# Iterative methods for passing duals between modules

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# Introduction

This document outlines iterative methods for combining multiple optimization problems and exchange dual variables between them. This problem shows up when modules must exchange prices that are calculated through dual variables.

This issue can be addressed by reformulating the original problem and/or by using nonlinear solvers. This document outlines techniques that can be used when neither way is possible or desirable.

# Combining Optimization Problems and Exchanging Dual Variables through an Iterative Procedure

Without loss of generality, consider the following two optimization problems:

$$\begin{aligned}&\min_x f(x)\\ &s.t\\ &g(x) = 0 \quad (\alpha)\end{aligned}$$

$$\begin{aligned}&\min_y p(y,\alpha)\\ &s.t\\ &q(y,\alpha) = 0 \quad (\beta)\end{aligned}$$

Where the dual variable $\alpha$ is calculated from the first optimization problem and inserted as a parameter into the second optimization problem. Think of this, for example, as a price that is calculated as a dual to a market clearing condition from one module and inserted into another module. Note the arguments here can work for inequality constraints and multiple variable transfers and optimization problems, for simplicity, we only show it for two problems with equality constraints and a one-way data transfer.

The equilibrium solution to these optimization problems is given by taking KKT conditions and solving the resulting conditions:

$$\begin{aligned}&\nabla f(x) + \alpha \nabla g(x) = 0\\ &\nabla p(y,\alpha) + \beta \nabla q(y,\alpha) = 0\\ &g(x) = 0\\ &q(y,\alpha) = 0\end{aligned}$$

Consider the combined optimization problem, where $\alpha$ is set as a fixed parameter $\alpha_0$

$$\begin{aligned}&\min_{x,y} f(x) + p(y,\alpha_0)\\ &s.t\\ &g(x) = 0 \quad (\alpha)\\ &q(y,\alpha_0) = 0 \quad (\beta)\end{aligned}$$

The KKT conditions of this optimization problem are:

$$
\begin{aligned}
&\nabla f(x) + \alpha \nabla g(x) = 0 \\
&\nabla p(y, \alpha_0) + \beta \nabla q(y, \alpha_0) = 0 \\
&g(x) = 0 \\
&q(y, \alpha_0) = 0
\end{aligned}
$$

Comparing the KKT conditions of the equilibrium solution with the KKT conditions of the combined optimization problem, we see that if we find an $\alpha = \alpha_0$ then the equilibrium solution will be equivalent to the solution of the combined optimization problem. Thus, if we use any iterative method to calculate $\alpha$ and update $\alpha_0$ then once both values coincide, we will end up with the equilibrium solution.