1. $(y - x + xy\cot x)dx + xdy = 0$

Gen Solution:

$$y e^{\int P(x)dx} = \int e^{\int P(x)dx} Q(x)\,dx + C$$

$$(y - x + xy\cot x)dx + xdy = 0$$

$$\frac{xdy + ydx - xdx + xy\cot x\,dx}{x} = 0$$

$$dy + \frac{1}{x}ydx - dx + y\cot x\,dx = 0$$

$$\frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right)y = 1$$

$$P(x) = \frac{1}{x} + \cot x$$
$$Q(x) = 1$$

$$\int\left(\frac{1}{x} + \cot x\right)dx = \ln|x| + \ln|\sin x| = \ln(x\sin x)$$

$$e^{\ln(x\sin x)} = x\sin x$$

$$y(x\sin x) = \int (x\sin x)(1)\,dx + C$$

$$\int x\sin x \longrightarrow -x\cos x + \sin x$$

$$uv - \int v\,du$$

$u = x$ $\quad$ $dv = \sin x$

$du = dx$ $\quad$ $v = -\cos x$

$$-x\cos x + \int \cos x\,dx$$

$$y\,x\sin x = -x\cos x + \sin x + c$$

2. $\left[y \ln y \, dx + (x - \ln y) dy = 0\right] \frac{1}{y \ln y}$

$$dx + \frac{(x - \ln y)}{y \ln y} dy = 0$$

$$\frac{dx}{dy} + \frac{1}{y \ln y} x = \frac{1}{y}$$

$$p(y) = \frac{1}{y \ln y}$$

$$Q(y) = \frac{1}{y}$$

$$\int \frac{1}{y \ln y} dy$$

$$\int \frac{1}{u} du$$

$$u = \ln y$$

$$du = \frac{1}{y} dy$$

$$\ln|\ln(y)|$$

$$e^{\cancel{\ln}|\ln(y)|}$$

$$x \ln(y) = \int \frac{\ln(y)}{y} dy + c$$

$$\int \frac{\ln(y)}{y} \quad u = \ln(y) \quad dv = \frac{1}{y}$$

$$du = \frac{1}{y} \quad v = \ln(y)$$

$$\int \frac{\ln(y)}{y} = \ln(y)\ln(y) - \int \frac{\ln(y)}{y}$$

$$2\int \frac{\ln(y)}{y} = \ln^2(y)$$

$$\int \frac{\ln(y)}{y} = \frac{\ln^2(y)}{2}$$

$$x\ln(y) = \frac{1}{2}\ln^2(y) + c$$

3. $(2xy+x^2+x^4)dx-(1+x^2)dy=0$

$$\frac{(2xy+x^2+x^4)dx-(1+x^2)dy=0}{-dx(1+x^2)}$$

$$\frac{dy}{dx}-\frac{2xy+x^2+x^4}{(1+x^2)}=0$$

$$\frac{dy}{dx}-\frac{2x}{1+x^2}y=x^2$$

$$P(x)=-\frac{2x}{1+x^2}$$

$$Q(x)=x^2$$

$$\int -\frac{2x}{1+x^2}dx=-2\int\frac{x}{1+x^2}dx$$

$$u=x^2+1$$

$$du=2x\,dx$$

$$=-\ln|x^2+1|$$

$$=\ln\left(\frac{1}{x^2+1}\right)$$

$$e^{\ln\left(\frac{1}{x^2+1}\right)}=\frac{1}{x^2+1}$$

$$\frac{y}{x^2+1}=\int\frac{x^2}{x^2+1}dx+c$$

$$x^2+1 \overline{\big) \; x^2 \;}^{\;1}$$

$$\frac{- \; x^2 - 1}{-1}$$

$$\frac{y}{x^2+1} = \int 1 - \frac{1}{x^2+1}\,dx + C$$

$$\frac{y}{x^2+1} = x - \tan^{-1}(x) + C$$

$$4.\left[\frac{d\theta}{dt}+(\tan\theta)t=\frac{e^{-\frac{t^2}{2}}}{\cos\theta}\right]\cos\theta\,dt$$

$$\frac{d\theta}{\cancel{dt}}+(\tan\theta)t=\frac{e^{-\frac{t^2}{2}}}{\cancel{\cos\theta}}\ \cancel{\cos\theta}\,\cancel{dt}$$

$$\cos\theta\,d\theta+t\,\overbrace{\cos\theta\,\tan\theta}^{\sin\theta}\,dt=e^{-\frac{t^2}{2}}\,dt$$

$$\cos\theta\,d\theta+t\sin\theta\,dt=e^{-\frac{t^2}{2}}\,dt$$

$$z=\sin\theta$$

$$dz=\cos\theta\,d\theta$$

$$\frac{dz+tz\,dt=e^{-\frac{t^2}{2}}\,dt}{dt}$$

$$\frac{dz}{dt}+tz=e^{-\frac{t^2}{2}}$$

$$P(t)=t$$

$$Q(t)=e^{-\frac{t^2}{2}}$$

$$\int P(t)\,dt=\int t\,dt=\frac{t^2}{2}$$

$$e^{\frac{t^2}{2}}=\int e^{\cancel{\frac{t^2}{2}}}e^{\cancel{-\frac{t^2}{2}}}\,dt+c$$

$$e^{t^2/2}=\int dt+c$$

$$z e^{t^2/2} = t + c$$

$$(\sin\theta) e^{t^2/2} = t + c$$

5. $((\cos y)(\sin 2x)\,dx + \cos^2 y\,dy - \cos^2 x\,dy = 0$

$$\left[((\cos y)(\sin 2x)\,dx + \cos^2 y\,dy - \cos^2 x\,dy = 0\right]\frac{1}{\cos y}$$

$$\sin 2x\,dx + \cos y\,dy - \frac{\cos^2 x\,dy}{\cos y} = 0$$

$\sin 2x = 2\sin x\cos x$ ← double angle formula

$$2\sin x\cos x\,dx + \cos y\,dy - \frac{\cos^2 x\,dy}{\cos y} = 0$$

double angle formula

$$\cos 2x = \cos(x+x) = \cos x\cos x - \sin x\sin x$$
$$= \cos^2 x - \sin^2 x$$
$$= \cos^2 x - (1-\cos^2 x)$$
$$= \cos^2 x - 1 + \cos^2 x$$
$$\cos 2x = 2\cos^2 x - 1$$
$$\frac{\cos 2x + 1}{2} = \cos^2 x$$

$$d(\cos^2 x) = d\left(\frac{\cos 2x + 1}{2}\right)$$
$$= \frac{1}{2}\left[d(\cos 2x) + d(1)\right]$$
$$= \frac{1}{2}(-2\sin 2x + 0)$$

$$= -\sin 2x$$

$$d(\cos^2 x) = -2\sin x \cos x$$

$$z = \cos^2 x$$

$$dz = -2\sin x \cos x \, dx$$

$$2\sin x \cos x \, dx + \cos y \, dy - \frac{\cos^2 x \, dy}{\cos y} = 0$$

$$\downarrow$$

$$-dz + \cos y \, dy - \frac{z}{\cos y} dy = 0$$

$$\frac{dz}{dy} - \cos y + \frac{z}{\cos y} = 0$$

$$\frac{dz}{dy} + \frac{1}{\cos y} z = \cos y$$

$$P(y) = \frac{1}{\cos y}$$

$$Q(y) = \cos y$$

$$\int P(y) \, dy = \int \frac{1}{\cos y} dy = \int \sec y \, dy$$

$$= \ln|\sec y + \tan y|$$

$$e^{\ln|\sec y + \tan y|} = \sec y + \tan y$$

$$z(\sec y+\tan y) = \int(\sec y+\tan y)\cos y\, dy + c$$

$$z(\sec y+\tan y) = \int(1+\sin y)dy + c$$

$$z(\sec y+\tan y) = y - \cos y + c$$

$$\boxed{\cos^2 x(\sec y+\tan y) = y-\cos y+c}$$