Minimal Polynomial Factorization Patterns

Prime numbers have exactly 2 irreducible factors in their minimal polynomial

- Prime
- Composite

Number of Irreducible Factors

Prime Threshold

Eigenvalue Distributions in Complex Plane

Eigenvalues form distinct Galois orbits for primes vs. composites

- n=97 (Prime)
- n=90 (Composite)

Imaginary Part

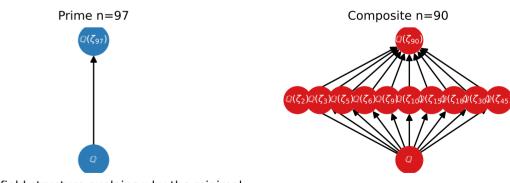Real Part

Cyclotomic Field Extension Structure

Field Extension Structure:

- For prime p, $\mathbb{Q}(\zeta_p)$ has no proper subfields containing roots of unity

- For composite n, $\mathbb{Q}(\zeta_n)$ contains multiple\n proper subfields $\mathbb{Q}(\zeta_d)$ for divisors d of n

Prime n=97

$\mathbb{Q}(\zeta_{97})$

$\mathbb{Q}$

Composite n=90

$\mathbb{Q}(\zeta_{90})$

$\mathbb{Q}(\zeta_2) \mathbb{Q}(\zeta_3) \mathbb{Q}(\zeta_5) \mathbb{Q}(\zeta_6) \mathbb{Q}(\zeta_9) \mathbb{Q}(\zeta_{10}) \mathbb{Q}(\zeta_{15}) \mathbb{Q}(\zeta_{18}) \mathbb{Q}(\zeta_{30}) \mathbb{Q}(\zeta_{45})$

$\mathbb{Q}$

This field structure explains why the minimal polynomial of C_n has exactly 2 irreducible factors for prime n, and more factors for composite n.