1. $y(6y^2 - x - 1)\,dx + 2x\,dy = 0$

$$y^{(1-n)} e^{\int (1-n) P(x)\,dx} = \int (1-n) Q(x) e^{\int (1-n) P(x)\,dx}\,dx + C$$

$$\frac{6y^3 - y(x+1)}{2x} + \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} + \frac{(x+1)y}{-2x} = -\frac{3y^3}{x}$$

$n = 3$

$(1-n) = -2$

$$P(x) = -\frac{x+1}{2x}$$

$$Q(x) = -\frac{3}{x}$$

$$\int (1-n) P(x)\,dx = \frac{-2}{-2}\int \frac{x+1}{x}$$

$$= \int 1 + \frac{1}{x}$$

$$= x + \ln x$$

$$e^{x + \ln x} = xe^x$$

$$y^{-2} xe^x = -2\int xe^x \left(-\frac{3}{x}\right) dx + C$$

$$= 6\int e^x\,dx + C$$

$$y^{-2} xe^x = 6e^x + C$$

$$xe^x = y^2(6e^x + C)$$

2. $xy\,dx + (y^4 - 3x^2)\,dy = 0$

$$\frac{xy\,dx + (y^4 - 3x^2)\,dy = 0}{(dy)(xy)}$$

$$\frac{dx}{dy} + \frac{3}{-y}x = -y\,x^{-1}$$

$n = -1$

$(1-n) = 2$

$P(y) = \frac{3}{-y}$

$Q(y) = -y^3$

$$\int 2\left(\frac{3}{-y}\right) = -6\ln|y| = \ln|y^{-6}|$$

$$x^2 y^{-6} = \int (2)(-y^3)\,y^{-6} + C$$

$$x^2 y^{-6} = -2\int y^{-3} + C$$

$$x^2 y^{-6} = \frac{1}{y^2} + C$$

$$\frac{x^2}{y^6} = \frac{1}{y^2} + C$$

$$x^2 = y^6\left(\frac{1}{y^2} + C\right) \rightarrow \boxed{x^2 = y^4(1 + Cy^2)}$$