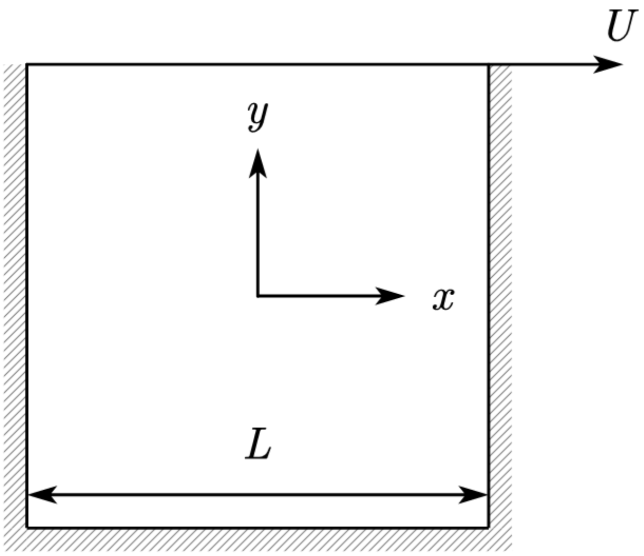
lid driven flow



$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\frac{\partial u}{\partial t}+\frac{\partial (uu)}{\partial x}+\frac{\partial (uv)}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial v}{\partial t}+\frac{\partial (vu)}{\partial x}+\frac{\partial (vv)}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

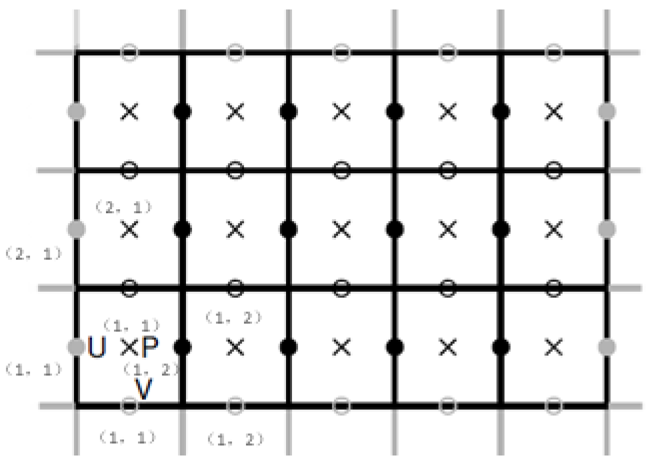
Boundary conditions：

$$U\big|_{x=0}=U\big|_{x=L}=U\big|_{y=0}=0 \qquad U\big|_{y=L}=1$$

$$V\big|_{x=0}=V\big|_{x=L}=V\big|_{y=0}=V\big|_{y=L}=0$$

$$\left.\frac{\partial P}{\partial x}\right|_{x=0}=\left.\frac{\partial P}{\partial x}\right|_{x=L}=\left.\frac{\partial P}{\partial y}\right|_{y=0}=\left.\frac{\partial P}{\partial y}\right|_{y=l}=0$$

create a staggered grid



Handling of Boundary Conditions

$$U\big|_{x=0}=U\big|_{x=L}=U\big|_{y=0}=0 \qquad U\big|_{y=L}=1$$

$$V\big|_{x=0}=V\big|_{x=L}=V\big|_{y=0}=V\big|_{y=L}=0$$

The boundary conditions of U and V are regarded as half of the sum of the virtual node and the internal layer of nodes is equal to zero, that is, the virtual node and the internal layer of nodes are opposite numbers.

$$\left.\frac{\partial P}{\partial x}\right|_{x=0} = \left.\frac{\partial P}{\partial x}\right|_{x=L} = \left.\frac{\partial P}{\partial y}\right|_{y=0} = \left.\frac{\partial P}{\partial y}\right|_{y=l} = 0$$

The boundary condition of P is expressed in the grid as the first layer and the second layer (or the last layer and the penultimate layer) are equal

finite difference

Substitute U and V assumed at the current moment into the difference equations (1) and (2) to obtain U* and V* at the current moment.

$$u_{i+1/2,j}^{n*} = u_{i+1/2,j}^{n} + A\Delta t - \frac{\Delta t}{\Delta x}(p_{i+1,j}^{n} - p_{i,j}^{n}) \qquad (1)$$

$$A = -\left[\frac{(u^2)_{i+3/2,j}^{n} - (u^2)_{i-1/2,j}^{n}}{2\Delta x} + \frac{u_{i+1/2,j+1}^{n}\bar{v} - u_{i+1/2,j-1}^{n}v}{2\Delta y}\right] + \frac{1}{\mathrm{Re}}\left[\frac{u_{i+3/2,j}^{n} - 2u_{i+1/2,j}^{n} + u_{i-1/2,j}^{n}}{(\Delta x)^2} + \frac{u_{i+1/2,j+1}^{n} - 2u_{i+1/2,j}^{n} + u_{i+1/2,j-1}^{n}}{(\Delta y)^2}\right]$$

$$\bar{v} = \frac{1}{2}(v_{i,j+1/2} + v_{i+1,j+1/2}), v = \frac{1}{2}(v_{i,j-1/2} + v_{i+1,j-1/2})$$

$$v_{i,j+1/2}^{n*} = v_{i,j+1/2}^{n} + B\Delta t - \frac{\Delta t}{\Delta y}(p_{i,j+1}^{n} - p_{i,j}^{n}) \qquad (2)$$

$$B = -\left[\frac{v_{i+1,j+1/2}^{n}\bar{u} - v_{i-1,j+1/2}^{n}u}{2\Delta x} + \frac{(v^2)_{i+1/2,j+3/2}^{n} - (v^2)_{i,j+3/2}^{n}}{2\Delta y}\right] + \frac{1}{\mathrm{Re}}\left[\frac{v_{i+1,j+1/2}^{n} - 2v_{i,j+1/2}^{n} + v_{i-1,j+1/2}^{n}}{(\Delta x)^2} + \frac{v_{i,j+3/2}^{n} - 2v_{i,j+1/2}^{n} + v_{i,j-1/2}^{n}}{(\Delta y)^2}\right]$$

$$u = \frac{1}{2}(u_{i-1/2,j} + v_{i-1/2,j+1}), \bar{u} = \frac{1}{2}(u_{i+1/2,j} + u_{i+1/2,j+1})$$

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Correction

Substitute U* and V* into the revised equation (3) to obtain P', and the program uses the Gauss-Seidel iteration method to obtain the equation system

$$P_{i,j}^{'} = -\frac{1}{a}(bP_{i+1,j}^{'} + bP_{i-1,j}^{'} + cP_{i,j+1}^{'} + cP_{i,j-1}^{'} + d) \qquad (3)$$

$$a = 2\left[\frac{\Delta t}{(\Delta x)^2} + \frac{\Delta t}{(\Delta y)^2}\right], b = -\frac{\Delta t}{(\Delta x)^2}, c = -\frac{\Delta t}{(\Delta y)^2}, d = \frac{1}{\Delta x}\left[(u^*)_{i+1/2,j} - (u^*)_{i-1/2,j}\right] + \frac{1}{\Delta y}\left[(v^*)_{i,j+1/2} - (v^*)_{i,j-1/2}\right]$$

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Substitute P' into P=P*+0.1×P' to get the accurate P value at the current moment.
Substitute P' into formula (4) to get the accurate U and V at the current moment.

$$U_{i+1/2,j}^{n} = U_{i+1/2,j}^{n*} - \frac{\Delta t}{\Delta x}(P_{i+1,j}^{'} - P_{i,j}^{'}), V_{i,j+1/2}^{n} = V_{i,j+1/2}^{n*} - \frac{\Delta t}{\Delta y}(P_{i,j+1}^{'} - P_{i,j}^{'}) \qquad (4)$$

Then substitute the accurate U and V at the current moment and the corrected P into the difference equation (1) to get the U and V at the next moment and start the cycle