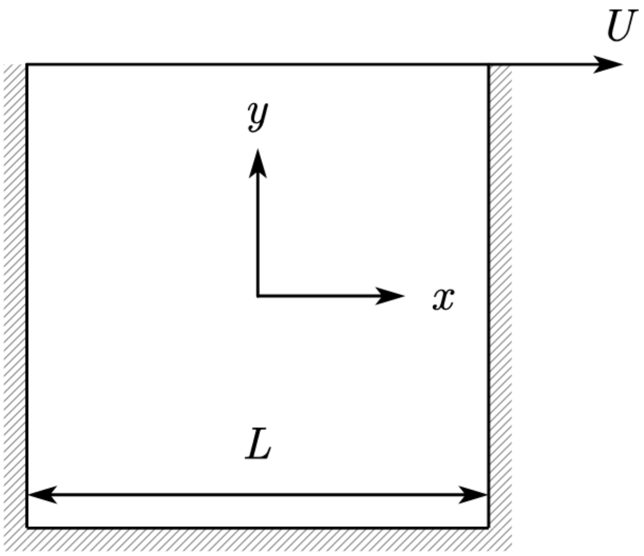
一、问题：方腔驱动流



$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\frac{\partial u}{\partial t}+\frac{\partial (uu)}{\partial x}+\frac{\partial (uv)}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})$$

$$\frac{\partial v}{\partial t}+\frac{\partial (vu)}{\partial x}+\frac{\partial (vv)}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})$$

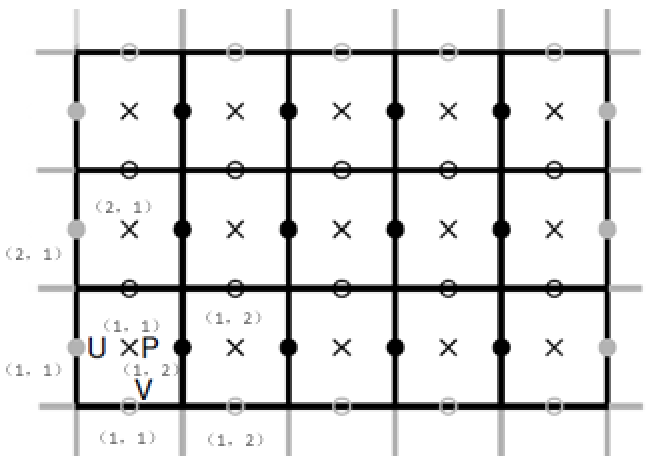
边界条件:

$$U\big|_{x=0}=U\big|_{x=L}=U\big|_{y=0}=0 \qquad U\big|_{y=L}=1$$

$$V\big|_{x=0}=V\big|_{x=L}=V\big|_{y=0}=V\big|_{y=L}=0$$

$$\left.\frac{\partial P}{\partial x}\right|_{x=0}=\left.\frac{\partial P}{\partial x}\right|_{x=L}=\left.\frac{\partial P}{\partial y}\right|_{y=0}=\left.\frac{\partial P}{\partial y}\right|_{y=l}=0$$

二、建立交错网格



三、边界条件的处理

$$U\big|_{x=0}=U\big|_{x=L}=U\big|_{y=0}=0 \qquad U\big|_{y=L}=1$$

$$V\big|_{x=0}=V\big|_{x=L}=V\big|_{y=0}=V\big|_{y=L}=0$$

U 与 V 的边界条件看成虚拟节点与内部一层节点之和的一半等于零，即虚拟节点与内部一层节点互为相反数。

$$\left.\frac{\partial P}{\partial x}\right|_{x=0}=\left.\frac{\partial P}{\partial x}\right|_{x=L}=\left.\frac{\partial P}{\partial y}\right|_{y=0}=\left.\frac{\partial P}{\partial y}\right|_{y=l}=0$$

P 的边界条件在网格中表现为第一层与第二层（或最后一层与倒数第二层）相等

四、有限差分

将当前时刻假设的 U 与 V 代入差分方程（1）与（2），求得当前时刻的 U*与 V*。

$$u_{i+1/2,j}^{n*}=u_{i+1/2,j}^{n}+A\Delta t-\frac{\Delta t}{\Delta x}(p_{i+1,j}^{n}-p_{i,j}^{n}) \tag{1}$$

$$A=-\left[\frac{(u^2)_{i+3/2,j}^{n}-(u^2)_{i-1/2,j}^{n}}{2\Delta x}+\frac{u_{i+1/2,j+1}^{n}\bar{v}-u_{i+1/2,j-1}^{n}v}{2\Delta y}\right]+\frac{1}{\mathrm{Re}}\left[\frac{u_{i+3/2,j}^{n}-2u_{i+1/2,j}^{n}+u_{i-1/2,j}^{n}}{(\Delta x)^2}+\frac{u_{i+1/2,j+1}^{n}-2u_{i+1/2,j}^{n}+u_{i+1/2,j-1}^{n}}{(\Delta y)^2}\right]$$

$$\bar{v}=\frac{1}{2}(v_{i,j+1/2}+v_{i+1,j+1/2}),v=\frac{1}{2}(v_{i,j-1/2}+v_{i+1,j-1/2})$$

$$v_{i,j+1/2}^{n*}=v_{i,j+1/2}^{n}+B\Delta t-\frac{\Delta t}{\Delta y}(p_{i,j+1}^{n}-p_{i,j}^{n}) \tag{2}$$

$$B=-\left[\frac{v_{i+1,j+1/2}^{n}\bar{u}-v_{i-1,j+1/2}^{n}u}{2\Delta x}+\frac{(v^2)_{i+1/2,j+3/2}^{n}-(v^2)_{i,j+3/2}^{n}}{2\Delta y}\right]+\frac{1}{\mathrm{Re}}\left[\frac{v_{i+1,j+1/2}^{n}-2v_{i,j+1/2}^{n}+v_{i-1,j+1/2}^{n}}{(\Delta x)^2}+\frac{v_{i,j+3/2}^{n}-2v_{i,j+1/2}^{n}+v_{i,j-1/2}^{n}}{(\Delta y)^2}\right]$$

$$u=\frac{1}{2}(u_{i-1/2,j}+v_{i-1/2,j+1}),\bar{u}=\frac{1}{2}(u_{i+1/2,j}+u_{i+1/2,j+1})$$

。

五、修正

将 U*与 V*代入修正方程（3）中，得到 P'，程序中利用高斯-塞德尔迭代法来求得方程组

$$P_{i,j}^{'}=-\frac{1}{a}(bP_{i+1,j}^{'}+bP_{i-1,j}^{'}+cP_{i,j+1}^{'}+cP_{i,j-1}^{'}+d) \tag{3}$$

$$a=2\left[\frac{\Delta t}{(\Delta x)^2}+\frac{\Delta t}{(\Delta y)^2}\right],b=-\frac{\Delta t}{(\Delta x)^2},c=-\frac{\Delta t}{(\Delta y)^2},d=\frac{1}{\Delta x}\left[(u^*)_{i+1/2,j}-(u^*)_{i-1/2,j}\right]+\frac{1}{\Delta y}\left[(v^*)_{i,j+1/2}-(v^*)_{i,j-1/2}\right]$$

,

将 P'代入 P=P*+0.1×P‘ 中，得到当前时刻准确的 P 值。
再将 P'代入公式（4）得到当前时刻准确的 U 与 V。

$$U_{i+1/2,j}^{n}=U_{i+1/2,j}^{n*}-\frac{\Delta t}{\Delta x}(P_{i+1,j}^{'}-P_{i,j}^{'}),V_{i,j+1/2}^{n}=V_{i,j+1/2}^{n*}-\frac{\Delta t}{\Delta y}(P_{i,j+1}^{'}-P_{i,j}^{'}) \tag{4}$$

再将当前时刻准确的 U 与 V 以及修正完毕的 P 代入差分方程（1）就可以得到下一时刻的 U 与 V 然后开始循环