

Advanced Programming

INFO135

Lecture 9: Greedy Algorithms

Mehdi Elahi

University of Bergen (UiB)

Binary Search Tree

- ❖ In the previous lecture we introduced **binary tree** structure.
- ❖ A special case of a binary tree that follows a specific **property** is called **Binary Search Tree**.
- ❖ Data stored in Binary Search Tree is more **efficient to find** (i.e., to search) than in an ordinary binary trees.

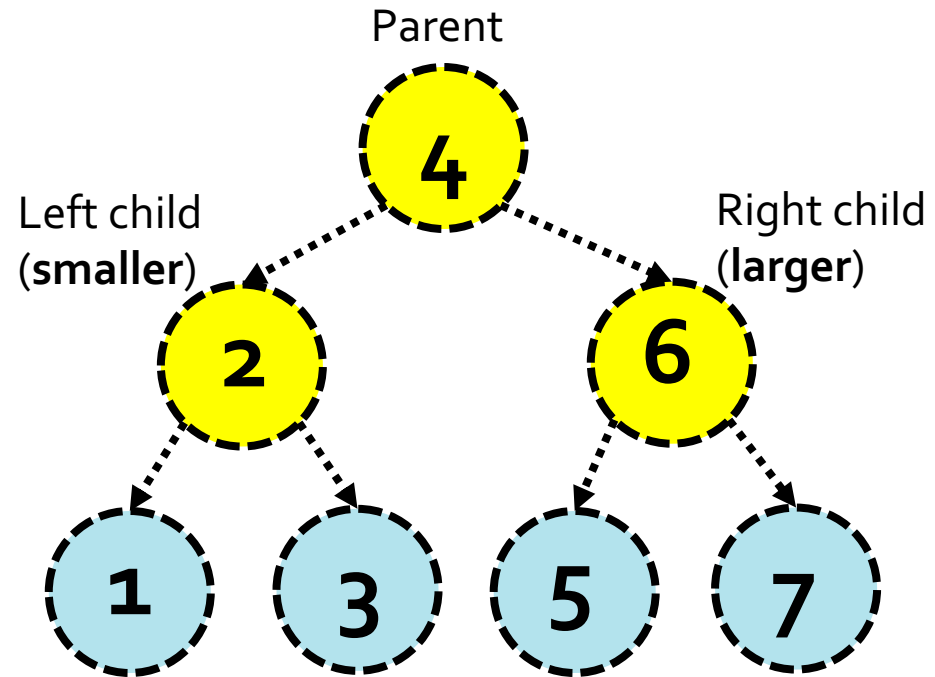
Binary Search Tree

- ❖ **Binary Search Tree (BST)** is a binary tree where:
 - ❖ values that are less than parent node are in **left** subtree
 - ❖ values that are greater than parent are in **right** subtree
- ❖ This is the **property** of Binary Search Tree. Value stored in a node is also called **Key**.
- ❖ Duplicates are **not allowed** in binary search tree (similar to Python Set).

Binary Search Tree

❖ Example:

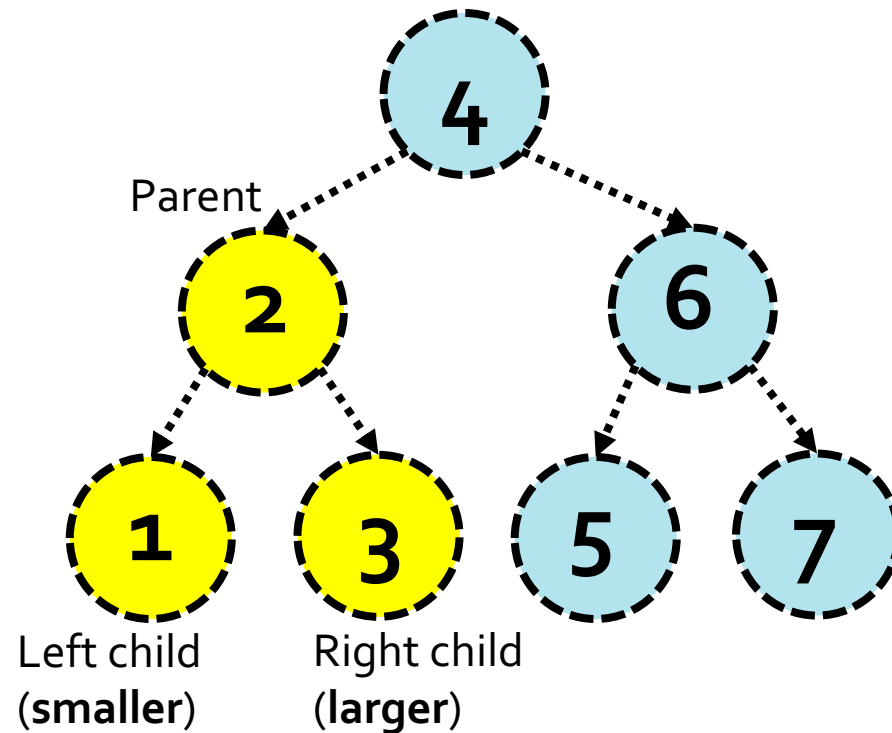
- check the highlighted nodes of this binary tree:
 - value of parent (4) **is larger than** left child (2)
 - value of parent (4) **is smaller than** right child (6)



Binary Search Tree

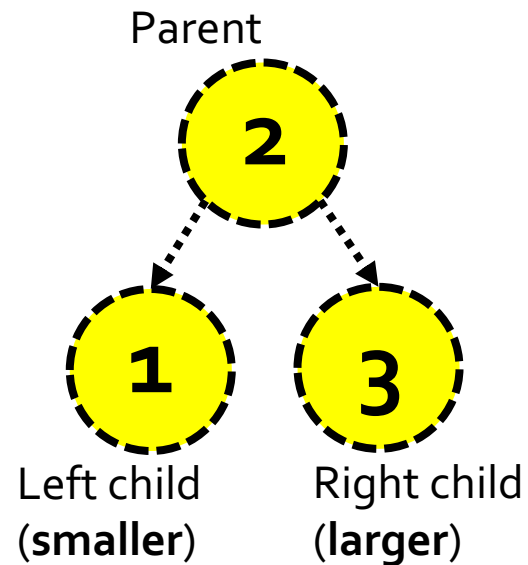
❖ Example:

- check the highlighted nodes of this binary tree:
 - value of parent (2) is **larger than** left child (1)
 - value of parent (2) is **smaller than** right child (3)



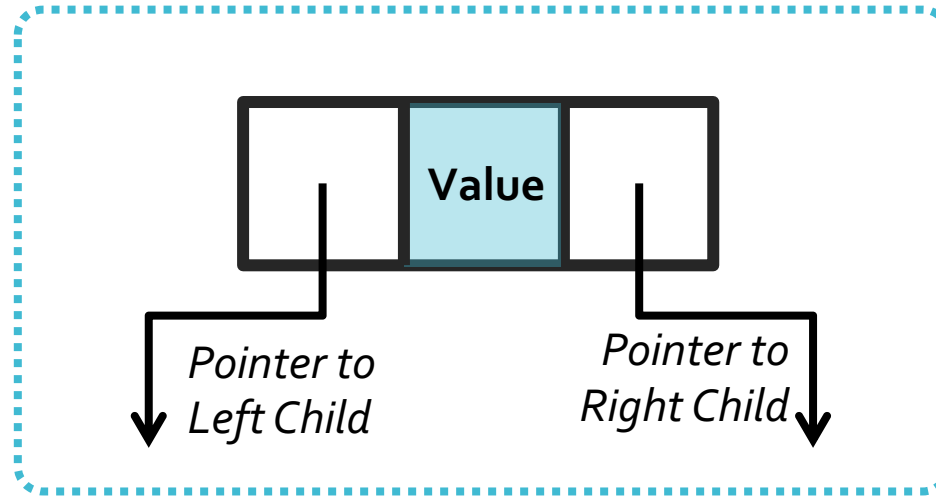
Binary Search Tree

❖ Lets check how this **Binary Search Tree** can be formed.

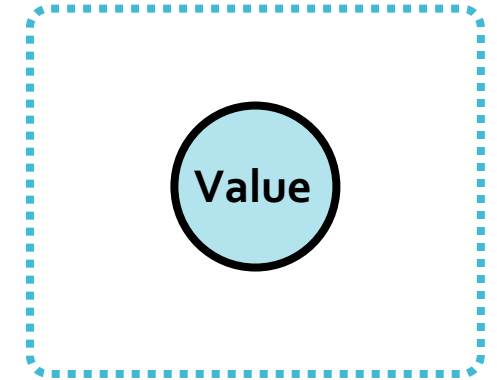


Binary Search Tree

- ❖ We **define a Node** structure, that has a value and a “pointer” to the left child & a “pointer” to right child.



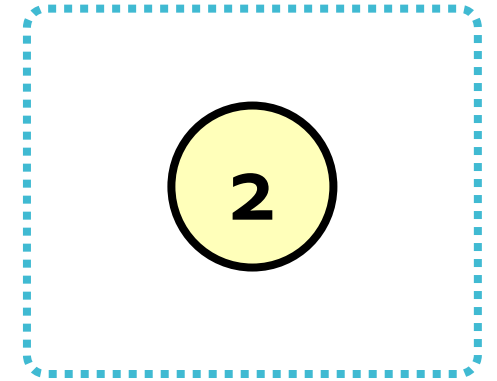
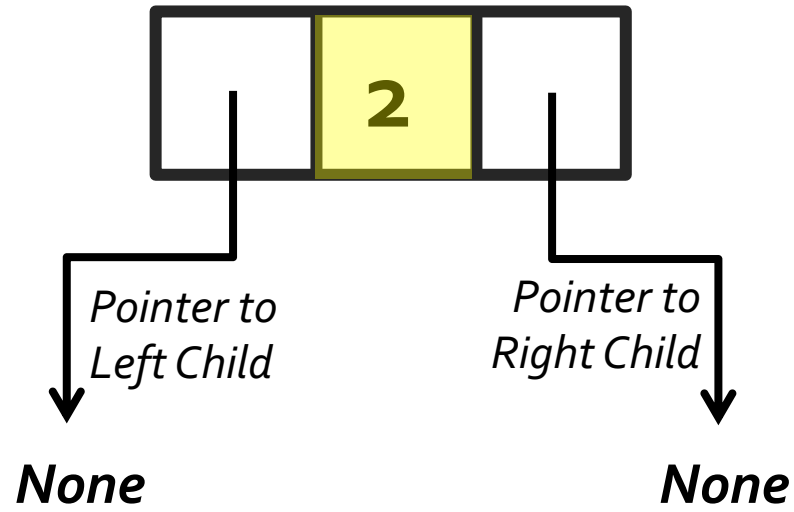
Node



Node

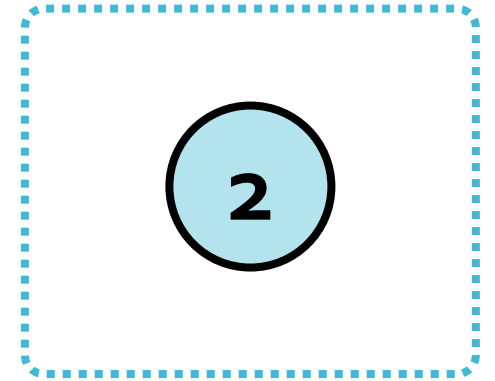
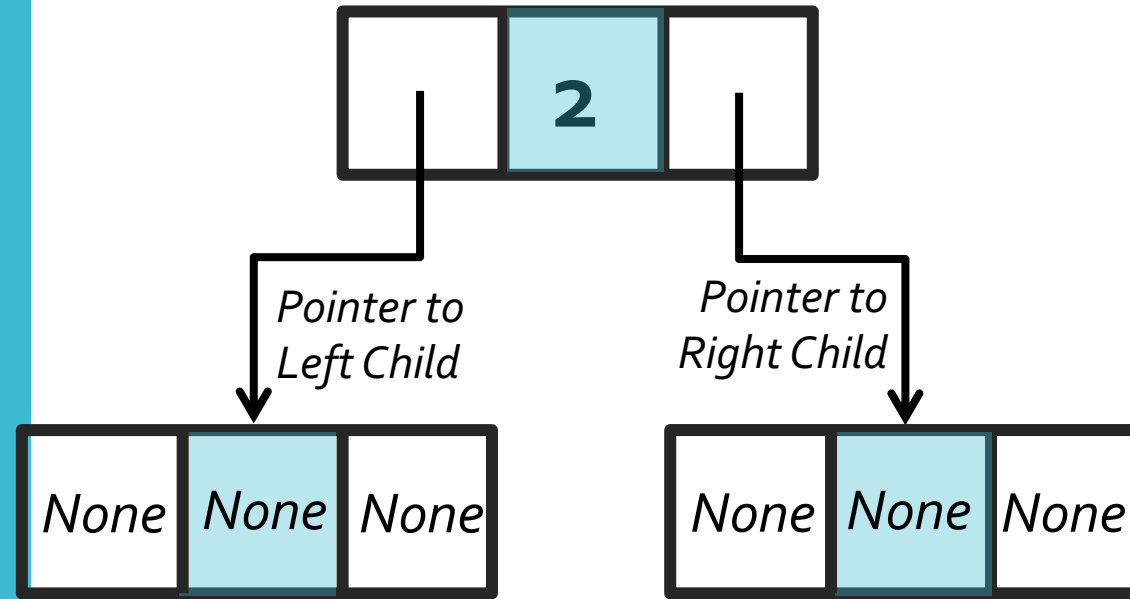
Binary Search Tree

❖ First node with **value 2** is created, but it does not have any (left or right) child node.



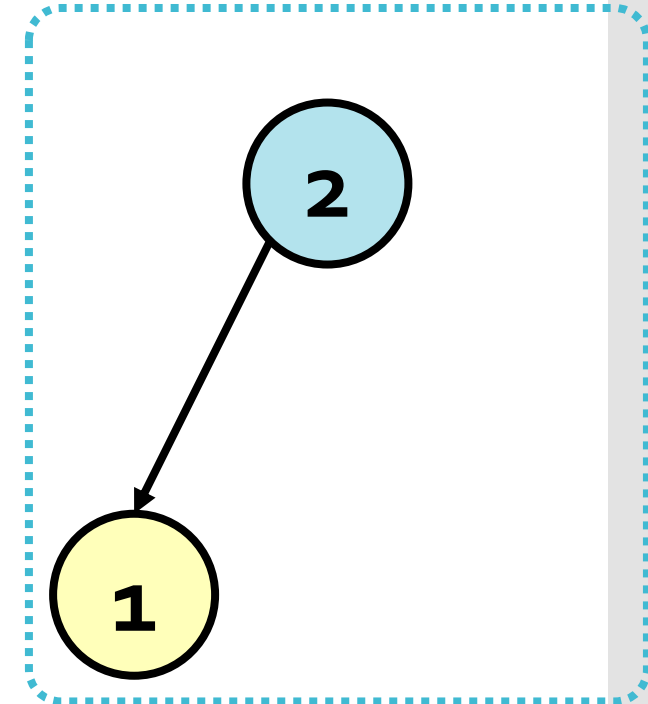
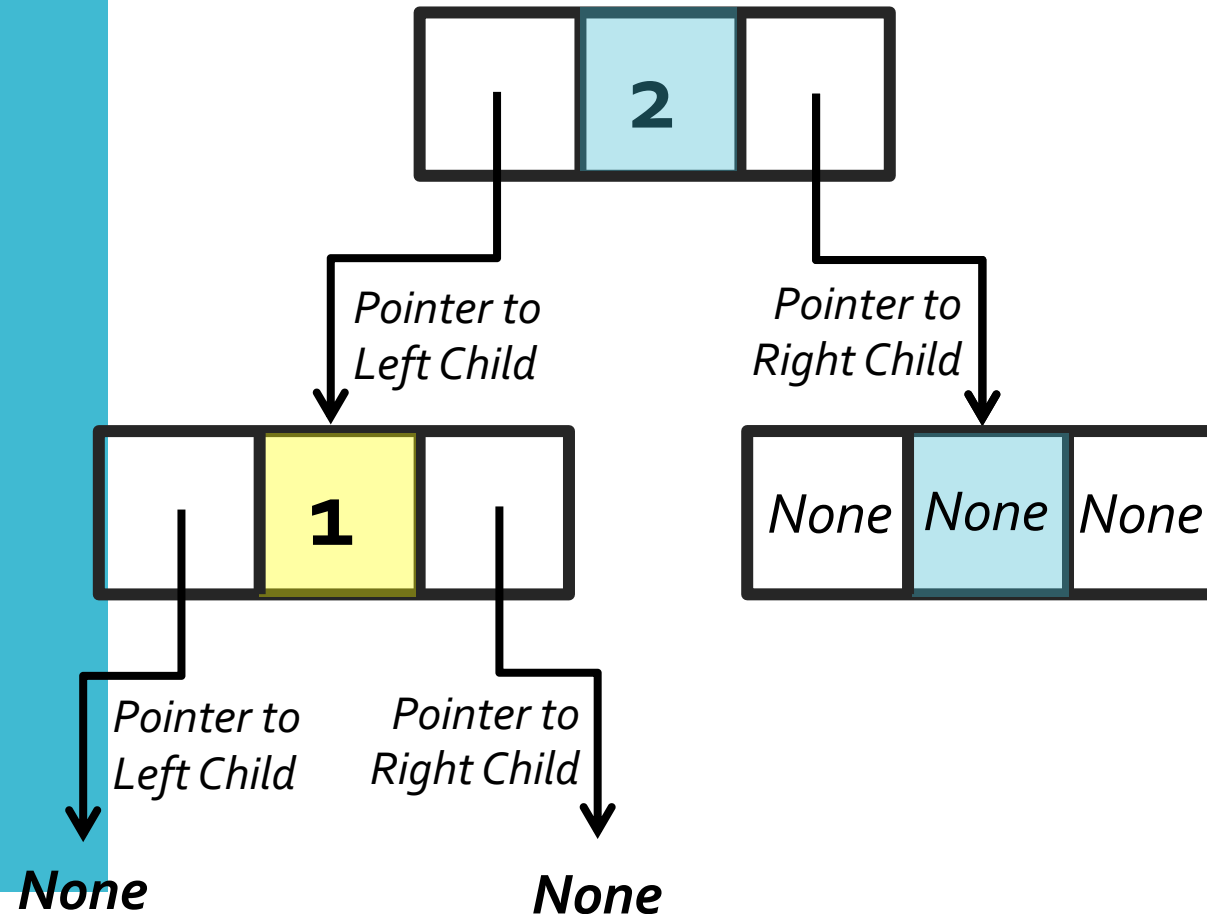
Binary Search Tree

- ❖ No child can be represented with an **empty node** where values are *None*.



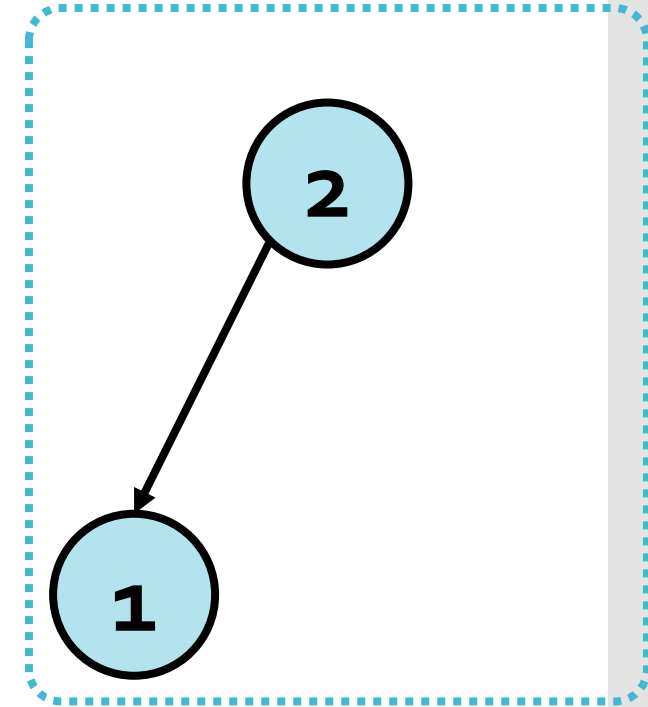
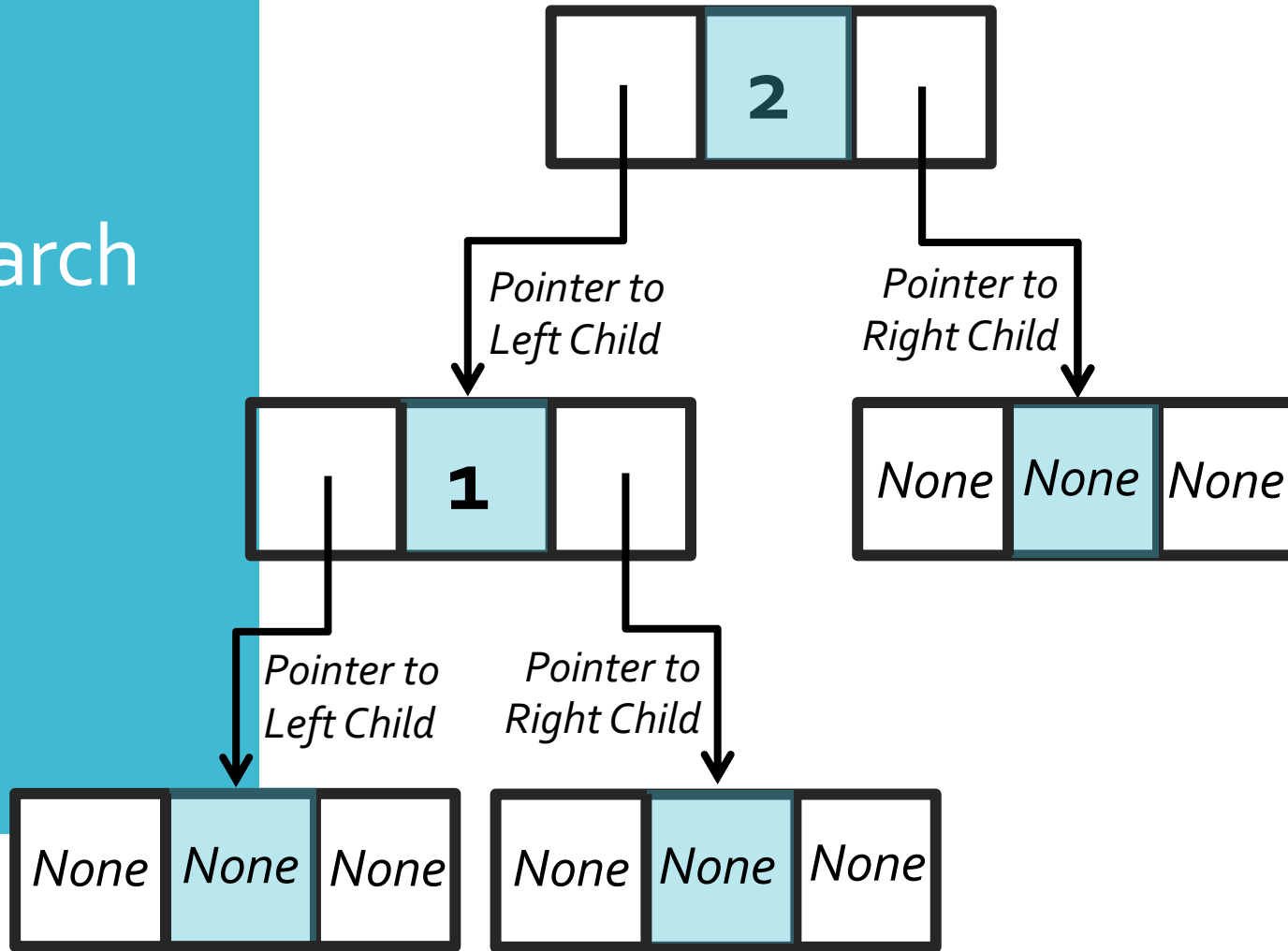
❖ Next node with **value 1** is created (to insert into the tree). This node has a value **smaller** than the first node and so, it is added as the **left child**.

Binary Search Tree



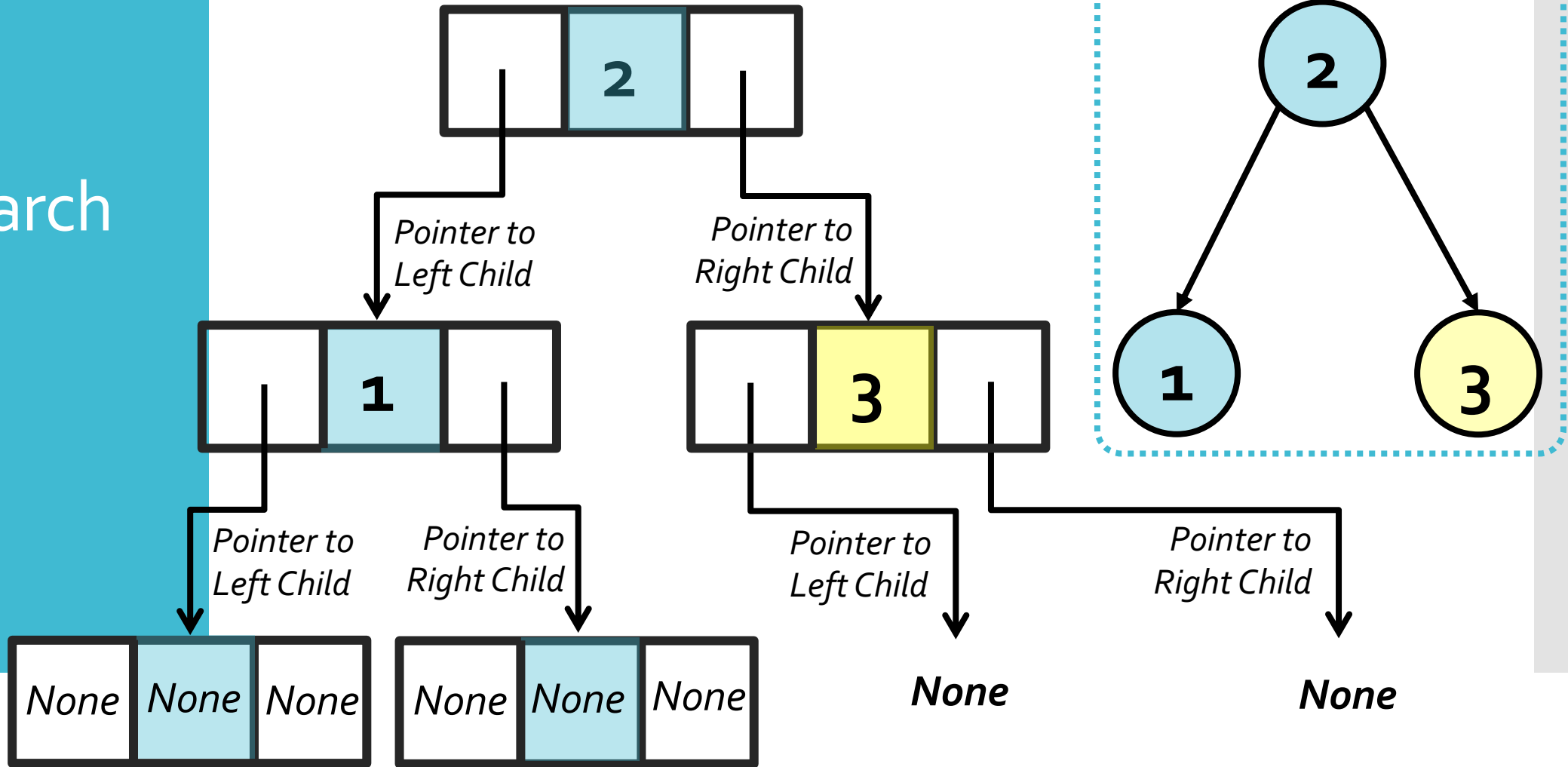
Binary Search Tree

❖ We create **empty child nodes** with values set to *None*.



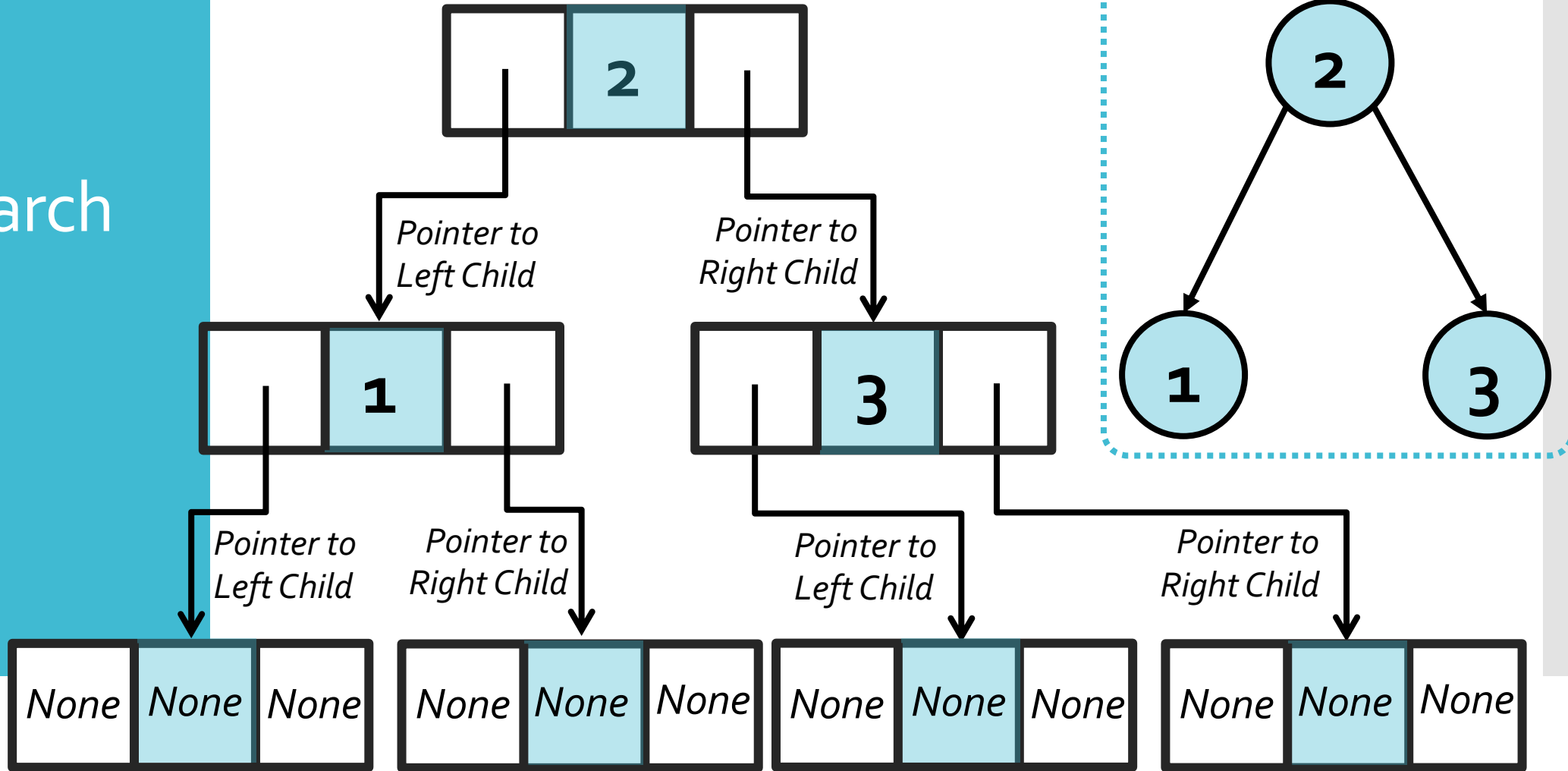
❖ The next is **value 3** & we create a node for it (to insert it into tree). This node has a value **larger** than root node (2) and so, it is added as the **right child**.

Binary Search Tree

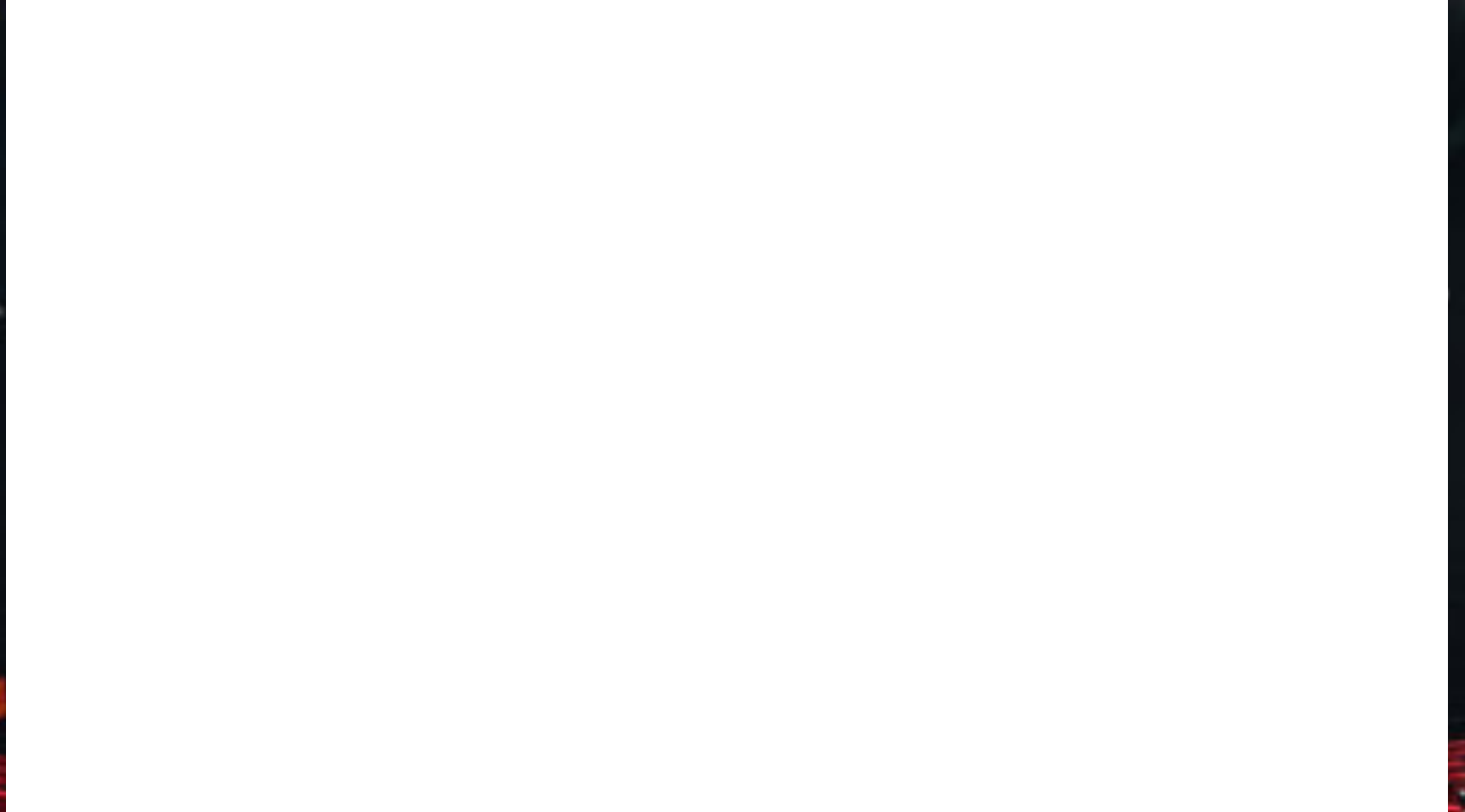


Binary Search Tree

❖ We create **empty child nodes** with values set to *None*.



Binary Search Tree



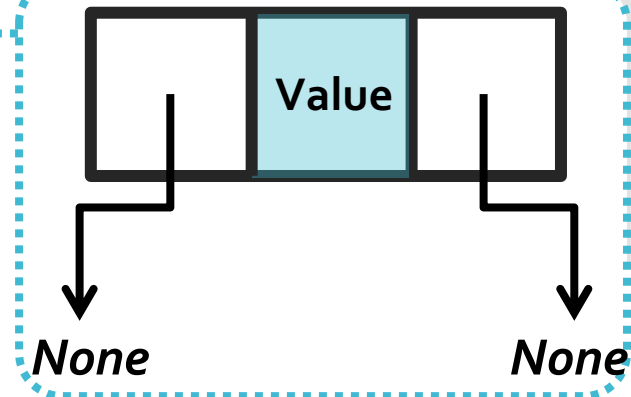
ref: youtu.be/7vw2ildqHlM

Binary Search Tree

❖ Implementing Binary Search Tree class (part 1)

```
class BinarySearchTree:

    def __init__(self, value=None):
        self.value = value
        if self.value:
            self.left_child = BinarySearchTree()
            self.right_child = BinarySearchTree()
        else:
            self.left_child = None
            self.right_child = None
```



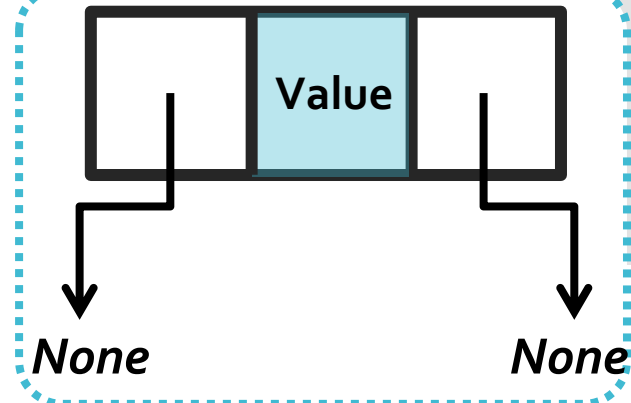
Binary Search Tree

❖ Implementing Binary Search Tree class (part 1)

```
class BinarySearchTree:
```

```
    def __init__(self, value=None):  
        self.value = value  
        if self.value:  
            self.left_child = BinarySearchTree()  
            self.right_child = BinarySearchTree()  
        else:  
            self.left_child = None  
            self.right_child = None
```

constructor



Binary Search Tree

❖ Implementing Binary Search Tree class (part 1)

```
class BinarySearchTree:
```

```
    def __init__(self, value=None):
```

```
        self.value = value
```

```
        if self.value:
```

```
            self.left_child = BinarySearchTree()
```

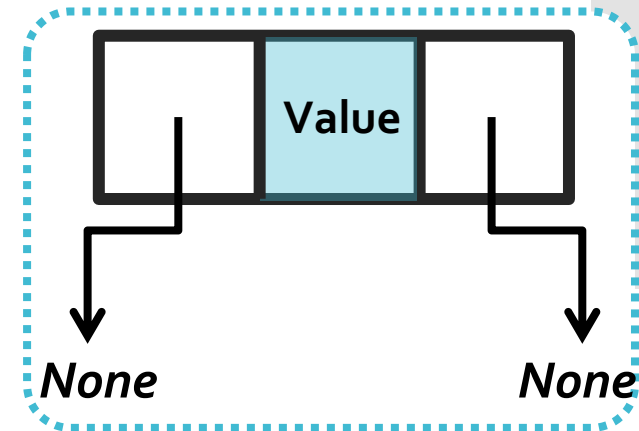
```
            self.right_child = BinarySearchTree()
```

```
        else:
```

```
            self.left_child = None
```

```
            self.right_child = None
```

Creates a node
with value



Binary Search Tree

❖ Implementing Binary Search Tree class (part 1)

```
class BinarySearchTree:
```

```
    def __init__(self, value=None
```

```
        self.value = value
```

```
        if self.value:
```

```
            self.left_child = BinarySearchTree()
```

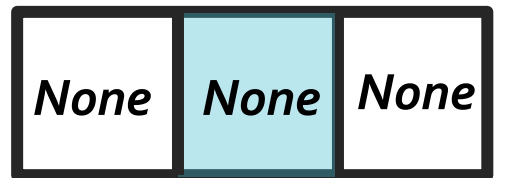
```
            self.right_child = BinarySearchTree()
```

```
        else:
```

```
            self.left_child = None
```

```
            self.right_child = None
```

Creates an
empty node



Binary Search Tree

❖ Implementing Binary Search Tree class (part 1)

```
class BinarySearchTree:
```

```
    def __init__(self, value=None):  
        self.value = value  
        if self.value:  
            self.left_child = BinarySearchTree()  
            self.right_child = BinarySearchTree()  
        else:  
            self.left_child = None  
            self.right_child = None
```

```
    def is_empty(self):  
        return self.value is None
```

checks if the
node is empty

Binary Search Tree

❖ Implementing Binary Search Tree class (part 2)

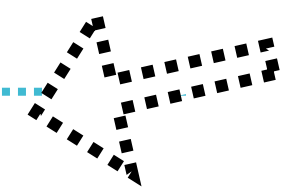
```
def insert(self, value):  
    if self.is_empty():  
        self.value = value  
        self.left_child = BinarySearchTree()  
        self.right_child = BinarySearchTree()  
  
    elif value < self.value:  
        self.left_child.insert(value)  
  
    elif value > self.value:  
        self.right_child.insert(value)
```

Binary Search Tree

❖ Implementing Binary Search Tree class (part 2)

```
def insert(self, value):  
    if self.is_empty():  
        self.value = value  
        self.left_child = BinarySearchTree()  
        self.right_child = BinarySearchTree()  
  
    elif value < self.value:  
        self.left_child.insert(value)  
  
    elif value > self.value:  
        self.right_child.insert(value)
```

if node is empty,
it adds a new
node



Binary Search Tree

❖ Implementing Binary Search Tree class (part 2)

```
def insert(self, value):  
    if self.is_empty():  
        self.value = value  
        self.left_child = BinarySearchTree()  
        self.right_child = BinarySearchTree()  
  
    elif value < self.value:  
        self.left_child.insert(value)  
  
    elif value > self.value:  
        self.right_child.insert(value)
```

if new value is less than current node, insert to left side

Binary Search Tree

❖ Implementing Binary Search Tree class (part 2)

```
def insert(self, value):  
    if self.is_empty():  
        self.value = value  
        self.left_child = BinarySearchTree()  
        self.right_child = BinarySearchTree()  
  
    elif value < self.value:  
        self.left_child.insert(value)  
  
    elif value > self.value:  
        self.right_child.insert(value)
```

if new value
is **larger** than current
node insert to right side

Binary Search Tree

❖ Implementing Binary Search Tree class (part 3)

```
def in_order(self):  
    if self.is_empty():  
        return []  
    else:  
        return self.left_child.in_order() + \  
               [self.value] + \  
               self.right_child.in_order()
```

```
def print_tree(self):  
    print(self.in_ord
```

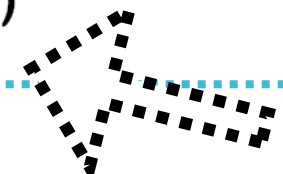
recursively
traverses the tree

Binary Search Tree

❖ Implementing Binary Search Tree class (part 3)

```
def in_order(self):  
    if self.is_empty():  
        return []  
    else:  
        return self.left_child.in_order() + \  
               [self.value] + \  
               self.right_child.in_order()
```

```
def print_tree(self):  
    print(self.in_order())
```



Prints
the nodes based
on in-order traverse

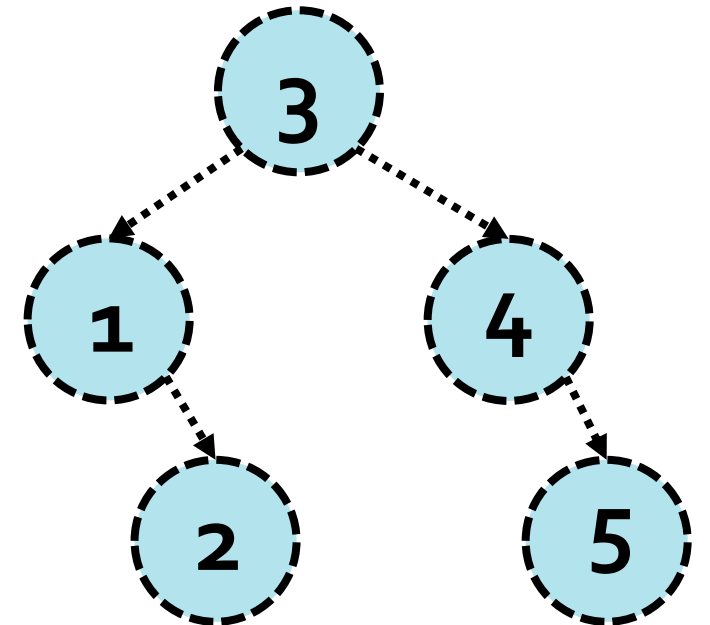
Binary Search Tree

❖ Testing the implementation.

```
my_tree = BinarySearchTree()  
my_tree.insert(3)  
my_tree.insert(1)  
my_tree.insert(4)  
my_tree.insert(2)  
my_tree.insert(5)  
  
my_tree.print_tree()
```

[Output:]

[1, 2, 3, 4, 5]



❖ Which Tree does the following code creates?



Quiz

```
my_tree = BinarySearchTree()
```

```
my_tree.insert(4)
```

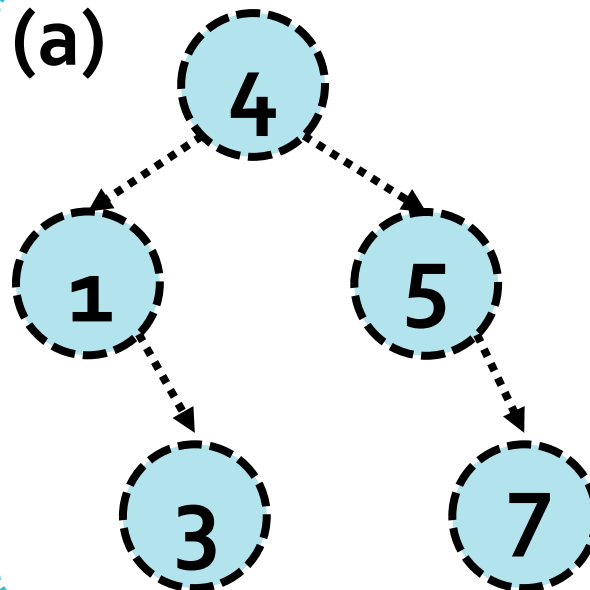
```
my_tree.insert(1)
```

```
my_tree.insert(5)
```

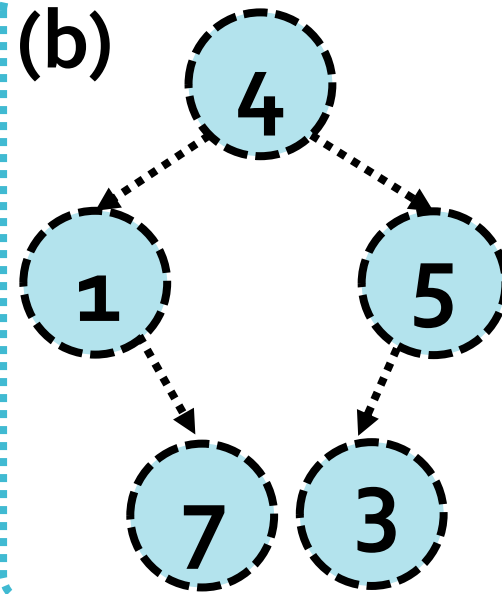
```
my_tree.insert(3)
```

```
my_tree.insert(7)
```

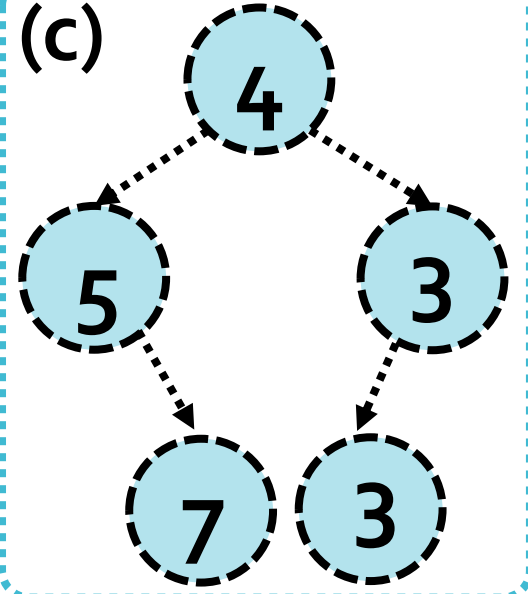
(a)



(b)



(c)



❖ Which Tree does the following code creates?



Answer

```
my_tree = BinarySearchTree()
```

```
my_tree.insert(4)
```

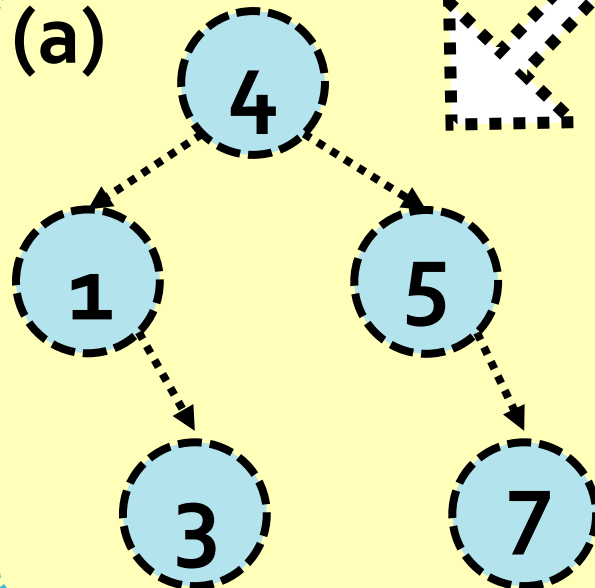
```
my_tree.insert(1)
```

```
my_tree.insert(5)
```

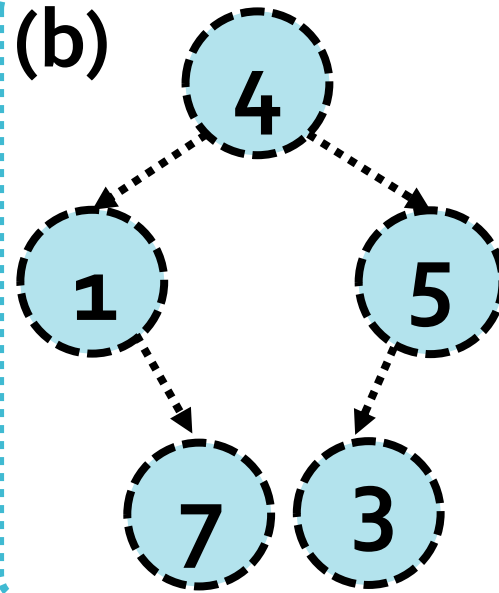
```
my_tree.insert(3)
```

```
my_tree.insert(7)
```

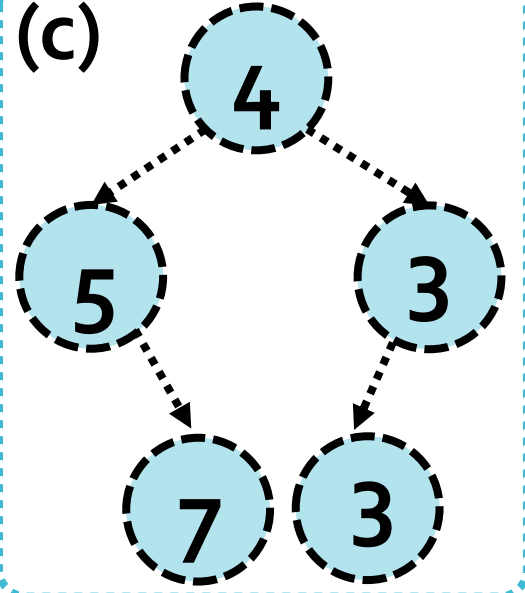
(a)



(b)



(c)



Binary Search Tree

- ❖ We need a **find(value)** method that searches the tree [-may be- recursively?] in case whether or not a value is found.
- ❖ Then we may ended up with these **conditions**:
 - ❖ we may **find** a node with matching value.
 - ❖ we may reach a non-matching **leaf node**.

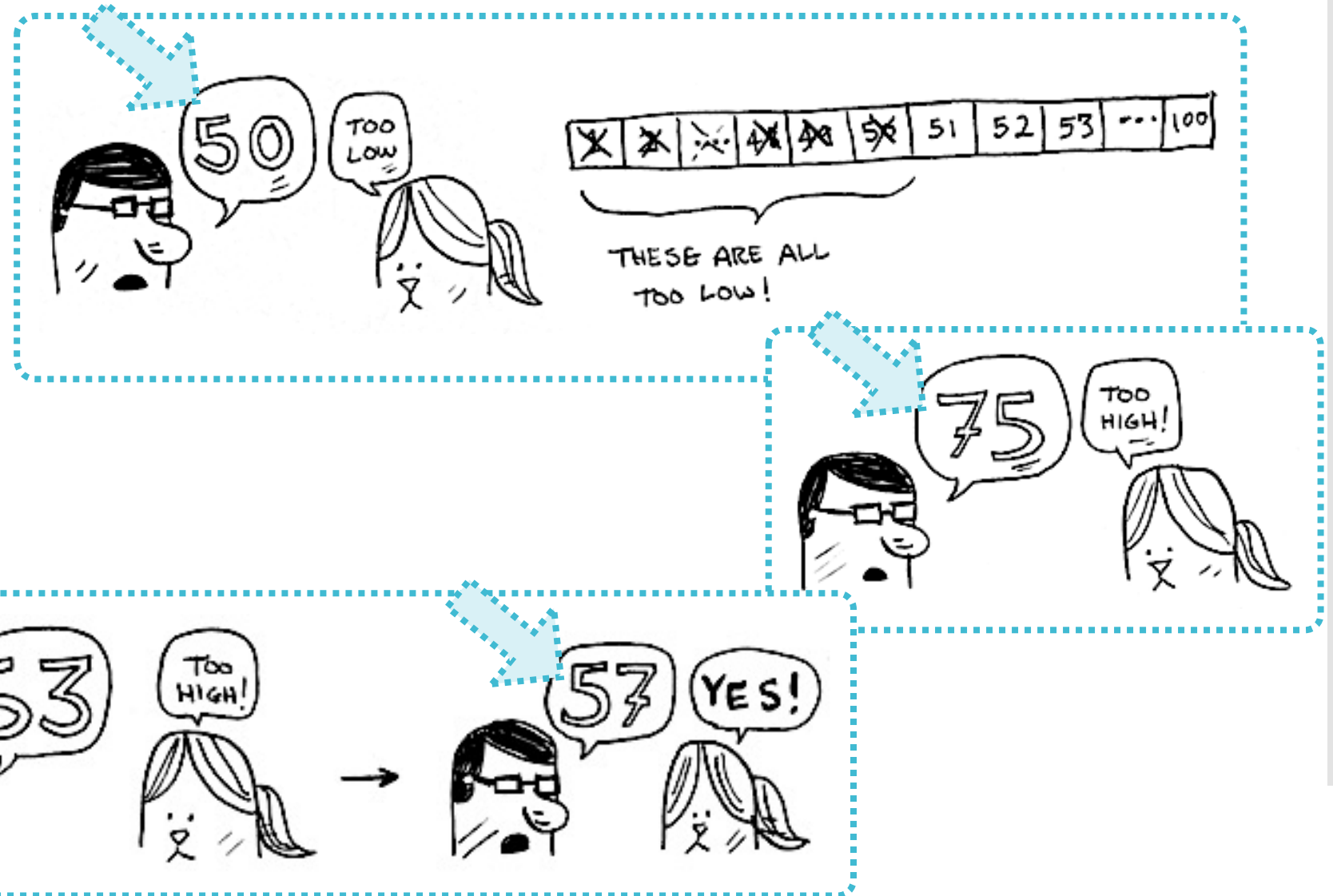
Binary Search Tree

- ❖ Suppose we are searching for value k , then we:
 - a) check the value of **current node** (v)
 - b) move to the **left** child if $k < v$
 - c) move to the **right** child if $k > v$

❖ Does it sound **familiar**?

Binary Search Tree

❖ Do you remember the game?



Binary Search Tree

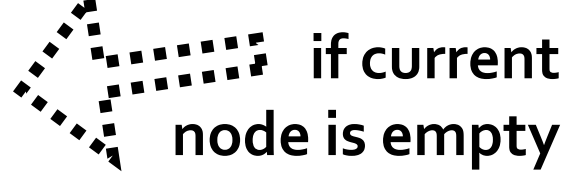
❖ Implementing **Binary Search Tree** class (part 4)

```
def find(self, value):  
    if self.is_empty():  
        return False  
  
    elif value == self.value:  
        return True  
  
    elif self.value > value:  
        return self.left_child.find(value)  
  
    elif self.value < value:  
        return self.right_child.find(value)
```


Binary Search Tree

❖ Implementing Binary Search Tree class (part 4)

```
def find(self, value):  
    if self.is_empty():  
        return False  
  
    elif value == self.value:  
        return True  
  
    elif self.value > value:  
        return self.left_child.find(value)  
  
    elif self.value < value:  
        return self.right_child.find(value)
```



Binary Search Tree

❖ Implementing Binary Search Tree class (part 4)

```
def find(self, value):  
    if self.is_empty():  
        return False
```

if current value
is what we are
searching for

```
elif value == self.value:  
    return True
```

```
elif self.value > value:  
    return self.left_child.find(value)
```

```
elif self.value < value:  
    return self.right_child.find(value)
```

Binary Search Tree

❖ Implementing Binary Search Tree class (part 4)

```
def find(self, value):  
    if self.is_empty():  
        return False  
  
    elif value == self.value:  
        return True  
  
    elif self.value > value:  
        return self.left_child.find(value)  
  
    elif self.value < value:  
        return self.right_child.find(value)
```

if current value
is larger than
search value

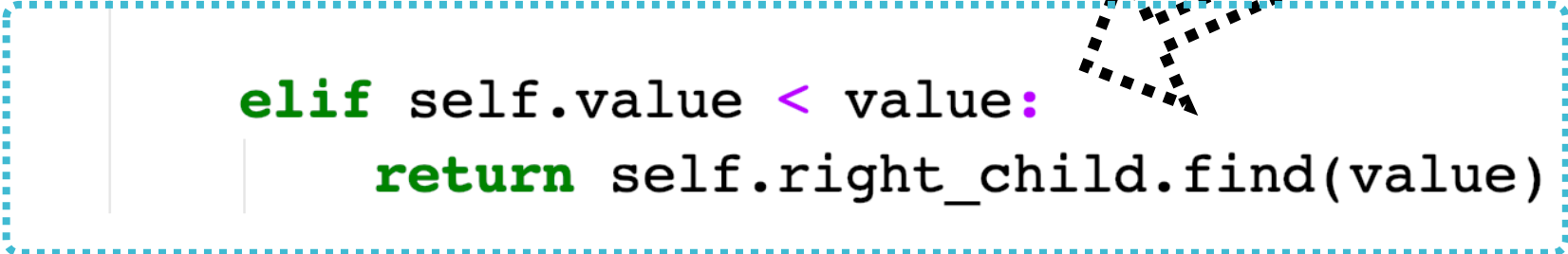


Binary Search Tree

❖ Implementing Binary Search Tree class (part 4)

```
def find(self, value):  
    if self.is_empty():  
        return False  
  
    elif value == self.value:  
        return True  
  
    elif self.value > value:  
        return self.left_child.find(value)  
  
    elif self.value < value:  
        return self.right_child.find(value)
```

if current value
is smaller than
search value



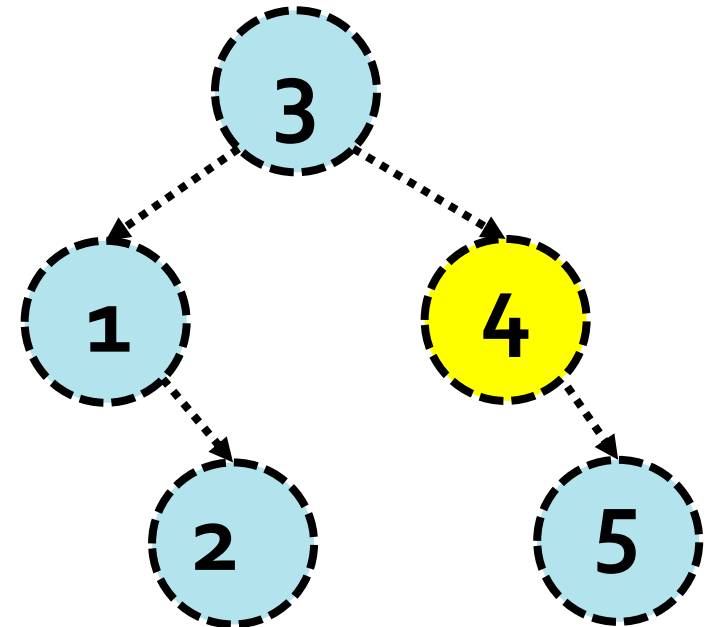
Binary Search Tree

❖ Testing the implementation.

```
my_tree = BinarySearchTree()  
my_tree.insert(3)  
my_tree.insert(1)  
my_tree.insert(4)  
my_tree.insert(2)  
my_tree.insert(5)  
  
print('Found 4?')  
print(my_tree.find(4))
```

[Output:]

Found 4?
True



Binary Search Tree

- ❖ We can extend the **Binary Search Tree** by implementing more methods.
- ❖ May be checking if we reached a **leaf node**.
- ❖ We can also implement a **copy method** for copy (for instance coping a child node).
- ❖ And also a **delete** method.

Binary Search Tree

❖ Implementing Binary Search Tree class (part 5)

```
def is_leaf(self):  
    return self.left_child.is_empty() and \  
           self.right_child.is_empty()  
  
def make_empty(self):  
    self.value = None  
    self.left_child = None  
    self.right_child = None  
  
def copy_child(self, child):  
    if child == 'left':  
        self.value = self.left_child.value  
        self.right_child = self.left_child.right_child  
        self.left_child = self.left_child.left_child  
    elif child == 'right':  
        self.value = self.right_child.value  
        self.left_child = self.right_child.left_child  
        self.right_child = self.right_child.right_child
```

Binary Search Tree

❖ Implementing Binary Search Tree class (part 5)

```
def is_leaf(self):  
    return self.left_child.is_empty() and \  
           self.right_child.is_empty()
```

```
def make_empty(self):  
    self.value = None  
    self.left_child = None  
    self.right_child = None
```

But why?
We will need it later!

```
def copy_child(self, child):  
    if child == 'left':  
        self.value = self.left_child.value  
        self.right_child = self.left_child.right_child  
        self.left_child = self.left_child.left_child  
    elif child == 'right':  
        self.value = self.right_child.value  
        self.left_child = self.right_child.left_child  
        self.right_child = self.right_child.right_child
```


Binary Search Tree

❖ Implementing Binary Search Tree class (part 5)

```
def is_leaf(self):  
    return self.left_child.is_empty() and \  
           self.right_child.is_empty()
```

```
def make_empty(self):  
    self.value = None  
    self.left_child = None  
    self.right_child = None
```

```
def copy_child(self, child):  
    if child == 'left':  
        self.value = self.left_child.value  
        self.right_child = self.left_child.right_child  
        self.left_child = self.left_child.left_child  
    elif child == 'right':  
        self.value = self.right_child.value  
        self.left_child = self.right_child.left_child  
        self.right_child = self.right_child.right_child
```

Binary Search Tree

❖ Implementing Binary Search Tree class (part 6)

```
def delete(self, value):  
    if self.is_empty():  
        print('Binary tree is empty.')  
  
    elif value < self.value:  
        self.left_child.delete(value)  
  
    elif value > self.value:  
        self.right_child.delete(value)  
  
    elif value == self.value:  
        if self.is_leaf():  
            self.make_empty()  
        elif self.left_child.is_empty():  
            self.copy_child('right')  
        else:  
            self.value = self.left_child.delete_max()
```

Binary Search Tree

❖ Implementing Binary Search Tree class (part 6)

```
def delete(self, value):  
    if self.is_empty():  
        print('Binary tree is empty.')  
    elif value < self.value:  
        self.left_child.delete(value)  
  
    elif value > self.value:  
        self.right_child.delete(value)  
  
    elif value == self.value:  
        if self.is_leaf():  
            self.make_empty()  
        elif self.left_child.is_empty():  
            self.copy_child('right')  
        else:  
            self.value = self.left_child.delete_max()
```

Binary Search Tree

❖ Implementing Binary Search Tree class (part 6)

```
def delete(self, value):  
    if self.is_empty():  
        print('Binary tree is empty.')  
  
    elif value < self.value:  
        self.left_child.delete(value)  
  
    elif value > self.value:  
        self.right_child.delete(value)  
  
    elif value == self.value:  
        if self.is_leaf():  
            self.make_empty()  
        elif self.left_child.is_empty():  
            self.copy_child('right')  
        else:  
            self.value = self.left_child.delete_max()
```

Binary Search Tree

❖ Implementing Binary Search Tree class (part 6)

```
def delete(self, value):  
    if self.is_empty():  
        print('Binary tree is empty.')  
  
    elif value < self.value:  
        self.left_child.delete(value)  
  
    elif value > self.value:  
        self.right_child.delete(value)  
  
    elif value == self.value:  
        if self.is_leaf():  
            self.make_empty()  
        elif self.left_child.is_empty():  
            self.copy_child('right')  
        else:  
            self.value = self.left_child.delete_max()
```

Binary Search Tree

❖ Implementing Binary Search Tree class (part 7)

```
def delete_max(self):  
    if self.right_child.is_empty():  
        max_val = self.value  
        if self.left_child.is_empty():  
            self.make_empty()  
        else:  
            self.copy_child('left')  
        return max_val  
    else:  
        return self.right_child.delete_max()
```

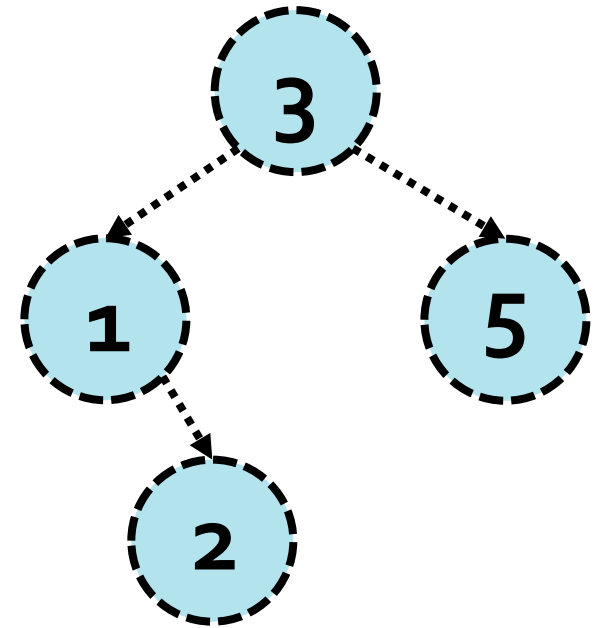
Binary Search Tree

❖ Testing the implementation.

```
my_tree.delete(4)
my_tree.print_tree()

print('Found 4?')
print(my_tree.find(4))
```

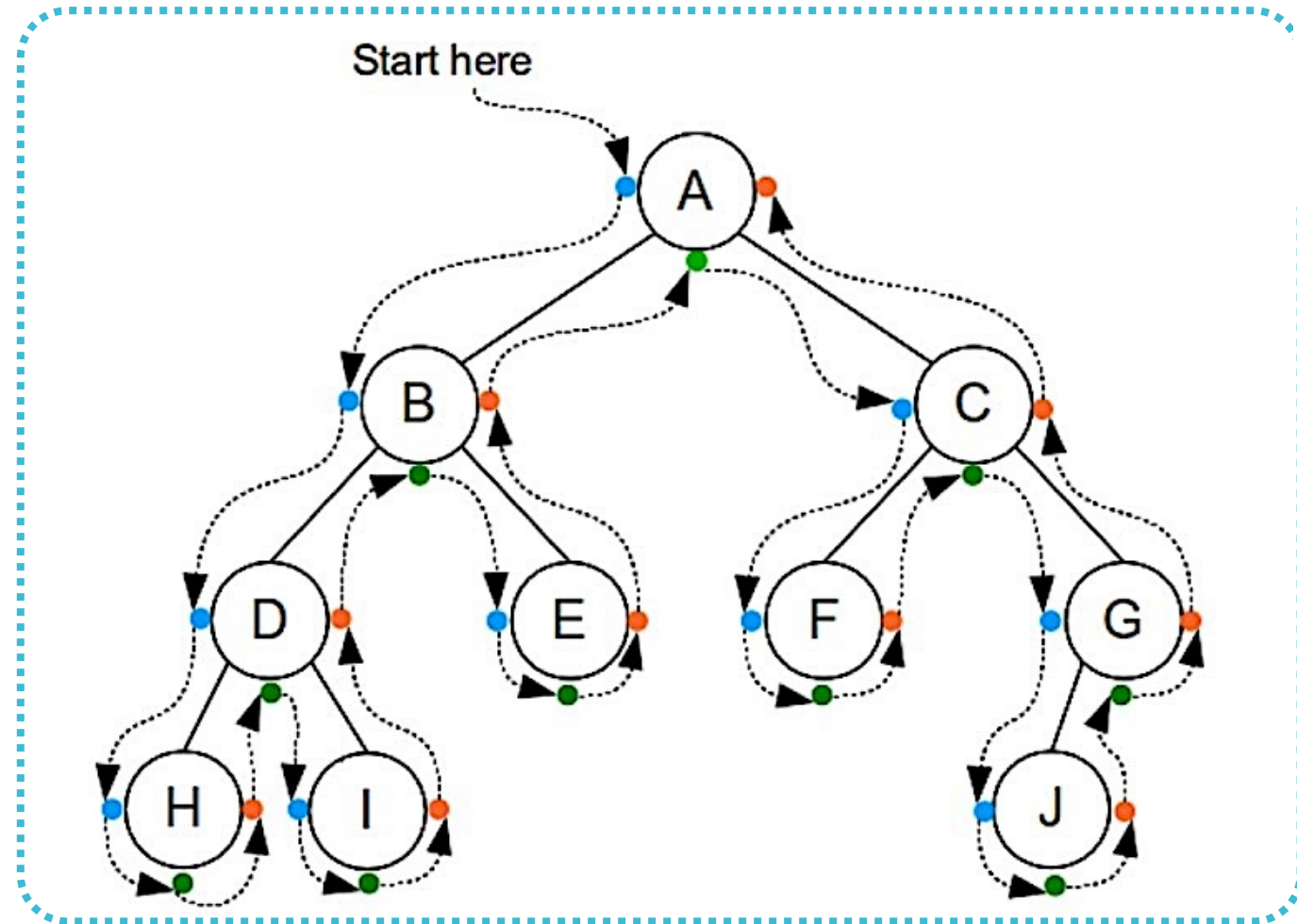
[Output:] [1, 2, 3, 5]
Found 4?
False



Preorder, Inorder, Postorder

- ❖ This implementation has one traverse method, i.e., **inorder**!
- ❖ **Inorder** recursively do a (inorder) traversal on the left subtree, then visit the root node, and finally do a recursive (inorder) traversal of the right subtree.
- ❖ But we can have **more** ways of traverse!

Quiz

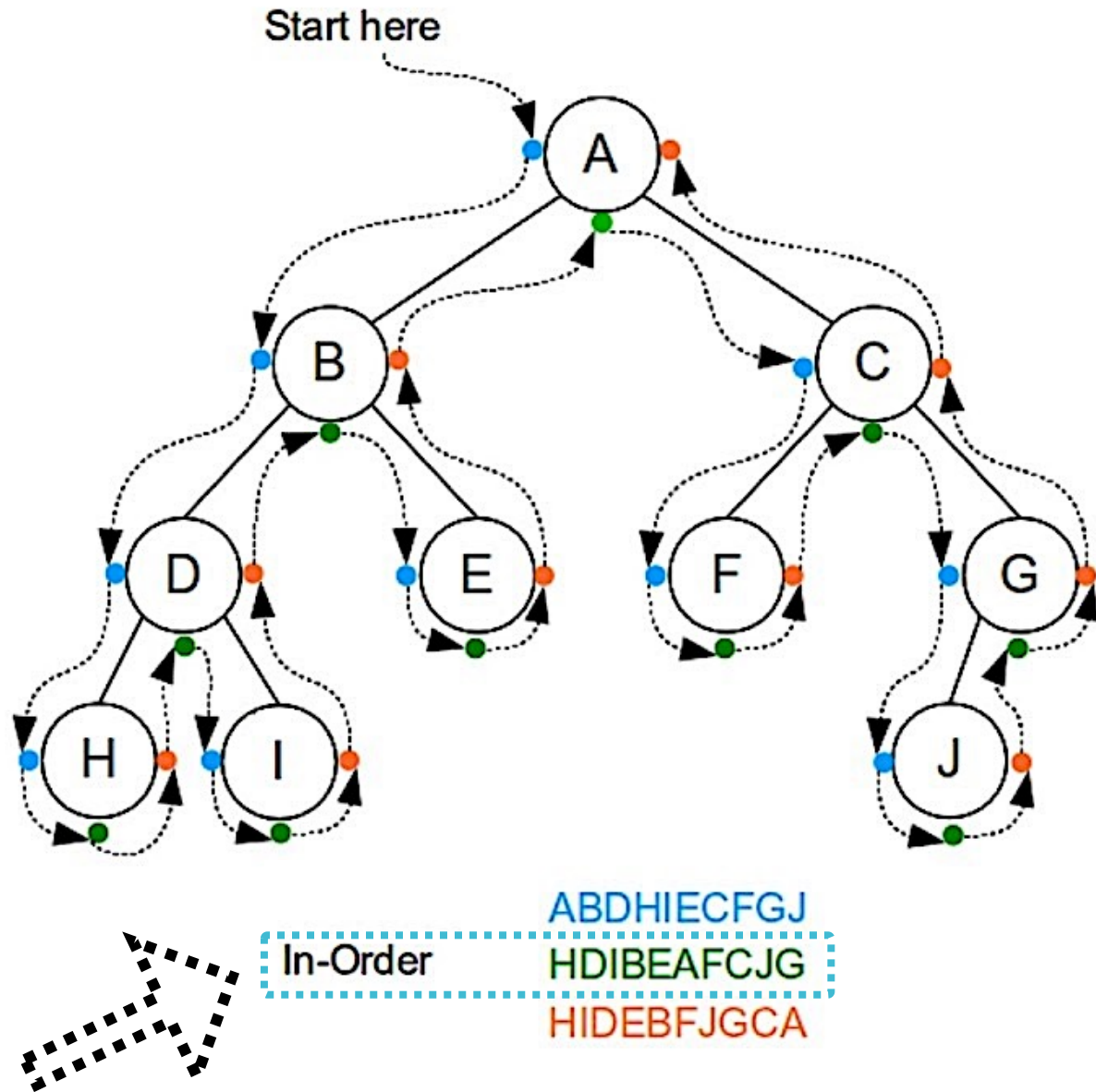


ABDHIECFGJ
HDIBEAFCJG
HIDEBFJGCA

?

Question: Which Color indicates In-order traverse?

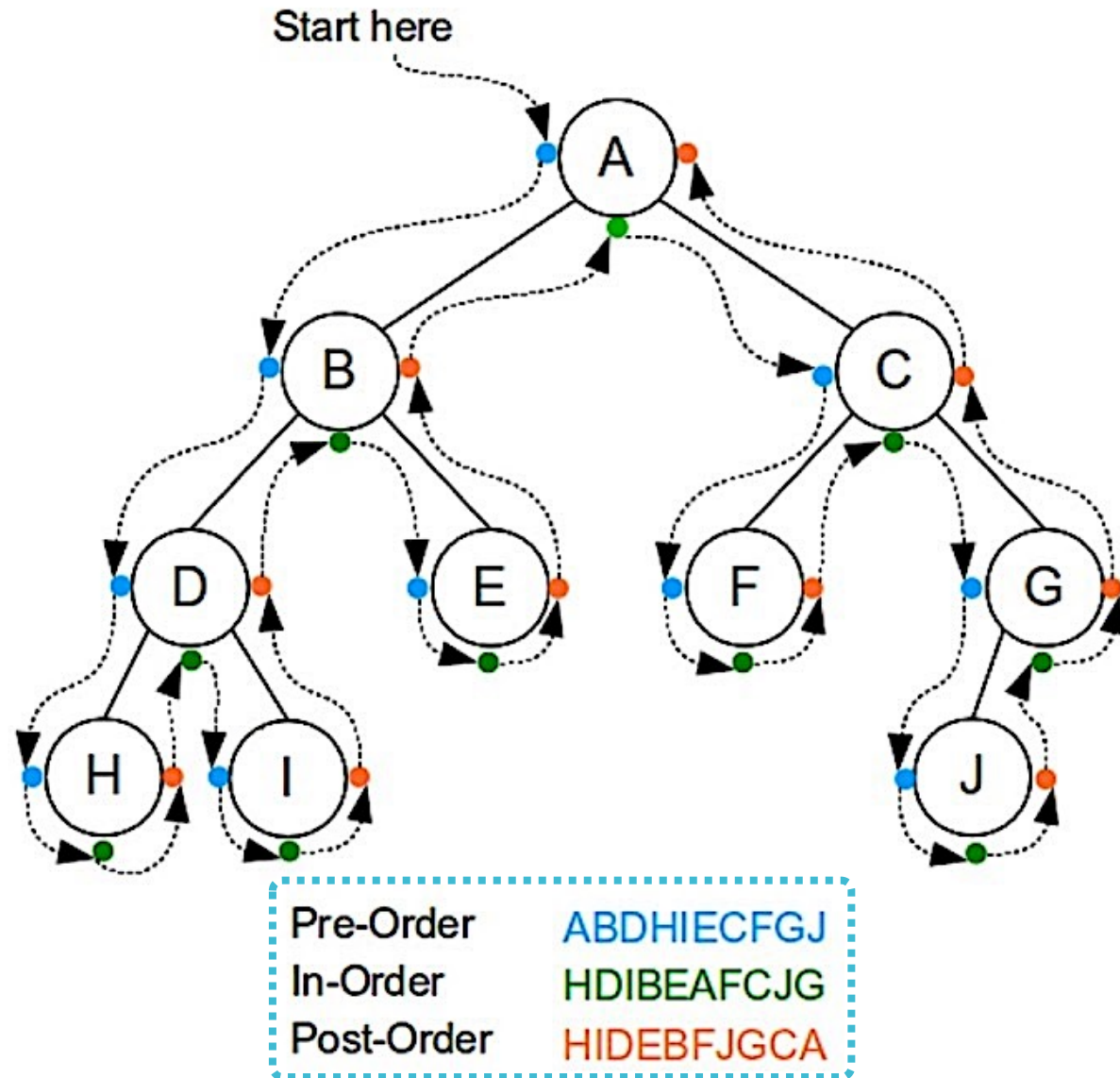
Answer



But what about the **other** colors?



Answer



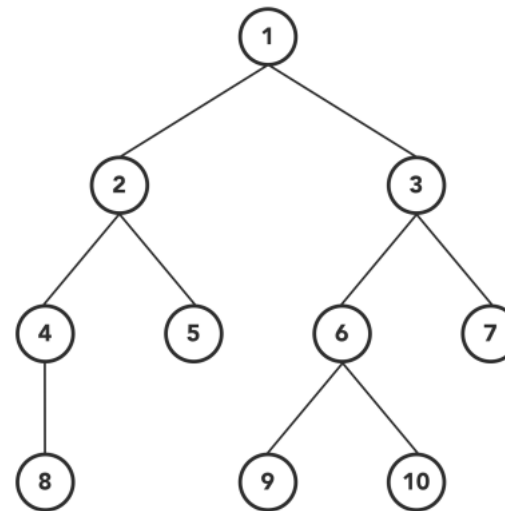
Preorder, Inorder, Postorder

- ❖ **Preorder:** visits the root node first, then recursively do a preorder traversal of the left subtree, followed by a recursive preorder traversal of the right subtree.
- ❖ **Postorder:** recursively do a postorder traversal of the left subtree and the right subtree followed by a visit to the root node.
- ❖ Now lets see them **traversing** an example tree!

Preorder, Inorder, Postorder

❖ Preorder:

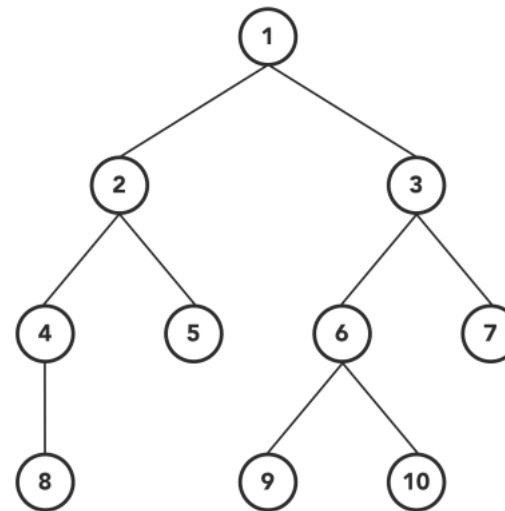
- ❖ visit root node,
- ❖ go to left-subtree,
- ❖ go to right-subtree



Preorder, Inorder, Postorder

❖ Inorder:

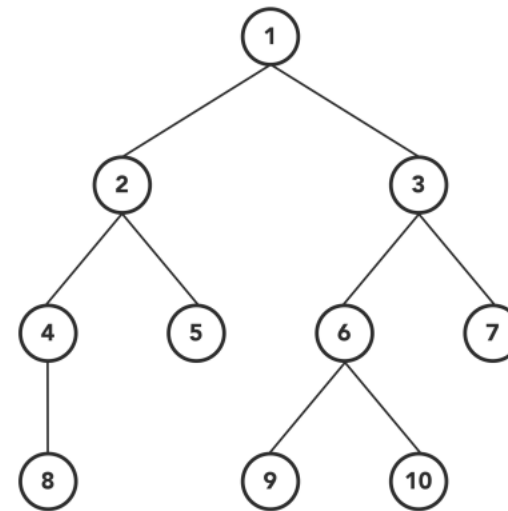
- ❖ go to left-subtree,
- ❖ visit root node,
- ❖ go to right-subtree



Preorder, Inorder, Postorder

❖ Postorder:

- ❖ go to left-subtree,
- ❖ go to right-subtree,
- ❖ visit root node

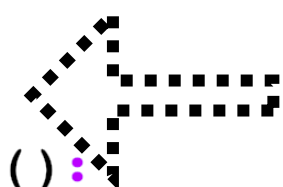


Preorder,
Inorder,
Postorder

❖ Implementation of Preorder and Postorder

```
class BinarySearchTree:
```

```
    def pre_order(self):  
        if self.is_empty():  
            return []  
        else:  
            return [self.value] + \  
                self.left_child.pre_order() + \  
                self.right_child.pre_order()
```



Preorder,
Inorder,
Postorder

❖ Implementation of Preorder and Postorder

```
class BinarySearchTree:
```

```
    def pre_order(self):
```

```
        if self.is_empty():
```

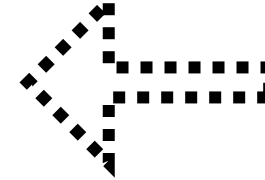
```
            return []
```

```
        else:
```

```
            return [self.value] + \
```

```
                self.left_child.pre_order() + \
```

```
                self.right_child.pre_order()
```



Preorder,
Inorder,
Postorder

❖ Implementation of Preorder and Postorder

```
class BinarySearchTree:
```

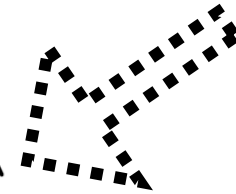
```
    def pre_order(self):
```

```
        if self.is_empty():
```

```
            return []
```

```
        else:
```

```
            return [self.value] + \
                    self.left_child.pre_order() + \
                    self.right_child.pre_order()
```



Preorder,
Inorder,
Postorder

❖ Implementation of Preorder and Postorder

```
class BinarySearchTree:
```

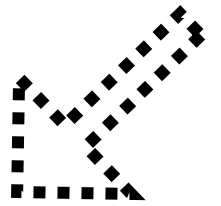
```
    def pre_order(self):
```

```
        if self.is_empty():
```

```
            return []
```

```
        else:
```

```
            return [self.value] + \
                    self.left_child.pre_order() + \
                    self.right_child.pre_order()
```



Preorder,
Inorder,
Postorder

❖ Implementation of Preorder and Postorder

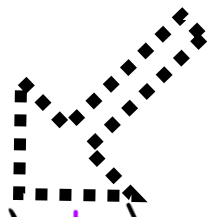
```
class BinarySearchTree:

    def pre_order(self):
        if self.is_empty():
            return []
        else:
            return [self.value] + \
                self.left_child.pre_order() + \
                self.right_child.pre_order()
```

❖ Implementation of Preorder and Postorder

Preorder,
Inorder,
Postorder

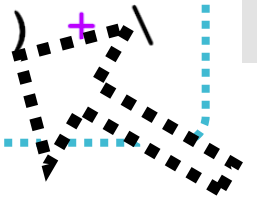
```
def post_order(self):  
    if self.is_empty():  
        return []  
    else:  
        return self.left_child.post_order() + \  
               self.right_child.post_order() + \  
               [self.value]
```



❖ Implementation of Preorder and Postorder

Preorder,
Inorder,
Postorder

```
def post_order(self):  
    if self.is_empty():  
        return []  
    else:  
        return self.left_child.post_order() + \  
               self.right_child.post_order() + \  
               [self.value]
```



❖ Implementation of Preorder and Postorder

Preorder,
Inorder,
Postorder

```
def post_order(self):  
    if self.is_empty():  
        return []  
    else:  
        return self.left_child.post_order() + \  
               self.right_child.post_order() + \  
               [self.value]
```

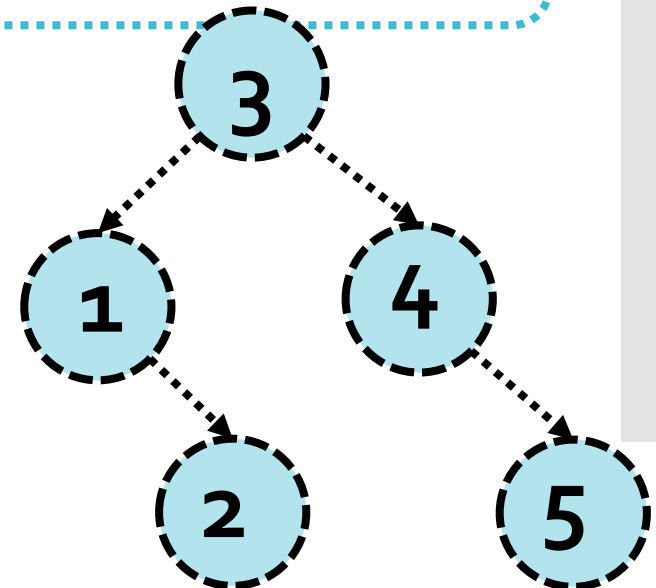
❖ Testing our implement

Preorder,
Inorder,
Postorder

```
my_tree = BinarySearchTree()  
my_tree.insert(3)  
my_tree.insert(1)  
my_tree.insert(4)  
my_tree.insert(2)  
my_tree.insert(5)  
  
print("Pre-order traversal:", my_tree.pre_order())  
print("In-order traversal:", my_tree.in_order())  
print("Post-order traversal:", my_tree.post_order())
```

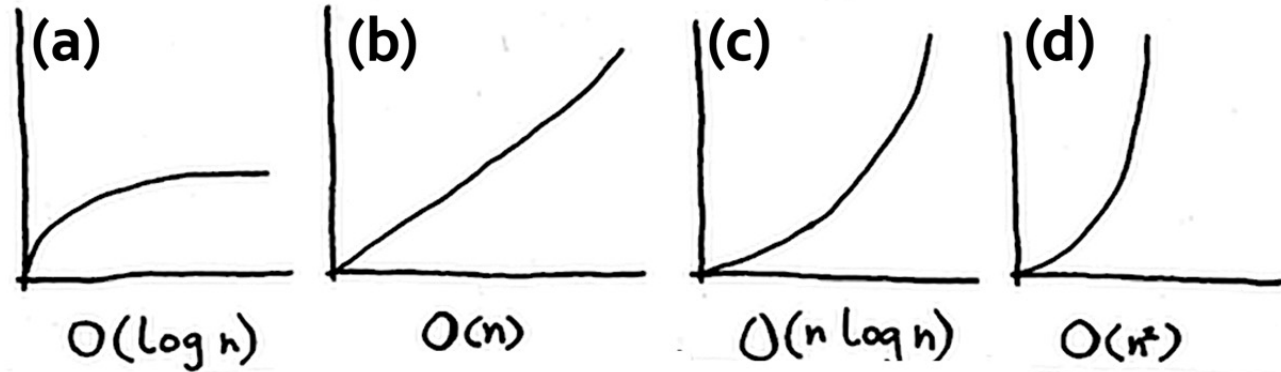
[Output:]

```
Pre-order traversal: [3, 1, 2, 4, 5]  
In-order traversal: [1, 2, 3, 4, 5]  
Post-order traversal: [2, 1, 5, 4, 3]
```



Quiz

- 1) Which of the images represent the Big O notation for **Preorder**, **Inorder**, **Postorder**?

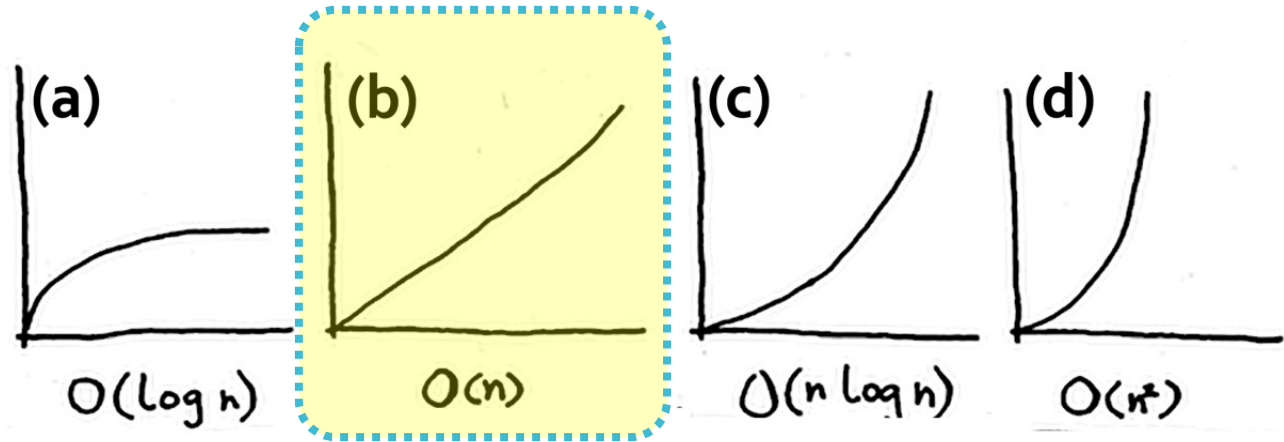


- 2) Which of the the tree traverse could be better:
- for making a **copy of a tree**
 - for **deleting a tree** (from leaf to root)



Answer

- 1) Which of the images represent the Big O notation for **Preorder**, **Inorder**, **Postorder**?



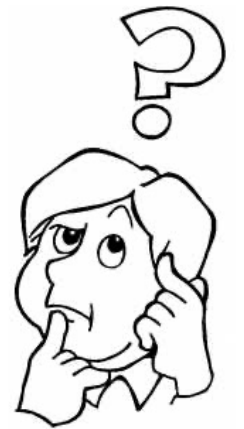
- Because we traverse each node **only once**.



Answer

❖ Which of the the tree traverse could be better:

- for making a **copy of a tree** --> Preorder
- for **deleting a tree** (from leaf to root) --> Postorder



Greedy Algorithms

- ❖ **Greedy Algorithms** follow an effective **strategy** to solve problems.
- ❖ **Greedy Algorithms:** make choices that seem to be the best solution at a moment (i.e., locally-optimal) hoping that it will lead to globally-optimal solution.
- ❖ If there is a (objective) function that **needs to be optimized** (such as cost to be minimized):
 - ❖ Greedy algorithm makes choices step-by-step to ensure that the objective function is optimized.

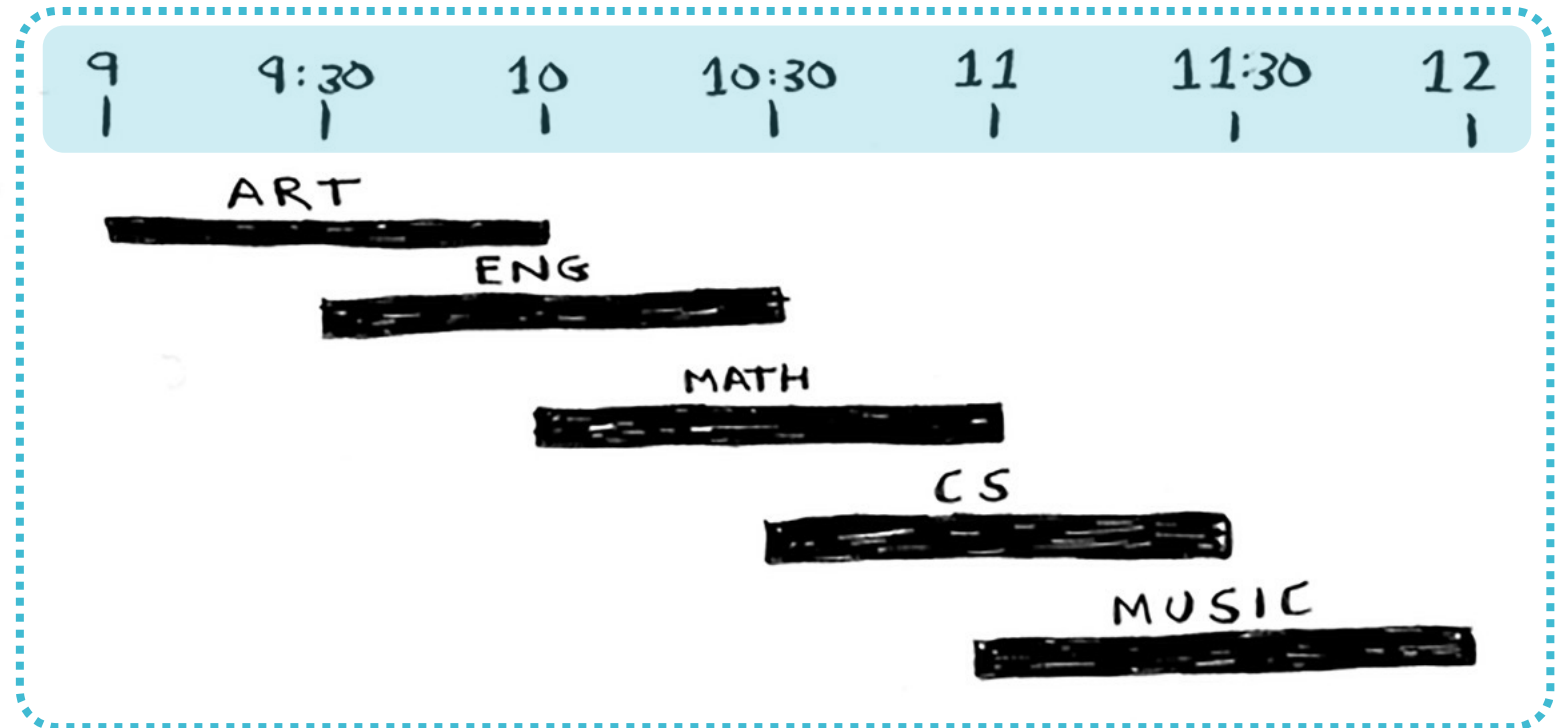
Greedy Algorithms

- ❖ Suppose you have a classroom and want to hold as **many classes here as possible**.
- ❖ This is the list of possible classes.

CLASS	START	END
ART	9 AM	10 AM
ENG	9:30 AM	10:30 AM
MATH	10 AM	11 AM
CS	10:30 AM	11:30 AM
MUSIC	11 AM	12 PM

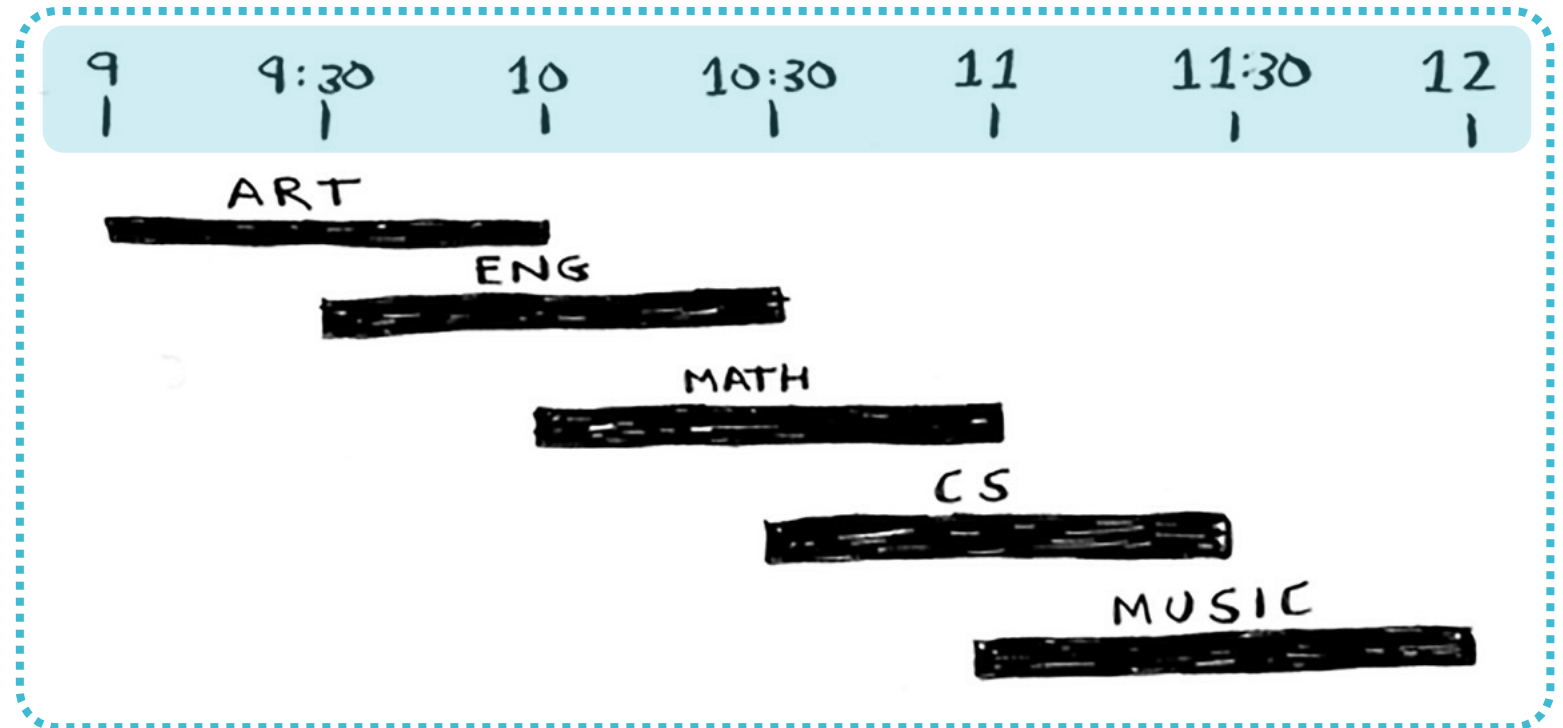
Greedy Algorithms

❖ You **can't hold all** of these classes in there, because some of them overlap.



Greedy Algorithms

❖ How do you pick what set of classes to hold, so that you get **the biggest set of classes** possible?



Greedy Algorithms

❖ The **algorithm** is the following:

1. First, pick the class that **ends the soonest**. This is the first class you'll hold.

Art ends the soonest (at 10:00 am) and we **pick it**.

CLASS	START	END
ART	9AM	10AM
ENG	4:30AM	10:30AM
MATH	10AM	11AM
CS	10:30AM	11:30AM
MUSIC	11AM	12PM

Greedy Algorithms

❖ The algorithm is the following:

2. Then, pick a class that **starts after the first class** and ends the **soonest**. This is the second class you'll hold.

English is out because it conflicts with Art, but Math works.

CLASS	START	END	
ART	9AM	10AM	✓
ENG	4:30AM	10:30AM	X
MATH	10AM	11AM	✓
CS	10:30AM	11:30AM	
MUSIC	11AM	12PM	

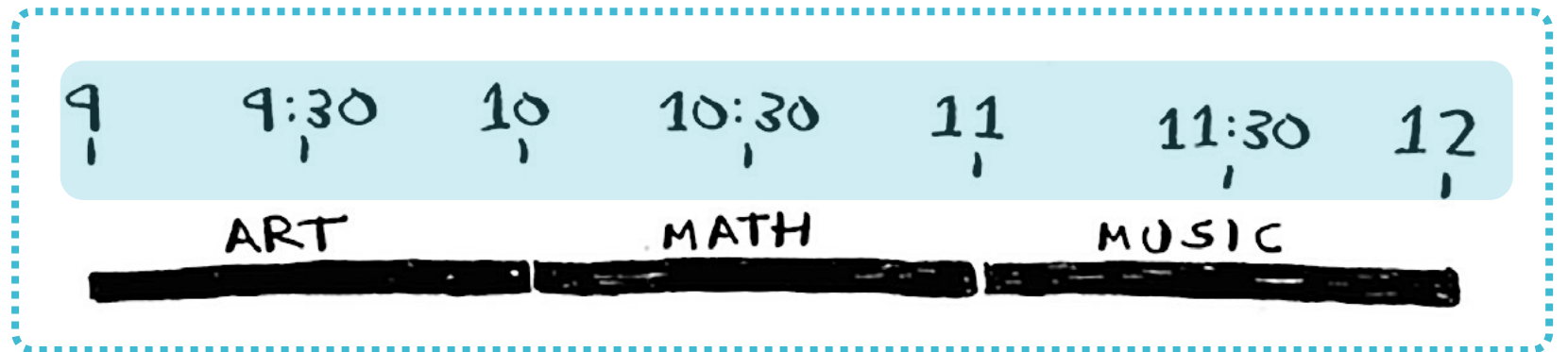
Greedy Algorithms

- ❖ The algorithm is the following:
 3. Finally, CS conflicts with Math, but Music works.

CLASS	START	END	
ART	9AM	10AM	✓
ENG	4:30AM	10:30AM	X
MATH	10AM	11AM	✓
CS	10:30AM	11:30AM	X
MUSIC	11AM	12PM	✓

Greedy Algorithms

❖ So these are the **three classes** you'll hold in this classroom.



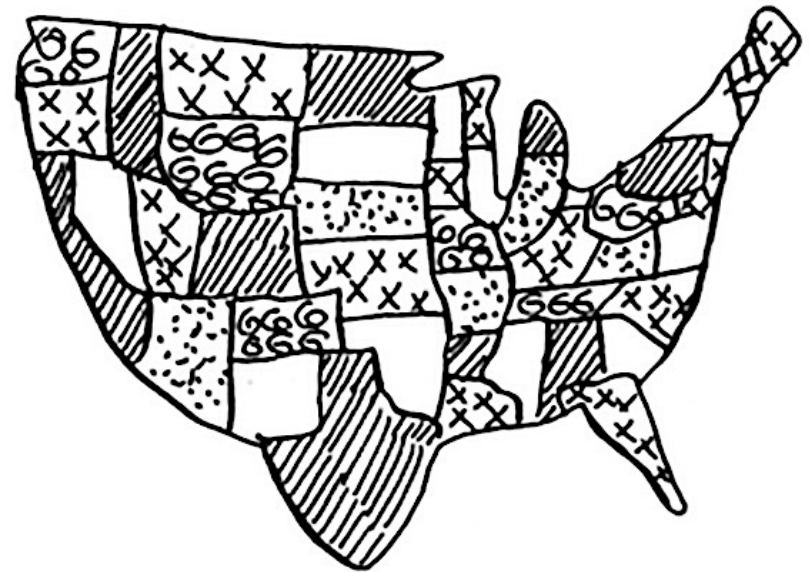
Greedy Algorithms

- ❖ This was a **Greedy Algorithm**, since:
 - ❖ at each step, it picks an **optimal move**.
 - ❖ in this case, it picks a class that **ends the soonest**.
- ❖ In technical terms:
 - ❖ at each step, it picks a **locally optimal** solution.
 - ❖ at the end, it reaches **globally optimal** solution.

Greedy Algorithms

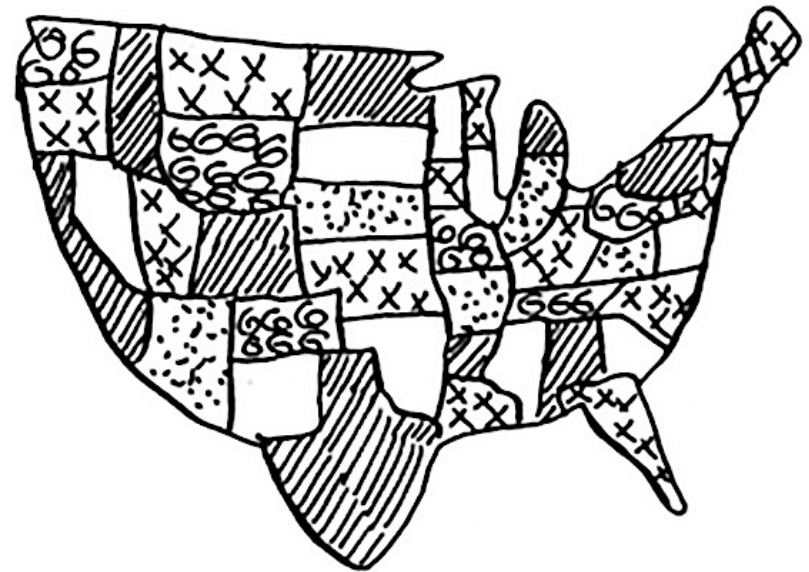
Suppose:

- ❖ You're starting a **radio show in USA**.
- ❖ You want to reach listeners in **all states**.
- ❖ But a station in a new state **will cost you**.
- ❖ You try to **minimize the cost** by choosing stations you will play on.



Greedy Algorithms

- ❖ Each station **covers a region**, and there's overlap. How to figure out the **smallest set of stations** you can play on to **cover all states**?
- ❖ Lets see how you can do it.



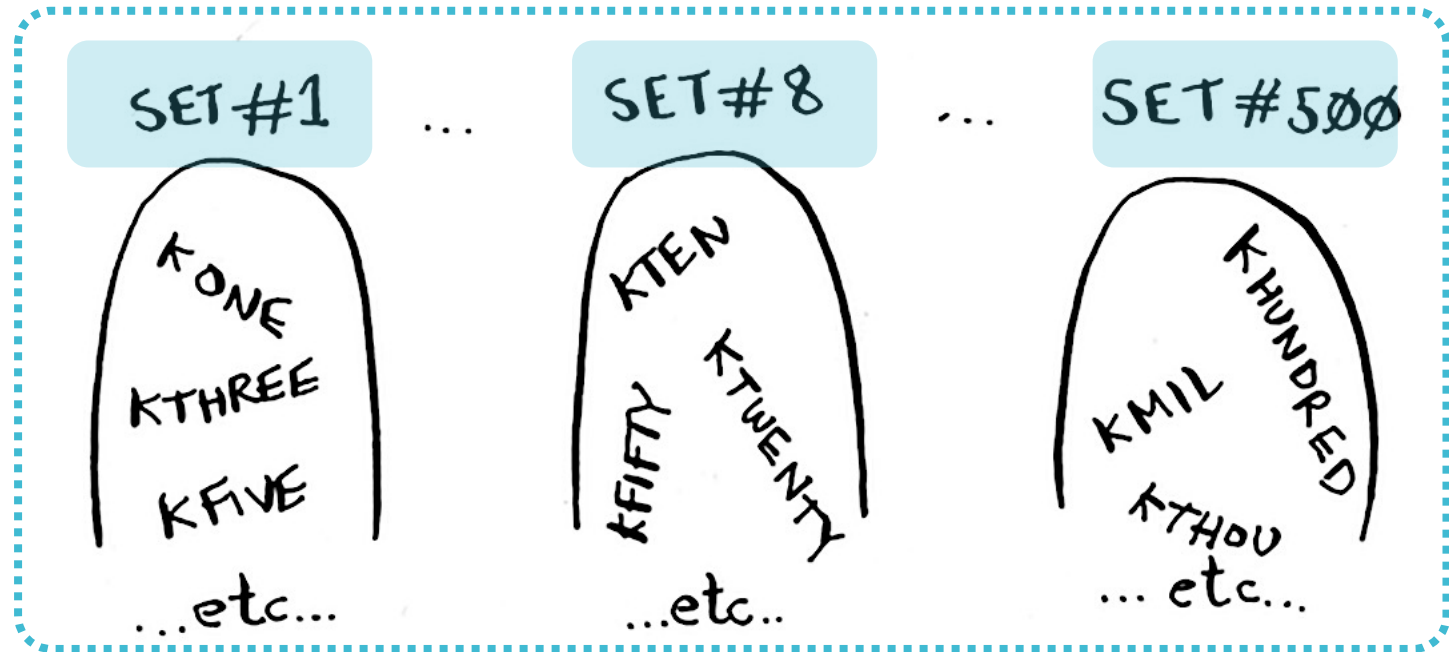
Greedy Algorithms

- 1) List every possible subset of stations. This is called **the power set**.

RADIO STATION	AVAILABLE IN
KONE	ID, NV, UT
KTWO	WA, ID, MT
KTHREE	OR, NV, CA
KFOUR	NV, UT
KFIVE	CA, AZ
...etc...	

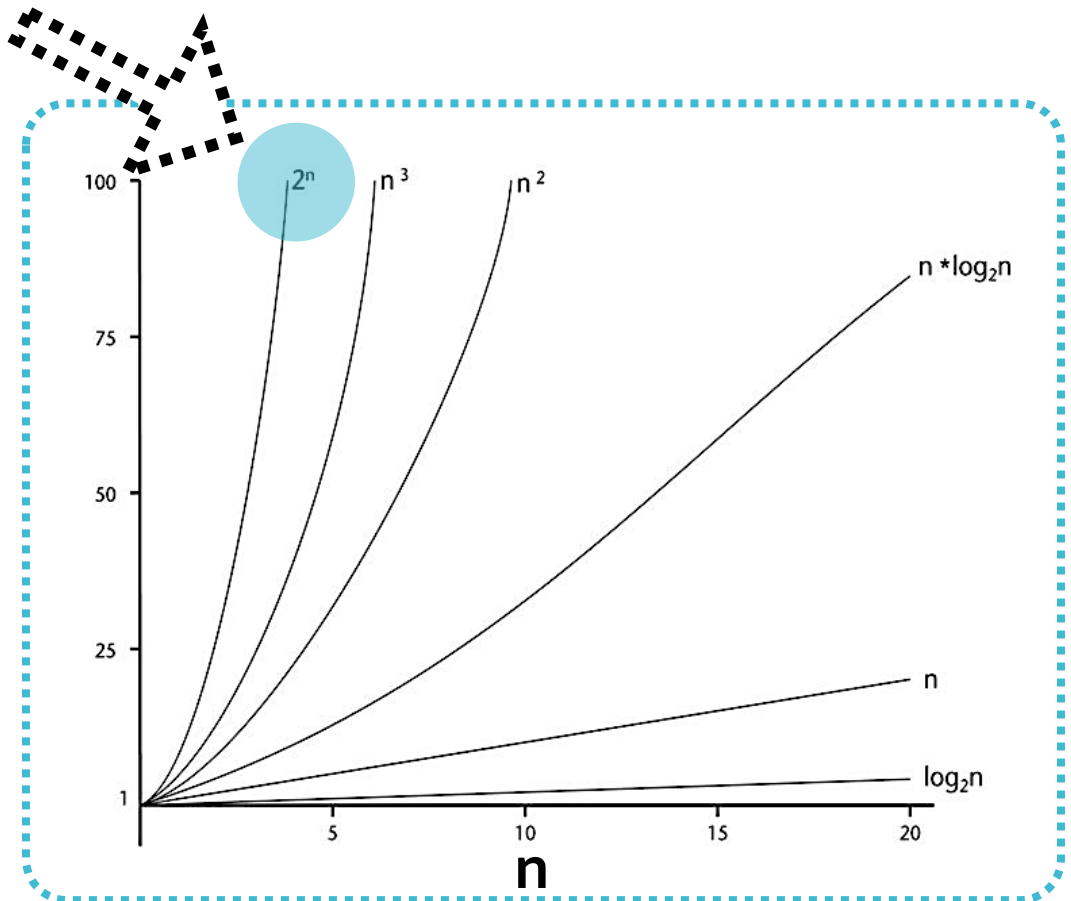
Greedy Algorithms

- 2) Then, pick the set with the **smallest number** of stations that **covers all states**.



Greedy Algorithms

- ❖ Can you say **how many subsets** we may have?
- ❖ There are 2^n possible subsets (**$=2^n$ stations**)!
- ❖ It will take **$O(2^n)$ time**. This really big.
- ❖ But how big?



Greedy Algorithms

- ❖ $O(2^n)$ time
- ❖ If you can calculate 10 subsets per second.

NUMBER OF STATIONS	TIME TAKEN
5	3.2 sec
10	102.4 sec
32	13.6 years
100	4×10^{21} years

Greedy Algorithms

- ❖ **Greedy algorithms** can be the rescue, again:
 - 1) pick the station that covers **the most** states, that haven't been covered yet.
 - 2) repeat until all the states are covered.
- ❖ This algorithm takes **$O(n^2)$ time**, where **n is the number of radio stations**.
- ❖ It is an ***approximation*** algorithm.
- ❖ If computing the exact solution takes too much time, such **approximation algorithm can help**.

Greedy Algorithms

- ❖ **Approximation** algorithms are judged by
 - ❖ how **fast** they are?
 - ❖ how **close** they are to the optimal solution?
- ❖ **Greedy algorithms** are good choices because:
 - ❖ they are **simple** to come up with.
 - ❖ they usually run **fast**.

Greedy Algorithms

❖ Lets **implement our Greedy Algorithm** to find the states, step-by-step.

Greedy Algorithms

- ❖ First, we build a **python dictionary** of stations:
 - ❖ each **key** is a **station** name
 - ❖ each **values** is the **set of states** that station covers
- ❖ **Example:** "k1" station covers:
 - ❖ {"id", "nv", "ut"}
 - ❖ that is Idaho (id), Nevada (nv), and Utah (ut).

```
states_needed = {"mt", "wa", "or", "id", "nv", "ut", "ca", "az"}
```

```
stations = dict()
```

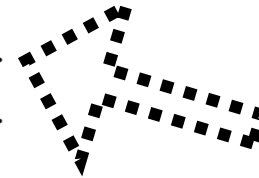
```
stations["k1"] = {"id", "nv", "ut"}
```

```
stations["k2"] = {"wa", "id", "mt"}
```

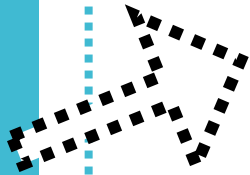
```
stations["k3"] = {"or", "nv", "ca"}
```

```
stations["k4"] = {"nv", "ut"}
```

```
stations["k5"] = {"ca", "az"}
```



Greedy Algorithms

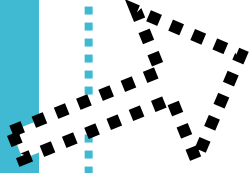


```
def find_states(states_needed, stations):  
    final_stations = set()  
    while states_needed:  
        best_station = None  
        states_covered = set()
```

final_stations()

❖ is a Python Set that keeps the final set of stations.

Greedy Algorithms



```
def find_states(states_needed, stations):  
    final_stations = set()  
    while states_needed:  
        best_station = None  
        states_covered = set()
```

while loop

- ❖ will go through all states and pick the station that covers **the most** uncovered states.
- ❖ we call this **best_station**

Greedy Algorithms

```
def find_states(states_needed, stations):  
    final_stations = set()  
    while states_needed:  
        best_station = None  
        states_covered = set()
```

best_station

❖ is initially set to **None**

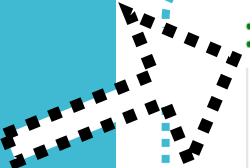
states_covered

❖ is the set of all states that **this station covers** and haven't yet been covered.

Greedy Algorithms

for loop

❖ will go through all station to check which one is the **best** station.



```
for station, states_for_station in stations.items():  
    covered = states_needed & states_for_station  
  
    if len(covered) > len(states_covered):  
        best_station = station  
        states_covered = covered
```

Greedy Algorithms

covered

❖ is the set of uncovered states that this station covers and it is the **intersection** of two sets:

❖ **states_needed** set

❖ **states_for_station** set

```
for station, states_for_station in stations.items():  
    covered = states_needed & states_for_station  
  
    if len(covered) > len(states_covered):  
        best_station = station  
        states_covered = covered
```

Greedy Algorithms

`if` clause checks whether `covered` station has more states than the current `best_station`

```
for station, states_for_station in stations.items():
    covered = states_needed & states_for_station

    if len(covered) > len(states_covered):
        best_station = station
        states_covered = covered
```

Greedy Algorithms

if so, then:

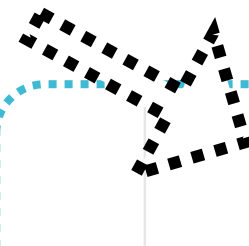
covered station is the new **best_station**.

```
for station, states_for_station in stations.items():  
    covered = states_needed & states_for_station  
  
    if len(covered) > len(states_covered):  
        best_station = station  
        states_covered = covered
```

Greedy Algorithms

states_needed

❖ is updated by removing the states that **aren't needed** anymore.



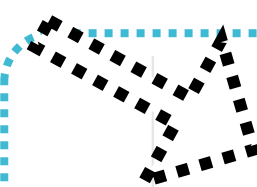
```
states_needed -= states_covered
final_stations.add(best_station)

print('Final stations are:', final_stations)
```

Greedy Algorithms

when **for** loop is over:

- ❖ we add **best_station** to the final list of stations, called **final_stations**
- ❖ and print the **final_stations**



```
states_needed -= states_covered
final_stations.add(best_station)

print('Final stations are:', final_stations)
```

Greedy Algorithms

❖ **Full implementation** of the Greedy Algorithm that can find the states to cover.

```
def find_states(states_needed, stations):  
    final_stations = set()  
    while states_needed:  
        best_station = None  
        states_covered = set()
```

Part 1

```
        for station, states_for_station in stations.items():  
            covered = states_needed & states_for_station  
  
            if len(covered) > len(states_covered):  
                best_station = station  
                states_covered = covered
```

Part 2

```
        states_needed -= states_covered  
        final_stations.add(best_station)  
  
    print('Final stations are:', final_stations)
```

Part 3

Greedy Algorithms

❖ Testing our implementation.

```
stations = dict()
```

```
stations["k1"] = {"id", "nv", "ut"}
```

```
stations["k2"] = {"wa", "id", "mt"}
```

```
stations["k3"] = {"or", "nv", "ca"}
```

```
stations["k4"] = {"nv", "ut"}
```

```
stations["k5"] = {"ca", "az"}
```

```
find_states(states_needed, stations)
```

[Output:]

Final stations are: {'k1', 'k3', 'k5', 'k2'}

Greedy Algorithms

❖ Greedy Algorithm vs Exact Algorithm

NUMBER OF STATIONS	$O(n!)$	$O(n^2)$
	EXACT ALGORITHM	GREEDY ALGORITHM
5	3.2 sec	2.5 sec
10	102.4 sec	10 sec
32	13.6 yrs	102.4 sec
100	4×10^{21} yrs	16.67 min

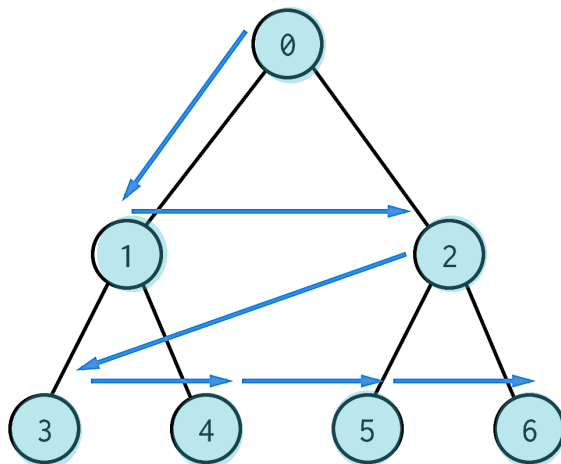


Quiz

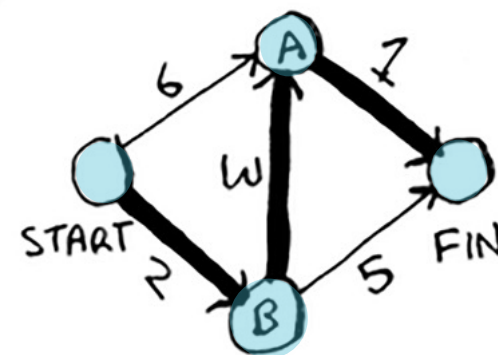
❖ Which of the following algorithms we discussed before, are **Greedy Algorithms**.

- a) Breadth-first search (BFS)
- b) Dijkstra's algorithm

Breadth-first search (BFS)



Dijkstra's

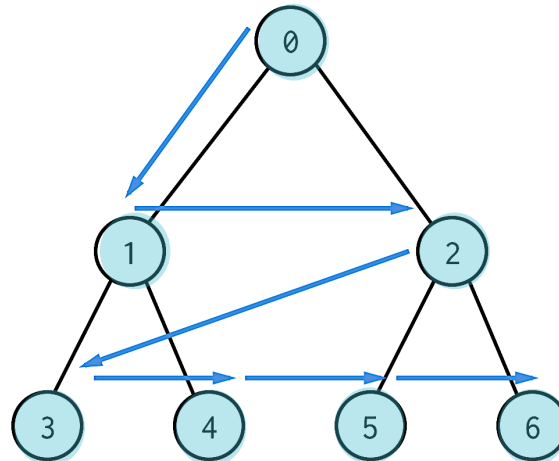


Answer

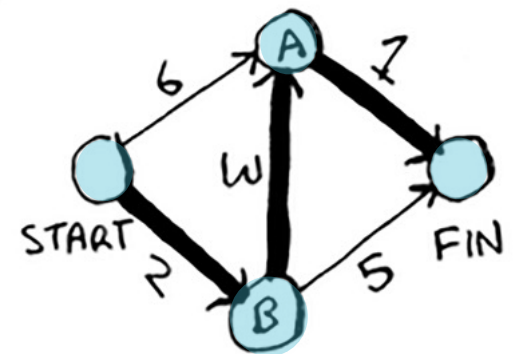
❖ Which of the following algorithms we discussed before, are Greedy Algorithms.

❖ Answer: Both of them.

Breadth-first search (BFS)



Dijkstra's



Next Lesson

❖ **Dynamic Programming**