University of California Riverside

ME170B Experimental Techniques: Lab 1
Linear and Radial Heat Conduction

Group A5
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Abstract:

The purpose of this experiment was to analyze linear and radial heat conduction in metals by using an apparatus designed to minimize multi-dimensional heat transfer and measure temperature gradients. It was hypothesized that between brass and stainless steel, brass would have a higher thermal conductivity *K*. This was accomplished by placing the metals in contact with a heater and running a water stream on either side, to create a stable temperature gradient for effective measurement. Temperature data collected at various points enabled the creation of a temperature profile along the metal lengths and radius, facilitating detailed analysis. Using Fourier's law, we calculated *K* values for brass and stainless steel as 134.56 W/m·K and 34.58 W/m·K, with error margins of 25.78% and 15.02%, respectively. Comparison with wattmeter readings for radial conduction at 30W and 40W showed errors of 0.667% and 1.59%. Additionally, the overall heat transfer coefficient U for a composite wall was determined to be approximately 787.573 W/m^2·K. This experiment confirmed the material-specific nature of thermal conductivity and validated Fourier's law, contributing to our understanding of thermal properties in engineering and materials science.

Introduction:

Heat conduction is fundamental to various applications, one of the most relatable being cooking, where heat from the stove transfers through the pan to the food. To deepen our understanding of conduction, it is important to examine heat transfer through different materials and applications, as the efficiency of heat transfer varies by material properties. This experiment explores heat conduction in steel and brass with different diameters in a linear setup and a brass plate in a radial setup. It is hypothesized that the measured thermal conductivity of brass would be higher than that of steel, and that the linear conduction experiment would reveal Fourier's law in the temperature distribution. It is also hypothesized that a steady rate of heat transfer would be observed for the radial experiment. The linear setup comprises a heated section on one end, heated via a power supply, and a cooled section on the other, maintained by a continuous water supply. The radial setup involves a brass disc with water circulating around its outer ring to regulate temperature, this can be shown in Figure 1. By measuring temperatures at various points, we tracked the heat distribution throughout each system to assess the metals' effectiveness and efficiency in heat transfer. The experiment aimed to verify whether steel and brass, given their different thermal properties and configurations, transfer heat as predicted, with each showing unique heat transfer coefficients.

Theory:

This lab is focused on conduction through linear and radial systems. The most important equations come from Fourier's Law:

$$k = \frac{-q}{A}\frac{dx}{dt} \tag{1}$$

The radial system is more complex, as the area changes with respect to the radius. To linearize this system, a logarithmic scale must be created with r, and the temperatures need to be based on the outer temperature T_o. Performing that and solving for the thermal conductivity, the equation for radial thermal conductivity becomes Equation 2:

$$k = \frac{q}{2\pi L} \cdot \frac{ln(\frac{r_2}{r_1})}{T_2 - T_1} \tag{2}$$

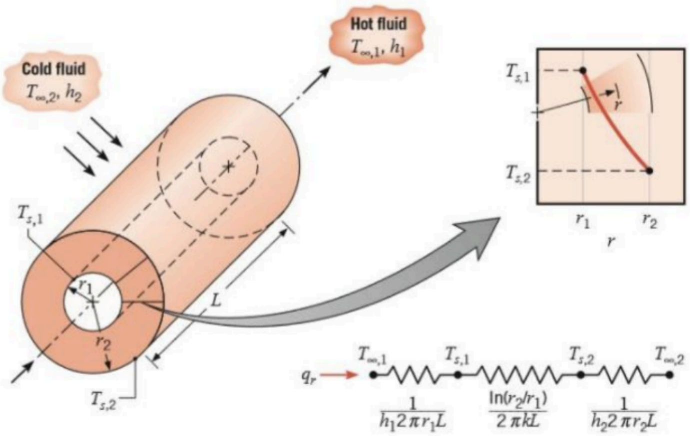


Figure 1: Hollow cylinder with heat being generated from the inner radius

When analyzing a linear heat conduction system composed of different materials with different thermal conductivities, it is often convenient to express the thermal conductivity as one; meaning an overall heat transfer coefficient. This allows for calculations to be performed more quickly, instead of having to account for each thermal conductivity constant and length of the materials present in the linear system.

$$U = (\frac{L_1}{k_b} + \frac{L_2}{k_s} + \frac{L_3}{k_b}) \tag{3}$$

Where b is the subscript for brass and s is the subscript for steel.

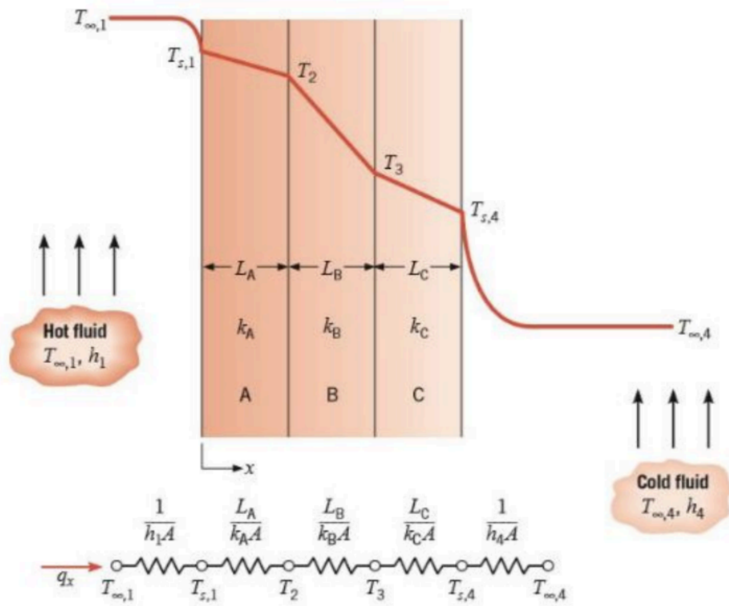


Figure 2: Equivalent thermal circuit for a series composite wall.

Furthermore, we can further derive the temperature profiles as a function of x and r. Where x can be the distance across the material in meters. Similarly, r can be the distance between the inner radius and outer radius. Equation 3 is derived from using Equation 1 by taking the integral and applying initial conditions:

$$T(x) = \frac{T_2 - T_1}{L}x + T_1 \tag{4}$$

We can also solve Fourier's law in a similar way for a tube with heat generation at the inner radius as:

$$T(r) = \frac{T_i - T_o}{ln(\frac{r_i}{r_o})} \cdot ln(r) + \frac{(T_i - T_o)ln(r_i)}{ln(\frac{r_i}{r_o})} \cdot T_1 \tag{5}$$

These are the theoretical profiles that can be used to check the temperature distribution across the metals. This will be used to validate experimental findings by ensuring heat was fully traveling through the metal during the experiment.

Experimental methods:

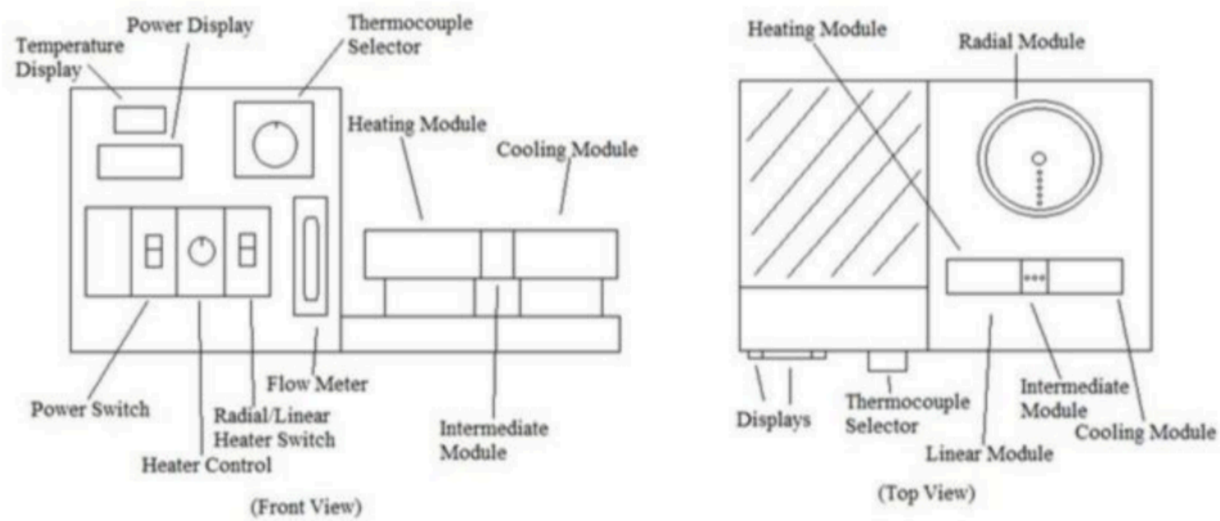


Figure 3: Heat conduction apparatus used for experiment with main components listed

For this experiment, no calibrations were needed or performed. Before beginning the experiment, measure the diameters of the three intermediate modules used: larger-diameter brass, small-diameter brass, and large-diameter stainless steel. The heat conduction apparatus consists of a linear and a radial module.

For the linear experiment, attach one of the intermediate sections in between the heater and cooling sections after applying conductive paste to the metal contact surfaces. Flip the switch to the linear heater setting and flip the power switch on. Using the heater control knob, set the power to 15W. Turn the knob of the water supply open so that the flow meter reads 23.7 GPH Water (1.5 L/min). Allow enough time for the temperatures to reach equilibrium before taking temperature readings. Make sure that the maximum temperature does not exceed 115 °C to prevent equipment damage. Use the thermocouple selector (OMEGA) to measure and record three temperatures (°C) for each section. Thermocouple channels 1, 2, and 3 correspond to the three temperature sensors of the heater section, and channels 5, 6, and 7 correspond to the cooling section. Attach a temperature probe (OMEGA) to the thermocouple labeled 04 on the heat conduction apparatus for measuring the three temperatures of the intermediate section. Apply the conductive paste to the probe tip before inserting it into the three holes on the intermediate section. Use channel 4 on the thermocouple selector for temperature readings of the intermediate section. Change the power wattage to 25W and repeat the above procedures for a total of 18 temperature readings. Change the intermediate section to an alternative one and repeat the procedure listed above to obtain the corresponding temperature readings for all the different materials and geometries.

For the radial experiment, flip the switch to the radial heater setting and increase the power to 30W. Once the temperatures reach equilibrium, record the temperatures from the thermocouple channels 8 through 12. Change the power wattage to 40W and repeat the above procedures for a total of 10 temperature readings. Upon completion of the experiment, flip the power switch off and turn off the water supply.

Results:

The results of the experiment found the thermal conductivity for brass and stainless steel as 134.56 W/m·K and 34.58 W/m·K, with error margins of 25.78% and 15.02%, respectively. Comparison with wattmeter readings for radial conduction at 30W and 40W showed errors of 0.667% and 1.59%. Additionally, the overall heat transfer coefficient U for a composite wall was determined to be approximately 787.573 W/m²·K. Each linear conduction test was powered under two different input loads: 15W and 25W. We began with the linear conduction setup using the large brass cylinder followed by the smaller brass cylinder, and the steel cylinder. We then observed radial heat conduction in the brass plate under two different input loads of 30W and 40W.

Figure 4 shows the system's temperature distribution relative to the heat travel distance. At the heat source origin, temperatures are 74.9°C for 25W input and 65.45°C for 15W input. At a distance of 40mm which is the start of the brass, the temperatures decrease to 33.6°C and 35.15°C for 25W and 15W, respectively. By 80mm, which is at the end of the brass cylinder, the temperatures further decline to 34.6°C for 25W and 31.05°C for 15W.

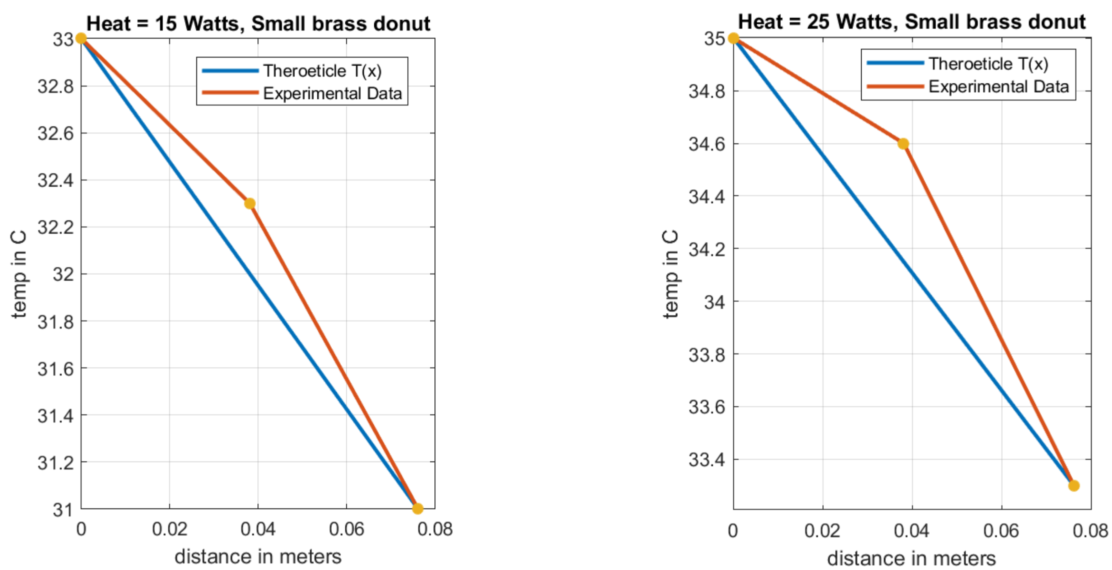


Figure 4: Heat distribution across the small brass cylinder

We repeated the experiment using a brass cylinder with a larger diameter. At the heat source origin, the system temperature was 98.7°C with a 25W input and 79.4°C with a 15W input. At 40mm from the origin, which is the start of the brass, the temperatures dropped to

47.8°C and 40.2°C for 25W and 15W, respectively. By 80mm, which is at the end of the brass cylinder, the temperatures further declined to 45.95°C for 25W and 38.2°C for 15W. When compared to the small brass donut, it is expected to see the ratio of the heat transfer rates under ideal conditions to be the ratio of areas. These results are summarized in Figure 5:

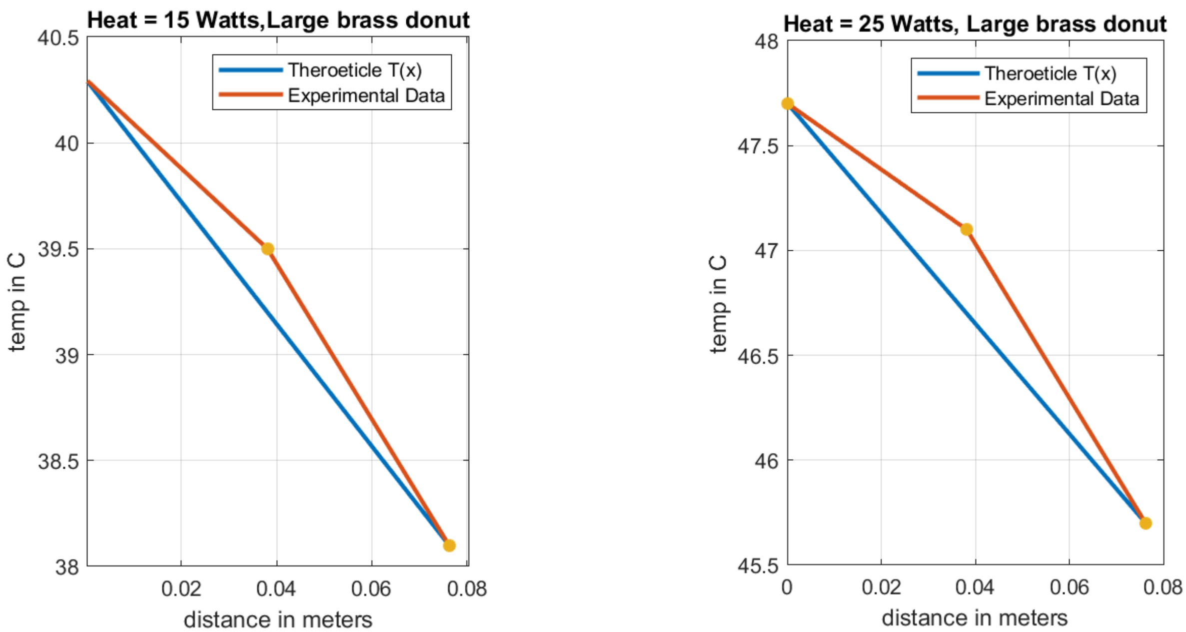


Figure 5: Heat distribution across the large brass cylinder

The same procedure was conducted using a steel cylinder, as shown in Figure 6. At the origin of the heat source, the temperature was 97.15°C with a 25W input and 84.9°C with a 15W input. At a distance of 40mm, which is the start of the steel cylinder, temperatures dropped to 62.55°C for 25W and 48.45°C for 15W. By 80mm, which is the end of the steel cylinder, the temperatures decreased further to 45.1°C and 40.95°C for 25W and 15W, respectively. This indicates that the steel was not transferring heat as efficiently as the brass donut.

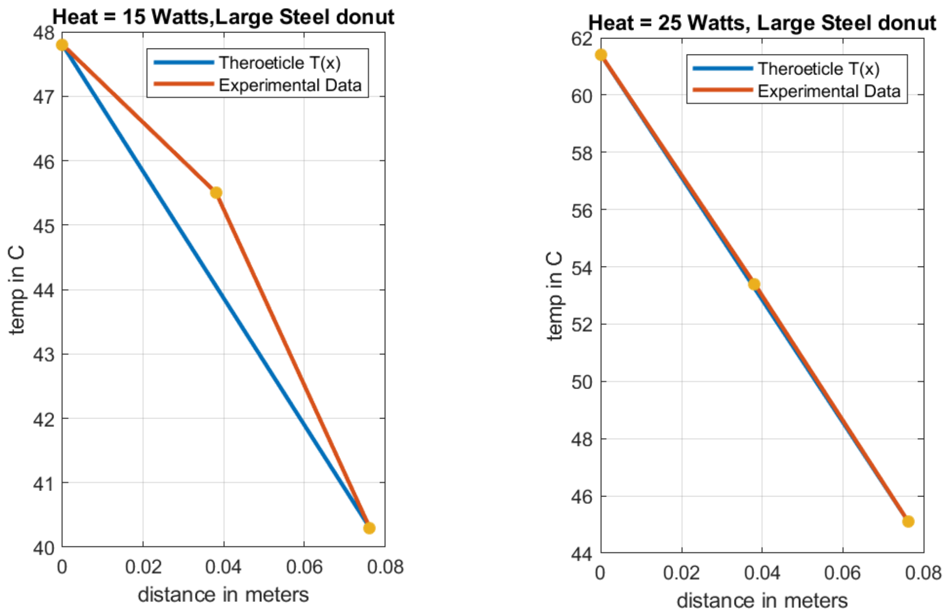


Figure 6: Heat distribution across the large steel cylinder

For the radial experiment, a logarithmic relationship between temperature and distance from the center was observed. For the 40-watt experiment, the innermost temperature was 39.7ºC, decreasing logarithmically to 24.3ºC at the outermost radius. Similarly, for the 30 Watt experiment the innermost temperature was measured to be 33.9ºC, decreasing logarithmically to 23.3C at the outermost radius These results are summarized in Figure 7:

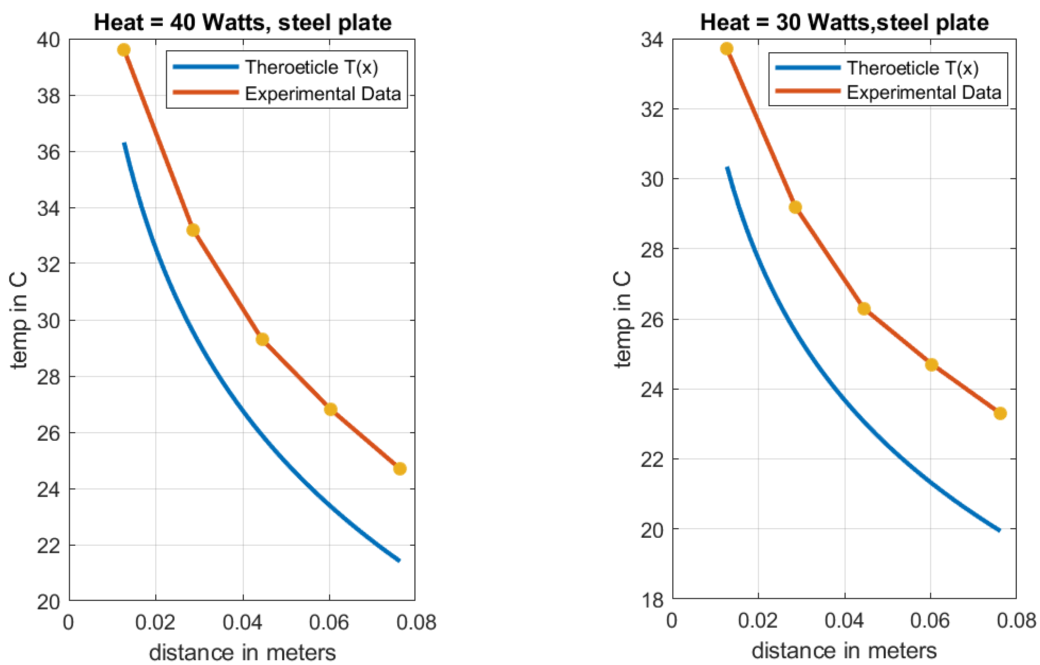


Figure 7: Radial temperature distribution across Brass* plate

Sources of uncertainty for this experiment include the calibration of thermocouples, which did not all read the same value, as well as the major assumption that heat transfer in the experiment was only occurring in a single direction.

Discussion:

The results of the experiment proved the validity of Fourier's Law of Conduction in linear and radial systems. The linear heat transfer experiments showed a linear relationship between temperature and distance given the limited data points. Likewise, the radial experiments showed a clear logarithmic relationship as predicted by the conduction law for radial systems. The hypothesis, that brass will have a higher thermal conductivity than stainless steel, was rendered valid by the results. Additionally, steady-state conditions were observed after a transient period for both linear and radial experiments as predicted by the hypothesis. These results implicate the validity of Fourier's law and also illustrate how the different thermal and geometric properties of metal objects can greatly impact their heat transfer capabilities.

While the results show a promising resemblance to the theoretical results predicted with Fourier's Law, this experiment had many sources of uncertainty that can explain the discrepancies between actual and predicted values. One major assumption leading to uncertainty is that heat transfer was occurring in a single direction within the metal cylinders. This approximation does not accurately represent the actual heat transfer occurring within the cylinder but is useful for simplifying theoretical calculations. Another source of uncertainty in the experiment is the actual contact between the donut, the heated cylinder and cooled cylinder. This contact was reinforced using thermal paste, but 100% conduction is only possible theoretically. The calibration of the thermocouples also resulted in experimental uncertainty as some of the thermocouples were reading different values than others, bringing into question their accuracy. When combined, these sources of uncertainty explain the discrepancy between the actual and theoretical values of thermal conductivities as well as the difference in temperature distributions.

Conclusion:

This experiment studied linear and radial heat conduction in an apparatus designed to reduce bi-directional heat flow by using insulation in order to reduce conduction and convection in undesired directions. By using brass and steel donuts to conduct heat, and measuring the corresponding temperature distributions, the thermal conductivities of brass and steel were found, with the thermal conductivity of brass being significantly higher. Additionally, this experiment allowed for the verification of Fourier's Law of conduction by creating temperature distributions in linear and radial systems. The linear system showed a linear temperature distribution while the radial showed a logarithmic relationship as predicted by Fourier's Law, indicating its validity.

Recommendations for improving the experiment would be to have a longer cylinder for the linear experiment so that more temperature measurements can be taken to find a more accurate temperature distribution. As data was collected for only 3 points on the actual cylinder, the analysis and calculation of the thermal conductivity were limited to the accuracy of these measurements. Likewise having a larger disk for the radial conduction experiment would result in a more logarithmic temperature distribution. Another recommendation would be to try several more metal cylinders with varying to observe the difference in both thermal conductivity and overall heat transfer rates.

Statement of Contributions:

Elijah Perez: Lab report (All), Data Analysis, Figures and Graphs, Experimental Design, Data collection

Soham Saha: Lab Report (Editing All, discussion, conclusion), Experimental Design

Alex Pham: Experimental Design, Data Collection, Conduction of Experiment

Appendix:

q = heat transfer rate

k = thermal conductivity

A = area of contact

T = temperature

r = radius

L = length of the disc in a radial system

U = heat transfer coefficient

Thermocouple	Temp ºC at 15 W	Temp ºC at 25W
1	46	54.1
2	45	52.9
3	44.7	52.5
4a	33	35
4b	32.3	34.6
4c	31	33.3
5	28	29.8
6	27.7	29.2
7	27.3	28.8
Slope (4)	-27.5	-21.25

***Table 1*:** Small brass Donut in Linear Heat conduction

Thermocouple	Temp ºC at 15 W	Temp ºC at 25W
1	52.5	65.8
2	50.3	62.4
3	48.5	59.8
4a	40.3	47.7
4b	39.5	47.1
4c	38.1	45.7
5	37.3	43.8
6	35.6	41.1
7	32.1	38.5

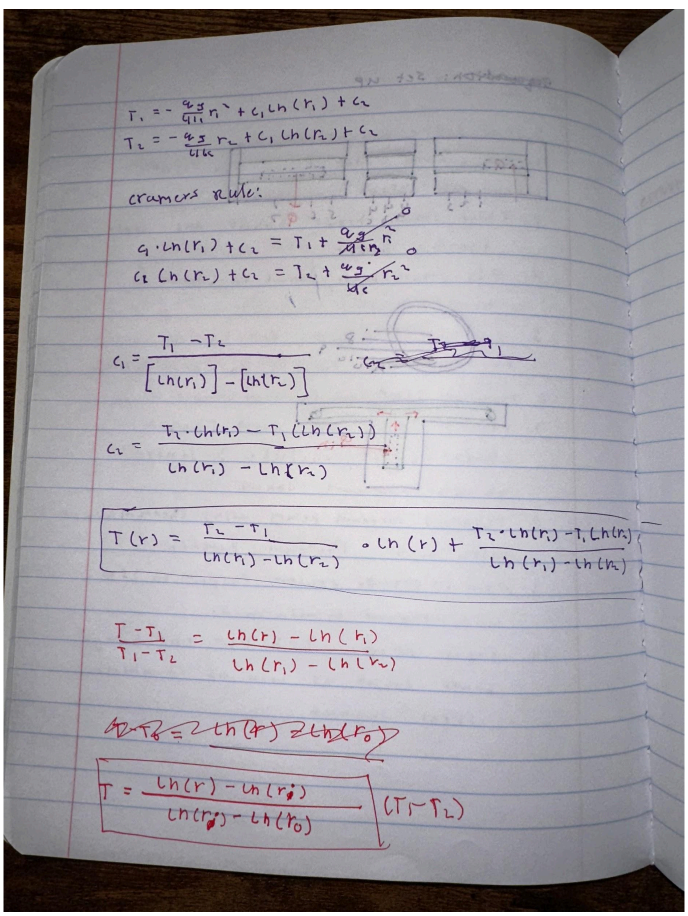
Table 2: Large Brass Donut in Linear Heat Conduction

Thermocouple	Temp ºC at 15 W	Temp ºC at 25W
1	72	85.2
2	70	82.6
3	68.7	81.1
4a	47.8	61.4
4b	45.5	53.4
4c	40.3	45.1
5	36.1	38.6
6	35.1	37.2
7	34.2	35.8

Table 3: Steel Donut in Linear Heat conduction

Thermocouple	Temp ºC at 30 W	Temp ºC at 40 W
8	33.7	39.6
9	29.2	33.2
10	26.3	29.3
11	24.7	26.8
12	23.3	24.7

Table 4: Radial Heat Conduction



$T_1 = -\frac{q_g}{4k} r_1^2 + c_1 \ln(r_1) + c_2$

$T_2 = -\frac{q_g}{4k} r_2 + c_1 \ln(r_2) + c_2$

cramers rule:

$c_1 \cdot \ln(r_1) + c_2 = T_1 + \cancel{\frac{q_g}{4k} r_1^2}$

$c_1 \ln(r_2) + c_2 = T_2 + \cancel{\frac{q_g}{4k} r_2^2}$

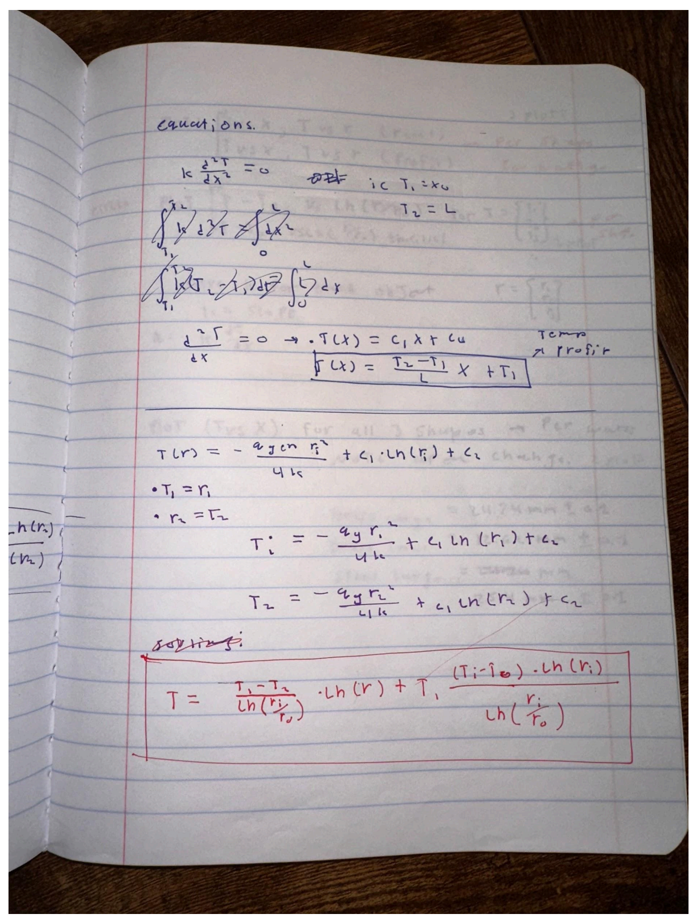
$c_1 = \frac{T_1 - T_2}{[\ln(r_1)] - [\ln(r_2)]}$

$c_2 = \frac{T_2 \cdot \ln(r_1) - T_1 (\ln(r_2))}{\ln(r_1) - \ln(r_2)}$

$\boxed{T(r) = \frac{T_2 - T_1}{\ln(r_1) - \ln(r_2)} \cdot \ln(r) + \frac{T_2 \cdot \ln(r_1) - T_1 \ln(r_2)}{\ln(r_1) - \ln(r_2)}}$

$\frac{T - T_1}{T_1 - T_2} = \frac{\ln(r) - \ln(r_1)}{\ln(r_1) - \ln(r_2)}$

$\boxed{T = \frac{\ln(r) - \ln(r_i)}{\ln(r_i) - \ln(r_0)}} (T_1 - T_2)$



equations.

$k \frac{d^2T}{dx^2} = 0$ $\quad$ ic $T_1 = x_0$

$T_2 = L$

$\frac{d^2T}{dx} = 0 \rightarrow \cdot T(x) = c_1 x + c_2$

$\boxed{T(x) = \frac{T_2 - T_1}{L} x + T_1}$ Temp profile

$T(r) = -\frac{q_{gen} r_i^2}{4k} + c_1 \cdot \ln(r_i) + c_2$

- $T_1 = r_1$
- $r_2 = T_2$

$T_i = -\frac{q_g r_1^2}{4k} + c_1 \ln(r_1) + c_2$

$T_2 = -\frac{q_g r_2^2}{4k} + c_1 \ln(r_2) + c_2$

~~solving:~~

$\boxed{T = \frac{T_1 - T_2}{\ln(\frac{r_i}{r_0})} \cdot \ln(r) + T_1 \frac{(T_i - T_0) \cdot \ln(r_i)}{\ln(\frac{r_i}{r_0})}}$