Report for Benjamin

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1 RC model

The schematic of the model can be seen in figure1 where $V_g(t)$ represents the difference in pressure between athmosphere and pleural pressure while $i_0(t)$ is the respiratory flow. R_0 is the resistance of the trachea, R_1 and R_2 the resistances of the bifurcations. The resistances can be gotten by Poiseuille: $R_i = \frac{8\eta L_i}{\pi r_i^4}$. C_1 and C_2 are the compliances of the 2 compartments. LKI

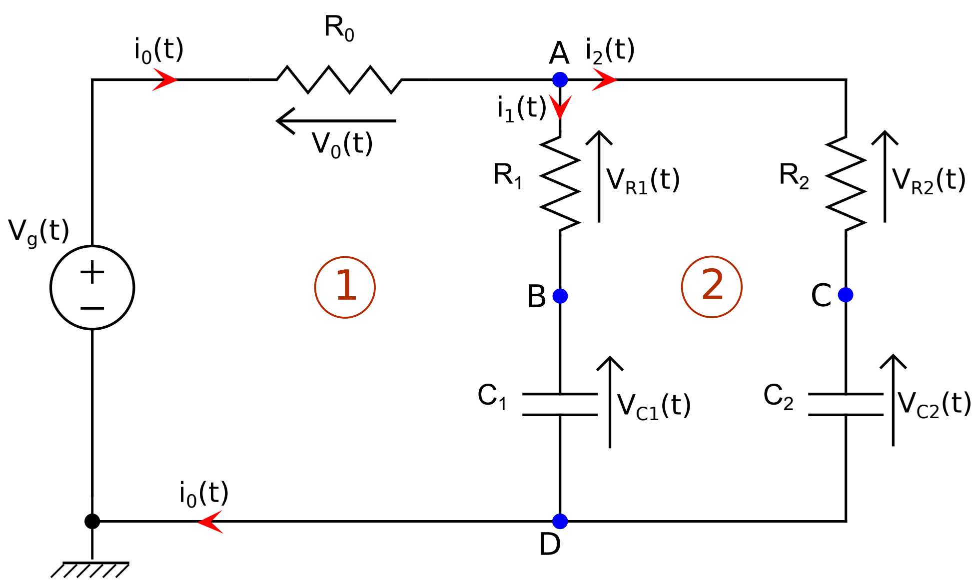

Figure 1: Electrical equivalent of a bifurcation. 2 compartments are studied.

equations:

$$i_0(t) = i_1(t) + i_2(t) \tag{1}$$

LKV equations:

$$\begin{cases} V_g(t) = V_0(t) + V_{R1}(t) + V_{C1}(t) \\ V_g(t) = V_0(t) + V_{R2}(t) + V_{C2}(t) \end{cases} \tag{2}$$

Components:

$$\begin{cases} i_1(t) &= C_1 \frac{dV_{C1}(t)}{dt} \\ i_1(t) &= \frac{V_{R1}(t)}{R_1} \\ i_2(t) &= C_2 \frac{dV_{C2}(t)}{dt} \\ i_2(t) &= \frac{V_{R2}(t)}{R_2} \end{cases} \tag{3}$$

So I can write:

$$\begin{cases} \frac{dV_{C1}(t)}{dt} &= \frac{V_{R1}(t)}{R_1 C_1} \\ \frac{dV_{C2}(t)}{dt} &= \frac{V_{R2}(t)}{R_2 C_2} \end{cases} \tag{4}$$

Calling $\tau_1 = R_1 C_1$ and $\tau_2 = R_2 C_2$ and sobstituting (4) in the LKV (2):

$$\begin{cases} V_g(t) &= V_0(t) + \frac{dV_{C1}(t)}{dt}\tau_1 + V_{C1}(t) \\ V_g(t) &= V_0(t) + \frac{dV_{C2}(t)}{dt}\tau_2 + V_{C2}(t) \end{cases} \tag{5}$$

which can be rewritten in canonic form as:

$$\boxed{\begin{cases} \frac{dV_{C1}(t)}{dt} + \frac{1}{\tau_1} V_{C1}(t) &= \frac{1}{\tau_1}(V_g(t) - V_0(t)) \\ \frac{dV_{C2}(t)}{dt} + \frac{1}{\tau_2} V_{C2}(t) &= \frac{1}{\tau_2}(V_g(t) - V_0(t)) \end{cases}} \tag{6}$$

Equation (6) represents the 2 ordinary differential equations got.

2 Questions

- *Is this actually correct?*

Also now I have gotten these 2 ODE. I tried to combine them by using (5)

$$\frac{dV_{C1}(t)}{dt}\tau_1 + V_{C1}(t) = \frac{dV_{C2}(t)}{dt}\tau_2 + V_{C2}(t) \tag{7}$$

and then using the LKI (1) where you can explicit the dipendence of dV_{C1}/dt from dV_{C2}/dt and i_0 explicitely. Moreover $i_0(t) = V_0(t)/R_0$. We get:

$$\begin{gathered} \frac{V_0(t)}{R_0} = C_1 \frac{dV_{C1}(t)}{dt} + C_2 \frac{dV_{C2}(t)}{dt} \\ \Rightarrow V_0(t) = R_0 C_1 \frac{dV_{C1}(t)}{dt} + R_0 C_2 \frac{dV_{C2}(t)}{dt} \end{gathered} \tag{8}$$

By sobstituting $V_0(t)$ in one of the 2 equations in (6) we finally get:

$$\boxed{\begin{cases} \frac{dV_{C1}(t)}{dt} + \frac{1}{\tau_1}V_{C1}(t) = \frac{1}{\tau_1}(V_g(t) - V_0(t)) \\ \frac{dV_{C2}(t)}{dt} + \frac{1}{\tau_2}V_{C2}(t) = \frac{1}{\tau_2}(V_g(t) - V_0(t)) \\ V_0(t) = R_0C_1\frac{dV_{C1}(t)}{dt} + R_0C_2\frac{dV_{C2}(t)}{dt} \end{cases}} \tag{9}$$

where the first 2 equations are from LKV and the last from LKI.
I'm not sure I can play with this system or if I'm actually missing something. Indeed I suppose the following variables are known:

$$R_0,\ R_1,\ R_2,\ C_1,\ C_2,\ i_0(t)$$

Hence since $i_0(t)$ and R_0 are known, also $V_0(t)$ is known.
However the variables I don't know are:

$$V_{C1},\ V_{C2},\ \frac{dV_{C1(t)}}{dt},\ \frac{dV_{C2(t)}}{dt}$$

I'm not too sure how to move from here.

- *Can I suppose to know $V_g(t) - V_0(t)$?* This would be the difference in pressure between the nodes **A** and **D**, assuming the pleural pressure as reference $\rightarrow V_D = 0$

3 While waiting for the meeting

I'll probably go ahead on math trying to get a proper system with a number of equation equal to the number of unknowns. I'm not sure I've already gotten it or not. Also I'm continuing to make myself a culture on lung modelling in literature and Pozin's thesis.