Variable Separable

1. $(xy+x)dx=(x^2y^2+x^2+y^2+1)dy$

$$\frac{x(y+1)dx=[x^2(y^2+1)+(y^2+1)]dy}{(y+1)(x^2+1)}$$

$$\frac{x}{x^2+1}dx=\frac{y^2+1}{y+1}dy$$

$$\begin{array}{r|l} & y \\ y+1 & y^2+1 \\ & -y^2-y \\ \hline & -y+1 \end{array}$$

$$\frac{x}{x^2+1}dx=\left(y+\frac{-y+1}{y+1}\right)dy$$

$$\begin{array}{r|l} & -1 \\ y+1 & -y+1 \\ & +y+1 \\ \hline & 2 \end{array}$$

$$\int\frac{x}{x^2+1}dx=\int\left(y+\frac{2}{y+1}-1\right)dy$$

$$\int\frac{x}{x^2+1}dx=\int y\,dy+\int\frac{2}{y+1}dy-\int dy$$

$$\frac{1}{2}\ln|x^2+1| = \left(\frac{y^2}{2} + 2\ln|y+1| - y\right)$$

$$\frac{1}{2}\ln|x^2+1| - \left(\frac{y^2}{2} + 2\ln|y+1| - y\right) = \ln c$$

$$\frac{1}{2}\ln|x^2+1| - \frac{y^2}{2} - 2\ln|y+1| + y = \ln c$$

$$\ln|x^2+1| - y^2 - 4\ln|y+1| + 2y = \ln c$$

$$\ln|x^2+1| - y^2 - \ln(y+1)^4 + 2y = c$$

$$\ln\frac{|x^2+1|}{(y+1)^4} - y^2 + 2y = c$$

$$y^2 - 2y = \ln\frac{|x^2+1|}{(y+1)^4} - \ln c$$

$$y^2 - 2y = \ln\frac{|x^2+1|}{c(y+1)^4}$$

2. $x\cos^2 y\,dx + \tan y\,dy = 0$

$$\frac{x\cos^2 y\,dx + \tan y\,dy = 0}{\cos^2 y}$$

$$x\,dx + \frac{\tan y}{\cos^2 y}\,dy = 0$$

$$\int x\,dx + \int \tan y \sec^2 y\,dy = 0$$

$$\frac{1}{2}(x^2 + \tan^2 y) = c$$

3. $y' = xe^{y-x^2}$ when: $x=0,\ y=0$

$$\frac{dy}{dx} = xe^{y-x^2}$$

$$dy = (xe^{y}e^{-x^2})dx$$

$$\int e^{-y}dy = \int x e^{-x^2}\,dx$$

$u=-y$
$du=-dy$

$$-e^{-y} = \int x e^{-x^2}\,dx$$

$u=-x^2$
$du=-2x\,dx$

$$-e^{-y} = -\frac{1}{2}e^{-x^2} + c$$

$$-e^{-y} + \frac{1}{2}e^{-x^2} = c \quad \Big\backslash \quad e^{-y} - \frac{1}{2}e^{-x^2} = c$$

$$-e^{0} + \frac{1}{2}e^{0} = c \rightarrow c = -1+\frac{1}{2} = -\frac{1}{2}$$

$$e^{0} - \frac{1}{2}e^{0} = c \rightarrow c = 1-\frac{1}{2} = \frac{1}{2}$$

$\rightarrow$ particular solution

$$e^{-y} - \frac{1}{2}e^{-x^2} = c$$

$$e^{-y} - \frac{1}{2}e^{-x^2} = \frac{1}{2}$$

$$e^{-y} = \frac{1}{2} + \frac{1}{2}e^{-x^2}$$

$$e^{-y} = \frac{1}{2}(1+\frac{1}{2}e^{-x^2})$$