1. $3xy\,dx + (x^2+y^2)\,dy = 0$

Homo test

$M(tx, ty) = 3(tx)(ty)$

$= t^2(3xy)$ homo 2nd degree

$N(tx, ty) = t^2x^2 + t^2y^2$

$= t^2(x^2+y^2)$ homo 2nd degree

M is simpler

$x = vy$

$dx = v\,dy + y\,dv$

$3(vy)\,y\,(v\,dy + y\,dv) + [(vy)^2 + y^2]\,dy = 0$

$3vy^2(v\,dy + y\,dv) + [v^2y^2 + y^2]\,dy = 0$

$3v^2y^2\,dy + 3vy^3\,dv + [v^2y^2 + y^2]\,dy = 0$

$3vy^3\,dv + (v^2y^2 \quad + y^2 \quad + 3v^2y^3)\,dy = 0$

$$\frac{3v(y^3)\,dv + y^2(v^2 + 1 + 3v^2)\,dy = 0}{(y^3)(v^2+1+3v^2)}$$

$$\frac{3v}{4v^2+1}\,dv + \frac{y^2}{y^3}\,dy$$

$$\int \frac{3v}{4v^2+1}\,dv + \int \frac{1}{y}\,dy = 0$$

$u = 4v^2 + 1$

$du = 8v\,dv$

$\frac{du}{8}$

$$\frac{3}{8}\ln|4v^2+1| + \ln|y| = c$$

$$3\ln|4v^2+1| + 8\ln|y| = c$$

$$\ln[(4v^2+1)^3 y^8] = c$$

$$(4v^2+1)^3 y^8 = C$$

$$\left(4\frac{x^2}{y^2}+1\right)^3 y^8 = C$$

$$\left(\frac{4x^2+y^2}{y^2}\right)^3 y^8 = C$$

$$\frac{(4x^2+y^2)^3}{y^6} y^8 = C$$

$$y^2(4x^2+y^2)^3 = C$$

2. $x^2y' = 4x^2 + 7xy + 2y^2$

$$x^2\frac{dy}{dx} = 4x^2 + 7xy + 2y^2$$

$$x^2\,dy - (4x^2 + 7xy + 2y^2)\,dx = 0$$

$M(tx, ty) = t^2x^2$ homo of 2nd degree

$N(tx, ty) = 4t^2x^2 + 7t^2xy + 2t^2y^2$

$= t^2(4x^2 + 7xy + 2y^2)$ homo of 2nd degree

M is simpler

$y = vx$

$dy = v\,dx + x\,dv$

$$x^2(v\,dx + x\,dv) - (4x^2 + 7x(vx) + 2(vx)^2)\,dx = 0$$

$$x^2v\,dx + x^3\,dv - (4x^2 + 7x^2v + 2x^2v^2)\,dx = 0$$

$$x^3\,dv + (x^2v - 4x^2 - 7x^2v - 2x^2v^2)\,dx = 0$$

$$\frac{x^3\,dv + x^2(v - 4 - 7v - 2v^2)\,dx = 0}{(x^3)(-2v^2 - 6v - 4)}$$

$$\int -\frac{dv}{2v^2 + 6v + 4} + \int\frac{dx}{x} = \int 0$$

$$\int -\frac{dv}{(2v+2)(v+2)} + \ln|x| = c$$

$$\frac{1}{(2v+2)(v+2)} = \frac{1}{2v+2} + \frac{-\frac{1}{2}}{v+2}$$

$$-\left(\int \frac{1}{2v+2} + -\frac{1}{2}\int \frac{1}{v+2}\right) + \ln|x| = c$$

$$-\frac{1}{2}\ln|v+1| + \frac{1}{2}\ln|v+2| + \ln|x| = c$$

$$-\ln|v+1| + \ln|v+2| + \ln|x^2| = c$$

$$\ln\left(\frac{x^2(v+2)}{v+1}\right) = c$$

$$\frac{x^2(v+2)}{v+1}$$

$$x^2(v+2) = c(v+1)$$

$$x^2\left(\frac{y+2x}{x}\right) = c\left(\frac{y+x}{x}\right)$$

$$x^2(y+2x) = c(y+x)$$

3. $x\,dx + \sin^2\left(\frac{y}{x}\right)(y\,dx - x\,dy) = 0$

$$x\,dx + \sin^2\left(\frac{y}{x}\right) y\,dx - \sin^2\left(\frac{y}{x}\right) x\,dy = 0$$

$$\left[x + \sin^2\left(\frac{y}{x}\right) y\right] dx - \sin^2\left(\frac{y}{x}\right) x\,dy = 0$$

$M(tx,ty) = tx + \sin^2\left(\frac{ty}{tx}\right)(ty)$ homo at 1

$N(tx,ty) =$ homo at 1

n is simpler

$$y = vx$$
$$dy = v\,dx + x\,dv$$

$$\left(x + \sin^2\left(\frac{v\cancel{x}}{\cancel{x}}\right) vx\right) dx - \left(\sin^2\left(\frac{v\cancel{x}}{\cancel{x}}\right) x\right) dy = 0$$

$$(x + \sin^2(v)\, vx)\, dx - (\sin^2(v)\, x)\, dy = 0$$

$$(x + \sin^2(v)\, vx)\, dx - (\sin^2(v)\, x)(v\,dx + x\,dv) = 0$$

$$[x \cancel{+ \sin^2(v) vx}]\, dx \cancel{- (\sin^2(v) vx)\, dx} - (\sin^2(v) x^2)\, dv = 0$$

$$\frac{x\,dx - x^2 \sin^2(v)\, dv = 0}{x^2}$$

$$\int \frac{dx}{x} - \int \sin^2(v)\,dv = 0$$

$$\ln|x| + \frac{1}{2}\left[\sin(2v) - v\right] = 0$$

$$\ln|x| - \frac{1}{4}\sin(2v) + \frac{1}{2}v = c$$

$$\ln|x| - \frac{1}{4}\sin\left(2\frac{y}{x}\right) + \frac{1}{2}\frac{y}{x} = c$$

4. $2(x+y)\,dx + y\,dy = 0$

$$y = vx$$
$$dy = v\,dx + x\,dv$$

$$2(x+vx)\,dx + vx(v\,dx + x\,dv) = 0$$

$$2x\,dx + 2vx\,dx + v^2x\,dx + vx^2\,dv = 0$$

$$\frac{x(2+2v+v^2)\,dx + vx^2\,dv = 0}{(2+2v+v^2)(x^2)}$$

$$\frac{dx}{x} + \frac{v}{v^2+2v+2}\,dv = 0$$

$$\int \frac{dx}{x} + \int \frac{(v+1)-1}{v^2+2v+2}\,dv = \int c$$

$u = v^2+2v+2$
$du = 2v+2$

$$\ln|x| + \frac{1}{2}\ln(v^2+2v+2) - \int \frac{1}{(v+1)^2+1} = c$$

$$\ln|x| + \frac{1}{2}\ln(v^2+2v+2) - \tan^{-1}(v+1) = c$$
$$\ln|x^2| + \ln(v^2+2v+2) - 2\tan^{-1}(v+1) = c$$

$$\ln|x^2| + \ln\left(\frac{y^2}{x^2} + 2\frac{y}{x} + 2\right) - 2\tan^{-1}\left(\frac{y}{x} + 1\right) = c$$

$$\ln(y^2 + 2xy + 2x^2) - 2\tan^{-1}\left(\frac{y}{x} + 1\right) = c$$

$$\ln(y^2 + 2xy + 2x^2) - 2\tan^{-1}\left(\frac{y+x}{x}\right) = c$$

$$\ln(y^2 + 2xy + 2x^2) = 2c\tan^{-1}\left(\frac{y+x}{x}\right)$$

correct answer