Project Part 1

1: (RK4) $\frac{dy}{dt} = f(t, y)$, IC, $y(\ ) = y_0$

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

- $k_1 = f(t_n, y_n)$
- $k_2 = f(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2})$
- $k_3 = f(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2})$
- $k_4 = f(t_n + h, y_n + h k_3)$

Part 2 Project

Q2: $ma = -F_{Drag} - F_{rolling} + F_{traction}$

$$m \cdot \frac{dV}{dt} = -\frac{1}{2} C_d \rho A V^2 - \mu_r mg + \mu_T \cdot mg$$

$$\frac{dV}{dt} = -\frac{1}{2m} C_d \rho A V^2 - \mu_r g + \mu_T g \quad (A) \quad \rightarrow$$

↓

ODE45

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$$\text{velocity}(t) + \frac{1}{2m} C_d \rho A V^2 + \mu_r g = F_T \quad (B)$$

MatLab: Tesla model S (2024)

- $C_d = 0.208$
- $A = (1.987 \cdot 1.431) \cdot 80\% = 2.27\ m^2$
- μ_T = Dry asfalt → 0.9
- μ_r = Typical for cars → 0.015
- $m = 1980$ kg
- $\rho = 1.293\ \frac{kg}{m^3}$
- $0 \rightarrow 60$ mph ; $0 \rightarrow 26.8224\ m/s$

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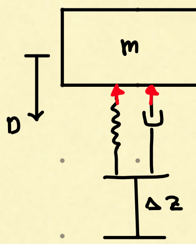
Project question 3



$$m\ddot{x} = -kx - c\dot{x} + F_{ext}$$

$$m\ddot{x} + c\dot{x} + kx = F_{ext}$$

$$\boxed{m\ddot{x} + c\dot{x} + kx = c\dot{z} + kz}$$

$$z(t) = z_0 e^{j\omega t}, \quad \dot{z}(t) = zj\omega e^{j\omega t}$$

$$m\ddot{x} + c\dot{x} + kx = c\left[zj\omega e^{j\omega t}\right] + k\left[z_0 e^{j\omega t}\right]$$

$$m\ddot{x} + c\dot{x} + kx = z_0 e^{j\omega t}\left(\left[c \cdot j\omega\right] + k\right)$$

- Let $x_p = Ae^{j\omega_0 t}$

$$\dot{x}_p = Aj\omega_0 e^{j\omega_0 t}$$

$$\ddot{x}_p = Aj^2\omega_0 e^{j\omega_0 t}$$

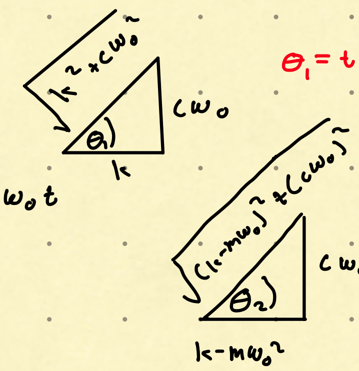
$$m\left[Aj^2\omega_0 \cancel{e^{j\omega_0 t}}\right] + c\left[Aj\omega_0 \cancel{e^{j\omega_0 t}}\right] + k\left[A\cancel{e^{j\omega_0 t}}\right] = z_0 \cancel{e^{j\omega t}}\left(\left[c \cdot j\omega\right] + k\right)$$

$$m\left[Aj^2\omega_0^2\right] + c\left[Aj\omega_0\right] + k\left[A\right] = z_0(cj\omega_0 + k)$$

$$A = \frac{z_0(cj\omega_0 + k)}{(k - m\omega_0^2) + cj\omega_0}$$

$$x(t) = \frac{z_0(cj\omega_0 + k)}{\underbrace{(k - m\omega_0^2)}_{x} + \underbrace{cj\omega_0}_{y}} \cdot e^{j\omega_0 t}$$

- Note, $x + jy = r \cdot e^{j\theta} \rightarrow x + jy = \sqrt{x^2 + y^2}\, e^{j\theta}$



$$\theta_1 = \tan^{-1}\left(\frac{c\omega_0}{k}\right)$$

$$\theta_2 = \tan^{-1}\left(\frac{c\omega_0}{k - m\omega_0^2}\right)$$

$$x(t) = \frac{z_0\sqrt{(c \cdot \omega_0)^2 + (k)^2} \cdot e^{j\theta_1}}{\sqrt{\left[(k - m\omega_0^2)\right]^2 + \left[(c \cdot \omega_0)\right]^2} \cdot e^{j\theta_2}} \cdot e^{j\omega_0 t}$$

$$x(t) = \frac{z_0\sqrt{(c \cdot \omega_0)^2 + (k)^2}}{\sqrt{\left[k - m\omega_0^2\right]^2 + \left[c \cdot \omega_0\right]^2}} \cdot e^{j\omega t + j\theta_1 - j\theta_2}$$

Phase

$$\phi = \omega t + \theta_1 - \theta_2$$

$$\downarrow$$

$$\phi = \omega t + \tan^{-1}\left(\frac{c\omega_0}{k}\right) - \tan^{-1}\left(\frac{c\omega_0}{k - m\omega_0^2}\right)$$

↓ amplitude

$$x(t) = \frac{z_0\sqrt{(c \cdot \omega_0)^2 + (k)^2}}{\sqrt{\left[k - m\omega_0^2\right]^2 + \left[c \cdot \omega_0\right]^2}}$$

Fourier series

$T = \frac{\lambda}{v_0}$, $\omega = \frac{2\pi v_0}{\lambda}$

$$a_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\,dt = \frac{v_0}{\lambda}\int_{-\frac{\lambda}{v_0 2}}^{\frac{\lambda}{v_0 2}} z_0 \cos(\omega t)\,dt$$

$$= \frac{2v_0}{\lambda}\int_0^{\frac{\lambda}{2v_0}} z_0 (\cos(\omega t)dt = \frac{2v_0}{\lambda}\left[z_0 v_0 \sin(\omega_0 t)\right]\Big|_0^{\frac{\lambda}{2v_0}}$$

$$= \frac{2v}{\lambda}\left[z_0 v_0 \sin(\overset{\frac{2\pi v_0}{\lambda}}{\omega_0}\,\frac{\lambda}{2v_0})\right] = 0 \qquad a_0 = 0$$

$$a_m = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos\left(\frac{m2\pi t}{T}\right)dt = \frac{2v_0 T}{\lambda}\int_{-\frac{\lambda}{v_0 2}}^{\frac{\lambda}{v_0 2}} z_0 \cos(\omega_0 t)\cos\left(\frac{m2\pi t \overset{\frac{\omega\lambda}{2\pi}}{v_0}}{\lambda}\right)db$$

$$= \frac{2v_0 T}{\lambda}\int_{-\frac{\lambda}{v_0 2}}^{\frac{\lambda}{v_0 2}} z_0 \cos(\omega_0 t)\cos(m\omega t)\,db = 0 \qquad \boxed{f(t) = 0}$$

• $f(t) = z_0|\cos(\omega_0 r)| = z_0|\cos(\omega_0 v t)|$ $\qquad T = \frac{\lambda}{v_0}$

$$a_0 = \frac{1}{T}\int_{\frac{T}{2}}^{\frac{T}{2}} f(t)\,dt \to \frac{2v_0}{\lambda}\int_0^{\frac{\lambda}{2v_0}} |z_0 \cos(\omega_0 v t)|\,dt = \frac{2v_0 z}{\lambda}\,\frac{\cos(\omega_0 v t)}{|\cos(\omega_0 v t)|}\int_0^{\frac{\lambda}{2v_0}} \cos(\omega v t)dt$$

$$a_0 = \frac{2v_0 z_0}{\lambda}\cdot\frac{\cos(\omega_0 v t)}{|\cos(\omega_0 v t)|}\cdot\frac{\sin(\omega_0 v t)}{v\omega_0}$$

$bm = 0$ - even function

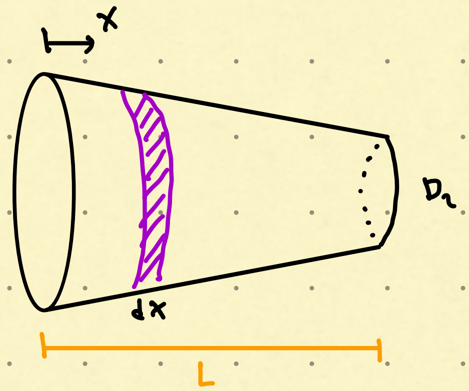
$$a_m = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cos\left(\frac{2\pi m t}{T}\right)db = \frac{2v_0}{\lambda}\int_{-\frac{\lambda}{2v_0}}^{\frac{\lambda}{2v_0}} z_0|\cos(\omega_0 v t)|\cdot\cos\left(\frac{2\pi v t}{T}\right)dt = 0$$

$bm = 0$ - since even

$$\boxed{f(t) = \frac{2v_0 z_0}{\lambda}\cdot\frac{\cos(2\omega_0 v t)}{|\cos(\omega_0 v t)|}\cdot\frac{\sin(\omega_0 v t)}{v\omega_0}}$$

Project Q4 Project

(A)



- $D_1 = 1.5$ cm
- $D_2 = 0.75$ cm
- $k = 200 \frac{W}{Mk}$
- $T_w = 150°c$
- $T_a = 20°C$
- $h = \frac{100\ W}{m^2 \cdot k}$

$A_s = 2\pi r \Delta x \rightarrow \pi D_i \Delta x$

$A_c = \pi r^2 \rightarrow \frac{\pi D_i^2}{4}$

Since D_i changes $\rightarrow D_1 + \frac{D_2 - D_1}{2} \cdot x$

- $\frac{-k\,dT}{dx} A_c = h(T - T_\infty)$

$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{cv}}{dt}$

$f = A_c h (T - T_\infty) + A_c k \frac{dT}{dx}$

$\dot{E}_{in} = (qA_c)_x \quad \dot{E}_{out} = (qA_c)_x + \Delta x$

so,

$(qA)_x - (qA_x) + \Delta x - A_s h (T - T_\infty) = \frac{d}{dt}(\rho A c_p T \Delta x)$

$- \frac{d}{dx}(qA\Delta x) - A_s h (T - T_\infty)$

$\frac{d}{dx}\left(-k\frac{dT}{dx} \cdot \frac{\pi}{4}(D^2(x))\Delta x\right) = A_s h (T - T_\infty)$

$\frac{d}{dx}\left(\frac{dT}{dx} D^2(x)\right) = \frac{4 D(x) h}{k}(T - T_\infty) - 2(D_1 + S_x) \cdot S_x \frac{dT}{dx}$

$\frac{d^2T}{dx^2} = \frac{4(h(T - T_\infty))}{k(D_1 + S_x)} - \frac{2 \cdot S_x \frac{dT}{dt}}{D_1 + S_x}$ $\rightarrow \frac{dT^2}{dx^2} = m^2(T - T_\infty)$

$; \frac{dT}{dx} = T_2$

- $T_\infty = 293k$
- $T_1 = T_w$

$$\frac{d^2T}{dx^2} = \frac{4(h(T_1 - T_\infty))}{k(D_1 + S_x)} - \frac{2 S_x T_2}{D_1 + S_x}$$