# Report for Benjamin

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## 1 RC model

The schematic of the model can be seen in figure1 where $V_g(t)$ represents the difference in pressure between athmosphere and pleural pressure while $i_0(t)$ is the respiratory flow. $R_0$ is the resistance of the trachea, $R_1$ and $R_2$ the resistances of the bifurcations. The resistances can be gotten by Poiseuille: $R_i = \frac{8\eta L_i}{\pi r_i^4}$. $C_1$ and $C_2$ are the compliances of the 2 compartments.

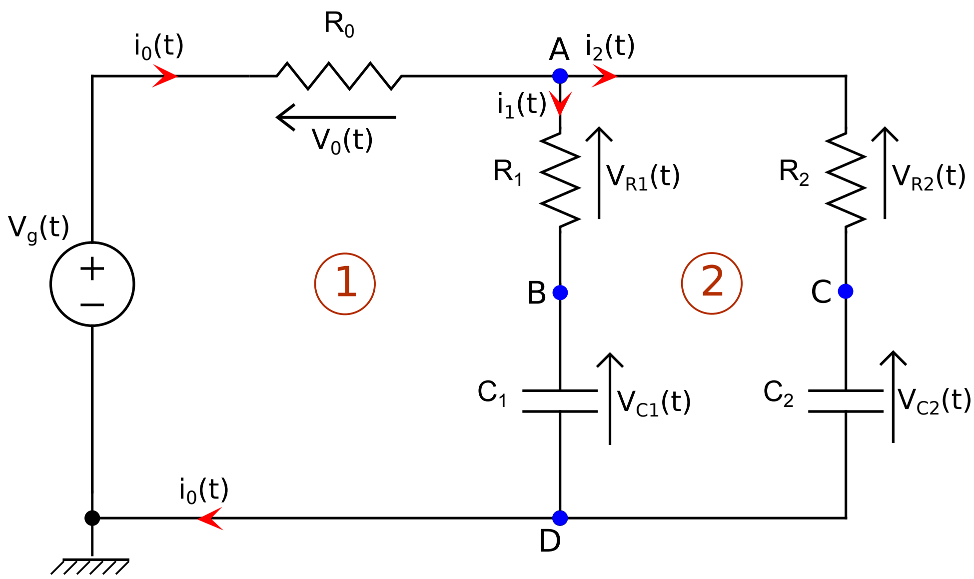


Figure 1: Electrical equivalent of a bifurcation. 2 compartments are studied.

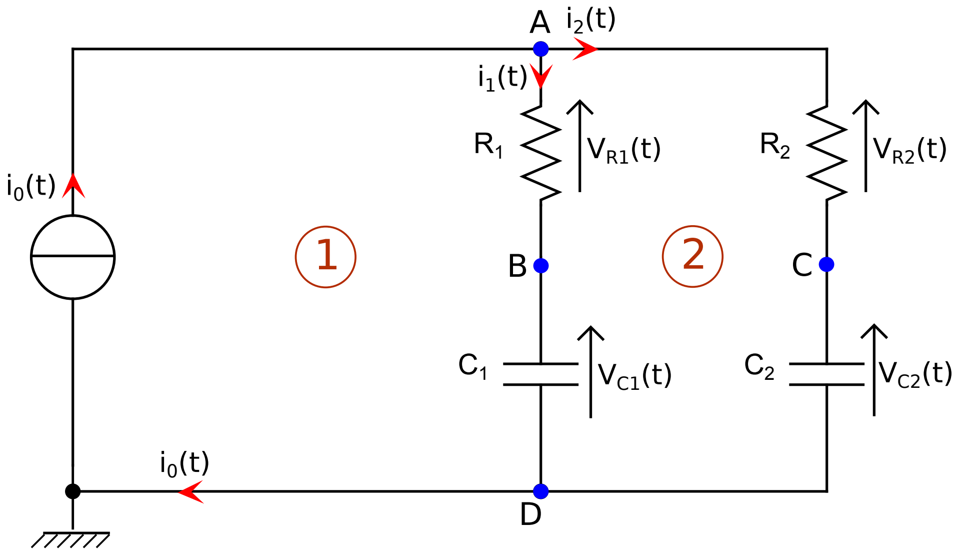


Figure 2: Thevenin equivalent of the circuit in figure 1. The voltage generator and $R_0$ are replaced in favor of a generator of current which simulate the airflow in the trachea.

## 2 Solving ODE in current

LKI and LKV equations:

$$\begin{cases} i_0 = i_1 + i_2 \\ V_{R_1} + V_{C_1} - V_{R_2} + V_{C_2} = 0 \end{cases} \tag{1}$$

Components:

$$\begin{cases} i_1 = V_{R_1}/R_1 = dQ_1/dt \\ i_2 = V_{R_2}/R_2 = dQ_2/dt \end{cases} \tag{2}$$

Combining the equations:

$$R_1 i_1 + \frac{1}{C_1}\int_{t_0}^{t_1} i_1(\tau)\mathrm{d}\tau - R_2 i_2 - \frac{1}{C_2}\int_{t_0}^{t_1} i_2(\tau)\mathrm{d}\tau = 0 \tag{3}$$

By deriving equation (3) over time:

$$\frac{di_1}{dt}R1 + \frac{i_1}{C_1} - \frac{di_2}{dt}R_2 - \frac{i_2}{C_2} = 0$$

Remembering the LKI and using the conservation of charge:

$$Q_0 = Q_1 + Q_2 \Rightarrow \frac{di_0}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\begin{cases} \frac{di_1}{dt}R_1 + \frac{i_1}{C_1} - \frac{di_2}{dt}R_2 - \frac{i_2}{C_2} = 0 \\ \frac{di_2}{dt}R_2 + \frac{i_2}{C_2} - \frac{di_1}{dt}R_1 - \frac{i_1}{C_1} = 0 \end{cases}$$

$$\begin{cases} \frac{di_1}{dt}(R_1 + R_2) + i_1(\frac{1}{C_1} + \frac{1}{C_2}) = R_2\frac{di_0}{dt} + \frac{i_0}{C_2} \\ \frac{di_2}{dt}(R_1 + R_2) + i_2(\frac{1}{C_1} + \frac{1}{C_2}) = R_1\frac{di_0}{dt} + \frac{i_0}{C_1} \end{cases}$$

$$\begin{cases} \frac{di_1}{dt} + i_1\frac{\frac{1}{C_1}+\frac{1}{C_2}}{R_1+R_2} = \frac{R_2\frac{di_0}{dt}+\frac{i_0}{C_2}}{R_1+R_2} \\ \frac{di_2}{dt} + i_2\frac{\frac{1}{C_1}+\frac{1}{C_2}}{R_1+R_2} = \frac{R_1\frac{di_0}{dt}+\frac{i_0}{C_1}}{R_1+R_2} \end{cases}$$

$$\boxed{\begin{cases} \frac{di_1}{dt} = \frac{R_2\frac{di_0}{dt}+\frac{i_0}{C_2}-i_1(\frac{1}{C_1}+\frac{1}{C_2})}{R_1+R_2} \\ \frac{di_2}{dt} = \frac{R_1\frac{di_0}{dt}+\frac{i_0}{C_1}-i_2(\frac{1}{C_2}+\frac{1}{C_1})}{R_1+R_2} \end{cases}} \tag{4}$$

Here we get a system of differential equations in which $R_1, R_2, C_1, C_2$ and $i_0$ are known and we can get the flow in the 2 different compartments.

## Testing the differential equations in Python