# 矩阵分解程序实现

## 一、 程序输入

文件列表：

| | | | | |
|---|---|---|---|---|
| 文档说明.pdf | 2018/12/26 12:08 | Adobe Acrobat ... | 704 KB |
| do_matrix.cpp | 2018/12/26 12:07 | C++ source file | 17 KB |
| do_matrix.exe | 2018/12/26 12:07 | 应用程序 | 1,300 KB |
| input.txt | 2018/12/26 12:07 | 文本文档 | 1 KB |

● 执行程序后，首先输入所要分解的矩阵 A 的尺寸 m*n，间隔为空格；

● 按行列输入该矩阵，间隔为空格；

● 接下来，会显示如下操作信息(以下操作可对矩阵 A 连续操作)：

```
please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
```

其中参数解释：

输入 0：打印该矩阵

输入 1：执行如 A=LU 的 LU 分解

输入 2：执行如 PA=LU 的 LU 分解

输入 3：执行施密特 QR 分解

输入 4：执行 Householder 分解

输入 5：执行 Givens 分解

输入其他字符：退出

```
please input integer m and n (m>0, n>0): 4 4
please the m*n matrix A:
1 2 -3 4
4 8 12 -8
2 3 2 1
-3 -1 1 -4
please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
```

## 二、 结果演示

### 1. LU

- A=LU

```
please input integer m and n (m>0, n>0): 3 3
please the m*n matrix A:
2 2 2
4 7 7
6 18 22
please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> 1
>>> The result of A=UL:
>>> L:
    1.0000000        0.0000000        0.0000000
    2.0000000        1.0000000        0.0000000
    3.0000000        4.0000000        1.0000000

>>> U:
    2.0000000        2.0000000        2.0000000
    0.0000000        3.0000000        3.0000000
    0.0000000        0.0000000        4.0000000

please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> 2
>>> The result of PA=UL:
>>> P:
    0.0000000        0.0000000        1.0000000
    0.0000000        1.0000000        0.0000000
    1.0000000        0.0000000        0.0000000

>>> L:
    1.0000000        4.0000000        1.0000000
    0.6666667        1.0000000        0.0000000
    0.3333333        0.8000000        1.0000000

>>> U:
    6.0000000       18.0000000       22.0000000
    0.0000000       -5.0000000       -7.6666667
    0.0000000        0.0000000        0.8000000
```

- PA=LU

```
please input integer m and n (m>0, n>0): 4 4
please the m*n matrix A:
1 2 -3 4
4 8 12 -8
2 3 2 1
-3 -1 1 -4
please input the operation:
        0. print the matrix
        1. LU Factorization(A=LU)
        2. LU Factorization(PA=LU)
        3. QR(Gram-Schmidt) Factorization
        4. Householder reduction
        5. Givens reduction
        Other. exit
>>> 0
>>> A:
    1.0000000       2.0000000      -3.0000000       4.0000000
    4.0000000       8.0000000      12.0000000      -8.0000000
    2.0000000       3.0000000       2.0000000       1.0000000
   -3.0000000      -1.0000000       1.0000000      -4.0000000

please input the operation:
        0. print the matrix
        1. LU Factorization(A=LU)
        2. LU Factorization(PA=LU)
        3. QR(Gram-Schmidt) Factorization
        4. Householder reduction
        5. Givens reduction
        Other. exit
>>> 1
Failed: Because there exists a zero pivot, this matrix can't be processed using LU Factorization!
please input the operation:
        0. print the matrix
        1. LU Factorization(A=LU)
        2. LU Factorization(PA=LU)
        3. QR(Gram-Schmidt) Factorization
        4. Householder reduction
        5. Givens reduction
        Other. exit
```

```
>>> 2
>>> The result of PA=UL:
>>> P:
    0.0000000       1.0000000       0.0000000       0.0000000
    0.0000000       0.0000000       0.0000000       1.0000000
    1.0000000       0.0000000       0.0000000       0.0000000
    0.0000000       0.0000000       1.0000000       0.0000000

>>> L:
    1.0000000       0.0000000       0.0000000       0.0000000
   -0.7500000       1.0000000       0.0000000       0.0000000
    0.2500000       0.0000000       1.0000000       0.0000000
    0.5000000      -0.2000000       0.3333333       1.0000000

>>> U:
    4.0000000       8.0000000      12.0000000      -8.0000000
    0.0000000       5.0000000      10.0000000     -10.0000000
    0.0000000       0.0000000      -6.0000000       6.0000000
    0.0000000       0.0000000       0.0000000       1.0000000
```

## 2. QR（Gram-Schmidt）

```
please input integer m and n (m>0, n>0): 3 3
please the m*n matrix A:
0 -20 -14
3 27 -4
4 11 -2
please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> 3
>>> The result of A=QR:
>>> Q:
        0.0000000      -0.8000000      -0.6000000
        0.6000000       0.4800000      -0.6400000
        0.8000000      -0.3600000       0.4800000

>>> R:
        5.0000000      25.0000000      -4.0000000
        0.0000000      25.0000000      10.0000000
        0.0000000       0.0000000      10.0000000

please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> .
请按任意键继续. . .
```

```
please input integer m and n (m>0, n>0): 4 3
please the m*n matrix A:
1 0 -1
1 2 1
1 1 -3
0 1 1
please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> 3
>>> The result of A=QR:
>>> Q:
        0.5773503      -0.5773503       0.4082483
        0.5773503       0.5773503       0.4082483
        0.5773503       0.0000000      -0.8164966
        0.0000000       0.5773503       0.0000000

>>> R:
        1.7320508       1.7320508      -1.7320508
        0.0000000       1.7320508       1.7320508
        0.0000000       0.0000000       2.4494897

please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> .
请按任意键继续. . .
```

```
please input integer m and n (m>0, n>0): 3 4
please the m*n matrix A:
4 2 -2 1
-3 -14 14 -7
4 -3 0 5
please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> 3
>>> The result of A=QR:
>>> Q:
        0.6246950      -0.1295131       0.7700535      -0.3300568
       -0.4685213      -0.8510860       0.2369396      -0.1499023
        0.6246950      -0.5088014      -0.5923489       0.9319827

>>> R:
        6.4031242       5.9346030      -7.8086881       7.0278193
        0.0000000      13.1825827     -11.6561784       3.2840820
        0.0000000       0.0000000       1.7770466      -3.8502677

please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> .
请按任意键继续. . .
```

## 3. Orthogonal Reduction

### a) Householder reduction

```
please input integer m and n (m>0, n>0): 3 3
please the m*n matrix A:
0 -20 -14
3 27 -4
4 11 -2
please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> 4
>>> The result of A=QR:
>>> Q:
      0.0000000      -0.8000000      -0.6000000
      0.6000000       0.4800000      -0.6400000
      0.8000000      -0.3600000       0.4800000

>>> R:
      5.0000000      25.0000000      -4.0000000
      0.0000000      25.0000000      10.0000000
      0.0000000       0.0000000      10.0000000

please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> .
请按任意键继续. . .
```

```
please input integer m and n (m>0, n>0): 4 3
please the m*n matrix A:
4 -3 4
2 -14 -3
-2 14 0
1 -7 15
please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> 4
>>> The result of A=QR:
>>> Q:
      0.8000000       0.6000000       0.0000000       0.0000000
      0.4000000      -0.5333333      -0.3333333      -0.6666667
     -0.4000000       0.5333333       0.1333333      -0.7333333
      0.2000000      -0.2666667       0.9333333      -0.1333333

>>> R:
      5.0000000     -15.0000000       5.0000000
      0.0000000      15.0000000       0.0000000
      0.0000000       0.0000000      15.0000000

please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> .
请按任意键继续. . .
```

```
please input integer m and n (m>0, n>0): 3 4
please the m*n matrix A:
4 2 -2 1
-3 -14 14 -7
4 -3 0 5
please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> 4
>>> The result of A=QR:
>>> Q:
      0.6246950      -0.1295131       0.7700535
     -0.4685213      -0.8510860       0.2369396
      0.6246950      -0.5088014      -0.5923489

>>> R:
      6.4031242       5.9346030      -7.8086881       7.0278193
      0.0000000      13.1825827     -11.6561784       3.2840820
      0.0000000       0.0000000       1.7770466      -3.8502677

please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> .
请按任意键继续. . .
```

### b) Givens reduction

```
please input integer m and n (m>0, n>0): 3 3
please the m*n matrix A:
0 -20 -14
3 27 -4
4 11 -2
please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> 5
>>> The result of A=QR:
>>> Q:
      0.0000000      -0.8000000      -0.6000000
      0.6000000       0.4800000      -0.6400000
      0.8000000      -0.3600000       0.4800000

>>> R:
      5.0000000      25.0000000      -4.0000000
      0.0000000      25.0000000      10.0000000
      0.0000000       0.0000000      10.0000000
```

```
please input integer m and n (m>0, n>0): 4 3
please the m*n matrix A:
4 -3 4
2 -14 -3
-2 14 0
1 -7 15
please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> 5
>>> The result of A=QR:
>>> Q:
      0.8000000       0.6000000       0.0000000       0.0000000
      0.4000000      -0.5333333      -0.3333333       0.6666667
     -0.4000000       0.5333333       0.1333333       0.7333333
      0.2000000      -0.2666667       0.9333333       0.1333333

>>> R:
      5.0000000     -15.0000000       5.0000000
      0.0000000      15.0000000       0.0000000
      0.0000000       0.0000000      15.0000000

please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> .
```

```
please input integer m and n (m>0, n>0): 3 4
please the m*n matrix A:
4 2 -2 1
-3 -14 14 -7
4 -3 0 5
please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> 5
>>> The result of A=QR:
>>> Q:
      0.6246950      -0.1295131       0.7700535
     -0.4685213      -0.8510860       0.2369396
      0.6246950      -0.5088014      -0.5923489
>>> R:
      6.4031242       5.9346030      -7.8086881       7.0278193
      0.0000000      13.1825827     -11.6561784       3.2840820
      0.0000000       0.0000000       1.7770466      -3.8502677

please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> .
请按任意键继续. . .
```

## 4. 连续执行

## a) 3*3 矩阵

```
please input integer m and n (m>0, n>0): 3 3      >>> 2                                      Other.exit
please the m*n matrix A:                          >>> The result of PA=UL:                    >>> 4
2 2 2                                             >>> P:                                      >>> The result of A=QR:
4 7 7                                                 0.0000000     0.0000000     1.0000000   >>> Q:
6 18 22                                               0.0000000     1.0000000     0.0000000       0.2672612    -0.5773503     0.7715167
please input the operation:                          1.0000000     0.0000000     0.0000000       0.5345225    -0.5773503    -0.6172134
        0.print the matrix                                                                        0.8017837     0.5773503     0.1543033
        1.LU Factorization(A=LU)                 >>> L:                                      >>> R:
        2.LU Factorization(PA=LU)                    1.0000000     0.0000000     0.0000000       7.4833148    18.7082869    21.9154218
        3.QR(Gram-Schmidt) Factorization             0.6666667     1.0000000     0.0000000       0.0000000     5.1961524     7.5055535
        4.Householder reduction                      0.3333333     0.8000000     1.0000000       0.0000000     0.0000000     0.6172134
        5.Givens reduction
        Other.exit                               >>> U:                                      please input the operation:
>>> 0                                                6.0000000    18.0000000    22.0000000           0.print the matrix
>>> A:                                                0.0000000    -5.0000000    -7.6666667           1.LU Factorization(A=LU)
    2.0000000     2.0000000     2.0000000            0.0000000     0.0000000     0.8000000           2.LU Factorization(PA=LU)
    4.0000000     7.0000000     7.0000000                                                            3.QR(Gram-Schmidt) Factorization
    6.0000000    18.0000000    22.0000000        please input the operation:                          4.Householder reduction
                                                         0.print the matrix                          5.Givens reduction
please input the operation:                              1.LU Factorization(A=LU)                     Other.exit
        0.print the matrix                               2.LU Factorization(PA=LU)              >>> 5
        1.LU Factorization(A=LU)                         3.QR(Gram-Schmidt) Factorization       >>> The result of A=QR:
        2.LU Factorization(PA=LU)                        4.Householder reduction                >>> Q:
        3.QR(Gram-Schmidt) Factorization                 5.Givens reduction                         0.2672612    -0.5773503     0.7715167
        4.Householder reduction                          Other.exit                                 0.5345225    -0.5773503    -0.6172134
        5.Givens reduction                       >>> 3                                              0.8017837     0.5773503     0.1543033
        Other.exit                               >>> The result of A=QR:
>>> 1                                            >>> Q:                                      >>> R:
>>> The result of A=UL:                              0.2672612    -0.5773503     0.7715167       7.4833148    18.7082869    21.9154218
>>> L:                                               0.5345225    -0.5773503    -0.6172134       0.0000000     5.1961524     7.5055535
    1.0000000     0.0000000     0.0000000            0.8017837     0.5773503     0.1543033       0.0000000     0.0000000     0.6172134
    2.0000000     1.0000000     0.0000000
    3.0000000     4.0000000     1.0000000        >>> R:                                      please input the operation:
                                                     7.4833148    18.7082869    21.9154218           0.print the matrix
>>> U:                                               0.0000000     5.1961524     7.5055535           1.LU Factorization(A=LU)
    2.0000000     2.0000000     2.0000000            0.0000000     0.0000000     0.6172134           2.LU Factorization(PA=LU)
    0.0000000     3.0000000     3.0000000                                                            3.QR(Gram-Schmidt) Factorization
    0.0000000     0.0000000     4.0000000        please input the operation:                          4.Householder reduction
                                                         0.print the matrix                          5.Givens reduction
please input the operation:                              1.LU Factorization(A=LU)                     Other.exit
        0.print the matrix                               2.LU Factorization(PA=LU)              >>> .
        1.LU Factorization(A=LU)                         3.QR(Gram-Schmidt) Factorization       请按任意键继续. . . _
        2.LU Factorization(PA=LU)                        4.Householder reduction
        3.QR(Gram-Schmidt) Factorization                 5.Givens reduction
        4.Householder reduction                          Other.exit
        5.Givens reduction
        Other.exit
```

## b) 4*3 矩阵

```
please input integer m and n (m>0, n>0): 4 3      >>> 3
please the m*n matrix A:                          >>> The result of A=QR:
1 0 -1                                            >>> Q:
1 2 1                                                 0.5773503    -0.5773503     0.4082483
1 1 -3                                                0.5773503     0.5773503     0.4082483
0 1 1                                                 0.5773503     0.0000000    -0.8164966
please input the operation:                           0.0000000     0.5773503     0.0000000
        0.print the matrix
        1.LU Factorization(A=LU)                 >>> R:
        2.LU Factorization(PA=LU)                    1.7320508     1.7320508    -1.7320508
        3.QR(Gram-Schmidt) Factorization             0.0000000     1.7320508     1.7320508
        4.Householder reduction                      0.0000000     0.0000000     2.4494897
        5.Givens reduction
        Other.exit                               please input the operation:
>>> 0                                                    0.print the matrix
>>> A:                                                   1.LU Factorization(A=LU)
    1.0000000     0.0000000    -1.0000000                2.LU Factorization(PA=LU)
    1.0000000     2.0000000     1.0000000                3.QR(Gram-Schmidt) Factorization
    1.0000000     1.0000000    -3.0000000                4.Householder reduction
    0.0000000     1.0000000     1.0000000                5.Givens reduction
                                                         Other.exit
please input the operation:                      >>> 4
        0.print the matrix                       >>> The result of A=QR:
        1.LU Factorization(A=LU)                 >>> Q:
        2.LU Factorization(PA=LU)                    0.5773503    -0.5773503     0.4082483     0.4082483
        3.QR(Gram-Schmidt) Factorization             0.5773503     0.5773503     0.4082483    -0.4082483
        4.Householder reduction                      0.5773503     0.0000000    -0.8164966     0.0000000
        5.Givens reduction                           0.0000000     0.5773503     0.0000000     0.8164966
        Other.exit
>>> 1                                            >>> R:
Failed: It's not a n*n matrix, and can't be processed by the LU Factorization!     1.7320508     1.7320508    -1.7320508
please input the operation:                          0.0000000     1.7320508     1.7320508
        0.print the matrix                           0.0000000     0.0000000     2.4494897
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)                please input the operation:
        3.QR(Gram-Schmidt) Factorization                 0.print the matrix
        4.Householder reduction                          1.LU Factorization(A=LU)
        5.Givens reduction                               2.LU Factorization(PA=LU)
        Other.exit                                       3.QR(Gram-Schmidt) Factorization
>>> 2                                                    4.Householder reduction
Failed: It's not a n*n matrix, and can't be processed by the LU Factorization!     5.Givens reduction
please input the operation:                              Other.exit
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
```

## c) 3*4 矩阵

```
>>> 5
>>> The result of A=QR:
>>> Q:
     0.6246950      -0.1295131       0.7700535
    -0.4685213      -0.8510860       0.2369396
     0.6246950      -0.5088014      -0.5923489

>>> R:
     6.4031242       5.9346030      -7.8086881       7.0278193
     0.0000000      13.1825827     -11.6561784       3.2840820
     0.0000000       0.0000000       1.7770466      -3.8502677

please input the operation:
        0.print the matrix
        1.LU Factorization(A=LU)
        2.LU Factorization(PA=LU)
        3.QR(Gram-Schmidt) Factorization
        4.Householder reduction
        5.Givens reduction
        Other.exit
>>> .
请按任意键继续. . .
```

## 三、 代码分析

### 1. LU

```cpp
// process the matrix using the LU Factorization
// for type=0, A=LU
// for type=1, PA=LU, using partial pivoting
bool doLU(int type, std::vector< std::vector<double> > input, std::vector<
std::vector<double> > *P, std::vector< std::vector<double> > *L, std::vector<
std::vector<double> > *U)
```

### 2. QR（Gram-Schmidt）

```cpp
// process the matrix using the QR(Gram-Schmidt)
bool doQR(std::vector< std::vector<double> > input, std::vector< std::vector<double> > *Q,
std::vector< std::vector<double> > *R)
```

### 3. Orthogonal Reduction

### a) Householder reduction

```cpp
// process the matrix using the Householder reduction
bool doHouseholder(std::vector< std::vector<double> > input, std::vector<
std::vector<double> > *P, std::vector< std::vector<double> > *Q, std::vector<
std::vector<double> > *R)
```

### b) Givens reduction

```cpp
// process the matrix using the Givens reduction
bool doGivens(std::vector< std::vector<double> > input, std::vector< std::vector<double> >
*P, std::vector< std::vector<double> > *Q, std::vector< std::vector<double> > *R)
```