1. $(2xy-3x^2)dx + (x^2+2y)dy = 0$

$\frac{\partial M}{\partial y}\Big|_{x=k} = 2x$ $\qquad$ $\frac{\partial N}{\partial x}\Big|_{y=k} = 2x$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\int 2xy = (2y)\left(\frac{x^2}{2}\right)$

$-\int 3x^2 = -x^3$

$$\int \frac{\partial F}{\partial x} = \int (2xy - 3x^2)dx + g(y)$$

$$F = yx^2 - x^3 + g(y)$$

$$\frac{\partial f}{\partial y} = \cancel{x^2} + g'(y) = \cancel{x^2} + 2y$$

$$g'(y) = 2y$$

$$\int g'(y) = \int 2y$$

$$g(y) = y^2$$

$$-x^3 + x^2y + y^2 = c$$

2. $(1-xy)^{-2}\,dx + [y^2 + x^2(1-xy)^{-2}]\,dy = 0$

$$\frac{\partial M}{\partial y} = (-2)(1-xy)^{-3}(-x) = \frac{2x}{(1-xy)^3}$$

$$\frac{\partial N}{\partial x} = \frac{(2)(x)(1-xy)^2 - (x^2)(2)(1-xy)(-y)}{(1-xy)^4}$$

$$= \frac{2x(1-xy)^2 + 2x^2y(1-xy)}{(1-xy)^4}$$

$$= \frac{(1-xy)[2x(1-xy) + 2x^2y]}{(1-xy)^4}$$

$$= \frac{2x(1-xy) + 2x^2y}{(1-xy)^3}$$

$$= \frac{\cancel{-2x^2y} + 2x + \cancel{2x^2y}}{(1-xy)^3}$$

$$= \frac{2x}{(1-xy)^3}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int dF = \int (1-xy)^{-2}\,dx + g(y) = (-1)\left(-\frac{1}{y}\right)(1-xy)^{-1}$$

$\int u^{-2}\,du \to u^{-1}$

$$= \frac{1}{y(1-xy)} + g(y)$$

$u = 1-xy$

$du = 0 - y\,dx = -y\,dx$

$$F = \frac{1}{y(1-xy)} + g(y)$$

$$\frac{\partial F}{\partial y} = \frac{\partial (y - xy^2)^{-1}}{\partial y} + g'(y)$$

$$\frac{\partial F}{\partial y} = (-1)(y-xy^2)^{-2}(1-2xy) + g'(y)$$

$$= -\frac{1-2xy}{(y-xy^2)^2} + g'(y) = y^2 + x^2(1-xy)^{-2}$$

$$g'(y) = y^2 + x^2(1-xy)^{-2} + \frac{1-2xy}{(y-xy^2)^2}$$

$$\int g'(y) = \int y^2 dy + \int x^2(1-xy)^{-2} dy + \int \frac{1-2xy}{(y-xy^2)^2} dy$$

$$g(y) = \frac{y^3}{3} + \frac{x}{1-xy} - \frac{1}{y-xy^2}$$

$u = y - xy^2$

$du = (1-2xy)dy$

$$F = \cancel{\frac{1}{y(1-xy)}} + \frac{y^3}{3} + \frac{x}{1-xy} - \cancel{\frac{1}{y-xy^2}}$$

$$\frac{y^3}{3} + \frac{x}{1-xy} = C$$

3. $[2x + y\cos(xy)]dx + x\cos(xy)dy = 0$

$$\frac{\partial M}{\partial y} = 0 + [(1)(\cos(xy)) + (y)(-\sin(xy)(x))$$
$$= \cos(xy) - xy\sin(xy)$$

$$\frac{\partial N}{\partial x} = (1)(\cos(xy)) + (x)(-\sin(xy))(y)$$
$$= \cos(xy) - xy\sin(xy)$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int dF = \int x\cos(xy)dy$$
$$F = \sin(xy) + f(x)$$
$$\frac{\partial F}{\partial x} = f'(x) + y\cos(xy)dx$$

$$\int f'(x) = \int 2x\,dx$$
$$f(x) = x^2$$
$$\boxed{\sin(xy) + x^2 = C}$$

4. $(4x^3y^3 - 2xy)dx + (3x^4y^2 - x^2)dy = 0$

$$\frac{\partial M}{\partial y}\Big|_{x=k} = 4x^3(3)y^2 - 2x$$
$$= 12x^3y^2 - 2x$$

$$\frac{\partial N}{\partial x}\Big|_{y=k} = 3y^2(4)x^3 - 2x$$
$$= 12x^3y^2 - 2x$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int dF = \int (3x^4y^2 - x^2)dy$$
$$= 3x^4(2)y - 0$$
$$F = 6x^4y + g(x)$$
$$\frac{\partial F}{\partial x} = (6y(4)x^3)dx + g'(x)$$
$$= 24x^3y + g'(x) = 4x^3y^3 - 2xy$$

$$\int g'(x) = \int (4x^3y^3 - 2xy - 24x^3y)\,dx$$
$$g(x) = x^4y^3 - x^2y - 6x^4y$$

$$\cancel{6x^4y} + x^4y^3 - x^2y \cancel{- 6x^4y}$$

$$x^4y^3 - x^2y = c$$

5. $(\cos 2y - 3x^2y^2)\,dx + (\cos 2y - 2x\sin 2y - 2x^3y)\,dy = 0$

$$\frac{\partial M}{\partial y} = -\sin(2y)(2) - 3x^2(2)(y)$$
$$= -2\sin(2y) - 6x^2y$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial N}{\partial x} = 0 - 2\sin 2y - 6x^2y$$
$$= -2\sin 2y - 6x^2y$$

$$\int dF = \int (\cos 2y - 3x^2y^2)\,dx$$
$$F = x\cos(2y) - x^3y^2 + g(y)$$
$$\frac{\partial F}{\partial y} = \cancel{-2x\sin(2y)} - \cancel{2x^3y} + g'(y) = \cos 2y - \cancel{2x\sin 2y} - \cancel{2x^3y}$$
$$\int g'(y) = \int \cos 2y$$
$$g(y) = \frac{\sin 2y}{2}$$

$$\boxed{x\cos(2y) - x^3y^2 + \frac{\sin 2y}{2} = C}$$