

Report for Benjamin

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1 RC model

The schematic of the model can be seen in figure1 where $V_g(t)$ represents the difference in pressure between atmosphere and pleural pressure while $i_0(t)$ is the respiratory flow. R_0 is the resistance of the trachea, R_1 and R_2 the resistances of the bifurcations. The resistances can be gotten by Poiseuille: $R_i = \frac{8\eta L_i}{\pi r_i^4}$. C_1 and C_2 are the compliances of the 2 compartments. LKI

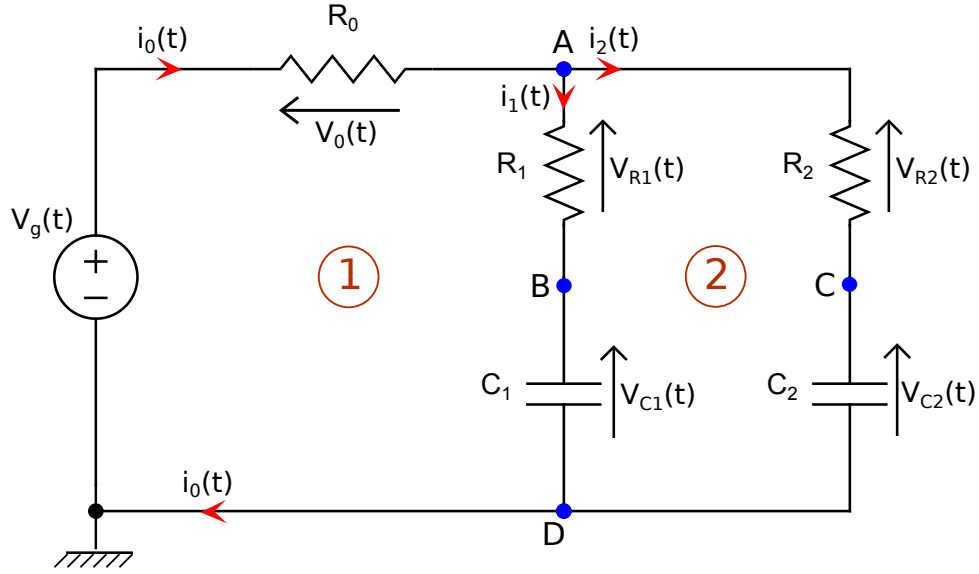


Figure 1: Electrical equivalent of a bifurcation. 2 compartments are studied.

and LKV equations:

$$\begin{cases} i_0 = i_1 + i_2 \\ V_{R1} + V_{C1} - V_{R2} + V_{C2} = 0 \end{cases} \quad (1)$$

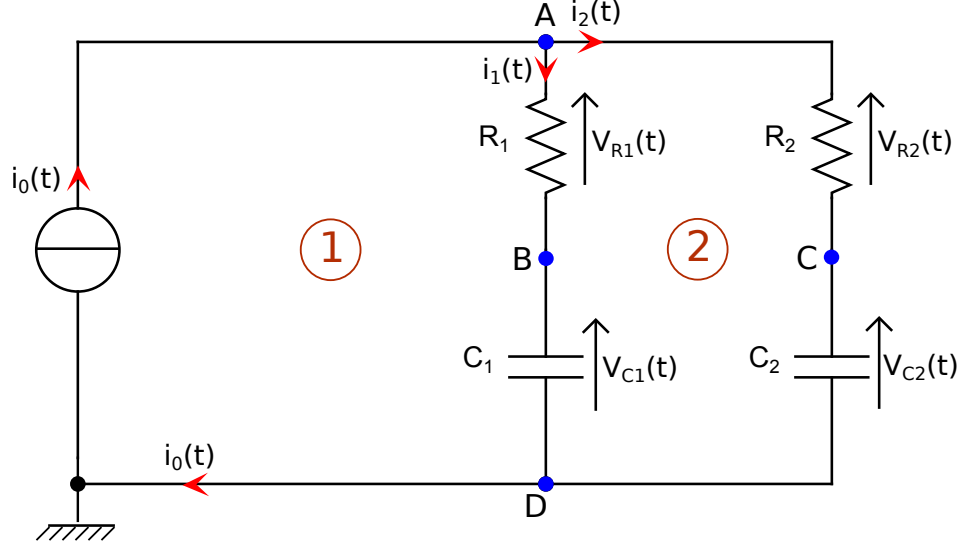


Figure 2: Thevenin equivalent of the circuit in figure 1. The voltage generator and R_0 are replaced in favor of a generator of current which simulate the airflow in the trachea.

Components:

$$\begin{cases} i_1 = V_{R1}/R_1 = dQ_1/dt \\ i_2 = V_{R2}/R_2 = dQ_2/dt \end{cases} \quad (2)$$

Combining the equations:

$$R_1 i_1 + \frac{1}{C_1} \int_{t_0}^{t_1} i_1(\tau) d\tau - R_2 i_2 - \frac{1}{C_2} \int_{t_0}^{t_1} i_2(\tau) d\tau = 0 \quad (3)$$

By deriving equation (3) over time:

$$\frac{di_1}{dt} R_1 + \frac{i_1}{C_1} - \frac{di_2}{dt} R_2 - \frac{i_2}{C_2} = 0$$

Remembering the LKI and using the conservation of charge:

$$Q_0 = Q_1 + Q_2 \Rightarrow \frac{di_0}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\begin{cases} \frac{di_1}{dt} R_1 + \frac{i_1}{C_1} - \frac{di_2}{dt} R_2 - \frac{i_2}{C_2} = 0 \\ \frac{di_2}{dt} R_2 + \frac{i_2}{C_2} - \frac{di_1}{dt} R_1 - \frac{i_1}{C_1} = 0 \end{cases}$$

$$\begin{cases} \frac{di_1}{dt} (R_1 R_2) + i_1 (\frac{1}{C_1} + \frac{1}{C_2}) = R_2 \frac{di_0}{dt} + \frac{i_0}{C_2} \\ \frac{di_2}{dt} (R_1 R_2) + i_2 (\frac{1}{C_1} + \frac{1}{C_2}) = R_1 \frac{di_0}{dt} + \frac{i_0}{C_1} \end{cases}$$

$$\begin{cases} \frac{di_1}{dt} + i_1 \frac{\frac{1}{C_1} + \frac{1}{C_2}}{R_1 + R_2} = \frac{R_2 \frac{di_0}{dt} + \frac{i_0}{C_2}}{R_1 + R_2} \\ \frac{di_2}{dt} + i_2 \frac{\frac{1}{C_1} + \frac{1}{C_2}}{R_1 + R_2} = \frac{R_1 \frac{di_0}{dt} + \frac{i_0}{C_1}}{R_1 + R_2} \end{cases}$$

(4)

Here we get a system of differential equations in which R_1, R_2, C_1, C_2 and i_0 are known and we can get the flow in the 2 different compartment.

Testing the differential equations in Python