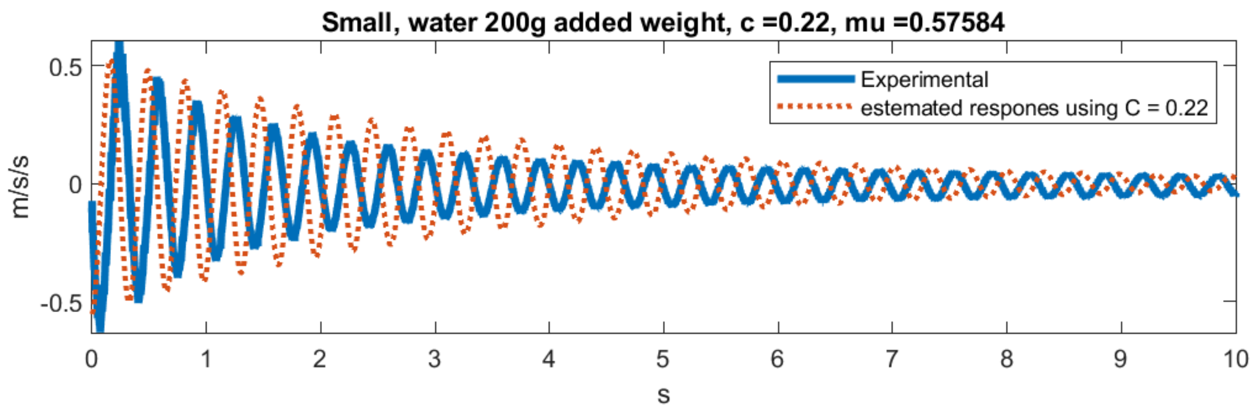

Figure 5: Small diameter in water

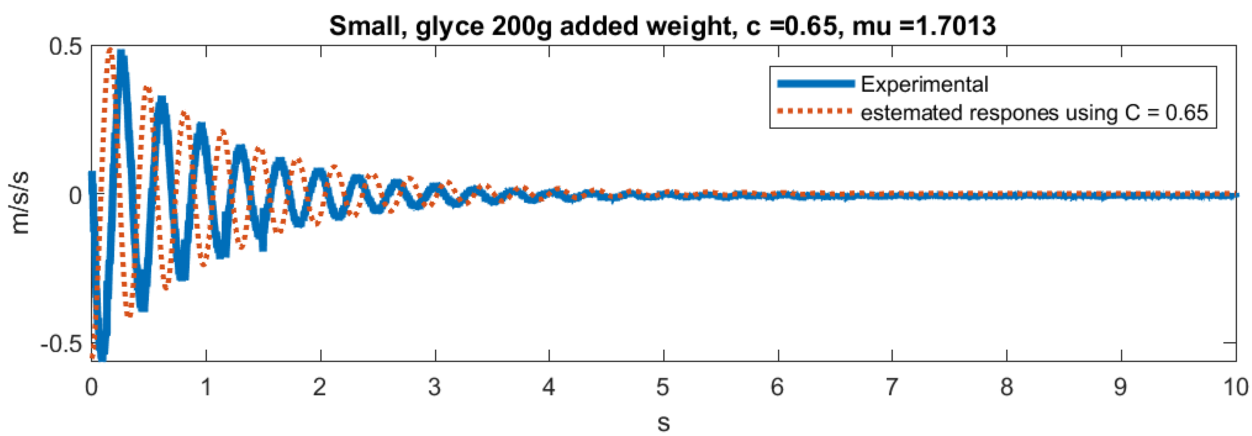

Figure 6: Small diameter in glycerine

Discussion:

Given the qualitative nature of the results, we can see that as expected, the damping effect for the more viscous fluid is more pronounced. That is the damping in water is greater than in air, and the damping in glycerin is more than in water. Additionally, we can see the effect that the size of the damper had on the damping coefficient, as larger dampers had a larger constant. This is also inline with Eq(14) which predicts the damping coefficient is proportional to the radius of a spherical damper. The significance of the results of this experiment is that the effect of the damping medium can be clearly seen. This supports the hypothesis that damping coefficient and fluid viscosity are proportional to each other. This implies that the viscosity of the fluid medium does indeed have a major influence on the damping coefficient for a mass spring damper system, and that there is a relationship between viscosity and damping that can be used to determine the viscosity of a fluid.

However, when viscosities were calculated, major error was observed for all three mediums. This can be attributed to the various assumptions that were made about the model. The first major assumption was that although the Stokes equation for viscosity is for a sphere, it can be directly applied to the disk shaped damper. While this assumption holds true for laminar flow scenarios, the assumption may not have been entirely appropriate within the context of this experiment. To account for this, it would have possibly been more appropriate to use a spherical shaped damper submerged in the fluid as opposed to a disk shaped damper. Although the visible damping effect may have been reduced, the Stokes equation could have been applied more accurately.

Another assumption that led to error was assuming that the system was restricted to motion in a single degree of freedom. While a majority of the motion of the system was vertical, the initial displacement conditions for each experiment likely caused some velocity in transverse directions, which can cause additional damping and energy loss of the spring mass damper system. On top of this, while the initial conditions for each trial were approximately similar, there is no doubt there was variation. The method of producing identical initial conditions was to force a displacement of 1” before releasing the system, but this was not trivial to accomplish with precision due to the experiment setup.

Conclusion and Recommendations:

This lab investigates how different fluids affect damping when using various surface area plates on a weighted spring and how oscillations change with different variables. By comparing air and water, it was observed that the spring’s energy dissipated more quickly in water due to the viscous forces acting on the plate’s surface area and the varying weights, which contribute to different levels of stored energy in the spring. Air, which provides minimal viscous force compared to water, was less effective at damping. At the end of the experiment, the spring constant was calculated to be 140 N/m. The damping coefficients for the 1.5”, 2.0”, and 2.5” diameter samples were $c = 0.053$, $c = 0.053$, and $c = 0.053$ in air. For water, we calculated the damping to be $c = 0.22$, $c = 0.35$, and $c = 0.7$. We also calculated the damping in the glycerin to be $c = 0.65$, $c = 1$, $c = 1.25$. The damping coefficient has units of Ns/m. The fluid viscosity of air was measured to be 0.244662 Pa*s, the fluid viscosity of water was measured to be 1.8322 Pa*s, and the fluid viscosity of glycerine was measured to be 2.6 Pa*s. The air calculation had a relative error of 3332.3333%, the water calculation had an error of 6387.67342%, and the glycerin measurement had an error of 83.8755304%.

This discrepancy is likely due to equipment limitations. For future experiments, we recommend improved control over wave dispersion and enhanced data acquisition for more accurate measurements, along with a system to have quantized initial displacements. Additionally, using a wider range of calculations and equations to verify results would strengthen the accuracy and reliability of findings. The assumptions made during the theoretical derivation process of the experiment likely had a major effect on the final results. Thus more improvements could be made by modifying the Stokes equation to fit the geometry of a disk shaped damper, or perhaps using a spherical damper for which the Stokes equation could directly apply.

References:

[1] Rao, S. S. Mechanical vibrations. Prentice Hall, 2011. 5th Edition.
[2] Nipun. "Difference Between Damped and Undamped Vibration." Pediaa.Com, 28 Oct. 2015, pediaa.com/difference-between-damped-and-undamped-vibration/
[3] Jor. (2015, October 1). How to Determine Viscosity using Damped Oscillations? [Online forum post]. Physics StackExchange. https://physics.stackexchange.com/questions/210206/how-to-determine-viscosity-using-damped-oscillations

Appendix:

F = **viscous damping force**
k = **spring constant**
x = **vertical displacement of the system**
m = **mass**
c = **coefficient of viscous damping**
C1 = **arbitrary constant one**
C2 = **arbitrary constant two**
t = **time**
ωn = **natural frequency**
cc = **critical damping constant**

ωd = frequency of damped vibration
ζ = damping ratio
μ = dynamic viscosity of fluid
A = cross-sectional area

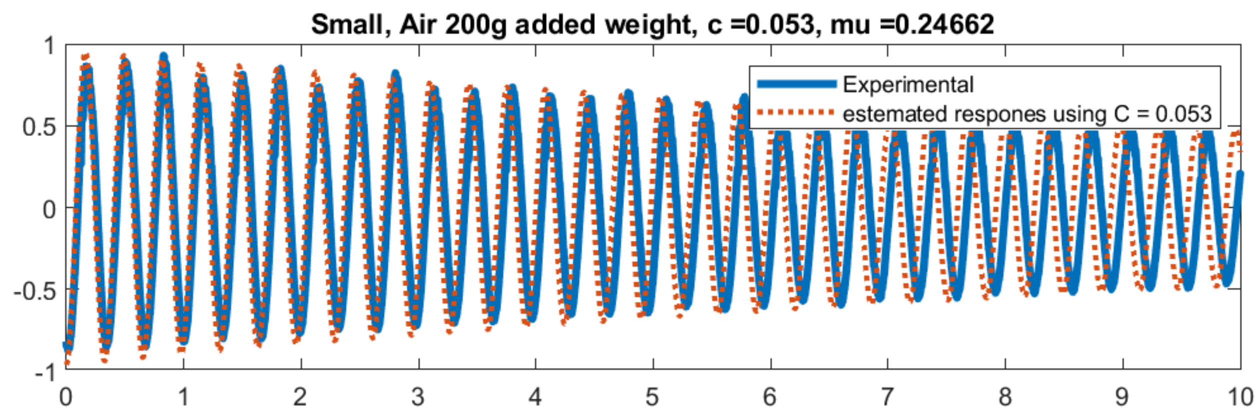

Figure 7: medium diameter in air

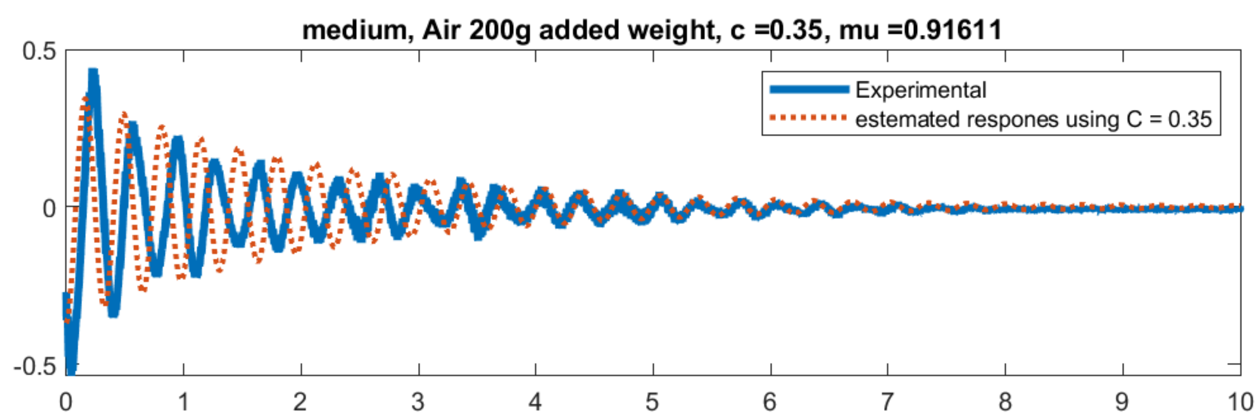

Figure 8: medium diameter in water

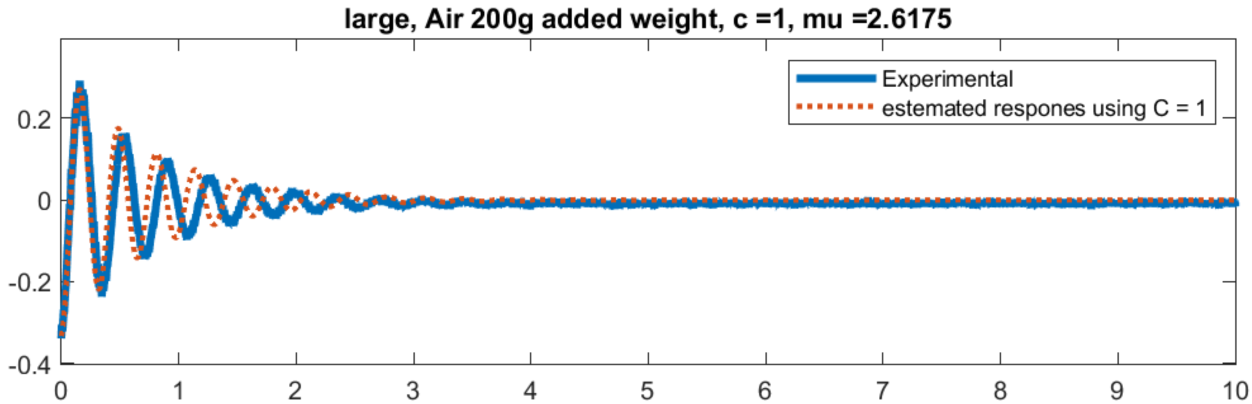

Figure 9: medium diameter in glycerine

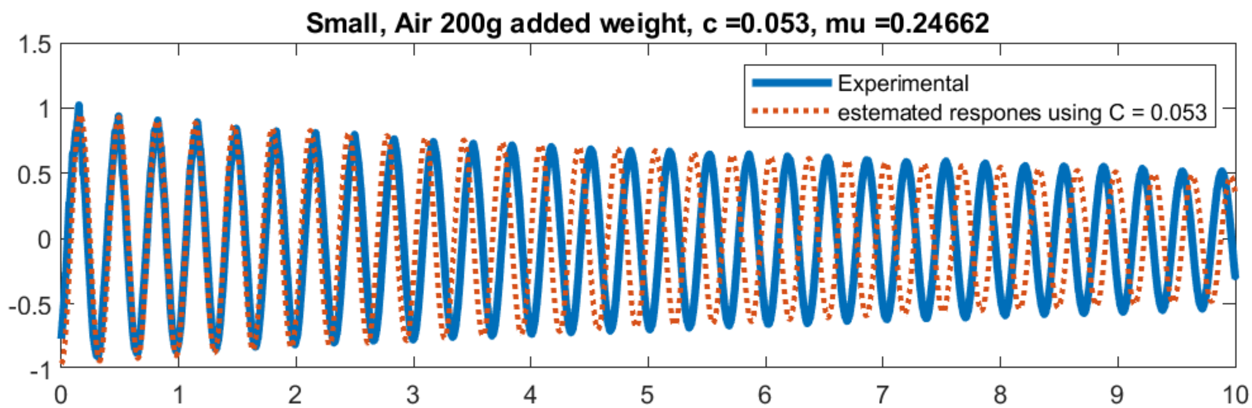

Figure 10: Large diameter in air

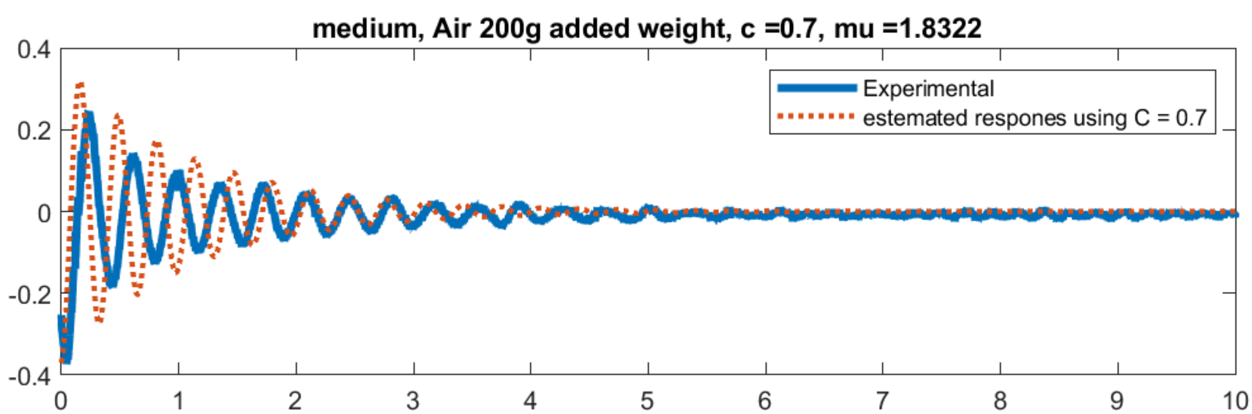

Figure 11: Large diameter in water

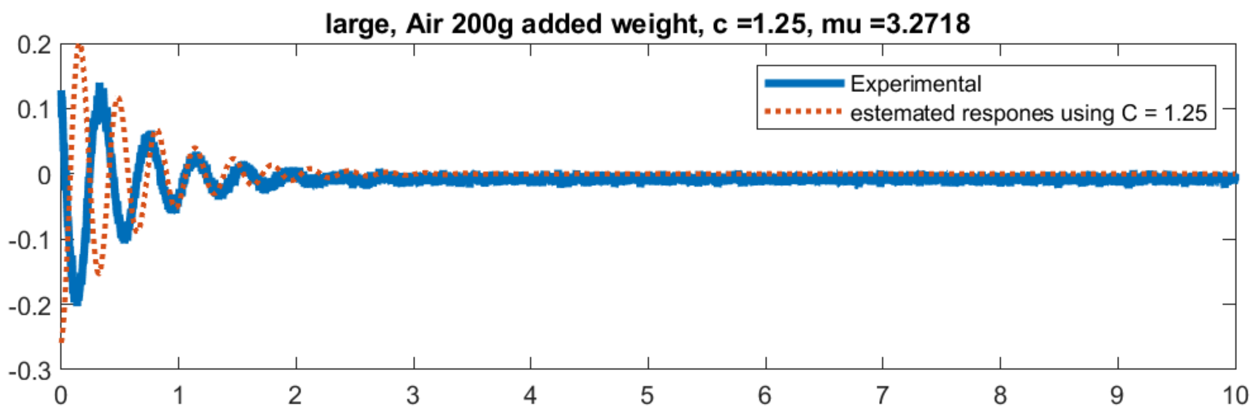

Figure 12: Large diameter in glycerine

Table 1: Calibration data for accelerometer for one single trial. This data is manually copied into LabView to determine the proper calibration for the accelerometer.

Angle (degrees)	**Output Voltage (mV)**
0	115.6
10	114.2
20	109.7
30	102.4
40	92.4
50	80.1
60	65.4
70	49.9
80	33.0
90	15.1

Table 2: Measured mass of all objects within the mass spring damper system.

1.5 Inch Disk	70.55g
2.0 Inch Disk	80.32g
2.5 Inch Disk	102.59g
Flat Disk	37.22g

Black Hook and Disk	109.87g
Spring and Hoop	7.54g
Accelerometer	23.57g

Table 3: Spring Constant calculation from measured deflection under load.

Weight of System (g)	Deflection (cm)	Calculated Spring Constant (N/m)
143.33g	1cm	140.61
243.33g	1.5cm	159.14
343.33g	2cm	168.40
443.33g	2.5cm	173.96
543.33	3cm	177.67

Statements of Contribution:

Elijah Perez: Experimental Design, Data processing, analysis and theoretical model, Lab report
Soham Saha: Experimental Design, Data Collection, Theory and Derivations, Lab report
Alex Pham: Lab Report Structure, Spring Constant Data Collection