

# 1 Surroundings

## 1.1 setup

```

1 測機 (test on C++ and Python)
2 AC : 好好寫
3 WA : cout << "0\n" / 結尾多印一行;
4 RE : 空間越界/除0
5 TLE : while(true);
6 CE : empty code
7 OLE : 瘋狂Hello World
8 NO Output : default code
9 待測 : stack深度、judge速度、陣列MAX
10 開賽
11 1. bash.rc打ac
12 2. 調gedit設定
13 3. 打default_code
14 4. 測試ac

```

## 1.2 bashrc

```

1 oj() {
2   ext=${1##*.}           #空格敏感
3   filename=${1##*/}      #空格敏感
4   filename=${filename%.*} #空格敏感
5   case $ext in
6     cpp ) g++ -o "/tmp/$filename" "$1" && "/tmp/$filename" ;;
7         #空格不敏感
8     py  ) python3 "$1" ;;
9         #空格不敏感
10
11   esac
12 }

```

## 1.3 vimrc

```

1 set tabstop=4
2 set shiftwidth=4
3 set softtabstop=4
4 set expandtab
5 set autoindent
6 set number

```

# 2 Data\_Structure

## 2.1 Sparse Table

```

1 // https://judge.yosupo.jp/problem/staticrmq 214 ms
2
3 template<typename T, int RANGE>
4 struct Sparse_Table {

```

```

5     struct Node {
6         T val;
7
8         Node(): val(INF) {}
9
10        Node operator +(const Node &rhs) {
11            Node ret;
12            ret.val = min(val, rhs.val);
13            return ret; // 視情況修改
14        }
15    };
16    vector<vector<Node>>> arr;
17
18    Sparse_Table() {
19        arr.resize(__lg(RANGE) + 1, vector<Node>(RANGE, Node()));
20    }
21
22    void build(auto &v) {
23        for (int i = 1; i <= n; i++) {
24            arr[0][i].val = v[i];
25        }
26        for (int i = 1; i <= __lg(n); i++)
27            for (int j = 1; j + (1 << (i - 1)) <= n; j++)
28                arr[i][j] = arr[i - 1][j] + arr[i - 1][j + (1 << (i - 1))];
29    }
30
31    Node query(int ql, int qr) {
32        int lg = __lg(qr - ql + 1);
33        return arr[lg][ql] + arr[lg][qr - (1 << lg) + 1];
34    }
35 };

```

## 2.2 Fenwick Tree

```

1 /** 普通 BIT ，為了加速打字只支援 1-based **/
2 const int MAXN = ? ; // 開全域加速打字
3 #define lowbit(x & (-x))
4
5
6 template<typename T>
7 struct Fenwick_Tree { // 1 based
8     // 二維：陣列開二維，修改跟查詢就是對 (x, y) 各自 +- lowbit
9     T arr[MAXN];
10    void init(int _n = MAXN) {
11        for (int i = 0; i < _n; i++)
12            arr[i] = 0;
13    }
14    void update(int i, T val) {
15        for (; i < MAXN; i += lowbit(i))
16            arr[i] += val;
17    }
18    T query(int i) {
19        T ret = 0;
20        for (; i != lowbit(i))
21            ret += arr[i];
22        return ret;
23    }
24 };

```

## 2.3 單點修改、區間查詢線段樹

```

1 // https://judge.yosupo.jp/problem/point_add_range_sum 331 ms
2 // https://judge.yosupo.jp/problem/staticrmq 359 ms
3 template<typename T, int RANGE>
4 struct Segment_Tree {
5     struct Node {
6         T val;
7
8         Node(): val(0) {} // mx: -INF, mn: INF, sum: 0, gcd: 1, lcm: 1
9
10        Node operator +(const Node &rhs) {
11            Node ret;
12            ret.val = val + rhs.val; // 對應不同操作修改
13
14            return ret;
15        }
16
17        void update(int _val) {
18            val += _val;
19        }
20    };
21
22    vector<Node> arr;
23
24    Segment_Tree() {
25        arr.resize(RANGE << 2);
26    }
27
28    void build(vector<int> &v, int i = 1, int l = 1, int r = n)
29    {
30        if (l == r) {
31            arr[i].val = v[l];
32            return;
33        }
34        int mid = (l + r) >> 1;
35        build(v, i << 1, l, mid);
36        build(v, i << 1 | 1, mid + 1, r);
37        arr[i] = arr[i << 1] + arr[i << 1 | 1];
38    }
39
40    void update(int pos, int val, int i = 1, int l = 1, int r = n)
41    {
42        if (l == r) {
43            arr[i].update(val);
44            return;
45        }
46        int mid = (l + r) >> 1;
47        if (pos <= mid) update(pos, val, i << 1, l, mid);
48        else update(pos, val, i << 1 | 1, mid + 1, r);
49        arr[i] = arr[i << 1] + arr[i << 1 | 1];
50    }
51
52    Node query(int ql, int qr, int i = 1, int l = 1, int r = n)
53    {
54        if (l > qr || r < ql)
55            return Node();
56        if (ql <= l && r <= qr)
57            return arr[i];
58        int mid = (l + r) >> 1;
59        return query(ql, qr, i << 1, l, mid) + query(ql, qr, i << 1 | 1, mid + 1, r);
60    }
61 };

```

## 2.4 最大區間和線段樹

```

1 /** 計算最大子區間連續和的線段樹，限定 1-based 。
2  * 複雜度 O(Q*log(N)) */
3 #define ls i << 1
4 #define rs i << 1 | 1
5 class MaxSumSegmentTree {
6     private:
7     struct node {
8         ll lss, rss, ss, ans;
9         void set(ll v) { lss = rss = ss = ans = v; }
10    };
11    int n;
12    vector<node> a; // 萬萬不可用普通陣列，要用 vector
13    vector<ll> z;
14    void pull(int i) {
15        a[i].ss = a[ls].ss + a[rs].ss;
16        a[i].lss = max(a[ls].lss, a[ls].ss + a[rs].lss);
17        a[i].rss = max(a[rs].rss, a[rs].ss + a[ls].rss);
18        a[i].ans = max(max(a[ls].ans, a[rs].ans),
19                       a[ls].rss + a[rs].lss);
20    }
21    void build(int i, int l, int r) {
22        if (l == r) return a[i].set(z[l]), void();
23        int m = (l + r) >> 1;
24        build(ls, l, m), build(rs, m + 1, r), pull(i);
25    }
26    void set(int i, int l, int r, int q, ll v) {
27        if (l == r) return a[i].set(v), void();
28        int m = (l + r) >> 1;
29        if (q <= m) set(ls, l, m, q, v);
30        else set(rs, m + 1, r, q, v);
31        pull(i);
32    }
33    node query(int i, int l, int r, int ql, int qr) {
34        if (ql <= l && r <= qr) return a[i];
35        int m = (l + r) >> 1;
36        if (qr <= m) return query(ls, l, m, ql, qr);
37        if (m < ql) return query(rs, m + 1, r, ql, qr);
38        node lo = query(ls, l, m, ql, qr),
39              ro = query(rs, m + 1, r, ql, qr), ans;
40        ans.ss = lo.ss + ro.ss;
41        ans.lss = max(lo.lss, lo.ss + ro.lss);
42        ans.rss = max(ro.rss, ro.ss + lo.rss);
43        ans.ans = max(max(lo.ans, ro.ans), lo.rss + ro.lss);
44        return ans;
45    }
46    public:
47    MaxSumSegmentTree(int n) : n(n) {
48        a.resize(n << 2), z.resize(n << 2);
49        build(1, 1, n);
50    }
51    // 單點設值。限定 1-based 。
52    inline void set(int i, ll v) { set(1, 1, n, i, v); }
53    // 問必區間 [l, r] 的最大子區間連續和。限定 1-based 。
54    inline ll query(int l, int r) {
55        return query(1, 1, n, l, r).ans;
56    }
57 }
58 };

```

## 2.5 懶標線段樹

```

1 struct Node {
2     int sum, tag;
3     Node(): sum(0), tag(0) {}
4
5     void update(int val, int l, int r) {
6         sum += (val) * (r - l + 1);
7         tag += val;
8     }
9     Node operator +(const Node rhs) {
10        Node ret;
11        ret.sum = sum + rhs.sum;
12        return ret;
13    }
14    void operator *=(const Node rhs) {
15        sum = rhs.sum;
16    }
17 };
18
19 template<typename T>
20 struct Segment_Tree {
21     vector<T> arr;
22
23     void init() {
24         arr.resize(MAXN << 2, Node());
25     }
26
27     void push(int i, int l, int r) {
28         if (l == r || arr[i].tag == 0)
29             return;
30         int mid = (l + r) / 2;
31         arr[i * 2].update(arr[i].tag, l, mid);
32         arr[i * 2 + 1].update(arr[i].tag, mid + 1, r);
33         arr[i].tag = 0;
34     }
35
36     void update(int ql, int qr, int val, int i = 1, int l = 1, int r = n) {
37         if (ql <= l && r <= qr) {
38             arr[i].update(val, l, r);
39             return;
40         }
41         if (l > qr || r < ql)
42             return;
43         int mid = (l + r) / 2;
44         push(i, l, r);
45         update(ql, qr, val, i * 2, l, mid);
46         update(ql, qr, val, i * 2 + 1, mid + 1, r);
47         arr[i].sum = (arr[i * 2] + arr[i * 2 + 1]).sum;
48     }
49
50     T query(int ql, int qr, int i = 1, int l = 1, int r = n) {
51         if (ql <= l && r <= qr)
52             return arr[i];
53         if (l > qr || r < ql)
54             return T();
55         push(i, l, r);
56         int mid = (l + r) / 2;
57         auto q1 = query(ql, qr, i * 2, l, mid);
58         auto q2 = query(ql, qr, i * 2 + 1, mid + 1, r);
59         return q1 + q2;
60     }
61 };

```

## 2.6 持久化線段樹

```

1 int a[maxn], b[maxn], root[maxn], cnt;
2 struct node {
3     int sum, L_son, R_son;
4 } tree[maxn << 5];
5 int create(int _sum, int _L_son, int _R_son) {
6     int idx = ++cnt;
7     tree[idx].sum = _sum, tree[idx].L_son = _L_son, tree[idx].R_son = _R_son;
8     return idx;
9 }
10 void Insert(int &root, int pre_rt, int pos, int L, int R) {
11     root = create(tree[pre_rt].sum+1, tree[pre_rt].L_son,
12                  tree[pre_rt].R_son);
13     if (L==R) return;
14     int M = (L+R)>>1;
15     if (pos<=M) Insert(tree[root].L_son, tree[pre_rt].L_son,
16                       pos, L, M);
17     else Insert(tree[root].R_son, tree[pre_rt].R_son, pos, M+1, R);
18 }
19
20 int query(int L_id, int R_id, int L, int R, int K) {
21     if (L==R) return L;
22     int M = (L+R)>>1;
23     int s = tree[tree[R_id].L_son].sum - tree[tree[L_id].L_son].sum;
24     if (K<=s) return query(tree[L_id].L_son, tree[R_id].L_son,
25                           L, M, K);
26     return query(tree[L_id].R_son, tree[R_id].R_son, M+1, R, K-s);
27 }
28
29 int main() {
30     int n, m; cin >> n >> m;
31     for (int i=1; i<=n; i++) {
32         cin >> a[i]; b[i] = a[i];
33     }
34     sort(b+1, b+1+n); // 離散化
35     int b_sz = unique(b+1, b+1+n) - (b+1);
36     cnt = root[0] = 0;
37     for (int i=1; i<=n; i++) {
38         int pos = lower_bound(b+1, b+1+b_sz, a[i]) - b;
39         Insert(root[i], root[i-1], pos, 1, b_sz);
40     }
41     while (m--) {
42         int l, r, k; cin >> l >> r >> k;
43         int pos = query(root[l-1], root[r], l, b_sz, k);
44         cout << b[pos] << endl;
45     }
46     return 0;
47 }

```

## 2.7 李超線段樹

```

1 template<typename T>
2 struct LiChao_SegTree {
3     T arr[MAXM << 2];
4
5     void init() {
6         for (int i = 0; i < (MAXM << 2); i++) {
7             arr[i] = {m, 0};
8         }
9     }
10 }

```

```

11 void insert(int i, int l, int r, T x) {
12     if (l == r) {
13         if (x(l) < arr[i](l)) {
14             arr[i] = x;
15         }
16         return;
17     }
18
19     if (arr[i].a > x.a) {
20         swap(arr[i], x);
21     }
22
23     int mid = (l + r) / 2;
24
25     if (x(mid) > arr[i](mid)) {
26         insert(i * 2, l, mid, x);
27     }
28     else {
29         swap(arr[i], x);
30         insert(i * 2 + 1, mid + 1, r, x);
31     }
32 }
33
34 int query(int i, int l, int r, int pos) {
35     if (l == r)
36         return arr[i](pos);
37     int mid = (l + r) / 2;
38     int res;
39     if (pos <= mid) {
40         res = query(i * 2, l, mid, pos);
41     }
42     else {
43         res = query(i * 2 + 1, mid + 1, r, pos);
44     }
45     return min(res, arr[i](pos));
46 }
47 };

```

## 2.8 Treap

```

1 // 支援區間加值、區間反轉、區間 rotate、區間刪除、插入元素、
  求區間
2 // 最小值的元素的 Treap。使用前建議 srand(time(0)); 除了 size
  ()
3 // 方法以外，所有操作都是 O(log N)。所有 public 方法各自獨
  立，請
4 // 斟酌要使用到哪些方法，有需要的才抄。
5 class Treap {
6     private:
7     struct Node {
8         int pri = rand(), size = 1;
9         ll val, mn, inc = 0;
10        bool rev = 0;
11        Node *lc = 0, *rc = 0;
12        Node(ll v) { val = mn = v; }
13    };
14    Node* root = 0;
15    void rev(Node* t) {
16        if (!t) return;
17        swap(t->lc, t->rc), t->rev ^= 1;
18    }
19    void update(Node* t, ll v) {

```

```

        if (!t) return;
        t->val += v, t->inc += v, t->mn += v;
    }
    void push(Node* t) {
        if (t->rev) rev(t->lc), rev(t->rc), t->rev = 0;
        update(t->lc, t->inc), update(t->rc, t->inc);
        t->inc = 0;
    }
    void pull(Node* t) {
        t->size = 1 + size(t->lc) + size(t->rc);
        t->mn = t->val;
        if (t->lc) t->mn = min(t->mn, t->lc->mn);
        if (t->rc) t->mn = min(t->mn, t->rc->mn);
    }
    // 看你要不要釋放記憶體
    void discard(Node* t) {
        if (!t) return;
        discard(t->lc), discard(t->rc);
        delete t;
    }
    void split(Node* t, Node*& a, Node*& b, int k) {
        if (!t) return a = b = 0, void();
        push(t);
        if (size(t->lc) < k) {
            a = t;
            split(t->rc, a->rc, b, k - size(t->lc) - 1);
            pull(a);
        } else {
            b = t;
            split(t->lc, a, b->lc, k);
            pull(b);
        }
    }
    Node* merge(Node* a, Node* b) {
        if (!a || !b) return a ? a : b;
        if (a->pri > b->pri) {
            push(a);
            a->rc = merge(a->rc, b);
            pull(a);
            return a;
        } else {
            push(b);
            b->lc = merge(a, b->lc);
            pull(b);
            return b;
        }
    }
    inline int size(Node* t) { return t ? t->size : 0; }
public:
    int size() { return size(root); }
    void add(int l, int r, ll val) {
        Node *a, *b, *c, *d;
        split(root, a, b, r);
        split(a, c, d, l - 1);
        update(d, val);
        root = merge(merge(c, d), b);
    }
    // 反轉區間 [l, r]
    void reverse(int l, int r) {
        Node *a, *b, *c, *d;
        split(root, a, b, r);
        split(a, c, d, l - 1);
        swap(d->lc, d->rc);
        d->rev ^= 1;
        root = merge(merge(c, d), b);
    }

```

```

    }
    // 區間 [l, r] 向右 rotate k 次，k < 0 表向左 rotate
    void rotate(int l, int r, int k) {
        int len = r - l + 1;
        Node *a, *b, *c, *d, *e, *f;
        split(root, a, b, r);
        split(a, c, d, l - 1);
        k = (k + len) % len;
        split(d, e, f, len - k);
        root = merge(merge(c, merge(f, e)), b);
    }
    // 插入一個元素 val 使其 index = i
    // 注意 i <= size
    void insert(int i, ll val) {
        if (i == size() + 1) {
            push_back(val);
            return;
        }
        assert(i <= size());
        Node *a, *b;
        split(root, a, b, i - 1);
        root = merge(merge(a, new Node(val)), b);
    }
    void push_back(ll val) {
        root = merge(root, new Node(val));
    }
    void remove(int l, int r) {
        int len = r - l + 1;
        Node *a, *b, *c, *d;
        split(root, a, b, l - 1);
        split(b, c, d, len);
        discard(c); // 看你要不要釋放記憶體
        root = merge(a, d);
    }
    ll minn(int l, int r) {
        Node *a, *b, *c, *d;
        split(root, a, b, r);
        split(a, c, d, l - 1);
        int ans = d->mn;
        root = merge(merge(c, d), b);
        return ans;
    }
};

```

## 2.9 Dynamic\_KD\_tree

```

1 template<typename T, size_t kd> // 有 kd 個維度
2 struct kd_tree {
3     struct point {
4         T d[kd];
5         T dist(const point &x) const {
6             T ret = 0;
7             for (size_t i = 0; i < kd; ++i) ret += abs(d[i] - x.d[i]);
8             return ret;
9         }
10    bool operator==(const point &p) {
11        for (size_t i = 0; i < kd; ++i)
12            if (d[i] != p.d[i]) return 0;
13        return 1;
14    }
15    bool operator<(const point &b) const {
16        return d[0] < b.d[0];

```

```

17     }
18 };
19 private:
20 struct node{
21     node *l,*r;
22     point pid;
23     int s;
24     node(const point &p):l(0),r(0),pid(p),s(1){}
25     ~node(){delete l,delete r;}
26     void up(){s=(l?l->s:0)+1+(r?r->s:0);}
27 }*root;
28 const double alpha,loga;
29 const T INF;//記得要給INF, 表示極大值
30 int maxn;
31 struct __cmp{
32     int sort_id;
33     bool operator()(const node*x,const node*y)const{
34         return operator()(x->pid,y->pid);
35     }
36     bool operator()(const point &x,const point &y)const{
37         if(x.d[sort_id]!=y.d[sort_id])
38             return x.d[sort_id]<y.d[sort_id];
39         for(size_t i=0;i<kd;++i)
40             if(x.d[i]!=y.d[i])return x.d[i]<y.d[i];
41         return 0;
42     }
43 }cmp;
44 int size(node *o){return o?o->s:0;}
45 vector<node*> A;
46 node* build(int k,int l,int r){
47     if(l>r) return 0;
48     if(k==kd) k=0;
49     int mid=(l+r)/2;
50     cmp.sort_id = k;
51     nth_element(A.begin()+l,A.begin()+mid,A.begin()+r+1,cmp);
52     node *ret=A[mid];
53     ret->l = build(k+1,l,mid-1);
54     ret->r = build(k+1,mid+1,r);
55     ret->up();
56     return ret;
57 }
58 bool isbad(node*o){
59     return size(o->l)>alpha*o->s||size(o->r)>alpha*o->s;
60 }
61 void flatten(node *u,typename vector<node*>::iterator &it){
62     if(!u)return;
63     flatten(u->l,it);
64     *it=u;
65     flatten(u->r,++it);
66 }
67 void rebuild(node*&u,int k){
68     if((int)A.size()<u->s)A.resize(u->s);
69     auto it=A.begin();
70     flatten(u,it);
71     u=build(k,0,u->s-1);
72 }
73 bool insert(node*&u,int k,const point &x,int dep){
74     if(!u) return u=new node(x), dep<=0;
75     ++u->s;
76     cmp.sort_id=k;
77     if(insert(cmp(x,u->pid)?u->l:u->r,(k+1)%kd,x,dep-1)){
78         if(!isbad(u))return 1;
79         rebuild(u,k);
80     }
81     return 0;

```

```

82 }
83 node *findmin(node*o,int k){
84     if(!o)return 0;
85     if(cmp.sort_id==k)return o->l?findmin(o->l,(k+1)%kd):o;
86     node *l=findmin(o->l,(k+1)%kd);
87     node *r=findmin(o->r,(k+1)%kd);
88     if(l&&!r)return cmp(l,o)?l:o;
89     if(!l&&r)return cmp(r,o)?r:o;
90     if(!l&&!r)return 0;
91     if(cmp(l,r))return cmp(l,o)?l:o;
92     return cmp(r,o)?r:o;
93 }
94 bool erase(node *&u,int k,const point &x){
95     if(!u)return 0;
96     if(u->pid==x){
97         if(u->r);
98         else if(u->l) u->r=u->l, u->l=0;
99         else return delete(u),u=0, 1;
100     }
101     --u->s;
102     cmp.sort_id=k;
103     u->pid=findmin(u->r,(k+1)%kd)->pid;
104     return erase(u->r,(k+1)%kd,u->pid);
105 }
106 cmp.sort_id=k;
107 if(erase(cmp(x,u->pid)?u->l:u->r,(k+1)%kd,x))
108     return --u->s, 1;
109 return 0;
110 }
111 T heuristic(const T h[])const{
112     T ret=0;
113     for(size_t i=0;i<kd;++i)ret+=h[i];
114     return ret;
115 }
116 int qM;
117 priority_queue<pair<T,point>> pQ;
118 void nearest(node *u,int k,const point &x,T *h,T &mndist){
119     if(u==0||heuristic(h)>=mndist)return;
120     T dist=u->pid.dist(x),old=h[k];
121     /*mndist=std::min(mndist,dist);*/
122     if(dist<mndist){
123         pQ.push(std::make_pair(dist,u->pid));
124         if((int)pQ.size()==qM+1)
125             mndist=pQ.top().first,pQ.pop();
126     }
127     if(x.d[k]<u->pid.d[k]){
128         nearest(u->l,(k+1)%kd,x,h,mndist);
129         h[k] = abs(x.d[k]-u->pid.d[k]);
130         nearest(u->r,(k+1)%kd,x,h,mndist);
131     }else{
132         nearest(u->r,(k+1)%kd,x,h,mndist);
133         h[k] = abs(x.d[k]-u->pid.d[k]);
134         nearest(u->l,(k+1)%kd,x,h,mndist);
135     }
136     h[k]=old;
137 }
138 vector<point>in_range;
139 void range(node *u,int k,const point&mi,const point&ma){
140     if(!u)return;
141     bool is=1;
142     for(int i=0;i<kd;++i)
143         if(u->pid.d[i]<mi.d[i]||ma.d[i]<u->pid.d[i])
144             { is=0;break; }
145     if(is) in_range.push_back(u->pid);
146     if(mi.d[k]<u->pid.d[k])range(u->l,(k+1)%kd,mi,ma);
147     if(ma.d[k]>u->pid.d[k])range(u->r,(k+1)%kd,mi,ma);

```

```

148 public:
149     kd_tree(const T &INF,double a=0.75):
150         root(0),alpha(a),loga(log2(1.0/a)),INF(INF),maxn(1){}
151     ~kd_tree(){delete root;}
152     void clear(){delete root,root=0,maxn=1;}
153     void build(int n,const point *p){
154         delete root,A.resize(maxn=n);
155         for(int i=0;i<n;++i)A[i]=new node(p[i]);
156         root=build(0,0,n-1);
157     }
158     void insert(const point &x){
159         insert(root,0,x,__lg(size(root))/loga);
160         if(root->s>maxn)maxn=root->s;
161     }
162     bool erase(const point &p){
163         bool d=erase(root,0,p);
164         if(root&&root->s<alpha*maxn)rebuild();
165         return d;
166     }
167     void rebuild(){
168         if(root)rebuild(root,0);
169         maxn=root->s;
170     }
171     T nearest(const point &x,int k){
172         qM=k;
173         T mndist=INF,h[kd]={};
174         nearest(root,0,x,h,mndist);
175         mndist=pQ.top().first;
176         pQ = priority_queue<pair<T,point>>();
177         return mndist;//回傳離x第k近的點的距離
178     }
179     const vector<point> &range(const point&mi,const point&ma){
180         in_range.clear();
181         range(root,0,mi,ma);
182         return in_range;//回傳介於mi到ma之間的點vector
183     }
184     int size(){return root?root->s:0;}
185 };

```

## 2.10 Heavy Light

```

1 #include<vector>
2 #define MAXN 100005
3 int siz[MAXN],max_son[MAXN],pa[MAXN],dep[MAXN];
4 int link_top[MAXN],link[MAXN],cnt;
5 vector<int> G[MAXN];
6 void find_max_son(int u){
7     siz[u]=1;
8     max_son[u]=-1;
9     for(auto v:G[u]){
10         if(v==pa[u])continue;
11         pa[v]=u;
12         dep[v]=dep[u]+1;
13         find_max_son(v);
14         if(max_son[u]==-1||siz[v]>siz[max_son[u]])max_son[u]=v;
15         siz[u]+=siz[v];
16     }
17 }
18 void build_link(int u,int top){
19     link[u]=++cnt;
20     link_top[u]=top;
21     if(max_son[u]==-1)return;

```

```

22 build_link(max_son[u],top);
23 for(auto v:G[u]){
24     if(v==max_son[u]||v==pa[u])continue;
25     build_link(v,v);
26 }
27 }
28 int find_lca(int a,int b){
29     //求LCA，可以在過程中對區間進行處理
30     int ta=link_top[a],tb=link_top[b];
31     while(ta!=tb){
32         if(dep[ta]<dep[tb]){
33             swap(ta,tb);
34             swap(a,b);
35         }
36         //這裡可以對a所在的鏈做區間處理
37         //區間為(link[ta],link[a])
38         ta=link_top[a=pa[ta]];
39     }
40     //最後a,b會在同一條鏈，若a!=b還要在進行一次區間處理
41     return dep[a]<dep[b]?a:b;
42 }

```

## 2.11 HLD By Koying

```

1 // https://cses.fi/problemset/task/1137/
2
3 struct HLD {
4     struct Info {
5         int sub, mxsub, dep, fa, root, id;
6     } arr[MAXN];
7
8     int index = 0;
9
10    void find_son(int i, int fa) {
11        pii mx(0, i);
12        arr[i].sub = 1;
13        for (auto it: G[i]) if (it != fa) {
14            arr[it].dep = arr[i].dep + 1;
15            arr[it].fa = i;
16            find_son(it, i);
17            cmax(mx, pii(arr[it].sub, it));
18            arr[i].sub += arr[it].sub;
19        }
20        arr[i].mxsub = mx.S;
21    }
22
23    void build(int i, int root) {
24        arr[i].root = root;
25        arr[i].id = ++index;
26        y[arr[i].id] = x[i];
27
28        if (arr[i].mxsub != i) {
29            build(arr[i].mxsub, root);
30            y[arr[i].id] += y[arr[arr[i].mxsub].id];
31        }
32
33        for (auto it: G[i]) if (it != arr[i].fa && it != arr[
34            i].mxsub) {
35            build(it, it);
36            y[arr[i].id] += y[arr[it].id];
37        }
38    }

```

```

38
39 void jump(int a, int b) { // from a to b (dep(a) > dep(b)
40     )
41     while (arr[a].root != arr[b].root) {
42         if (arr[arr[a].root].dep < arr[arr[b].root].dep)
43             a = arr[arr[a].root].fa;
44     }
45     if (arr[a].dep < arr[b].dep)
46         swap(a, b);
47
48     return mx;
49 }
50 } HLD;

```

## 2.12 Link Cut Tree

```

1 struct splay_tree{
2     int ch[2],pa; //子節點跟父母
3     bool rev; //反轉的懶惰標記
4     splay_tree():pa(0),rev(0){ch[0]=ch[1]=0;}
5 };
6 vector<splay_tree> nd;
7 //有的時候用vector會TLE，要注意
8 //這邊以node[0]作為null節點
9 bool isroot(int x){ //判斷是否為這棵splay tree的根
10     return nd[nd[x].pa].ch[0]!=x&&nd[nd[x].pa].ch[1]!=x;
11 }
12 void down(int x){ //懶惰標記下推
13     if(nd[x].rev){
14         if(nd[x].ch[0])nd[nd[x].ch[0]].rev^=1;
15         if(nd[x].ch[1])nd[nd[x].ch[1]].rev^=1;
16         swap(nd[x].ch[0],nd[x].ch[1]);
17         nd[x].rev=0;
18     }
19 }
20 void push_down(int x){ //所有祖先懶惰標記下推
21     if(!isroot(x))push_down(nd[x].pa);
22     down(x);
23 }
24 void up(int x){ //將子節點的資訊向上更新
25 void rotate(int x){ //旋轉，會自行判斷轉的方向
26     int y=nd[x].pa,z=nd[y].pa,d=(nd[y].ch[1]==x);
27     nd[x].pa=z;
28     if(!isroot(y))nd[z].ch[nd[z].ch[1]==y]=x;
29     nd[y].ch[d]=nd[x].ch[d^1];
30     nd[nd[y].ch[d]].pa=y;
31     nd[y].pa=x,nd[x].ch[d^1]=y;
32     up(y),up(x);
33 }
34 void splay(int x){ //將x伸展到splay tree的根
35     push_down(x);
36     while(!isroot(x)){
37         int y=nd[x].pa;
38         if(!isroot(y)){
39             int z=nd[y].pa;
40             if((nd[z].ch[0]==y)^(nd[y].ch[0]==x))rotate(y);
41             else rotate(x);
42         }
43         rotate(x);
44     }

```

```

45 }
46 int access(int x){
47     int last=0;
48     while(x){
49         splay(x);
50         nd[x].ch[1]=last;
51         up(x);
52         last=x;
53         x=nd[x].pa;
54     }
55     return last; //access後splay tree的根
56 }
57 void access(int x,bool is=0){ //is=0就是一般的access
58     int last=0;
59     while(x){
60         splay(x);
61         if(is&&nd[x].pa){
62             //printf("%d\n",max(nd[last].ma,nd[nd[x].ch[1]].ma));
63         }
64         nd[x].ch[1]=last;
65         up(x);
66         last=x;
67         x=nd[x].pa;
68     }
69 }
70 void query_edge(int u,int v){
71     access(u);
72     access(v,1);
73 }
74 void make_root(int x){
75     access(x),splay(x);
76     nd[x].rev^=1;
77 }
78 void make_root(int x){
79     nd[access(x)].rev^=1;
80     splay(x);
81 }
82 void cut(int x,int y){
83     make_root(x);
84     access(y);
85     splay(y);
86     nd[y].ch[0]=0;
87     nd[x].pa=0;
88 }
89 void cut_parents(int x){
90     access(x);
91     splay(x);
92     nd[nd[x].ch[0]].pa=0;
93     nd[x].ch[0]=0;
94 }
95 void link(int x,int y){
96     make_root(x);
97     nd[x].pa=y;
98 }
99 int find_root(int x){
100     x=access(x);
101     while(nd[x].ch[0])x=nd[x].ch[0];
102     splay(x);
103     return x;
104 }
105 int query(int u,int v){
106     //傳回uv路徑splay tree的根結點
107     //這種寫法無法求LCA
108     make_root(u);
109     return access(v);

```

```

110 }
111 int query_lca(int u,int v){
112 //假設求鏈上點權的總和，sum是子樹的權重和，data是節點的權重
113 access(u);
114 int lca=access(v);
115 splay(u);
116 if(u==lca){
117 //return nd[lca].data+nd[nd[lca].ch[1]].sum
118 }else{
119 //return nd[lca].data+nd[nd[lca].ch[1]].sum+nd[u].sum
120 }
121 }
122 struct EDGE{
123 int a,b,w;
124 }e[10005];
125 int n;
126 vector<pair<int,int>> G[10005];
127 //first表示子節點，second表示邊的編號
128 int pa[10005],edge_node[10005];
129 //pa是父母節點，暫存用的，edge_node是每個編被存在哪個點裡面的
    陣列
130 void bfs(int root){
131 //在建構的時候把每個點都設成一個splay tree
132 queue<int > q;
133 for(int i=1;i<=n;++i)pa[i]=0;
134 q.push(root);
135 while(q.size()){
136 int u=q.front();
137 q.pop();
138 for(auto P:G[u]){
139 int v=P.first;
140 if(v!=pa[u]){
141 pa[v]=u;
142 nd[v].pa=u;
143 nd[v].data=e[P.second].w;
144 edge_node[P.second]=v;
145 up(v);
146 q.push(v);
147 }
148 }
149 }
150 }
151 void change(int x,int b){
152 splay(x);
153 //nd[x].data=b;
154 up(x);
155 }

```

## 3 DP

### 3.1 LCIS

```

1 vector<int> LCIS(vector<int> a, vector<int> b) {
2     int n = a.size(), m = b.size();
3     int dp[LEN][LEN] = {}, pre[LEN][LEN] = {};
4     for(int i=1; i<=n; i++) {
5         int p = 0;
6         for(int j=1; j<=m; j++)
7             if(a[i-1]!=b[j-1]) {
8                 dp[i][j] = dp[i-1][j], pre[i][j] = j;

```

```

9         if( a[i-1]>b[j-1] && dp[i-1][j]>dp[i-1][p] )
10             p = j;
11     } else {
12         dp[i][j] = dp[i-1][p]+1, pre[i][j] = p;
13     }
14 }
15 int len = 0, p = 0;
16 for(int j=1; j<=m; j++)
17     if(dp[n][j]>len) len = dp[n][j], p = j;
18 vector<int> ans;
19 for(int i=n; i>=1; i--) {
20     if(a[i-1]==b[p-1]) ans.push_back(b[p-1]);
21     p = pre[i][p];
22 }
23 reverse(ans.begin(), ans.end());
24 return ans;
25 }

```

### 3.2 Bounded\_Knapsack

```

1 namespace {
2     static const int MAXW = 1000005;
3     static const int MAXN = 1005;
4     struct BB {
5         int w, v, c;
6         BB(int w = 0, int v = 0, int c = 0): w(w), v(v), c(c) {}
7     };
8     bool operator<(const BB &x) const {
9         return w * c < x.w * x.c;
10    };
11    static int run(BB A[], int dp[], int W, int N) {
12        static int MQ[MAXW][2];
13        for (int i = 0, sum = 0; i < N; i++) {
14            int w = A[i].w, v = A[i].v, c = A[i].c;
15            sum = min(sum + w*c, W);
16            for (int j = 0; j < w; j++) {
17                int l = 0, r = 0;
18                MQ[l][0] = 0, MQ[l][1] = dp[j];
19                for (int k = 1, tw = w+j, tv = v; tw <= sum
20                    && k <= c; k++, tw += w, tv += v) {
21                    int dpv = dp[tw] - tv;
22                    while (l <= r && MQ[r][1] <= dpv) r--;
23                    MQ[r][0] = k, MQ[r][1] = dpv;
24                    dp[tw] = max(dp[tw], MQ[l][1] + tv);
25                }
26                for (int k = c+1, tw = (c+1)*w+j, tv = (c+1)*
27                    v; tw <= sum; k++, tw += w, tv += v) {
28                    if (k - MQ[l][0] > c) l++;
29                    int dpv = dp[tw] - tv;
30                    while (l <= r && MQ[r][1] <= dpv) r--;
31                    MQ[r][0] = k, MQ[r][1] = dpv;
32                    dp[tw] = max(dp[tw], MQ[l][1] + tv);
33                }
34            }
35        }
36    }
37    static int knapsack(int C[][3], int N, int W) { // O(WN)
38        vector<BB> A;
39        for (int i = 0; i < N; i++) {
40            int w = C[i][0], v = C[i][1], c = C[i][2];

```

```

41         A.push_back(BB(w, v, c));
42     }
43     assert(N < MAXN);
44     static int dp1[MAXW+1], dp2[MAXW+1];
45     BB Ar[2][MAXN];
46     int ArN[2] = {};
47     memset(dp1, 0, sizeof(dp1[0])*(W+1));
48     memset(dp2, 0, sizeof(dp2[0])*(W+1));
49     sort(A.begin(), A.end());
50     int sum[2] = {};
51     for (int i = 0; i < N; i++) {
52         int ch = sum[1] < sum[0];
53         Ar[ch][ArN[ch]] = A[i];
54         ArN[ch]++;
55         sum[ch] = min(sum[ch] + A[i].w*A[i].c, W);
56     }
57     run(Ar[0], dp1, W, ArN[0]);
58     run(Ar[1], dp2, W, ArN[1]);
59     int ret = 0;
60     for (int i = 0, j = W, mx = 0; i <= W; i++, j--) {
61         mx = max(mx, dp2[i]);
62         ret = max(ret, dp1[j] + mx);
63     }
64     return ret;
65 }
66 }
67 int main() {
68     int W, N;
69     assert(scanf("%d %d", &W, &N) == 2);
70     int C[MAXN][3];
71     for (int i = 0; i < N; i++)
72         assert(scanf("%d %d %d", &C[i][1], &C[i][0], &C[i][2]) == 3);
73     printf("%d\n", knapsack(C, N, W));
74     return 0;
75 }

```

### 3.3 1D1D

```

1 int t, n, L, p;
2 char s[MAXN][35];
3 ll sum[MAXN] = {0};
4 long double dp[MAXN] = {0};
5 int prevd[MAXN] = {0};
6 long double pw(long double a, int n) {
7     if (n == 1) return a;
8     long double b = pw(a, n/2);
9     if (n & 1) return b*b*a;
10    else return b*b;
11 }
12 long double f(int i, int j) {
13     // cout << (sum[i] - sum[j]+i-j-1-L) << endl;
14     return pw(abs(sum[i] - sum[j]+i-j-1-L), p) + dp[j];
15 }
16 struct INV {
17     int L, R, pos;
18 };
19 INV stk[MAXN*10];
20 int top = 1, bot = 1;
21 void update(int i) {
22     while (top > bot && i < stk[top].L && f(stk[top].L, i) <
23         f(stk[top].L, stk[top].pos) ) {
24         stk[top - 1].R = stk[top].R;

```



```

24     top--;
25 }
26 int lo = stk[top].L, hi = stk[top].R, mid, pos = stk[top]
    ].pos;
27 // if ( i >= lo ) lo = i + 1;
28 while ( lo != hi ) {
29     mid = lo + (hi - lo) / 2;
30     if ( f(mid, i) < f(mid, pos) ) hi = mid;
31     else lo = mid + 1;
32 }
33 if ( hi < stk[top].R ) {
34     stk[top + 1] = (INV) { hi, stk[top].R, i };
35     stk[top++].R = hi;
36 }
37 }
38 int main() {
39     cin >> t;
40     while ( t-- ) {
41         cin >> n >> L >> p;
42         dp[0] = sum[0] = 0;
43         for ( int i = 1; i <= n; i++ ) {
44             cin >> s[i];
45             sum[i] = sum[i-1] + strlen(s[i]);
46             dp[i] = numeric_limits<long double>::max();
47         }
48         stk[top] = (INV) {1, n + 1, 0};
49         for ( int i = 1; i <= n; i++ ) {
50             if ( i >= stk[bot].R ) bot++;
51             dp[i] = f(i, stk[bot].pos);
52             update(i);
53             // cout << (ll) f(i, stk[bot].pos) << endl;
54         }
55         if ( dp[n] > 1e18 ) {
56             cout << "Too hard to arrange" << endl;
57         } else {
58             vector<PI> as;
59             cout << (ll)dp[n] << endl;
60         }
61     } return 0;
62 }

```

## 4 Graph

### 4.1 Dijkstra

```

1  /** 問某點到所有圖上的點的最短距離。0/1-based 都安全。 edge
    要
2  * 是 {cost, dest} 格式。回傳的陣列若含有 -1 表示 src 到該位
    置
3  * 不連通 **/
4  typedef pair<ll, int> pii;
5  vector<ll> dijkstra(int src, vector<vector<pii>>& edge) {
6      vector<ll> sum(edge.size(), -1);
7      priority_queue<pii, vector<pii>, greater<pii>> q;
8      q.emplace(0, src);
9      while (q.size()) {
10         int v = q.top().second; ll d = q.top().first;
11         q.pop();
12         if (sum[v] != -1) continue;
13         sum[v] = d;

```

```

14         for (auto& e : edge[v])
15             if (sum[e.second] == -1)
16                 q.emplace(d + e.first, e.second);
17         } return sum;
18     }

```

### 4.2 Bellman Ford

```

1  vector<pii> G[maxn];
2  int dis[maxn];
3  bool BellmanFord(int n, int s) {
4      for(int i=1; i<=n; i++) dis[i] = INF;
5      dis[s] = 0;
6      bool relax;
7      for(int r=1; r<=n; r++) { //O(VE)
8          relax = false;
9          for(int i=1; i<=n; i++)
10             for(pii e:G[i])
11                 if( dis[i] + e.second < dis[e.first] )
12                     dis[e.first] = dis[i] + e.second, relax =
                        true;
13         }
14         return relax; //有負環
15     }

```

### 4.3 SPFA

```

1  vector<pii> G[maxn]; int dis[maxn];
2  void SPFA(int n, int s) { //O(kE) k~2.
3      for(int i=1; i<=n; i++) dis[i] = INF;
4      dis[s] = 0;
5      queue<int> q; q.push(s);
6      bool inque[maxn] = {};
7      while(!q.empty()) {
8          int u = q.front(); q.pop();
9          inque[u] = false;
10         for(pii e:G[u]) {
11             int v = e.first, w = e.second;
12             if( dis[u] + w < dis[v] ) {
13                 if(!inque[v]) q.push(v), inque[v] = true;
14                 dis[v] = dis[u] + w;
15             }
16         }
17     }
18 }

```

### 4.4 Prim

```

1  /** 0/1-based 安全， n 是節點數量 (必須剛好) 。 edge 格式為
2  * {cost, dest}，回傳 -1 表示圖不連通。 **/
3  typedef pair<ll, int> pii;
4  ll minpath(vector<vector<pii>>& edge, int n) {
5      vector<bool> vis(n + 1);
6      priority_queue<pii, vector<pii>, greater<pii>> q;
7      q.emplace(0, 1);
8      ll ret = 0; int nvis = 0;

```

```

9      while (nvis < n && q.size()) {
10         ll d = q.top().first;
11         int v = q.top().second; q.pop();
12         if (vis[v]) continue;
13         vis[v] = 1; ret += d;
14         if (++nvis == n) return ret;
15         for (auto& e : edge[v])
16             if (!vis[e.second]) q.push(e);
17     } return -1;
18 }

```

### 4.5 Mahattan MST

```

1  #define REP(i,n) for(int i=0;i<n;i++)
2  typedef long long LL;
3  const int N=200100;
4  int n,m;
5  struct PT {int x,y,z,w,id;} p[N];
6  inline int dis(const PT &a,const PT &b){return abs(a.x-b.x)+
    abs(a.y-b.y);}
7  inline bool cpx(const PT &a,const PT &b)
8  {return a.x!=b.x? a.x>b.x:a.y>b.y;}
9  inline bool cpz(const PT &a,const PT &b){return a.z<b.z;}
10 struct E{int a,b,c;}e[8*N];
11 bool operator<(const E&a,const E&b){return a.c<b.c;}
12 struct Node{ int L,R,key; } node[4*N];
13 int s[N];
14 int F(int x) {return s[x]==x? x : s[x]=F(s[x]); }
15 void U(int a,int b) {s[F(b)]=F(a);}
16 void init(int id,int L,int R) {
17     node[id] = (Node){L,R,-1};
18     if(L==R)return;
19     init(id*2,L,(L+R)/2);
20     init(id*2+1,(L+R)/2+1,R);
21 }
22 void ins(int id,int x) {
23     if(node[id].key==-1 || p[node[id].key].w>p[x].w)
24         node[id].key=x;
25     if(node[id].L==node[id].R) return;
26     if(p[x].z<=(node[id].L+node[id].R)/2) ins(id*2,x);
27     else ins(id*2+1,x);
28 }
29 int Q(int id,int L,int R){
30     if(R<node[id].L || L>node[id].R)return -1;
31     if(L<=node[id].L && node[id].R<=R)return node[id].key;
32     int a=Q(id*2,L,R),b=Q(id*2+1,L,R);
33     if(b==-1 || (a!=-1 && p[a].w<p[b].w)) return a;
34     else return b;
35 }
36 void calc() {
37     REP(i,n) {
38         p[i].z = p[i].y-p[i].x;
39         p[i].w = p[i].x+p[i].y;
40     }
41     sort(p,p+n,cpz);
42     int cnt = 0, j, k;
43     for(int i=0; i<n; i=j){
44         for(j=i+1; p[j].z==p[i].z && j<n; j++);
45         for(k=i, cnt++; k<j; k++) p[k].z = cnt;
46     }
47     init(1,1,cnt);
48     sort(p,p+n,cpx);
49     REP(i,n) {

```

```

50     j=Q(1,p[i].z,cnt);
51     if(j!=-1) e[m++] = (E){p[i].id, p[j].id, dis(p[i],p[j]
52         )});
53     ins(1,i);
54 }
55 LL MST() {
56     LL r=0;
57     sort(e, e+m);
58     REP(i, m) {
59         if(F(e[i].a)==F(e[i].b)) continue;
60         U(e[i].a, e[i].b);
61         r += e[i].c;
62     }
63     return r;
64 }
65 int main() {
66     int ts;
67     scanf("%d", &ts);
68     while (ts--) {
69         m = 0;
70         scanf("%d",&n);
71         REP(i,n) {scanf("%d",&p[i].x,&p[i].y);p[i].id=s[i]=
72             i;}
73         calc();
74         REP(i,n)p[i].y= -p[i].y;
75         calc();
76         REP(i,n)swap(p[i].x,p[i].y);
77         calc();
78         REP(i,n)p[i].x=-p[i].x;
79         calc();
80         printf("%lld\n",MST()*2);
81     }
82     return 0;
83 }

```

## 4.6 LCA

```

1  /** 所有 LCA 都是 0/1-based 安全的。建構式 edge 表示 adj
2  * 邊資訊。 只支援無向樹。這三個類別各有優缺點。*/
3
4  /** 最快的 LCA  $O(N+Q)$ ，但非常吃記憶體  $O(N^2)$ 。支援非離線。*
5  */
6  class SsadjTarjan {
7  private:
8      int n;
9      vector<int> par, dep; vector<vector<int>> ca;
10     int dfs(int u, vector<vector<int>>& edge, int d) {
11         dep[u] = d;
12         for (int a = 0; a < n; a++)
13             if (dep[a] != -1)
14                 ca[a][u] = ca[u][a] = parent(a);
15         for (int a : edge[u]) {
16             if (dep[a] != -1) continue;
17             dfs(a, edge, d + 1);
18             par[a] = u;
19         }
20     }
21     int parent(int x) {
22         if (par[x] == x) return x;
23         return par[x] = parent(par[x]);
24     }
25 }

```

```

24 public:
25     SsadjTarjan(vector<vector<int>>& edge, int root)
26         : n(edge.size()) {
27         dep.assign(n, -1); par.resize(n);
28         ca.assign(n, vector<int>(n));
29
30         for (int i = 0; i < n; i++) par[i] = i;
31         dfs(root, edge, 0);
32     }
33     int lca(int a, int b) { return ca[a][b]; }
34     int dist(int a, int b) {
35         return dep[a] + dep[b] - 2 * dep[ca[a][b]];
36     }
37 }
38
39 /** 最快的 LCA  $O(N+Q)$  且最省記憶體  $O(N+Q)$ 。但必須離線。*/
40 #define x first // 加速
41 #define y second
42 class OfflineTarjan {
43 private:
44     vector<int> par, anc, dep, ans, rank;
45     vector<vector<pii>> qry;
46     // 出於安全考量你可以把 & 去掉
47     vector<vector<int>>& edge;
48     int root, n;
49
50     void merge(int a, int b) {
51         a = parent(a), b = parent(b);
52         if (rank[a] < rank[b]) swap(a, b);
53         par[b] = a;
54         if (rank[a] == rank[b]) rank[a]++;
55     }
56
57     void dfs(int u, int d) {
58         anc[parent(u)] = u, dep[u] = d;
59         for (int a : edge[u]) {
60             if (dep[a] != -1) continue;
61             dfs(a, d + 1);
62             merge(a, u);
63             anc[parent(u)] = u;
64         }
65         for (auto q : qry[u]) {
66             if (dep[q.first] != -1)
67                 ans[q.second] = anc[parent(q.first)];
68         }
69     }
70     int parent(int x) {
71         if (par[x] == x) return x;
72         return par[x] = parent(par[x]);
73     }
74
75     void solve(vector<pii>& query) {
76         dep.assign(n, -1), rank.assign(n, 0);
77         par.resize(n), anc.resize(n);
78         for (int i = 0; i < n; i++) anc[i] = par[i] = i;
79         ans.resize(query.size());
80         qry.resize(n);
81         for (int i = 0; i < query.size(); i++) {
82             auto& q = query[i];
83             qry[q.first].emplace_back(q.second, i);
84             qry[q.second].emplace_back(q.first, i);
85         }
86         dfs(root, 0);
87     }
88 public:

```

```

89     // edge 是傳 reference，完成所有查詢前萬萬不可以改。
90     OfflineTarjan(vector<vector<int>>& edge, int root)
91         : edge(edge), root(root), n(edge.size()) {}
92     // 離線查詢，query 陣列包含所有詢問 {src, dst}。呼叫一
93     // 次無
94     // 論 query 量多少，複雜度都是  $O(N)$ 。所以應盡量只呼叫一
95     // 次。
96     vector<int> lca(vector<pii>& query) {
97         solve(query);
98         return ans;
99     }
100     vector<int> dist(vector<pii>& query) {
101         solve(query);
102         for (int i = 0; i < query.size(); i++) {
103             auto& q = query[i];
104             ans[i] = dep[q.first] + dep[q.second] -
105                 2 * dep[ans[i]];
106         }
107         return ans;
108     }
109
110 /** 威達的 LCA，時間普通  $O(Q \log(N))$ ，記憶體需求也普通
111 *  $O(N \log(N))$ 。支援非離線。*/
112 class SparseTableTarjan {
113 private:
114     int maxlg;
115     vector<vector<int>> anc;
116     vector<int> dep;
117
118     void dfs(int u, vector<vector<int>>& edge, int d) {
119         dep[u] = d;
120         for (int i = 1; i < maxlg; i++)
121             if (anc[u][i - 1] == -1) break;
122             else anc[u][i] = anc[anc[u][i - 1]][i - 1];
123         for (int a : edge[u]) {
124             if (dep[a] != -1) continue;
125             anc[a][0] = u;
126             dfs(a, edge, d + 1);
127         }
128     }
129 public:
130     SparseTableTarjan(vector<vector<int>>& edge, int root) {
131         int n = edge.size();
132         maxlg = ceil(log2(n));
133         anc.assign(n, vector<int>(maxlg, -1));
134         dep.assign(n, -1);
135         dfs(root, edge, 0);
136     }
137     int lca(int a, int b) {
138         if (dep[a] > dep[b]) swap(a, b);
139         for (int k = 0; dep[b] - dep[a]; k++)
140             if (((dep[b] - dep[a]) >> k) & 1) b = anc[b][k];
141
142         if (a == b) return a;
143         for (int k = maxlg - 1; k >= 0; k--)
144             if (anc[a][k] != anc[b][k])
145                 a = anc[a][k], b = anc[b][k];
146         return anc[a][0];
147     }
148     int dist(int a, int b) {
149         return dep[a] + dep[b] - 2 * dep[lca(a, b)];
150     }
151 }

```



## 4.7 Tarjan

```

1 割點
2 點 u 為割點 if and only if 滿足 1. or 2.
3 1. u 為樹根，且 u 有多於一個子樹。
4 2. u 不為樹根，且滿足存在 (u,v) 為樹枝邊（或稱父子邊，即 u 為
   v 在搜索樹中的父親），使得 DFN(u) <= Low(v)。
5 -----
6 橋
7 一條無向邊 (u,v) 是橋 if and only if (u,v) 為樹枝邊，且滿足
   DFN(u) < Low(v)。
8 // 0 base
9 struct TarjanSCC{
10     static const int MAXN = 1000006;
11     int n, dfn[MAXN], low[MAXN], scc[MAXN], scn, count;
12     vector<int> G[MAXN];
13     stack<int> stk;
14     bool ins[MAXN];
15     void tarjan(int u) {
16         dfn[u] = low[u] = ++count;
17         stk.push(u);
18         ins[u] = true;
19         for(auto v:G[u]) {
20             if(!dfn[v]) {
21                 tarjan(v);
22                 low[u] = min(low[u], low[v]);
23             } else if(ins[v]) {
24                 low[u] = min(low[u], dfn[v]);
25             }
26         }
27         if(dfn[u] == low[u]) {
28             int v;
29             do {
30                 v = stk.top(); stk.pop();
31                 scc[v] = scn;
32                 ins[v] = false;
33             } while(v != u);
34             scn++;
35         }
36     }
37     void getSCC(){
38         memset(dfn,0,sizeof(dfn));
39         memset(low,0,sizeof(low));
40         memset(ins,0,sizeof(ins));
41         memset(scc,0,sizeof(scc));
42         count = scn = 0;
43         for(int i = 0; i < n; i++) {
44             if(!dfn[i]) tarjan(i);
45         }
46     } SCC;

```

## 4.8 BCC\_edge

```

1 邊雙連通
2 任意兩點間至少有兩條不重疊的路徑連接，找法：
3 1. 標記出所有的橋
4 2. 對全圖進行 DFS，不走橋，每一次 DFS 就是一個新的邊雙連通
5 // from BCW
6 struct BccEdge {
7     static const int MXN = 100005;

```

```

8     struct Edge { int v,eid; };
9     int n,m,step,par[MXN],dfn[MXN],low[MXN];
10    vector<Edge> E[MXN];
11    DisjointSet djs;
12    void init(int _n) {
13        n = _n; m = 0;
14        for (int i=0; i<n; i++) E[i].clear();
15        djs.init(n);
16    }
17    void add_edge(int u, int v) {
18        E[u].PB({v, m});
19        E[v].PB({u, m});
20        m++;
21    }
22    void DFS(int u, int f, int f_eid) {
23        par[u] = f;
24        dfn[u] = low[u] = step++;
25        for (auto it:E[u]) {
26            if (it.eid == f_eid) continue;
27            int v = it.v;
28            if (dfn[v] == -1) {
29                DFS(v, u, it.eid);
30                low[u] = min(low[u], low[v]);
31            } else {
32                low[u] = min(low[u], dfn[v]);
33            }
34        }
35    }
36    void solve() {
37        step = 0;
38        memset(dfn, -1, sizeof(int)*n);
39        for (int i=0; i<n; i++) {
40            if (dfn[i] == -1) DFS(i, i, -1);
41        }
42        djs.init(n);
43        for (int i=0; i<n; i++) {
44            if (low[i] < dfn[i]) djs.uni(i, par[i]);
45        }
46    }
47 } graph;

```

## 4.9 最小平均環

```

1 #include<cstdio> //for DBL_MAX
2 int dp[MAXN][MAXN]; // 1-base,0(NM)
3 vector<tuple<int,int,int>> edge;
4 double mmc(int n){ //allow negative weight
5     const int INF = 0x3f3f3f3f;
6     for(int t=0; t<n; ++t){
7         memset(dp[t+1],0x3f,sizeof(dp[t+1]));
8         for(const auto &e:edge) {
9             int u, v, w; tie(u,v,w) = e;
10            dp[t+1][v] = min(dp[t+1][v],dp[t][u]+w);
11        }
12    }
13    double res = DBL_MAX;
14    for(int u=1; u<=n; ++u) {
15        if(dp[n][u]==INF) continue;
16        double val = -DBL_MAX;
17        for(int t=0;t<n;++t)
18            val = max(val,(dp[n][u]-dp[t][u])*1.0/(n-t));
19        res = min(res,val);
20    } return res;

```

21 }

## 4.10 2-SAT

```

1 const int MAXN = 2020;
2 struct TwoSAT{
3     static const int MAXv = 2*MAXN;
4     vector<int> GO[MAXv],BK[MAXv],stk;
5     bool vis[MAXv];
6     int SC[MAXv];
7     void imply(int u,int v){ // u imply v
8         GO[u].push_back(v);
9         BK[v].push_back(u);
10    }
11    int dfs(int u,vector<int>*G,int sc){
12        vis[u]=1, SC[u]=sc;
13        for (int v:G[u])if (!vis[v])
14            dfs(v,G,sc);
15        if (G==GO) stk.push_back(u);
16    }
17    int scc(int n=MAXv){
18        memset(vis,0,sizeof(vis));
19        for (int i=0; i<n; i++)
20            if (!vis[i]) dfs(i,GO,-1);
21        memset(vis,0,sizeof(vis));
22        int sc=0;
23        while (!stk.empty()){
24            if (!vis[stk.back()])
25                dfs(stk.back(),BK,sc++);
26            stk.pop_back();
27        }
28    } SAT;
29    int main(){
30        SAT.scc(2*n);
31        bool ok = 1;
32        for (int i=0; i<n; i++){
33            if (SAT.SC[2*i]==SAT.SC[2*i+1]) ok = 0;
34        }
35        if (ok) {
36            for (int i=0; i<n; i++)
37                if (SAT.SC[2*i]>SAT.SC[2*i+1])
38                    cout << i << endl;
39        }
40        else puts("NO");
41    }
42 }
43 void warshall(){
44     bitset<2003> d[2003];
45     for (int k=0; k<n; k++)
46         for (int i=0; i<n; i++)
47             if (d[i][k]) d[i] |= d[k];
48 }

```

## 4.11 生成樹數量

```

1 // D : degree-matrix
2 // A : adjacent-matrix
3 // 無向圖
4 // (u,v)

```

```

5 // A[u][v]++, A[v][u]++
6 // D[u][u]++, D[v][v]++
7 // G = D-A
8 // abs(det(G去掉i-col和i-row))
9 // 生成樹的數量
10 // 有向圖
11 // A[u][v]++
12 // D[v][v]++ (in-deg)
13 // 以i為root的樹形圖數量
14 // 所有節點都能到達root

```

## 5 Flow\_Matching

### 5.1 Dinic

```

1 // 一般來說複雜度遠低於  $O(EV^2)$ ，二分圖約  $O(E * \sqrt{V})$ 。
2 // 0/1-based 都安全。
3 class Dinic {
4     struct edge {
5         int d, r; ll c;
6         edge(int d, ll c, int r) : d(d), c(c), r(r){};
7     };
8     private:
9     vector<vector<edge>> adj; vector<int> lv, ve; int n;
10     bool mklv(int s, int d) {
11         lv.assign(n, -1); lv[s] = 0;
12         queue<int> q; q.push(s);
13         while (!q.empty()) {
14             int v = q.front(); q.pop();
15             for (auto& e : adj[v]) {
16                 if (e.c == 0 || lv[e.d] != -1) continue;
17                 lv[e.d] = lv[v] + 1, q.push(e.d);
18             }
19         }
20         return lv[d] > 0;
21     }
22     ll aug(int v, ll f, int d) {
23         if (v == d) return f;
24         for (; ve[v] < adj[v].size(); ve[v]++) {
25             auto& e = adj[v][ve[v]];
26             if (lv[e.d] != lv[v] + 1 || !e.c) continue;
27             ll sent = aug(e.d, min(f, e.c), d);
28             if (sent > 0) {
29                 e.c -= sent, adj[e.d][e.r].c += sent;
30                 return sent;
31             }
32         }
33         return 0;
34     }
35     public:
36     // 建立空圖，n 是節點 (包含 source, sink) 數量
37     Dinic(int n) : n(n + 1) { clear(); }
38     // 清空整個圖，這需要重複使用 dinic 時 (如二分搜) 很方便
39     void clear() { adj.assign(n, vector<edge>()); }
40     // 加有向邊 src->dst，cap 是容量
41     void add_edge(int src, int dst, ll cap) {
42         edge ss(dst, cap, adj[dst].size());
43         edge dd(src, 0, adj[src].size());
44         adj[src].push_back(ss), adj[dst].push_back(dd);

```

```

45     }
46     ll max_flow(int s, int d) {
47         ll ret = 0;
48         while (mklv(s, d)) {
49             ve.assign(n, 0);
50             while (ll f = aug(s, 9e18, d)) ret += f;
51         }
52         return ret;
53     }
54 };

```

### 5.2 Min Cost Max Flow

```

1 /** Min cost max flow。0/1-based 都安全。 */
2 class MCMF {
3     private:
4     struct edge { int to, r; ll rest, c; };
5     int n; ll f = 0, c = 0;
6     vector<vector<edge>> g;
7     vector<int> pre, prel;
8     bool run(int s, int t) {
9         vector<ll> dis(n, INF); vector<bool> vis(n);
10        dis[s] = 0; queue<int> q; q.push(s);
11        while (q.size()) {
12            int u = q.front(); q.pop(); vis[u] = 0;
13            for (int i = 0; i < g[u].size(); i++) {
14                int v = g[u][i].to; ll w = g[u][i].c;
15                if (g[u][i].rest <= 0 ||
16                    dis[v] <= dis[u] + w)
17                    continue;
18                pre[v] = u, prel[v] = i;
19                dis[v] = dis[u] + w;
20                if (!vis[v]) vis[v] = 1, q.push(v);
21            }
22        }
23        if (dis[t] == INF) return 0;
24        ll tf = INF;
25        for (int v = t, u = 1; v != s; v = u) {
26            u = pre[v], l = prel[v];
27            tf = min(tf, g[u][l].rest);
28        }
29        for (int v = t, u = 1; v != s; v = u) {
30            u = pre[v], l = prel[v], g[u][l].rest -= tf;
31            g[v][g[u][l].r].rest += tf;
32        }
33        c += tf * dis[t], f += tf;
34        return 1;
35     }
36     public:
37     // 建立空圖，n 是節點數量 (包含 source 和 sink)
38     MCMF(int n) : n(n + 1), g(n + 1), pre(n + 1), prel(n + 1) {}
39     // 加有向邊 u->v，cap 容量 cost 成本
40     void add_edge(int u, int v, ll cap, ll cost) {
41         g[u].push_back({v, (int)g[v].size(), cap, cost});
42         g[v].push_back({u, (int)g[u].size() - 1, 0, -cost});
43     }
44     pair<ll, ll> query(int src, int sink) {
45         while (run(src, sink))
46             return {f, c}; // {min cost, max flow}
47     }
48 };

```

### 5.3 Ford Fulkerson

```

1 const int maxn = 1e5 + 10, INF = 1e9;
2 const long long INF64 = 1e18;
3 struct edge { int to, cap, rev; };
4 vector<edge> G[maxn];
5 int n, m, s, t, a, b, c;
6 bool vis[maxn];
7 int dfs(int v, int t, int f) {
8     cout << v << ' ' << t << ' ' << f << '\n';
9     if (v == t) return f;
10    vis[v] = true;
11    for (edge &e: G[v]) {
12        if (!vis[e.to] && e.cap > 0) {
13            int d = dfs(e.to, t, min(f, e.cap));
14            if (d > 0) {
15                e.cap -= d, G[e.to][e.rev].cap += d;
16                return d;
17            }
18        }
19    }
20    return 0;
21 }
22 int ford_fulkerson(int s, int t) {
23     int flow = 0, f;
24     for (int i = 0; i < n; i++) {
25         cout << i << " : ";
26         for (edge e: G[i])
27             cout << '(' << e.to << ', ' << e.cap << ')' << ' ';
28         cout << '\n';
29     }
30     do {
31         memset(vis, false, sizeof(vis));
32         f = dfs(s, t, INF);
33         for (int i = 0; i < n; i++) {
34             cout << i << " : ";
35             for (edge e: G[i])
36                 cout << '(' << e.to << ', ' << e.cap << ')' << ' ';
37             cout << '\n';
38         }
39         cout << f << '\n';
40         flow += f;
41     } while (f > 0);
42     return flow;
43 }
44 void init(int n) {
45     for (int i = 0; i < n; i++) G[i].clear();
46 }
47 int main() {
48     cin >> n >> m >> s >> t;
49     init(n);
50     while (m--) {
51         cin >> a >> b >> c;
52         G[a].push_back((edge){b, c, (int)G[b].size()});
53         G[b].push_back((edge){a, 0, (int)G[a].size() - 1});
54     }
55     cout << ford_fulkerson(s, t) << '\n';
56     return 0;
57 }

```

## 5.4 KM

```

1 /** 二分圖最大權值匹配 KM 演算法，複雜度  $O(n^3)$  */
2 #define inf 5e18
3 class KM {
4 private:
5     const vector<vector<ll>>& e;
6     int xx, yy;
7     vector<ll> cx, cy, wx, wy;
8     vector<bool> vx, vy;
9     ll z;
10
11     bool dfs(int u) {
12         vx[u] = 1;
13         for (int v = 0; v < yy; v++) {
14             if (vy[v] || e[u][v] == inf) continue;
15             ll t = wx[u] + wy[v] - e[u][v];
16             if (t == 0) {
17                 vy[v] = 1;
18                 if (cy[v] == -1 || dfs(cy[v])) {
19                     cx[u] = v, cy[v] = u;
20                     return 1;
21                 }
22             } else if (t > 0) {
23                 z = min(z, t);
24             }
25         }
26         return 0;
27     }
28 public:
29     // 問最大匹配權重。
30     ll max_weight() {
31         for (int i = 0; i < xx; i++)
32             for (int j = 0; j < yy; j++) {
33                 if (e[i][j] == inf) continue;
34                 wx[i] = max(wx[i], e[i][j]);
35             }
36         for (int i = 0; i < xx; i++) {
37             while (1) {
38                 z = inf, vx.assign(xx, 0), vy.assign(yy, 0);
39                 if (dfs(i)) break;
40                 for (int j = 0; j < xx; j++)
41                     if (vx[j]) wx[j] -= z;
42                 for (int j = 0; j < yy; j++)
43                     if (vy[j]) wy[j] += z;
44             }
45         }
46         ll ans = 0;
47         for (int i = 0; i < xx; i++)
48             if (cx[i] != -1) ans += e[i][cx[i]];
49         return ans;
50     }
51     // 給他  $n * m$  的權重表 ( $n \leq m$ )，求最大完全匹配權重，權重
52     // 可以
53     // 是負數。注意  $n > m$  會導致無窮迴圈。
54     KM(vector<vector<ll>>& e) : e(e) {
55         xx = e.size(), yy = e[0].size(); // xx 要  $\leq$  yy !!
56         cx.assign(xx, -1), cy.assign(yy, -1);
57         wx.assign(xx, 0), wy.assign(yy, 0);
58     }
59 };

```

## 5.5 Hopcroft Karp

```

1 int n, m, vis[maxn], level[maxn], pr[maxn], pr2[maxn];
2 vector<int> edge[maxn]; // for Left
3 bool dfs(int u) {
4     vis[u] = true;
5     for (vector<int>::iterator it = edge[u].begin();
6         it != edge[u].end(); ++it) {
7         int v = pr2[*it];
8         if (v == -1 ||
9             (!vis[v] && level[u] < level[v] && dfs(v))) {
10             pr[u] = *it, pr2[*it] = u;
11             return true;
12         }
13     }
14     return false;
15 }
16 int hopcroftKarp() {
17     memset(pr, -1, sizeof(pr));
18     memset(pr2, -1, sizeof(pr2));
19     for (int match = 0;;) {
20         queue<int> Q;
21         for (int i = 1; i <= n; ++i) {
22             if (pr[i] == -1) {
23                 level[i] = 0;
24                 Q.push(i);
25             } else
26                 level[i] = -1;
27         }
28         while (!Q.empty()) {
29             int u = Q.front();
30             Q.pop();
31             for (vector<int>::iterator it = edge[u].begin();
32                 it != edge[u].end(); ++it) {
33                 int v = pr2[*it];
34                 if (v != -1 && level[v] < 0) {
35                     level[v] = level[u] + 1;
36                     Q.push(v);
37                 }
38             }
39         }
40         for (int i = 1; i <= n; ++i) vis[i] = false;
41         int d = 0;
42         for (int i = 1; i <= n; ++i)
43             if (pr[i] == -1 && dfs(i)) ++d;
44         if (d == 0) return match;
45         match += d;
46     }
47 }

```

## 5.6 SW-MinCut

```

1 // all pair min cut
2 // global min cut
3 struct SW { //  $O(V^3)$ 
4     static const int MXN = 514;
5     int n, vst[MXN], del[MXN];
6     int edge[MXN][MXN], wei[MXN];
7     void init(int _n) {
8         n = _n; FZ(edge); FZ(del);
9     }
10    void addEdge(int u, int v, int w) {

```

```

11        edge[u][v] += w; edge[v][u] += w;
12    }
13    void search(int &s, int &t) {
14        FZ(vst); FZ(wei);
15        s = t = -1;
16        while (true) {
17            int mx = -1, cur = 0;
18            for (int i = 0; i < n; i++)
19                if (!del[i] && !vst[i] && mx < wei[i])
20                    cur = i, mx = wei[i];
21            if (mx == -1) break;
22            vst[cur] = 1;
23            s = t; t = cur;
24            for (int i = 0; i < n; i++)
25                if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
26        }
27    }
28    int solve() {
29        int res = 2147483647;
30        for (int i = 0, x, y; i < n-1; i++) {
31            search(x, y);
32            res = min(res, wei[y]);
33            del[y] = 1;
34            for (int j = 0; j < n; j++)
35                edge[x][j] = (edge[j][x] += edge[y][j]);
36        }
37        return res;
38    }
39 } graph;

```

## 5.7 Stable Marriage

```

1 // 演算法筆記
2 1. N位男士各自向自己最喜愛的女士求婚。
3 2. N位女士各自從自己的求婚者中，挑最喜愛的那位男士訂婚，但是
4     往後可背約。
5     沒有求婚者的女士，就只好等等。
6 3. 失敗的男士們，只好各自向自己次喜愛的女士求婚。
7 4. N位女士各自從自己的求婚者中，挑最喜歡的那位男士訂婚，但是
8     往後可背約。
9     已訂婚卻有更喜愛的男士求婚的女士，就毀約，改為與此男士訂
10    婚。
11    沒有求婚者的女士，就只好再等等。
12 5. 重複3. 4.直到形成N對伴侶為止。
13 // Jinkela
14 queue<int> Q;
15 for ( i : 所有考生 ) {
16     設定在第0志願;
17     Q.push(考生i);
18 }
19 while(Q.size()){
20     當前考生=Q.front();Q.pop();
21     while ( 此考生未分發 ) {
22         指標移到下一志願;
23         if ( 已經沒有志願 or 超出志願總數 ) break;
24         計算該考生在該科系加權後的總分;
25         if ( 不符科系需求 ) continue;
26         if ( 目前科系有餘額 ) {

```

```

24         依加權後分數高低順序將考生id加入科系錄取名單中;
25         break;
26     }
27     if ( 目前科系已額滿 ) {
28         if ( 此考生成績比最低分數還高 ) {
29             依加權後分數高低順序將考生id加入科系錄取名單;
30             Q.push(被踢出的考生);
31         }
32     }
33 }
34 }

```

## 6 Math

### 6.1 快速冪

```

1 const int P = 1e9 + 7;
2 #define ll long long
3 ll fpow(int a, int b) {
4     ll ret = 1;
5     while (b) {
6         if (b & 1)
7             ret = ret * a % P;
8         a = a * a % P;
9     }
10    return ret;
11 }

```

### 6.2 模逆元

```

1 // 解 (ax == 1) mod p 。p 必須是質數，a 是正整數。
2 ll modinv(ll a, ll p) {
3     if (p == 1) return 0;
4     ll pp = p, y = 0, x = 1;
5     while (a > 1) {
6         ll q = a / p, t = p;
7         p = a % p, a = t, t = y, y = x - q * y, x = t;
8     }
9     if (x < 0) x += pp;
10    return x;
11 }
12 // 解 (ax == b) mod p 。p 必須是質數，a 和 b 是正整數。
13 ll modinv(ll a, ll b, ll p) {
14     ll ret = modinv(a, p);
15     return ret * b % p;
16 }

```

### 6.3 離散根號

```

1 int order(ll b, ll p) {
2     if (__gcd(b, p) != 1) return -1;
3     int ret = 2;
4     while (++ret)
5         if (fastpow(b, ret, p) == 1) break;

```

```

6     return ret;
7 }
8 // 把 fastpow 也抄過來，會用到。
9 // 問 (x^2 = y) mod p 的解。回傳 -1 表示 x 無解。
10 ll dsqrt(ll y, ll p) {
11     if (__gcd(y, p) != 1) return -1;
12     if (fastpow(y, (p - 1) / 2, p) == p - 1) return -1;
13     int e = 0;
14     ll s = p - 1;
15     while (!(s & 1)) s >>= 1, e++;
16     int q = 2;
17     while (1)
18         if (fastpow(q, (p - 1) / 2, p) == p - 1)
19             break;
20     else q++;
21     ll x = fastpow(y, (s + 1) / 2, p);
22     ll b = fastpow(y, s, p);
23     ll g = fastpow(q, s, p);
24     while (1) {
25         int m;
26         for (m = 0; m < e; m++) {
27             int o = order(p, b);
28             if (o == -1) return -1;
29             if (o == fastpow(2, m, p)) break;
30         }
31         if (m == 0) return x;
32         x = x * fastpow(g, fastpow(2, e - m - 1, p) % p);
33         g = fastpow(g, fastpow(2, e - m, p), p);
34         b = b * g % p;
35         if (b == 1) return x;
36         e = m;
37     }
38 }

```

### 6.4 外星模運算

```

1 //a[0]^(a[1]^a[2]^...)
2 #define maxn 1000000
3 int euler[maxn+5];
4 bool is_prime[maxn+5];
5 void init_euler(){
6     is_prime[1] = 1; //一不是質數
7     for(int i=1; i<=maxn; i++) euler[i]=i;
8     for(int i=2; i<=maxn; i++) {
9         if(!is_prime[i]) { //是質數
10             euler[i]--;
11             for(int j=i<<1; j<=maxn; j+=i) {
12                 is_prime[j]=1;
13                 euler[j] = euler[j]/i*(i-1);
14             }
15         }
16     }
17 }
18 LL pow(LL a, LL b, LL mod) { //a^b%mod
19     LL ans=1;
20     for(; b; a=a*a%mod, b>>=1)
21         if(b&1) ans = ans*a%mod;
22     return ans;
23 }
24 bool isless(LL *a, int n, int k) {
25     if(*a==1)return k>1;
26     if(--n==0)return *a<k;

```

```

27     int next=0;
28     for(LL b=1;b<k;++next)
29         b *= *a;
30     return isless(a+1, n, next);
31 }
32 LL high_pow(LL *a, int n, LL mod){
33     if(*a==1||--n==0)return *a%mod;
34     int k = 0, r = euler[mod];
35     for(LL tma=1;tma!=pow(*a,k+r,mod);++k)
36         tma = tma*(*a)%mod;
37     if(isless(a+1,n,k))return pow(*a,high_pow(a+1,n,k),mod);
38     int tmd = high_pow(a+1,n,r), t = (tmd-k+r)%r;
39     return pow(*a,k+t,mod);
40 }
41 LL a[1000005]; int t,mod;
42 int main(){
43     init_euler();
44     scanf("%d", &t);
45     #define n 4
46     while(t--){
47         for(int i=0;i<n;++i)scanf("%lld", &a[i]);
48         scanf("%d", &mod);
49         printf("%lld\n", high_pow(a,n,mod));
50     }
51     return 0;
52 }

```

### 6.5 SG

```

1 Anti Nim (取走最後一個石子者敗) :
2 先手必勝 if and only if
3 1. 「所有」堆的石子數都為 1 且遊戲的 SG 值為 0。
4 2. 「有些」堆的石子數大於 1 且遊戲的 SG 值不為 0。
5 -----
6 Anti-SG (決策集合為空的遊戲者贏) :
7 定義 SG 值為 0 時，遊戲結束，
8 則先手必勝 if and only if
9 1. 遊戲中沒有單一遊戲的 SG 函數大於 1 且遊戲的 SG 函數為 0。
10 2. 遊戲中某個單一遊戲的 SG 函數大於 1 且遊戲的 SG 函數不為 0。
11 -----
12 Sprague-Grundy :
13 1. 雙人、回合制
14 2. 資訊完全公開
15 3. 無隨機因素
16 4. 可在有限步內結束
17 5. 沒有和局
18 6. 雙方可採取的行動相同
19
20 SG(S) 的值為 0：後手(P)必勝
21 不為 0：先手(N)必勝
22 int mex(set S) {
23     // find the min number >= 0 that not in the S
24     // e.g. S = {0, 1, 3, 4} mex(S) = 2
25 }
26 state = []
27 int SG(A) {
28     if (A not in state) {
29         S = sub_states(A)
30         if( len(S) > 1 ) state[A] = reduce(operator.xor, [SG(B)
31             for B in S])

```

```

31     else state[A] = mex(set(SG(B) for B in next_states(A)))
32 } return state[A]
33 }

```

## 6.6 Matrix

```

1 struct Matrix {
2     int r, c;
3     vector<vector<ll>> m;
4     Matrix(int r, int c): r(r), c(c), m(r, vector<ll>(c)) {}
5     vector<ll> &operator[](int i) { return m[i]; }
6     Matrix operator +(const Matrix &a) {
7         Matrix rev(r, c);
8         for (int i = 0; i < r; ++i)
9             for (int j = 0; j < c; ++j)
10                 rev[i][j] = m[i][j] + a.m[i][j];
11         return rev;
12     }
13     Matrix operator -(const Matrix &a) {
14         Matrix rev(r, c);
15         for (int i = 0; i < r; ++i)
16             for (int j = 0; j < c; ++j)
17                 rev[i][j] = m[i][j] - a.m[i][j];
18         return rev;
19     }
20     Matrix operator *(const Matrix &a) {
21         Matrix rev(r, a.c);
22         Matrix tmp(a.c, a.r);
23         for (int i = 0; i < a.r; ++i)
24             for (int j = 0; j < a.c; ++j)
25                 tmp[j][i] = a.m[i][j];
26         for (int i = 0; i < r; ++i)
27             for (int j = 0; j < a.c; ++j)
28                 for (int k = 0; k < c; ++k)
29                     rev.m[i][j] += m[i][k] * tmp[j][k];
30         return rev;
31     }
32     // 回傳反矩陣。注意這是 const 方法所以原矩陣不受影響。
33     Matrix inverse() const {
34         Matrix t(r, r + c);
35         for (int y = 0; y < r; y++) {
36             t.m[y][c + y] = 1;
37             for (int x = 0; x < c; x++) t.m[y][x] = m[y][x];
38         }
39         if (!t.gauss()) return Matrix(0, 0);
40         Matrix ret(c, r);
41         for (int y = 0; y < r; y++)
42             for (int x = 0; x < c; x++)
43                 ret[y][x] = t.m[y][c + x] / t.m[y][y];
44         return ret;
45     }
46     // 做高斯消去 (最高次係數應置於最左, 常數應置於最右) 並回傳 det
47     // 行列式值。複雜度  $O(n^3)$ 。如果不是方陣, 回傳值無意義。
48     ll gauss() {
49         vector<ll> lazy(r, 1);
50         bool sign = false;
51         for (int i = 0; i < r; ++i) {
52             if (m[i][i] == 0) {
53                 int j = i + 1;
54                 while (j < r && !m[j][i]) j++;
55                 if (j == r) continue;

```

```

56         m[i].swap(m[j]); sign = !sign;
57     }
58     for (int j = 0; j < r; ++j) {
59         if (i == j) continue;
60         lazy[j] = lazy[j] * m[i][i];
61         ll mx = m[j][i];
62         for (int k = 0; k < c; ++k)
63             m[j][k] =
64                 m[j][k] * m[i][i] - m[i][k] * mx;
65     }
66     ll det = sign ? -1 : 1;
67     for (int i = 0; i < r; ++i) {
68         det = det * m[i][i] / lazy[i];
69         for (auto &j : m[i]) j /= lazy[i];
70     }
71     return det;
72 }
73 };
74 };

```

## 6.7 Karatsuba

```

1 // N is power of 2
2 template<typename Iter>
3 void DC(int N, Iter tmp, Iter A, Iter B, Iter res){
4     fill(res, res+2*N, 0);
5     if (N<=32){
6         for (int i=0; i<N; i++)
7             for (int j=0; j<N; j++)
8                 res[i+j] += A[i]*B[j];
9         return;
10    }
11    int n = N/2;
12    auto a = A+n, b = A;
13    auto c = B+n, d = B;
14    DC(n, tmp+n, a, c, res+2*N);
15    for (int i=0; i<N; i++){
16        res[i+n] += res[2*N+i];
17        res[i+n] -= res[2*N+i];
18    }
19    DC(n, tmp+n, b, d, res+2*N);
20    for (int i=0; i<N; i++){
21        res[i] += res[2*N+i];
22        res[i+n] -= res[2*N+i];
23    }
24    auto x = tmp;
25    auto y = tmp+n;
26    for (int i=0; i<n; i++) x[i] = a[i]+b[i];
27    for (int i=0; i<n; i++) y[i] = c[i]+d[i];
28    DC(n, tmp+n, x, y, res+2*N);
29    for (int i=0; i<n; i++)
30        res[i+n] += res[2*N+i];
31 }
32 // DC(1<=16, tmp.begin(), A.begin(), B.begin(), res.begin());

```

## 6.8 Euler Function

```

1 // 查詢 phi(x) 亦即比 x 小且與 x 互質的數的數量。
2 int phi(int x) {
3     int r = x;

```

```

4     for (int p = 2; p * p <= x; p++) {
5         if (x % p == 0) {
6             while (x % p == 0) x /= p;
7             r -= r / p;
8         }
9     }
10    if (x > 1) r -= r / x;
11    return r;
12 }
13 // 查詢所有 phi(x), 且 x in [0, n)。注意右開區間, 回傳陣列。
14 vector<int> phi_in(int n) {
15     vector<bool> p(n, 1); vector<int> r(n);
16     p[0] = p[1] = 0;
17     for (int i = 0; i < n; i++) r[i] = i;
18     for (int i = 2; i < n; i++) {
19         if (!p[i]) continue;
20         r[i]--;
21         for (int j = i * 2; j < n; j += i)
22             p[j] = 0, r[j] = r[j] / i * (i - 1);
23     }
24     r[1] = 0;
25     return r;
26 }

```

## 6.9 Miller Rabin

```

1 typedef long long LL;
2 inline LL mul(LL a, LL b, LL m) { // a*b%m
3     return (a%m)*(b%m)%m;
4 }
5
6 template<typename T> bool isprime(T n, int num=3) { // num = 3,7
7     int sprp[3] = {2,7,61}; //int範圍可解
8     //int llsprp[7] = {2,325,9375,28178,450775,9780504,1795265022}; //至少
9     //unsigned long long範圍
10    if (n==2) return true;
11    if (n<2 || n%2==0) return false;
12    //n-1 = u * 2^t
13    int t = 0; T u = n-1;
14    while (u%2==0) u >>= 1, t++;
15    for (int i=0; i<num; i++) {
16        T a = sprp[i]%n;
17        if (a==0 || a==1 || a==n-1) continue;
18        T x = fpow(a, u, n);
19        if (x==1 || x==n-1) continue;
20        for (int j=1; j<t; j++) {
21            x = mul(x, x, n);
22            if (x==1) return false;
23            if (x==n-1) break;
24        }
25        if (x!=n-1) return false;
26    }
27    return true;
28 }

```

## 6.10 質因數分解

```

1 typedef __int128 ll;
2 vector<ll> vv;
3
4 /* fastoi here */
5
6 ll abs(ll x){
7     return (x>0?-x);
8 }
9 ll func(ll t,ll c,ll x) {
10    return (t*t+c)%x;
11 }
12 ll Pollard_Rho(ll x) {
13     ll t = 0;
14     ll c = rand() % (x - 1) + 1;
15     for (int i = 1; i < 1145; ++i) t = func(t, c, x);
16     ll s = t;
17     int step = 0, goal = 1;
18     ll val = 1;
19     for (goal = 1;; goal <= 1, s = t, val = 1) {
20         for (step = 1; step <= goal; ++step) {
21             t = func(t, c, x);
22             val = val * abs(t - s) % x;
23             if (!val) return x;
24             if (step % 127 == 0) {
25                 ll d = __gcd(val, x);
26                 if (d > 1) return d;
27             }
28         }
29         ll d = __gcd(val, x);
30         if (d > 1) return d;
31     }
32 }
33 void prefactor(ll &n, vector<ll> &v) {
34     ll prime[12] = {2,3,5,7,11,13,17,19,23,29,31,37};
35     for(int i=0;i<12;++i) {
36         while(n%prime[i]==0) {
37             v.push_back(prime[i]);
38             n/=prime[i];
39         }
40     }
41 }
42 void comfactor(const ll &n, vector<ll> &v) {
43     if(isPrime(n,15)) { // MillerRabin
44         v.push_back(n);
45         return;
46     }
47     ll d = Pollard_Rho(n);
48     comfactor(d,v);
49     comfactor(n/d,v);
50 }
51 void Factor(const ll &x, vector<ll> &v) {
52     ll n = x;
53     if(n==1) { puts("Factor 1"); return; }
54     prefactor(n,v);
55     if(n==1) return;
56     comfactor(n,v);
57     sort(v.begin(),v.end());
58 }
59 void AllFactor(const ll &n,vector<ll> &v) {
60     vector<ll> tmp;
61     Factor(n,tmp);
62     v.clear();
63     v.push_back(1);
64     ll len;
65     ll now=1;
66     ll lentmp = tmp.size();

```

```

67     for(int i=0;i<lentmp;++i) {
68         if(i==0 || tmp[i]!=tmp[i-1]) {
69             len = v.size();
70             now = 1;
71         }
72         now*=tmp[i];
73         for(int j=0;j<len;++j)
74             v.push_back(v[j]*now);
75     }
76 }
77 void prime_factorization(){
78     srand(time(NULL));
79     ll n = read();
80     AllFactor(n,vv);
81     sort(vv.begin(),vv.end());
82     for(auto i:vv){
83         print(i); putchar(' ');
84     }
85 }

```

## 6.11 質數

1	12721	13331	14341	75577
2	123457	222557	556679	880301
3	999983	1e6+99	1e9+9	2e9+99
4	1e12+39	1e15+37	1e9+7	1e7+19
5	1097774749	1076767633	100102021	
6	999997771	1001010013	1000512343	
7	987654361	999991231	999888733	
8	98789101	987777733	999991921	
9	1010101333	1010102101		
10	2305843009213693951		4611686018427387847	
11	9223372036854775783		18446744073709551557	

## 6.12 實根

```

1 // an*x^n + ... + a1x + a0 = 0;
2 int sign(double x){
3     return x < -eps ? -1 : x > eps;
4 }
5
6 double get(const vector<double>&coef, double x){
7     double e = 1, s = 0;
8     for(auto i : coef) s += i*e, e *= x;
9     return s;
10 }
11 double find(const vector<double>&coef, int n, double lo,
12             double hi){
13     double sign_lo, sign_hi;
14     if( !(sign_lo = sign(get(coef,lo))) ) return lo;
15     if( !(sign_hi = sign(get(coef,hi))) ) return hi;
16     if(sign_lo * sign_hi > 0) return INF;
17     for(int stp = 0; stp < 100 && hi - lo > eps; ++stp){
18         double m = (lo+hi)/2.0;
19         int sign_mid = sign(get(coef,m));
20         if(!sign_mid) return m;
21         if(sign_lo*sign_mid < 0) hi = m;
22         else lo = m;
23     }
24     return (lo+hi)/2.0;
25 }

```

```

24 vector<double> cal(vector<double>coef, int n){
25     vector<double>res;
26     if(n == 1){
27         if(sign(coef[1])) res.pb(-coef[0]/coef[1]);
28         return res;
29     }
30     vector<double>dcoef(n);
31     for(int i = 0; i < n; ++i) dcoef[i] = coef[i+1]*(i+1);
32     vector<double>droot = cal(dcoef, n-1);
33     droot.insert(droot.begin(), -INF);
34     droot.pb(INF);
35     for(int i = 0; i+1 < droot.size(); ++i){
36         double tmp = find(coef, n, droot[i], droot[i+1]);
37         if(tmp < INF) res.pb(tmp);
38     }
39     return res;
40 }
41 int main () {
42     vector<double>ve;
43     vector<double>ans = cal(ve, n);
44     // 視情況把答案 +eps, 避免 -0
45 }

```

## 6.13 FFT

```

1 template<typename T,typename VT=vector<complex<T> > >
2 struct FFT{
3     const T pi;
4     FFT(const T pi=acos((T)-1)):pi(pi){}
5     unsigned bit_reverse(unsigned a,int len){
6         a=((a&0x55555555U)<<1)|((a&0xAAAAAAAAU)>>1);
7         a=((a&0x33333333U)<<2)|((a&0xCCCCCCCCU)>>2);
8         a=((a&0x0F0F0F0FU)<<4)|((a&0xF0F0F0FU)>>4);
9         a=((a&0xFF0FF0FFU)<<8)|((a&0xFFFF0FFFU)>>8);
10        a=((a&0x0000FFFFU)<<16)|((a&0xFFFF0000U)>>16);
11        return a>>(32-len);
12    }
13    void fft(bool is_inv,VT &in,VT &out,int N){
14        int bitlen=__lg(N),num=is_inv?-1:1;
15        for(int i=0;i<N;++i) out[bit_reverse(i,bitlen)]=in[i];
16
17        for(int step=2; step<=N; step<=1){
18            const int mh = step>>1;
19            for(int i=0; i<mh; ++i){
20                complex<T> wi = exp(complex<T>(0,i*num*pi/mh));
21                for(int j=i; j<N; j+=step){
22                    int k = j+mh;
23                    complex<T> u = out[j], t = wi*out[k];
24                    out[j] = u+t;
25                    out[k] = u-t;
26                }
27            }
28        }
29        if(is_inv) for(int i=0;i<N;++i) out[i]/=N;
30    };
31 };

```

## 6.14 NTT



```

1 template<typename T,typename VT=std::vector<T> >
2 struct NTT{
3     const T P,G;
4     NTT(T p=(1<<23)*7*17+1,T g=3):P(p),G(g){}
5     inline unsigned int bit_reverse(unsigned int a,int len){
6         a=((a&0x55555555U)<<1)|((a&0xAAAAAAAAU)>>1);
7         a=((a&0x33333333U)<<2)|((a&0xCCCCCCCCU)>>2);
8         a=((a&0x0F0F0F0FU)<<4)|((a&0xF0F0F0F0U)>>4);
9         a=((a&0x00FF00FFU)<<8)|((a&0xFF00FF00U)>>8);
10        a=((a&0x0000FFFFU)<<16)|((a&0xFFFF0000U)>>16);
11        return a>>(32-len);
12    }
13    inline T pow_mod(T n,T k,T m){
14        T ans=1;
15        for(n=(n>=m?n%m:n);k;k>=1){
16            if(k&1)ans=ans*n%m;
17            n=n*n%m;
18        } return ans;
19    }
20    inline void ntt(bool is_inv,VT &in,VT &out,int N){
21        int bitlen=std::lg(N);
22        for(int i=0;i<N;++i)out[bit_reverse(i,bitlen)]=in[i];
23        for(int step=2,id=1;step<=N;step<=1,++id){
24            T wn=pow_mod(G,(P-1)>>id,P),wi=1,u,t;
25            const int mh=step>>1;
26            for(int i=0;i<mh;++i){
27                for(int j=i;j<N;j+=step){
28                    u = out[j], t = wi*out[j+mh]%P;
29                    out[j] = u+t;
30                    out[j+mh] = u-t;
31                    if(out[j]>=P)out[j]-=P;
32                    if(out[j+mh]<0)out[j+mh]+=P;
33                }
34                wi = wi*wn%P;
35            }
36        }
37        if(is_inv){
38            for(int i=1;i<N/2;++i)std::swap(out[i],out[N-i]);
39            T invn=pow_mod(N,P-2,P);
40            for(int i=0;i<N;++i)out[i]=out[i]*invn%P;
41        }
42    }
43 };
44 #endif

```

## 6.15 Simplex

```

1 /*target:
2   max \sum_{j=1}^n A_{0,j}*x_j
3 condition:
4   \sum_{j=1}^n A_{i,j}*x_j <= A_{i,0} | i=1~m
5   x_j >= 0 | j=1~n
6 VDB = vector<double>*/
7 template<class VDB>
8 VDB simplex(int m,int n,vector<VDB> a){
9     vector<int> left(m+1), up(n+1);
10    iota(left.begin(), left.end(), n);
11    iota(up.begin(), up.end(), 0);
12    auto pivot = [&](int x, int y){
13        swap(left[x], up[y]);
14        auto k = a[x][y]; a[x][y] = 1;
15        vector<int> pos;
16        for(int j = 0; j <= n; ++j){

```

```

17            a[x][j] /= k;
18            if(a[x][j] != 0) pos.push_back(j);
19        }
20        for(int i = 0; i <= m; ++i){
21            if(a[i][y]==0 || i == x) continue;
22            k = a[i][y], a[i][y] = 0;
23            for(int j : pos) a[i][j] -= k*a[x][j];
24        }
25    };
26    for(int x,y;;){
27        for(int i=x+1; i <= m; ++i)
28            if(a[i][0]<a[x][0]) x = i;
29        if(a[x][0]>=0) break;
30        for(int j=y+1; j <= n; ++j)
31            if(a[x][j]<a[x][y]) y = j;
32        if(a[x][y]>=0) return VDB(); //infeasible
33        pivot(x, y);
34    }
35    for(int x,y;;){
36        for(int j=y+1; j <= n; ++j)
37            if(a[0][j] > a[0][y]) y = j;
38        if(a[0][y]<=0) break;
39        x = -1;
40        for(int i=1; i<=m; ++i) if(a[i][y] > 0)
41            if(x == -1 || a[i][0]/a[i][y]
42               < a[x][0]/a[x][y]) x = i;
43        if(x == -1) return VDB(); //unbounded
44        pivot(x, y);
45    }
46    VDB ans(n + 1);
47    for(int i = 1; i <= m; ++i)
48        if(left[i] <= n) ans[left[i]] = a[i][0];
49    ans[0] = -a[0][0];
50    return ans;
51 }

```

## 6.16 Expression

```

1 /**
2  * 支援處理四則運算的工具。給四則運算的字串，檢查格式並計算其
3   值。如果
4  * 格式不合法，會丟出錯誤。複雜度 O(字串長度)。支援的符號有
5   四則運算
6  * 和求餘數，先乘除後加減。可以使用括號、或前置正負號。數字開
7   頭可以為
8  * 零或禁止為零。可以兼容或禁止多重前置號 (例如 --1 視為 1、
9   +-+1
10  * 視為 -1)。空字串視為不合法。運算範圍限於 long long。如果
11   試圖除
12  * 以零或對零求餘也會丟出錯誤。
13  */
14 void req(bool b) { if (!b) throw ""; }
15 const int B = 2; // 可以調整成 B 進位
16 class Expr {
17 private:
18     deque<char> src;
19     Expr(const string& s) : src(s.begin(), s.end()) {}
20     inline char top() {
21         return src.empty() ? '\0' : src.front();
22     }
23     inline char pop() {

```

```

19         char c = src.front(); src.pop_front(); return c;
20     }
21     ll n() {
22         ll ret = pop() - '0';
23         // 若要禁止數字以 0 開頭，加上這行
24         // req(ret || !isdigit(top()));
25         while (isdigit(top())) ret = B * ret + pop() - '0';
26         return ret;
27     }
28     ll fac() {
29         if (isdigit(top())) return n();
30         if (top() == '-') { pop(); return -fac(); }
31         if (top() == '(') {
32             pop();
33             ll ret = expr(1);
34             req(pop() == ')');
35             return ret;
36         }
37         // 若要允許前置正號，加上這行
38         // if(top() == '+') { pop(); return fac(); }
39         throw "";
40     }
41     ll term() {
42         ll ret = fac(); char c = top();
43         while (c == '*' || c == '/' || c == '%') {
44             pop();
45             if (c == '*') ret *= fac();
46             else {
47                 ll t = fac(); req(t);
48                 if (c == '/') ret /= t; else ret %= t;
49             }
50             c = top();
51         } return ret;
52     }
53     ll expr(bool k) {
54         ll ret = term();
55         while (top() == '+' || top() == '-')
56             if (pop() == '+') ret += term();
57             else ret -= term();
58         req(top() == (k ? ')' : '\0'));
59         return ret;
60     }
61 public:
62     // 給定數學運算的字串，求其值。若格式不合法，丟出錯誤。
63     static ll eval(const string& s) {
64         // 若要禁止多重前置號，加上這四行
65         // req(s.find("--") == -1); // 禁止多重負號
66         // req(s.find("-+") == -1);
67         // req(s.find("+-") == -1);
68         // req(s.find("++") == -1);
69         return Expr(s).expr(0);
70     }
71 };

```

## 6.17 Pick's Theorem

```

1 /* i:number of integer points interior to the polygon
2  b:the number of integer points on its boundary (including
3   both vertices and points along the sides).
4  Then the area A of this polygon is: A = i + b/2 - 1 */

```

```

5 pair<ll, ll> operator-(const pair<ll, ll>& a, const pair<ll,
    ll>& b) {
6     return {a.first - b.first, a.second - b.second};
7 }
8
9 int n;
10 pair<ll, ll> p[100010];
11
12 ll Pick() {
13     cin >> n;
14     for(int i = 0; i < n; ++i)
15         cin >> p[i].first >> p[i].second;
16     p[n] = p[0];
17     ll area = 0;
18     for(int i = 0; i < n; ++i)
19         area += p[i].first * p[i + 1].second - p[i].second * p[i
            + 1].first;
20     area = abs(area);
21     ll b = 0;
22     for(int i = 0; i < n; ++i) {
23         pair<ll, ll> v = p[i + 1] - p[i];
24         b += abs(__gcd(v.first, v.second));
25     }
26     ll a = (area + 2 - b) / 2;
27     return a;
28 }

```

## 6.18 擴展歐幾里德

```

1 // 給 a,b , 解 ax+by=gcd(a,b)
2 typedef pair<ll, ll> pii;
3 pii extgcd(ll a, ll b) {
4     if (b == 0) return {1, 0};
5     ll k = a / b;
6     pii p = extgcd(b, a - k * b);
7     return {p.second, p.first - k * p.second};
8 }

```

## 6.19 線性篩

```

1 int prime[MAXN];
2 vector<int> p;
3 void sieve(int n){
4     fill(prime+2, prime+n+1, 1);
5     for(int i=2; i<=n; ++i){
6         if(prime[i]==1) p.push_back(i);
7         for(int j:p){
8             if(i*j>n) break;
9             prime[i*j]=j; //順便紀錄最小的質因數是誰
10            if(i%j==0) break; //表示後面的質數都大於最小質因
                數了
11        }
12    }
13 }

```

## 7 String

### 7.1 Rolling Hash

```

1 // 問 pat 在 str 第一次出現的開頭 index 。 -1 表示找不到。
2 int rollhash(string& str, string& pat) {
3     const ll x = 1e6 + 99; // 隨意大質數，建議 1e6
4     const ll m = 1e9 + 9; // 隨意大質數，建議 1e9
5     assert(pat.size()); // pat 不能是空字串
6     ll xx = 1, sh = 0;
7     for (char c : pat)
8         sh = (sh * x + c) % m, xx = xx * x % m;
9     deque<ll> hash = {0};
10    int ret = 0;
11    for (char c : str) {
12        hash.push_back((hash.back() * x + c) % m);
13        if (hash.size() <= pat.size()) continue;
14        ll h = hash.back() - hash.front() * xx;
15        h = (h % m + m) % m;
16        if (h == sh) return ret;
17        hash.pop_front();
18        ret++;
19    }
20    return -1;
21 }

```

### 7.2 Trie

```

1 class Trie {
2 private:
3     struct Node {
4         int cnt = 0, sum = 0;
5         Node *tr[128] = {};
6         ~Node() {
7             for (int i = 0; i < 128; i++)
8                 if (tr[i]) delete tr[i];
9         }
10    };
11    Node *root;
12 public:
13    void insert(char *s) {
14        Node *ptr = root;
15        for (; *s; s++) {
16            if (!ptr->tr[*s]) ptr->tr[*s] = new Node();
17            ptr = ptr->tr[*s];
18            ptr->sum++;
19        }
20        ptr->cnt++;
21    }
22    inline int count(char *s) {
23        Node *ptr = find(s);
24        return ptr ? ptr->cnt : 0;
25    }
26    Node *find(char *s) {
27        Node *ptr = root;
28        for (; *s; s++) {
29            if (!ptr->tr[*s]) return 0;
30            ptr = ptr->tr[*s];
31        }
32        return ptr;
33    }
34    bool erase(char *s) {

```

```

34    Node *ptr = find(s);
35    if (!ptr) return false;
36    int num = ptr->cnt;
37    if (!num) return false;
38    ptr = root;
39    for (; *s; s++) {
40        Node *tmp = ptr;
41        ptr = ptr->tr[*s];
42        ptr->sum -= num;
43        if (!ptr->sum) {
44            delete ptr;
45            tmp->tr[*s] = 0;
46            return true;
47        }
48    }
49    }
50    Trie() { root = new Node(); }
51    ~Trie() { delete root; }
52 };

```

### 7.3 AC 自動機

```

1 template<char L='a',char R='z'>
2 class ac_automaton{
3     struct joe{
4         int next[R-L+1], fail, efl, ed, cnt_dp, vis;
5         joe():ed(0),cnt_dp(0),vis(0){
6             for(int i=0; i<=R-L; i++) next[i]=0;
7         }
8     };
9 public:
10    std::vector<joe> S;
11    std::vector<int> q;
12    int qs,qe,vt;
13    ac_automaton():S(1),qs(0),qe(0),vt(0){}
14    void clear(){
15        q.clear();
16        S.resize(1);
17        for(int i=0; i<=R-L; i++) S[0].next[i] = 0;
18        S[0].cnt_dp = S[0].vis = qs = qe = vt = 0;
19    }
20    void insert(const char *s){
21        int o = 0;
22        for(int i=0;id; s[i]; i++){
23            id = s[i]-L;
24            if(!S[o].next[id]){
25                S.push_back(joe());
26                S[o].next[id] = S.size()-1;
27            }
28            o = S[o].next[id];
29        }
30        ++S[o].ed;
31    }
32    void build_fail(){
33        S[0].fail = S[0].efl = -1;
34        q.clear();
35        q.push_back(0);
36        ++qe;
37        while(qs!=qe){
38            int pa = q[qs++], id, t;
39            for(int i=0; i<=R-L; i++){
40                t = S[pa].next[i];
41                if(!t) continue;

```

```

42     id = S[pa].fail;
43     while(~id && !S[id].next[i]) id = S[id].fail;
44     S[t].fail = ~id ? S[id].next[i] : 0;
45     S[t].efl = S[S[t].fail].ed ? S[t].fail : S[S[t].fail
46         ].efl;
47     q.push_back(t);
48     ++qe;
49 }
50 }
51 /*DP出每個前綴在字串s出現的次數並傳回所有字串被s匹配成功的
    次數O(N*M)*/
52 int match_0(const char *s){
53     int ans = 0, id, p = 0, i;
54     for(i=0; s[i]; i++){
55         id = S[i]-L;
56         while(!S[p].next[id] && p) p = S[p].fail;
57         if(!S[p].next[id])continue;
58         p = S[p].next[id];
59         ++S[p].cnt_dp; /*匹配成功則它所有後綴都可以被匹配(DP計算)*/
60     }
61     for(i=qe-1; i>=0; --i){
62         ans += S[q[i]].cnt_dp * S[q[i]].ed;
63         if(~S[q[i]].fail) S[S[q[i]].fail].cnt_dp += S[q[i]].
64             cnt_dp;
65     }
66     return ans;
67 }
68 /*多串匹配走efl邊並傳回所有字串被s匹配成功的次數O(N*M^1.5)
    */
69 int match_1(const char *s)const{
70     int ans = 0, id, p = 0, t;
71     for(int i=0; s[i]; i++){
72         id = S[i]-L;
73         while(!S[p].next[id] && p) p = S[p].fail;
74         if(!S[p].next[id])continue;
75         p = S[p].next[id];
76         if(S[p].ed) ans += S[p].ed;
77         for(t=S[p].efl; ~t; t=S[t].efl){
78             ans += S[t].ed; /*因為都走efl邊所以保證匹配成功*/
79         }
80     }
81     return ans;
82 }
83 /*枚舉(s的子字串⊆A)的所有相異字串各恰一次並傳回次數O(N*M
    ^{(1/3)})*/
84 int match_2(const char *s){
85     int ans=0, id, p=0, t;
86     ++vt;
87     /*把數記vt+=1, 只要vt沒溢位, 所有S[p].vis==vt就會變成
        false
88     這種利用vt的方法可以O(1)歸零vis陣列*/
89     for(int i=0; s[i]; i++){
90         id = S[i]-L;
91         while(!S[p].next[id]&&p) p = S[p].fail;
92         if(!S[p].next[id])continue;
93         p = S[p].next[id];
94         if(S[p].ed && S[p].vis!=vt){
95             S[p].vis = vt;
96             ans += S[p].ed;
97         }
98         for(t=S[p].efl; ~t && S[t].vis!=vt; t=S[t].efl){
99             S[t].vis = vt;

```

```

99         ans += S[t].ed; /*因為都走efl邊所以保證匹配成功*/
100     }
101 }
102 return ans;
103 }
104 /*把AC自動機變成真的自動機*/
105 void evolution(){
106     for(qs=1; qs!=qe;){
107         int p = q[qs++];
108         for(int i=0; i<=R-L; i++)
109             if(S[p].next[i]==0) S[p].next[i] = S[S[p].fail].next[
110                 i];
111     }
112 }

```

## 7.4 KMP

```

1 // KMP fail function.
2 int* kmp_fail(string& s) {
3     int* f = new int[s.size()]; int p = f[0] = -1;
4     for (int i = 1; s[i]; i++) {
5         while (p != -1 && s[p + 1] != s[i]) p = f[p];
6         if (s[p + 1] == s[i]) p++;
7         f[i] = p;
8     }
9     return f;
10 }
11 // 問 sub 在 str 中出現幾次。
12 int kmp_count(string& str, string& sub) {
13     int* fail = kmp_fail(sub); int p = -1, ret = 0;
14     for (int i = 0; i < str.size(); i++) {
15         while (p != -1 && sub[p + 1] != str[i]) p = fail[p];
16         if (sub[p + 1] == str[i]) p++;
17         if (p == sub.size() - 1) p = fail[p], ret++;
18     }
19     delete[] fail; return ret;
20 }
21 // 問 sub 在 str 第一次出現的開頭 index 。-1 表示找不到。
22 int kmp(string& str, string& sub) {
23     int* fail = kmp_fail(sub);
24     int i, j = 0;
25     while (i < str.size() && j < sub.size()) {
26         if (sub[j] == str[i]) i++, j++;
27         else if (j == 0) i++;
28         else j = fail[j - 1] + 1;
29     }
30     delete[] fail;
31     return j == sub.size() ? (i - j) : -1;
32 }

```

## 7.5 Z

```

1 void z_build(string &s, int *z) {
2     int bst = z[0] = 0;
3     for (int i = 1; s[i]; i++) {
4         if (z[bst] + bst < i) z[i] = 0;
5         else z[i] = min(z[bst] + bst - i, z[i - bst]);
6         while (s[z[i]] == s[i + z[i]]) z[i]++;

```

```

7         if (z[i] + i > z[bst] + bst) bst = i;
8     }
9 }
10 // Queries how many times s appears in t
11 int z_match(string &s, string &t) {
12     int ans = 0;
13     int lens = s.length(), lent = t.length();
14     int z[lens + lent + 5];
15     string st = s + "$" + t;
16     z_build(st, z);
17     for (int i = lens + 1; i <= lens + lent; i++)
18         if (z[i] == lens) ans++;
19     return ans;
20 }

```

## 7.6 BWT

```

1 const int N = 8; // 字串長度
2 int s[N+N+1] = "suffixes"; // 字串, 後面預留一倍空間。
3 int sa[N]; // 後綴陣列
4 int pivot;
5 cmp(const void* i, const void* j) {
6     return strcmp(s+*(int*)i, s+*(int*)j, N);
7 }
8 // 此處便宜行事, 採用 O(N^2logN) 的後綴陣列演算法。
9 void BWT() {
10     strncpy(s + N, s, N);
11     for (int i=0; i<N; ++i) sa[i] = i;
12     qsort(sa, N, sizeof(int), cmp);
13     // 當輸入字串的所有字元都相同, 必須當作特例處理。
14     // 或者改用stable sort。
15     for (int i=0; i<N; ++i)
16         cout << s[(sa[i] + N-1) % N];
17     for (int i=0; i<N; ++i)
18         if (sa[i] == 0) {
19             pivot = i;
20             break;
21         }
22 }
23 // Inverse BWT
24 const int N = 8; // 字串長度
25 char t[N+1] = "xuffessi"; // 字串
26 int pivot;
27 int next[N];
28 void IBWT() {
29     vector<int> index[256];
30     for (int i=0; i<N; ++i)
31         index[t[i]].push_back(i);
32     for (int i=0, n=0; i<256; ++i)
33         for (int j=0; j<index[i].size(); ++j)
34             next[n++] = index[i][j];
35     int p = pivot;
36     for (int i=0; i<N; ++i)
37         cout << t[p = next[p]];
38 }

```

## 7.7 Suffix\_Array\_LCP

```

1 #define radix_sort(x,y){
2   for(i=0;i<A;++i) c[i] = 0;
3   for(i=0;i<n;++i) c[x[y[i]]]++;
4   for(i=1;i<A;++i) c[i] += c[i-1];
5   for(i=n-1;~i;--i) sa[--c[x[y[i]]]] = y[i];
6 }
7 #define AC(r,a,b) r[a]!=r[b]||a+k>n||r[a+k]!=r[b+k]
8 void suffix_array(const char *s,int n,int *sa,int *rank,int *
9   tmp,int *c){
10   int A='z'+1,i,k,id=0;
11   for(i=0; i<n; ++i)rank[tmp[i]]=s[i];
12   radix_sort(rank,tmp);
13   for(k=1; id<n-1; k<=<1){
14     for(id=0,i=n-k; i<n; ++i) tmp[id++]=i;
15     for(i=0; i<n; ++i)
16       if(sa[i]>=k) tmp[id++]=sa[i]-k;
17     radix_sort(rank,tmp);
18     swap(rank,tmp);
19     for(rank[sa[0]]=id=0,i=1; i<n; ++i)
20       rank[sa[i]] = id+=AC(tmp,sa[i-1],sa[i]);
21     A = id+1;
22   }
23 }
24 //h:高度數組 sa:後綴數組 rank:排名
25 void suffix_array_lcp(const char *s,int len,int *h,int *sa,
26   int *rank){
27   for(int i=0; i<len; ++i)rank[sa[i]]=i;
28   for(int i=0,k=0; i<len; ++i){
29     if(rank[i]==0)continue;
30     if(k--<0)
31       while(s[i+k]==s[sa[rank[i]-1]+k])++k;
32     h[rank[i]]=k;
33   }
34 }

```

## 7.8 LPS

```

1 char t[1001]; // 原字串
2 char s[1001 * 2]; // 穿插特殊字元之後的t
3 int z[1001 * 2], L, R; // 源自Gusfield's Algorithm
4 // 由a往左、由b往右，對稱地作字元比對。
5 int extend(int a, int b) {
6   int i = 0;
7   while (a-i>0 && b+i<N && s[a-i] == s[b+i]) i++;
8   return i;
9 }
10 void longest_palindromic_substring() {
11   int N = strlen(t);
12   // t穿插特殊字元，存放到s。
13   // (實際上不會這麼做，都是細算索引值。)
14   memset(s, '.', N*2+1);
15   for (int i=0; i<N; ++i) s[i*2+1] = t[i];
16   N = N*2+1;
17   // s[N] = '\0'; // 可做可不做
18   // Manacher's Algorithm
19   z[0] = 1; L = R = 0;
20   for (int i=1; i<N; ++i) {
21     int ii = L - (i - L); // i的映射位置
22     int n = R + 1 - i;
23     if (i > R) {

```

```

24     z[i] = extend(i, i);
25     L = i;
26     R = i + z[i] - 1;
27   } else if (z[ii] == n) {
28     z[i] = n + extend(i-n, i+n);
29     L = i;
30     R = i + z[i] - 1;
31   } else z[i] = min(z[ii], n);
32 }
33 // 尋找最長迴文子字串的長度。
34 int n = 0, p = 0;
35 for (int i=0; i<N; ++i)
36   if (z[i] > n) n = z[p = i];
37 // 記得去掉特殊字元。
38 cout << "最長迴文子字串的長度是" << (n-1) / 2;
39 // 印出最長迴文子字串，記得別印特殊字元。
40 for (int i=p-z[p]+1; i<=p+z[p]-1; ++i)
41   if (i & 1) cout << s[i];
42 }

```

## 7.9 Edit Distance

```

1 // 問從 src 到 dst 的最小 edit distance
2 // ins 插入一個字元的成本
3 // del 刪除一個字元的成本
4 // sst 替換一個字元的成本
5 ll edd(string& src, string& dst, ll ins, ll del, ll sst) {
6   ll dp[src.size() + 1][dst.size() + 1]; // 不用初始化
7   for (int i = 0; i <= src.size(); i++) {
8     for (int j = 0; j <= dst.size(); j++) {
9       if (i == 0) dp[i][j] = ins * j;
10      else if (j == 0) dp[i][j] = del * i;
11      else if (src[i - 1] == dst[j - 1])
12        dp[i][j] = dp[i - 1][j - 1];
13      else
14        dp[i][j] = min(dp[i][j - 1] + ins,
15          min(dp[i - 1][j] + del,
16            dp[i - 1][j - 1] + sst));
17    }
18  }
19  return dp[src.size()][dst.size()];
20 }

```

## 8 Geometry

### 8.1 Geometry

```

1 //Copy from Jinkela
2 const double PI=atan2(0.0,-1.0);
3 template<typename T>
4 struct point{
5   T x,y;
6   point(){
7     point(const T&x,const T&y):x(x),y(y){
8     point operator+(const point &b)const{
9       return point(x+b.x,y+b.y);

```

```

10   point operator-(const point &b)const{
11     return point(x-b.x,y-b.y);
12   }
13   point operator*(const T &b)const{
14     return point(x*b,y*b);
15   }
16   point operator/(const T &b)const{
17     return point(x/b,y/b);
18   }
19   bool operator==(const point &b)const{
20     return x==b.x&&y==b.y;
21   }
22   T dot(const point &b)const{
23     return x*b.x+y*b.y;
24   }
25   T cross(const point &b)const{
26     return x*b.y-y*b.x;
27   }
28   point normal()const{//求法向量
29     return point(-y,x);
30   }
31   T abs2()const{//向量長度的平方
32     return dot(*this);
33   }
34   T rad(const point &b)const{//兩向量的弧度
35     return fabs(atan2(fabs(cross(b)),dot(b)));
36   }
37   T getA()const{//對x軸的弧度
38     T A=atan2(y,x);
39     if(A<=-PI/2)A+=PI*2;
40     return A;
41   }
42 }
43 };
44 template<typename T>
45 struct line{
46   line(){
47     point<T> p1,p2;
48     T a,b,c;//ax+by+c=0
49     line(const point<T>&x,const point<T>&y):p1(x),p2(y){
50     void pton()const{//轉成一般式
51       a=p1.y-p2.y;
52       b=p2.x-p1.x;
53       c=-a*p1.x-b*p1.y;
54     }
55     T ori(const point<T> &p)const{//點和有向直線的關係，>0左
56       //邊、=0在線上、<0右邊
57       return (p2-p1).cross(p-p1);
58     }
59     T btw(const point<T> &p)const{//點投影落在線段上<=0
60       return (p1-p).dot(p2-p);
61     }
62     bool point_on_segment(const point<T>&p)const{//點是否在線段
63       //上
64       return ori(p)==0&&btw(p)<=0;
65     }
66     T dis2(const point<T> &p,bool is_segment=0)const{//點跟直線
67       //線段的距離平方
68       point<T> v=p2-p1,v1=p-p1;
69       if(is_segment){
70         point<T> v2=p-p2;
71         if(v.dot(v1)<=0)return v1.abs2();
72         if(v.dot(v2)>=0)return v2.abs2();
73       }
74       T tmp=v.cross(v1);
75       return tmp*tmp/v.abs2();
76     }
77   }
78   T seg_dis2(const line<T> &l1)const{//兩線段距離平方
79     return min({dis2(l1.p1,1),dis2(l1.p2,1),l1.dis2(p1,1),l1.dis2
80       (p2,1)});
81   }
82 }
83 point<T> projection(const point<T> &p)const{//點對直線的投
84   //影

```

```

68     point<T> n=(p2-p1).normal();
69     return p-n*(p-p1).dot(n)/n.abs2();
70 }
71 point<T> mirror(const point<T> &p)const{
72     //點對直線的鏡射，要先呼叫pton轉成一般式
73     point<T> R;
74     T d=a*b*b;
75     R.x=(b*b*p.x-a*a*p.x-2*a*b*p.y-2*a*c)/d;
76     R.y=(a*a*p.y-b*b*p.y-2*a*b*p.x-2*b*c)/d;
77     return R;
78 }
79 bool equal(const line &l)const{//直線相等
80     return ori(l.p1)==0&&ori(l.p2)==0;
81 }
82 bool parallel(const line &l)const{
83     return (p1-p2).cross(l.p1-l.p2)==0;
84 }
85 bool cross_seg(const line &l)const{
86     return (p2-p1).cross(l.p1-p1)*(p2-p1).cross(l.p2-p1)<=0;
87     //直線是否交線段
88 }
89 int line_intersect(const line &l)const{//直線相交情況，-1無
90     //限多點、1交於一點、0不相交
91     return parallel(l)?(ori(l.p1)==0?-1:0):1;
92 }
93 int seg_intersect(const line &l)const{
94     T c1=ori(l.p1), c2=ori(l.p2);
95     T c3=l.ori(p1), c4=l.ori(p2);
96     if(c1==0&&c2==0){//共線
97         bool b1=btw(l.p1)>=0,b2=btw(l.p2)>=0;
98         T a3=l.btw(p1),a4=l.btw(p2);
99         if(b1&&b2&&a3==0&&a4==0) return 2;
100         if(b1&&b2&&a3>0&&a4==0) return 3;
101         if(b1&&b2&&a3>0&&a4>0) return 0;
102         return -1;//無限交點
103     }else if(c1*c2<=0&&c3*c4<=0)return 1;
104     return 0;//不相交
105 }
106 point<T> line_intersection(const line &l)const{/*直線交點*/
107     point<T> a=p2-p1,b=l.p2-l.p1,s=l.p1-p1;
108     //if(a.cross(b)==0)return INF;
109     return p1+a*(s.cross(b)/a.cross(b));
110 }
111 point<T> seg_intersection(const line &l)const{//線段交點
112     int res=seg_intersect(l);
113     if(res<=0) assert(0);
114     if(res==2) return p1;
115     if(res==3) return p2;
116     return line_intersection(l);
117 }
118 };
119 template<typename T>
120 struct polygon{
121     polygon(){
122         vector<point<T> > p;
123         //逆時針順序
124         T area()const{//面積
125             T ans=0;
126             for(int i=p.size()-1,j=0;j<(int)p.size();i=j++){
127                 ans+=p[i].cross(p[j]);
128             }
129             return ans/2;
130         }
131         T a=p[i].cross(p[j]);
132         cx+=(p[i].x+p[j].x)*a;
133         cy+=(p[i].y+p[j].y)*a;
134         w+=a;
135     }
136     return point<T>(cx/3/w,cy/3/w);
137 }
138 char ahas(const point<T>& t)const{//點是否在簡單多邊形內，
139     //是的話回傳1、在邊上回傳-1、否則回傳0
140     bool c=0;
141     for(int i=0,j=p.size()-1;i<p.size();j=i++){
142         if(line<T>(p[i],p[j]).point_on_segment(t))return -1;
143         else if((p[i].y>t.y)!=p[j].y>t.y)&&
144             t.x<(p[j].x-p[i].x)*(t.y-p[i].y)/(p[j].y-p[i].y)+p[i].x)
145             c=!c;
146     }
147     return c;
148 }
149 char point_in_convex(const point<T>&x)const{
150     int l=1,r=(int)p.size()-2;
151     while(l<r){
152         //點是否在凸多邊形內，是的話回傳1、在邊上回傳
153         // -1、否則回傳0
154         int mid=(l+r)/2;
155         T a1=(p[mid]-p[0]).cross(x-p[0]);
156         T a2=(p[mid+1]-p[0]).cross(x-p[0]);
157         if(a1>0&&a2<=0){
158             T res=(p[mid+1]-p[mid]).cross(x-p[mid]);
159             return res>0?1:(res>0?-1:0);
160         }else if(a1<0)r=mid-1;
161         else l=mid+1;
162     }
163     return 0;
164 }
165 vector<T> getA()const{//凸包邊對x軸的夾角
166     vector<T>res;//一定是遞增的
167     for(size_t i=0;i<p.size();i++){
168         res.push_back((p[(i+1)%p.size()]-p[i]).getA());
169     }
170     return res;
171 }
172 bool line_intersect(const vector<T>&A,const line<T> &l)
173     const{//O(logN)
174     int f1=upper_bound(A.begin(),A.end(),(l.p1-l.p2).getA())-
175         A.begin();
176     int f2=upper_bound(A.begin(),A.end(),(l.p2-l.p1).getA())-
177         A.begin();
178     return l.cross_seg(line<T>(p[f1],p[f2]));
179 }
180 polygon cut(const line<T> &l)const{//凸包對直線切割，得到直
181     //線l左側的凸包
182     polygon ans;
183     for(int n=p.size(),i=n-1,j=0;j<n;i=j++){
184         if(l.ori(p[i])>=0){
185             ans.p.push_back(p[i]);
186             if(l.ori(p[j])<0)
187                 ans.p.push_back(l.line_intersection(line<T>(p[i],p[
188                     j])));
189             }else if(l.ori(p[j])>0)
190                 ans.p.push_back(l.line_intersection(line<T>(p[i],p[j
191                     ])));
192         }
193     }
194     return ans;
195 }
196 static bool graham_cmp(const point<T>& a,const point<T>& b)
197     {
198         //凸包排序函數
199         return (a.x<b.x)||((a.x==b.x&&a.y<b.y);
200     }
201 void graham(vector<point<T> > &s){//凸包
202     sort(s.begin(),s.end(),graham_cmp);
203     p.resize(s.size()+1);
204     int m=0;
205     for(size_t i=0;i<s.size();i++){
206         while(m>=2&&(p[m-1]-p[m-2]).cross(s[i]-p[m-2])<=0)--m;
207         p[m++]=s[i];
208     }
209     for(int i=s.size()-2,t=m+1;i>0;--i){
210         while(m>=2&&(p[m-1]-p[m-2]).cross(s[i]-p[m-2])<=0)--m;
211         p[m++]=s[i];
212     }
213     if(s.size()>1)--m;
214     p.resize(m);
215 }
216 T diam()//直徑
217 int n=p.size(),t=1;
218 T ans=0;p.push_back(p[0]);
219 for(int i=0;i<n;i++){
220     point<T> now=p[i+1]-p[i];
221     while(now.cross(p[t+1]-p[i])>now.cross(p[t]-p[i]))t=(t
222         +1)%n;
223     ans=max(ans,(p[i]-p[t]).abs2());
224 }
225 return p.pop_back(),ans;
226 }
227 T min_cover_rectangle()//最小覆蓋矩形
228 int n=p.size(),t=1,r=1,l;
229 if(n<3)return 0;//也可以做最小周長矩形
230 T ans=1e99;p.push_back(p[0]);
231 for(int i=0;i<n;i++){
232     point<T> now=p[i+1]-p[i];
233     while(now.cross(p[t+1]-p[i])>now.cross(p[t]-p[i]))t=(t
234         +1)%n;
235     while(now.dot(p[r+1]-p[i])>now.dot(p[r]-p[i]))r=(r+1)%n;
236     if(!l)l=r;
237     while(now.dot(p[l+1]-p[i])<now.dot(p[l]-p[i]))l=(l+1)%n;
238     T d=now.abs2();
239     T tmp=now.cross(p[t]-p[i])*(now.dot(p[r]-p[i])-now.dot(
240         p[l]-p[i]))/d;
241     ans=min(ans,tmp);
242 }
243 return p.pop_back(),ans;
244 }
245 T dis2(polygon &p1){//凸包最近距離平方
246     vector<point<T> > &P=p,&Q=p1.p;
247     int n=P.size(),m=Q.size(),l=0,r=0;
248     for(int i=0;i<n;++i)if(P[i].y<P[l].y)l=i;
249     for(int i=0;i<m;++i)if(Q[i].y<Q[r].y)r=i;
250     P.push_back(P[0]),Q.push_back(Q[0]);
251     T ans=1e99;
252     for(int i=0;i<n;++i){
253         while((P[l]-P[l+1]).cross(Q[r+1]-Q[r])<0)r=(r+1)%m;
254         ans=min(ans,line<T>(P[l],P[l+1]).seg_dis2(line<T>(Q[r],
255             Q[r+1])));
256         l=(l+1)%n;
257     }
258     return P.pop_back(),Q.pop_back(),ans;
259 }
260 static char sign(const point<T>&t){
261     return (t.y==0?t.x:t.y)<0;
262 }

```



```

243 }
244 static bool angle_cmp(const line<T>& A,const line<T>& B){
245     point<T> a=A.p2-A.p1,b=B.p2-B.p1;
246     return sign(a)<sign(b)||((sign(a)==sign(b)&&a.cross(b)>0);
247 }
248 int halfplane_intersection(vector<line<T> > &s){//半平面交
249     sort(s.begin(),s.end(),angle_cmp);//線段左側為該線段半平面
250     int L,R,n=s.size();
251     vector<point<T> > px(n);
252     vector<line<T> > q(n);
253     q[L=R=0]=s[0];
254     for(int i=1;i<n;++i){
255         while(L<R&&s[i].ori(px[R-1])<=0)--R;
256         while(L<R&&s[i].ori(px[L])<=0)++L;
257         q[++R]=s[i];
258         if(q[R].parallel(q[R-1])){
259             --R;
260             if(q[R].ori(s[i].p1)>0)q[R]=s[i];
261         }
262         if(L<R)px[R-1]=q[R-1].line_intersection(q[R]);
263     }
264     while(L<R&&q[L].ori(px[R-1])<=0)--R;
265     p.clear();
266     if(R-L<=1)return 0;
267     px[R]=q[R].line_intersection(q[L]);
268     for(int i=L;i<=R;++i)p.push_back(px[i]);
269     return R-L+1;
270 }
271 };
272 template<typename T>
273 struct triangle{
274     point<T> a,b,c;
275     triangle(){
276         triangle(const point<T> &a,const point<T> &b,const point<T> &c):a(a),b(b),c(c){}
277     }
278     T area()const{
279         T t=(b-a).cross(c-a)/2;
280         return t>0?t:-t;
281     }
282     point<T> barycenter()const{//重心
283         return (a+b+c)/3;
284     }
285     point<T> circumcenter()const{//外心
286         static line<T> u,v;
287         u.p1=(a+b)/2;
288         u.p2=point<T>(u.p1.x-a.y+b.y,u.p1.y+a.x-b.x);
289         v.p1=(a+c)/2;
290         v.p2=point<T>(v.p1.x-a.y+c.y,v.p1.y+a.x-c.x);
291         return u.line_intersection(v);
292     }
293     point<T> incenter()const{//內心
294         T A=sqrt((b-c).abs2()),B=sqrt((a-c).abs2()),C=sqrt((a-b).abs2());
295         return point<T>(A*a.x+B*b.x+C*c.x,A*a.y+B*b.y+C*c.y)/(A+B+C);
296     }
297     point<T> perpcenter()const{//垂心
298         return barycenter()*3-circumcenter()*2;
299     }
300 };
301 template<typename T>
302 struct point3D{
303     T x,y,z;
304     point3D(){
305         point3D(const T&x,const T&y,const T&z):x(x),y(y),z(z){}
306     }
307     point3D operator+(const point3D &b)const{
308         return point3D(x+b.x,y+b.y,z+b.z);}
309     point3D operator-(const point3D &b)const{
310         return point3D(x-b.x,y-b.y,z-b.z);}
311     point3D operator*(const T &b)const{
312         return point3D(x*b,y*b,z*b);}
313     point3D operator/(const T &b)const{
314         return point3D(x/b,y/b,z/b);}
315     bool operator==(const point3D &b)const{
316         return x==b.x&&y==b.y&&z==b.z;}
317     T dot(const point3D &b)const{
318         return x*b.x+y*b.y+z*b.z;}
319     point3D cross(const point3D &b)const{
320         return point3D(y*b.z-z*b.y,z*b.x-x*b.z,x*b.y-y*b.x);}
321     T abs2()const{//向量長度的平方
322         return dot(*this);}
323     T area2(const point3D &b)const{//和b、原點圍成面積的平方
324         return cross(b).abs2()/4;}
325 };
326 template<typename T>
327 struct line3D{
328     point3D<T> p1,p2;
329     line3D(){
330         line3D(const point3D<T> &p1,const point3D<T> &p2):p1(p1),p2(p2){}
331     }
332     T dis2(const point3D<T> &p,bool is_segment=0)const{//點跟直線/線段的距離平方
333         point3D<T> v=p2-p1,v1=p-p1;
334         if(is_segment){
335             point3D<T> v2=p-p2;
336             if(v.dot(v1)<=0)return v1.abs2();
337             if(v.dot(v2)>=0)return v2.abs2();
338         }
339         point3D<T> tmp=v.cross(v1);
340         return tmp.abs2()/v.abs2();
341     }
342     pair<point3D<T>,point3D<T>> closest_pair(const line3D<T> &l)const{
343         point3D<T> v1=(p1-p2),v2=(l.p1-l.p2);
344         point3D<T> N=v1.cross(v2),ab(p1-l.p1);
345         //if(N.abs2()==0)return NULL;平行或重合
346         T tmp=N.dot(ab),ans=tmp*tmp/N.abs2();//最近點對距離
347         point3D<T> d1=p2-p1,d2=l.p2-l.p1,D=d1.cross(d2),G=l.p1-p1;
348         T t1=(G.cross(d2)).dot(D)/D.abs2();
349         T t2=(G.cross(d1)).dot(D)/D.abs2();
350         return make_pair(p1+d1*t1,l.p1+d2*t2);
351     }
352     bool same_side(const point3D<T> &a,const point3D<T> &b)const{
353         return (p2-p1).cross(a-p1).dot((p2-p1).cross(b-p1))>0;
354     }
355 };
356 template<typename T>
357 struct plane{
358     point3D<T> p0,n;//平面上的點和法向量
359     plane(){
360         plane(const point3D<T> &p0,const point3D<T> &n):p0(p0),n(n){}
361     }
362     T dis2(const point3D<T> &p)const{//點到平面距離的平方
363         T tmp=(p-p0).dot(n);
364         return tmp*tmp/n.abs2();
365     }
366 };
367 point3D<T> projection(const point3D<T> &p)const{
368     return p-n*(p-p0).dot(n)/n.abs2();
369 }
370 point3D<T> line_intersection(const line3D<T> &l1)const{
371     T tmp=n.dot(l.p2-l.p1);//等於0表示平行或重合該平面
372     return l.p1+(l.p2-l.p1)*(n.dot(p0-l.p1)/tmp);
373 }
374 line3D<T> plane_intersection(const plane &p1)const{
375     point3D<T> e=n.cross(p1.n),v=n.cross(e);
376     T tmp=p1.n.dot(v);//等於0表示平行或重合該平面
377     point3D<T> q=p0+(v*(p1.n.dot(p0-p0))/tmp);
378     return line3D<T>(q,q+e);
379 }
380 };
381 template<typename T>
382 struct triangle3D{
383     point3D<T> a,b,c;
384     triangle3D(){
385         triangle3D(const point3D<T> &a,const point3D<T> &b,const point3D<T> &c):a(a),b(b),c(c){}
386     }
387     bool point_in(const point3D<T> &p)const{//點在該平面上的投影在三角形中
388         return line3D<T>(b,c).same_side(p,a)&&line3D<T>(a,c).same_side(p,b)&&line3D<T>(a,b).same_side(p,c);
389     }
390 };
391 template<typename T>
392 struct tetrahedron{//四面體
393     point3D<T> a,b,c,d;
394     tetrahedron(){
395         tetrahedron(const point3D<T> &a,const point3D<T> &b,const point3D<T> &c,const point3D<T> &d):a(a),b(b),c(c),d(d){}
396     }
397     T volume6()const{//體積的六倍
398         return (d-a).dot((b-a).cross(c-a));
399     }
400     point3D<T> centroid()const{
401         return (a+b+c+d)/4;
402     }
403     bool point_in(const point3D<T> &p)const{
404         return triangle3D<T>(a,b,c).point_in(p)&&triangle3D<T>(c,d,a).point_in(p);
405     }
406 };
407 template<typename T>
408 struct convexhull3D{
409     static const int MAXN=1005;
410     struct face{
411         int a,b,c;
412         face(int a,int b,int c):a(a),b(b),c(c){}
413     };
414     vector<point3D<T>> pt;
415     vector<face> ans;
416     int fid[MAXN][MAXN];
417     void build(){
418         int n=pt.size();
419         ans.clear();
420         memset(fid,0,sizeof(fid));
421         ans.emplace_back(0,1,2);//注意不能共線
422         ans.emplace_back(2,1,0);
423         int ftop = 0;
424         for(int i=3,ftop=1;i<n; ++i,++ftop){
425             vector<face> next;
426             for(auto &f:ans){

```



```

420 T d=(pt[i]-pt[f.a]).dot((pt[f.b]-pt[f.a]).cross(pt[f.
    c]-pt[f.a]));
421 if(d<=0) next.push_back(f);
422 int ff=0;
423 if(d>0) ff=ftop;
424 else if(d<0) ff=-ftop;
425 fid[f.a][f.b]=fid[f.b][f.c]=fid[f.c][f.a]=ff;
426 }
427 for(auto &f:ans){
428     if(fid[f.a][f.b]>0 && fid[f.a][f.b]!=fid[f.b][f.a])
429         next.emplace_back(f.a,f.b,i);
430     if(fid[f.b][f.c]>0 && fid[f.b][f.c]!=fid[f.c][f.b])
431         next.emplace_back(f.b,f.c,i);
432     if(fid[f.c][f.a]>0 && fid[f.c][f.a]!=fid[f.a][f.c])
433         next.emplace_back(f.c,f.a,i);
434 }
435 ans=next;
436 }
437 }
438 point3D<T> centroid()const{
439     point3D<T> res(0,0,0);
440     T vol=0;
441     for(auto &f:ans){
442         T tmp=pt[f.a].dot(pt[f.b].cross(pt[f.c]));
443         res=res+(pt[f.a]+pt[f.b]+pt[f.c])*tmp;
444         vol+=tmp;
445     }
446     return res/(vol*4);
447 }
448 };

```

## 8.2 旋轉卡尺

```

1 typedef pair<ll, ll> pii;
2 #define x first
3 #define y second
4 #define ii (i + 1) % n // 打字加速!
5 inline pii operator-(const pii& a, const pii& b) {
6     return {a.x - b.x, a.y - b.y};
7 } // const 不可省略
8 inline ll operator*(const pii& a, const pii& b) {
9     return a.x * b.y - a.y * b.x;
10 }
11 inline ll crzf(const pii& o, const pii& a, const pii& b) {
12     return (a - o) * (b - o)
13 }
14 inline ll dd(const pii& a, const pii& b) {
15     ll dx = a.x - b.x, dy = a.y - b.y;
16     return dx * dx + dy * dy;
17 }
18 // 給平面上任意個點，求其凸包。返回順序為逆時針。此方法會移除
    重複點。
19 #define jud \
20     crzf(ret[ret.size() - 2], ret.back(), pp[i]) <= 0
21 vector<pii> makepoly(vector<pii>& pp) {
22     int n = pp.size();
23     sort(pp.begin(), pp.end());
24     pp.erase(unique(pp.begin(), pp.end()), pp.end());
25     vector<pii> ret;
26     for (int i = 0; i < n; i++) {
27         while (ret.size() >= 2 && jud) ret.pop_back();
28         ret.push_back(pp[i]);

```

```

    }
    for (int i = n - 2, t = ret.size() + 1; i >= 0; i--) {
        while (ret.size() >= t && jud) ret.pop_back();
        ret.push_back(pp[i]);
    }
    if (n >= 2) ret.pop_back();
    return ret;
}
// (shoelace formula)
// 給凸包，問其面積「的兩倍」。若凸包少於三個點，回傳零。
ll area(vector<pii>& poly) {
    int n = poly.size();
    ll ret = 0;
    for (int i = 0; i < n; i++)
        ret += (poly[i].x * poly[i+1].y);
    for (int i = 0; i < n; i++)
        ret -= (poly[i].y * poly[i+1].x);
    return ret;
}
// 給凸包，問其兩點最遠距離「的平方」。若要問平面上任意個點的
    兩點最遠
49 // 距離，請先轉成凸包。若凸包少於兩個點，回傳零。
50 #define kk (k + 1) % n
51 ll maxdist(vector<pii>& poly) {
52     int k = 1, n = poly.size();
53     if (n < 2) return 0;
54     if (n == 2) return dd(poly[0], poly[1]);
55     ll ret = 0;
56     for (int i = 0; i < n; i++) {
57         while (abs(crzf(poly[kk], poly[i], poly[i+1])) >=
58             abs(crzf(poly[k], poly[i], poly[i+1])))
59             k = kk;
60         ret = max(ret, max(dd(poly[i], poly[k]),
61             dd(poly[i+1], poly[k])));
62     }
63     return ret;
64 }

```

## 8.3 最近點對

```

1 typedef pair<ll, ll> pii;
2 #define x first
3 #define y second
4 ll dd(const pii& a, const pii& b) {
5     ll dx = a.x - b.x, dy = a.y - b.y;
6     return dx * dx + dy * dy;
7 }
8 const ll inf = 1e18;
9 ll dac(vector<pii>& p, int l, int r) {
10     if (l >= r) return inf;
11     int m = (l + r) / 2;
12     ll d = min(dac(p, l, m), dac(p, m + 1, r));
13     vector<pii> t;
14     for (int i = m; i >= l && p[m].x - p[i].x < d; i--)
15         t.push_back(p[i]);
16     for (int i = m + 1; i <= r && p[i].x - p[m].x < d; i++)
17         t.push_back(p[i]);
18     sort(t.begin(), t.end());
19     for (pii& a, pii& b) { return a.y < b.y; };
20     int n = t.size();
21     for (int i = 0; i < n - 1; i++)
22         for (int j = 1; j < 4 && i + j < n; j++)

```

```

23 // 這裡可以知道是哪兩點是最小點對
24 d = min(d, dd(t[i], t[i + j]));
25 return d;
26 }
27 // 給一堆點，求最近點對的距離「的平方」。
28 ll closest_pair(vector<pii>& pp) {
29     sort(pp.begin(), pp.end());
30     return dac(pp, 0, pp.size() - 1);
31 }

```

## 8.4 最小覆蓋圓

```

1 using PT = point<T>;
2 using CPT = const PT;
3 PT circumcenter(CPT &a, CPT &b, CPT &c) {
4     PT u = b-a, v = c-a;
5     T c1 = u.abs2()/2, c2 = v.abs2()/2;
6     T d = u.cross(v);
7     return PT(a.x+(v.y*c1-u.y*c2)/d, a.y+(u.x*c2-v.x*c1)/d);
8 }
9 void solve(PT p[], int n, PT &c, T &r2){
10     random_shuffle(p,p+n);
11     c = p[0]; r2 = 0; // c,r2 = 圓心,半徑平方
12     for(int i=1; i<n; i++){
13         if( (p[i]-c).abs2() > r2) {
14             c=p[i]; r2=0;
15             for(int j=0; j<i; j++){
16                 if( (p[j]-c).abs2() > r2) {
17                     c.x = (p[i].x+p[j].x)/2;
18                     c.y = (p[i].y+p[j].y)/2;
19                     r2 = (p[j]-c).abs2();
20                     for(int k=0; k<j; k++){
21                         if( (p[k]-c).abs2() > r2) {
22                             c = circumcenter(p[i], p[j], p[k]);
23                             r2 = (p[i]-c).abs2();
24                         }
25                     }
26                 }
27             }

```

## 8.5 Rectangle Union Area

```

1 const int maxn = 1e5 + 10;
2 struct rec{
3     int t, b, l, r;
4 } r[maxn];
5 int n, cnt[maxn << 2];
6 long long st[maxn << 2], ans = 0;
7 vector<int> x, y;
8 vector<pair<pair<int, int>, pair<int, int>>> v;
9 void modify(int t, int l, int r, int ql, int qr, int v) {
10     if (ql <= l && r <= qr) cnt[t] += v;
11     else {
12         int m = (l + r) >> 1;
13         if (qr <= m) modify(t << 1, l, m, ql, qr, v);
14         else if (ql >= m) modify(t << 1 | 1, m, r, ql, qr, v);
15         else modify(t << 1, l, m, ql, m, v), modify(t << 1 |
            1, m, r, m, qr, v);

```

```

16 }
17 if (cnt[t]) st[t] = y[r] - y[l];
18 else if (r - l == 1) st[t] = 0;
19 else st[t] = st[t << 1] + st[t << 1 | 1];
20 }
21 int main() {
22     cin >> n;
23     for (int i = 0; i < n; i++) {
24         cin >> r[i].l >> r[i].r >> r[i].b >> r[i].t;
25         if (r[i].l > r[i].r) swap(r[i].l, r[i].r);
26         if (r[i].b > r[i].t) swap(r[i].b, r[i].t);
27         x.push_back(r[i].l);
28         x.push_back(r[i].r);
29         y.push_back(r[i].b);
30         y.push_back(r[i].t);
31     }
32     sort(x.begin(), x.end());
33     sort(y.begin(), y.end());
34     x.erase(unique(x.begin(), x.end()), x.end());
35     y.erase(unique(y.begin(), y.end()), y.end());
36     for (int i = 0; i < n; i++) {
37         r[i].l = lower_bound(x.begin(), x.end(), r[i].l) - x.
            begin();
38         r[i].r = lower_bound(x.begin(), x.end(), r[i].r) - x.
            begin();
39         r[i].b = lower_bound(y.begin(), y.end(), r[i].b) - y.
            begin();
40         r[i].t = lower_bound(y.begin(), y.end(), r[i].t) - y.
            begin();
41         v.emplace_back(make_pair(r[i].l, 1), make_pair(r[i].b
            , r[i].t));
42         v.emplace_back(make_pair(r[i].r, -1), make_pair(r[i].
            b, r[i].t));
43     }
44     sort(v.begin(), v.end(), [](pair<pair<int, int>, pair<int
        , int>> a, pair<pair<int, int>, pair<int, int>> b){
45         if (a.first.first != b.first.first) return a.first.
            first < b.first.first;
46         return a.first.second > b.first.second;
47     });
48     for (int i = 0; i < v.size(); i++) {
49         if (i) ans += (x[v[i].first.first] - x[v[i - 1].first
            .first]) * st[1];
50         modify(1, 0, y.size(), v[i].second.first, v[i].second
            .second, v[i].first.second);
51     }
52     cout << ans << '\n';
53     return 0;
54 }

```

## 9 Other

### 9.1 Fastio

```

1 inline ll read(){
2     ll x=0,f=0;
3     char ch = getchar();
4     if(ch==EOF)
5         return 0;
6     while(ch<'0' || ch>'9') f|=ch=='-',ch=getchar();

```

```

7     while(ch>='0'&&ch<='9') x=(x<<3)+(x<<1)+(ch^48),ch=getchar
8     ();
9     return f?-x:x;
10 }
11 inline void print(ll x,bool bk = false) {
12     if(x<0){
13         putchar('-');
14         x = -x;
15     }
16     if(x==0){
17         if(!bk)putchar('0');
18         return;
19     }
20     print(x/10,true);
21     putchar((x-10*(x/10))^'0');
22 }

```

### 9.2 pbds

```

1 #include<bits/extc++.h>
2 using namespace __gnu_pbds;
3
4 // hash_table : 用法和map差不多 //均攤O(1)
5 gp_hash_table <string,int> mp;
6 mp.find(); mp[];
7 mp.insert(make_pair())
8
9 // heaps
10 priority_queue<int, greater<int>, TAG> Q;
11 /*
12 Tag                | push | pop | join | modify |
13 pairing_heap_tag   | 0(1) | 0(lgN) | 0(1) | 0(lgN) |
14 thin_heap_tag      | 0(lgN) | 0(lgN) | 慢 | 慢 |
15 binomial_heap_tag  | 0(1) | 0(lgN) | 0(lgN) | 0(lgN) |
16 rc_binomial_heap_tag | 0(1) | 0(lgN) | 0(lgN) | 0(lgN) |
17 binary_heap_tag    | 0(1) | 0(lgN) | 慢 | 0(lgN) |
18 */ //可以用迭代器遍歷
19 Q.push(x); Q.pop(); Q.top();
20 Q.join(b); //merge two heap
21 Q.empty(); Q.size();
22 Q.modify(it, 6); Q.erase(it);
23
24 // k-th
25 typedef tree<int,null_type,less<int>,rb_tree_tag,
26     tree_order_statistics_node_update> set_t;
27 set_t s; s.insert(12); s.insert(505);
28 assert(*s.find_by_order(0) == 12);
29 assert(*s.find_by_order(3) == 505);
30 assert(s.order_of_key(12) == 0);
31 assert(s.order_of_key(505) == 1);
32 s.erase(12);
33 assert(*s.find_by_order(0) == 505);
34 assert(s.order_of_key(505) == 0);

```

### 9.3 BuiltIn

```

1 //gcc專用
2 //unsigned int ffs

```

```

3 //unsigned long ffs1
4 //unsigned long long ffs11
5 unsigned int x; scanf("%u",&x)
6 printf("右起第一個1:的位置");
7 printf("%d\n",__builtin_ffs(x));
8 printf("左起第一個1之前0的個數:");
9 printf("%d\n",__builtin_clz(x));
10 printf("右起第一個1之後0的個數:");
11 printf("%d\n",__builtin_ctz(x));
12 printf("1的個數:");
13 printf("%d\n",__builtin_popcount(x));
14 printf("1的個數的奇偶性:");
15 printf("%d\n",__builtin_parity(x));

```

### 9.4 莫隊算法-區間眾數

```

1 using namespace std;
2 const int maxn = 1e6 + 10;
3 struct query { int id, bk, l, r; };
4 int arr[maxn], cnt[maxn], d[maxn], n, m, bk, mx;
5 pair<int,int> ans[maxn];
6 vector<query> q;
7 bool cmp(query x,query y) {
8     return (x.bk < y.bk || (x.bk == y.bk) && x.r < y.r);
9 }
10 void add(int pos) {
11     d[cnt[arr[pos]]]--;
12     cnt[arr[pos]]++;
13     d[cnt[arr[pos]]]++;
14     if(d[mx + 1] > 0) mx++;
15 }
16 void del(int pos) {
17     d[cnt[arr[pos]]]--;
18     cnt[arr[pos]]--;
19     d[cnt[arr[pos]]]++;
20     if(d[mx] == 0) mx--;
21 }
22 void mo(int n, int m) {
23     sort(q.begin(), q.end(), cmp);
24     for(int i = 0, cl = 1, cr = 0; i < m; i++) {
25         while(cr < q[i].r) add(++cr);
26         while(cl > q[i].l) add(--cl);
27         while(cr > q[i].r) del(cr--);
28         while(cl < q[i].l) del(cl--);
29         ans[q[i].id] = make_pair(mx, d[mx]);
30     }
31 }
32 int main(){
33     cin >> n >> m;
34     bk = (int)sqrt(n + 0.5);
35     for(int i = 1; i <= n; i++) cin >> arr[i];
36     q.resize(m);
37     for(int i = 0; i < m; i++) {
38         cin >> q[i].l >> q[i].r;
39         q[i].id = i,q[i].bk = (q[i].l - 1) / bk;
40     }
41     mo(n, m);
42     for(int i = 0; i < m; i++)
43         cout << ans[i].first << ' ' << ans[i].second << '\n';
44     return 0;
45 }

```

## 9.5 CNF

```

1 #define MAXN 55
2 struct CNF{
3     int s,x,y;//s->xy | s->x, if y==--1
4     int cost;
5     CNF(){}
6     CNF(int s,int x,int y,int c):s(s),x(x),y(y),cost(c){}
7 };
8 int state;//規則數量
9 map<char,int> rule;//每個字元對應到的規則，小寫字母為終端字符
10 vector<CNF> cnf;
11 void init(){
12     state=0;
13     rule.clear();
14     cnf.clear();
15 }
16 void add_to_cnf(char s,const string &p,int cost){
17     //加入一個s -> <p>的文法，代價為cost
18     if(rule.find(s)==rule.end())rule[s]=state++;
19     for(auto c:p)if(rule.find(c)==rule.end())rule[c]=state++;
20     if(p.size()==1){
21         cnf.push_back(CNF(rule[s],rule[p[0]],-1,cost));
22     }else{
23         int left=rule[s];
24         int sz=p.size();
25         for(int i=0;i<sz-2;++i){
26             cnf.push_back(CNF(left,rule[p[i]],state,0));
27             left=state++;
28         }
29         cnf.push_back(CNF(left,rule[p[sz-2]],rule[p[sz-1]],cost));
30     }
31 }
32 vector<long long> dp[MAXN][MAXN];
33 vector<bool> neg_INF[MAXN][MAXN]; //如果花費是負的可能會有無限
34 // 小的情形
35 void relax(int l,int r,const CNF &c,long long cost,bool neg_c
36 =0){
37     if(!neg_INF[l][r][c.s]&&(neg_INF[l][r][c.x]||cost<dp[l][r][
38         c.s])){
39         if(neg_c||neg_INF[l][r][c.x]){
40             dp[l][r][c.s]=0;
41             neg_INF[l][r][c.s]=true;
42         }else dp[l][r][c.s]=cost;
43     }
44 }
45 void bellman(int l,int r,int n){
46     for(int k=1;k<=state;++k)
47         for(auto c:cnf)
48             if(c.y==--1)relax(l,r,c,dp[l][r][c.x]+c.cost,k==n);
49 }
50 void cyk(const vector<int> &tok){
51     for(int i=0;i<(int)tok.size();++i){
52         for(int j=0;j<(int)tok.size();++j){
53             dp[i][j]=vector<long long>(state+1,INT_MAX);
54             neg_INF[i][j]=vector<bool>(state+1,false);
55         }
56         dp[i][i][tok[i]]=0;
57         bellman(i,i,tok.size());
58     }
59     for(int r=1;r<(int)tok.size();++r){
60         for(int l=r-1;l>0;--l){
61             for(int k=1;k<r;++k)

```

```

59         for(auto c:cnf)
60             if(~c.y)relax(l,r,c,dp[l][k][c.x]+dp[k+1][r][c.y]+c
61                 .cost);
62         bellman(l,r,tok.size());
63     }
64 }

```

## 9.6 提醒事項

```

1 Debug List:
2 1. Long Long !!
3 2. python3 整數除法 "/"
4 3. connected / unconnected
5 4. 範圍看清楚
6 5. eps 夠小嗎!!
7 6. 可多生 case 測
8 7. 找不用胖資結的其他作法 e.g. multiset -> 單調對列
9 8. 離散化
10 9. 鴿籠原理
11 10. TLE 後找人多想

```

```

12 -----
13 Lucas's Theorem
14 For non-negative integer n,m and prime P,
15  $C(m,n) \bmod P = C(m/M,n/M) * C(m\%M,n\%M) \bmod P$ 
16  $= \text{mult\_i} ( C(m\_i,n\_i) )$ 
17 where  $m\_i$  is the i-th digit of m in base P.
18 -----
19 Kirchhoff's theorem
20  $A_{\{ii\}} = \deg(i), A_{\{ij\}} = (i,j) \setminus \text{in } E ? -1 : 0$ 
21 Deleting any one row, one column, and cal the det(A)
22 -----
23 Nth Catalan recursive function:
24  $C_0 = 1, C_{n+1} = C_n * 2(2n+1)/(n+2)$ 
25 -----
26 Mobius Formula
27  $u(n) = 1$  , if  $n = 1$ 
28  $(-1)^m$  , 若  $n$  無平方數因數，且  $n = p_1 * p_2 * p_3 * \dots * p_k$ 
29  $0$  , 若  $n$  有大於 1 的平方數因數
30 -----
31 - Property
32 1. (積性函數)  $u(a)u(b) = u(ab)$ 
33 2.  $\sum_{d|n} u(d) = [n == 1]$ 
34 -----
35 Mobius Inversion Formula
36 if  $f(n) = \sum_{d|n} g(d)$ 
37 then  $g(n) = \sum_{d|n} u(n/d)f(d)$ 
38  $= \sum_{d|n} u(d)f(n/d)$ 
39 -----
40 - Application
41 the number/power of gcd(i, j) = k
42 - Trick
43 分塊,  $O(\sqrt{n})$ 
44 -----
45 Chinese Remainder Theorem ( $m_i$  兩兩互質)
46  $x = a_1 \pmod{m_1}$ 
47  $x = a_2 \pmod{m_2}$ 
48 ....
49  $x = a_i \pmod{m_i}$ 
50 construct a solution:
51 Let  $M = m_1 * m_2 * m_3 * \dots * m_n$ 
52 Let  $M_i = M / m_i$ 

```

```

52 t_i = 1 / M_i
53 t_i * M_i = 1 (mod m_i)
54 solution x = a_1 * t_1 * M_1 + a_2 * t_2 * M_2 + ... + a_n
55 * t_n * M_n + k * M
56 = k*M + \sum a_i * t_i * M_i, k is positive integer.
57 under mod M, there is one solution x = \sum a_i * t_i * M_i
58 -----
59 Burnside's lemma
60 |G| * |X/G| = sum( |X^g| ) where g in G
61 總方法數：每一種旋轉下不動點的個數總和 除以 旋轉的方法數
62 -----
63 Linear Algebra
64 trace : tr(A) = 對角線和
65 eigen vector : Ax = cx => (A-cI)x = 0
66 -----
67 Josephus Problem
68 f(n,k) = (f(n-1,k)+k)%(mod n)
69 f(1,k) = 0

```

## 9.7 霍夫曼樹



# NTHU- ELEPHANTGANG CODEBOOK

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