

Vector

1 Terminology

Line

Ray

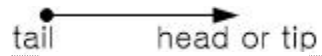
Line Segment



[Fig] Line Segment, Ray and Line

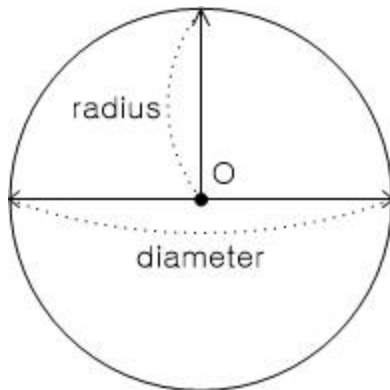
Begin, End

Tail, Head

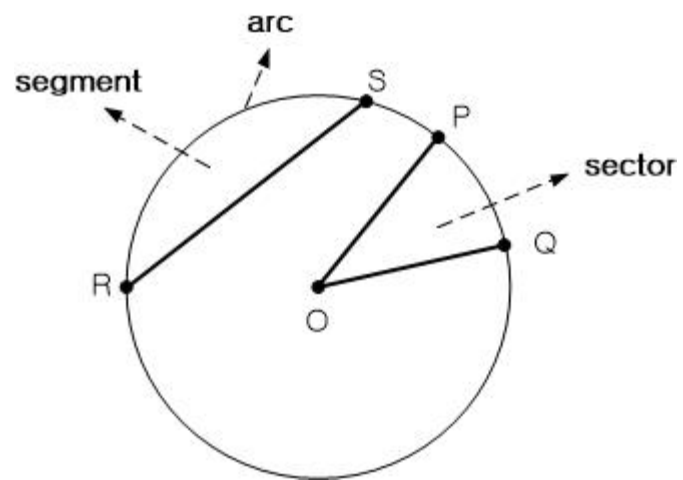


[Fig] Tail and Head

Circle

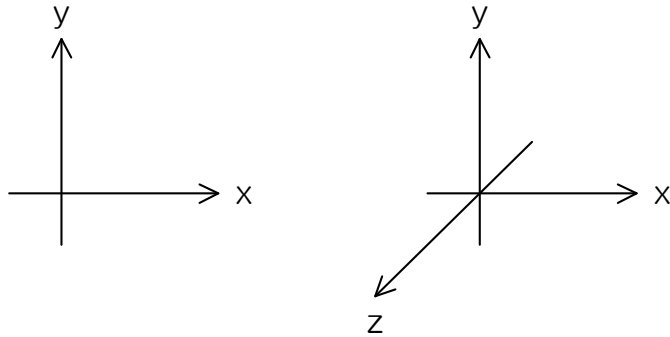


[Fig] Radius and Diameter



[Fig] Sector, Segment and Arc

Coordinate System



[Fig] Standard Axis

Right Hand Coordinate System

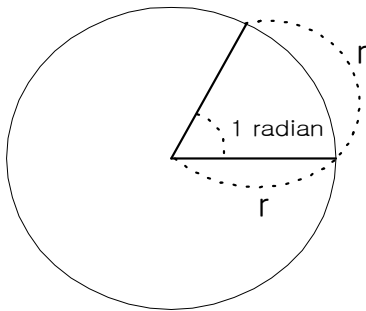
Right Hand Rule

2 Basic Concepts

radian

pi, π

3.141592

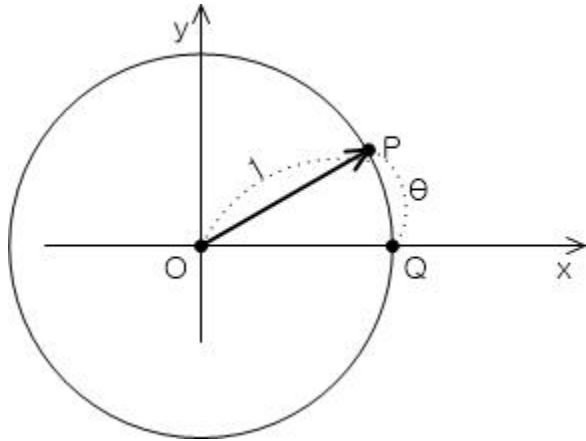


[Fig] Radian

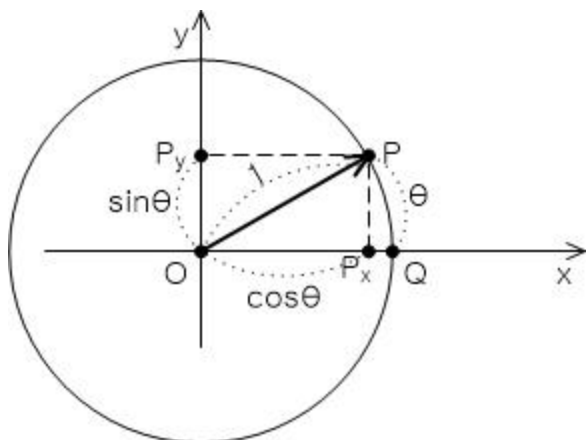
Pi and Degree

$$\pi \text{ radian} = 180 \text{ degree}$$

Trigonometric Functions



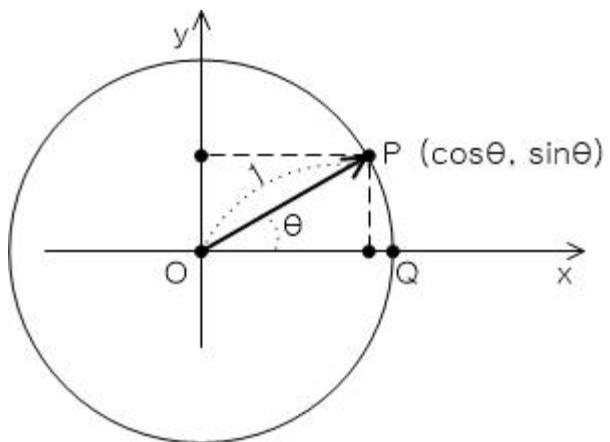
[Fig] unit circle



[Fig] Length of the projected line segment

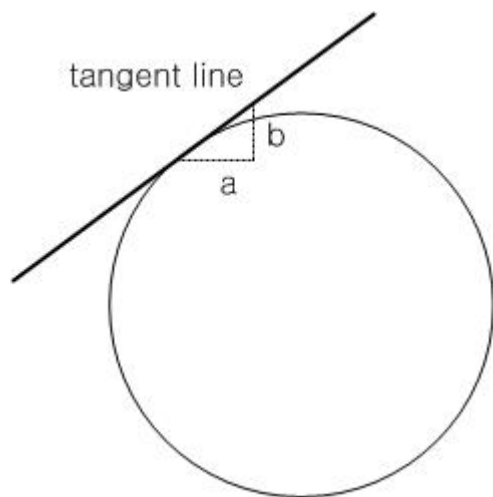
Sine

Cosine

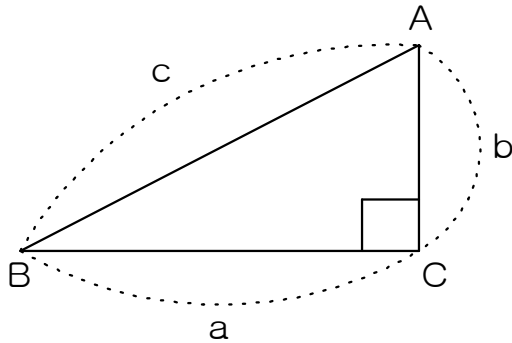


[Fig] Coordiante of P

$(\cos(\theta), \sin(\theta))$



[Fig] tangent line: Slope quotient is b/a



[Fig] Trigonometric Functions

Pythagorean Theorem

$$c^2 = a^2 + b^2, c = \sqrt{a^2 + b^2}$$

$$1^2 = \sin^2\theta + \cos^2\theta, 1 = \sin^2\theta + \cos^2\theta$$

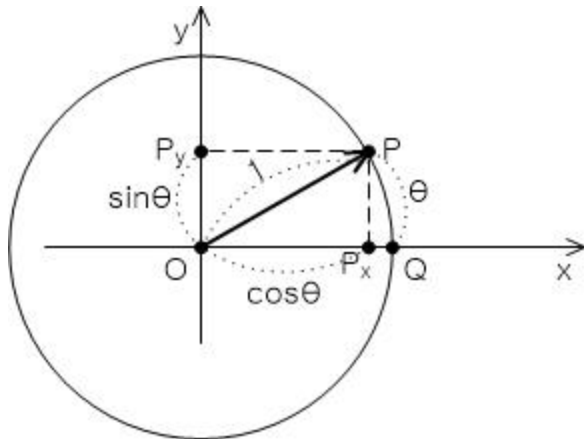
Taylor series

Trigonometric functions

$\sin()$, $\cos()$

Inverse Function

$\text{asin}()$, $\text{acos}()$



[Fig] $\cos()$ 의 역함수의 정의

Length of Arc θ

Length of OP_x

arc cosine

$$\text{acos}(\cos \theta) = \theta$$

3 Vector

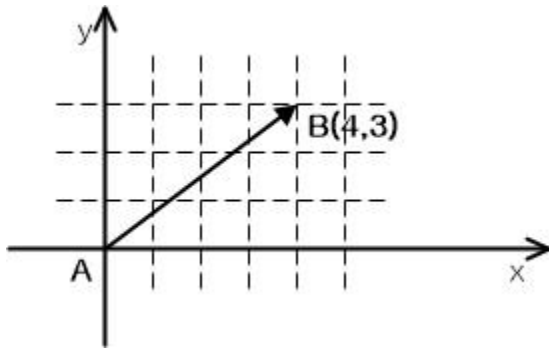
magnitude and direction

scalar

Speed and Velocity

initial point A, terminal point B

$$v = \overrightarrow{AB}$$



[Fig] Vector: $v = \overrightarrow{AB}$ is expressed as (4,3) or $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$$v = \overrightarrow{AB}$$

row vector (4,3)

column vector $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

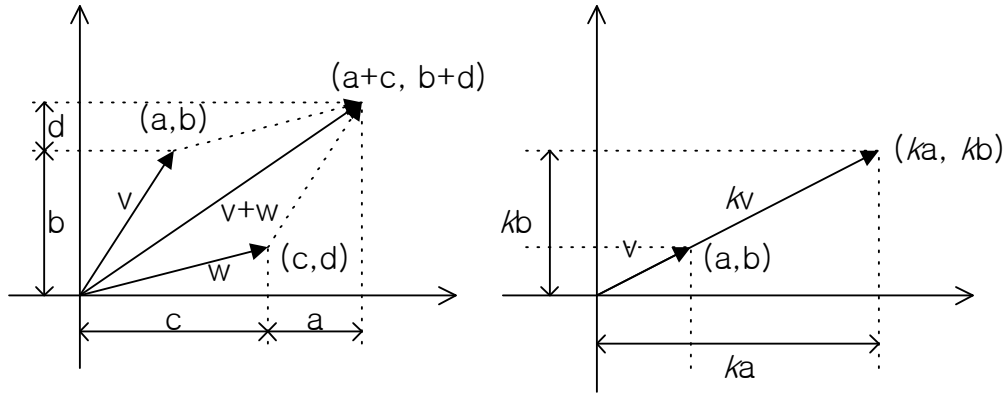
Vector valued funtion f() and g()

apply result of $f\left(\begin{bmatrix} 4 \\ 3 \end{bmatrix}\right)$ to g()

$$g\left(f\left(\begin{bmatrix} 4 \\ 3 \end{bmatrix}\right)\right)$$

$$g\left(f\left(\begin{bmatrix} 4 \\ 3 \end{bmatrix}\right)\right)$$

Vector Operations



[Fig] Vector Addition and Scalar Multiplication

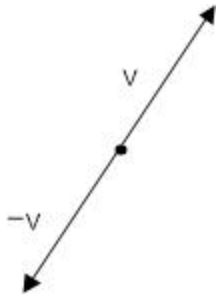
$$\vec{v} = (a, b), \vec{w} = (c, d)$$

$$\vec{v} + \vec{w} = (a, b) + (c, d) = (a + c, b + d)$$

$$\vec{v} = (a, b), \text{ real number } k$$

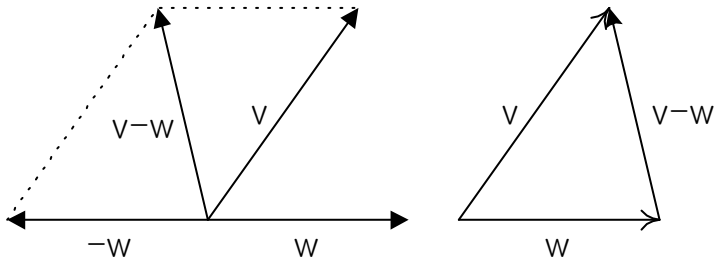
$$k\vec{v} = k(a, b) = (ka, kb)$$

Negative Vector



[Fig] Negative Vector

$$v = (a, b), \quad -v = (-a, -b)$$



[Fig]Vector Subtraction

$$\vec{v} - \vec{w} = (a - c, b - d)$$

length or norm

$$|v| = \sqrt{a^2 + b^2}$$

normalization

4 Implementing class KVector2

```
class KVector2
{
public:
    float    x;
    float    y;

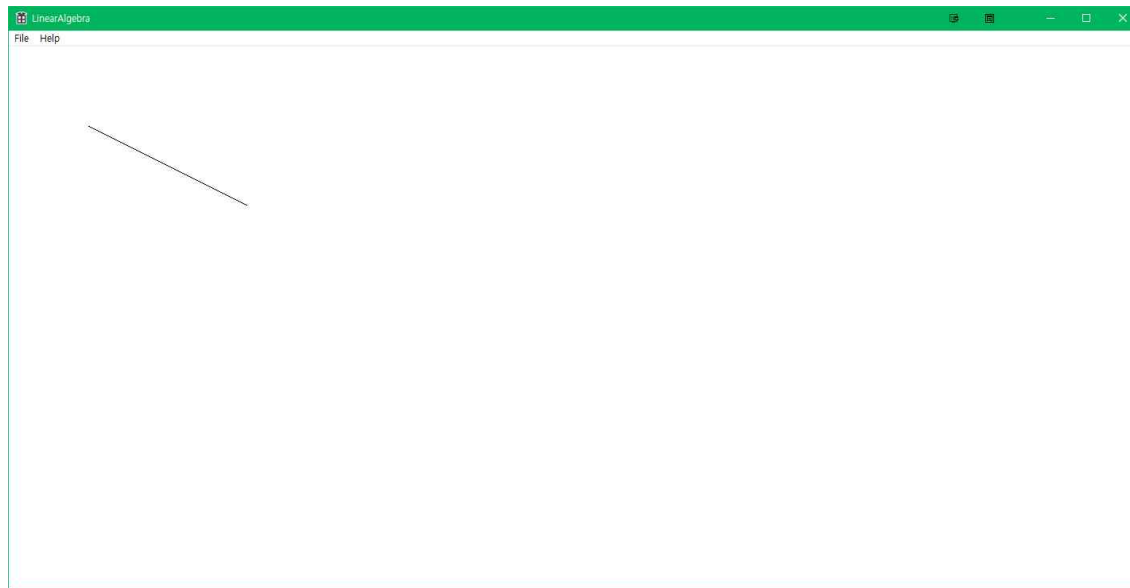
public:
    KVector2(float tx, float ty) { x = tx; y = ty; }
    KVector2(int tx, int ty) { x = (float)tx; y = (float)ty; }
};

inline KVector2 operator+(const KVector2& lhs, const KVector2& rhs)
{
```



```
    KVector2 temp(lhs.x + rhs.x, lhs.y + rhs.y);  
    return temp;  
}  
  
inline KVector2 operator*(float scalar, const KVector2& rhs)  
{  
    KVector2 temp(scalar*rhs.x, scalar*rhs.y);  
    return temp;  
}  
  
inline KVector2 operator*(const KVector2& lhs, float scalar)  
{  
    KVector2 temp(scalar*lhs.x, scalar*lhs.y);  
    return temp;  
}
```

```
void KVectorUtil::DrawLine(HDC hdc, const KVector2& v0, const KVector2& v1)  
{  
    MoveToEx(hdc, (int)v0.x, (int)v0.y, nullptr);  
    LineTo(hdc, (int)v1.x, (int)v1.y);  
}
```



[Fig] LinearAlgebra_Step02 Vectors

Exercise

1. Implement `Normalize()` member function of class `KVector2`
2. Draw a line segment of length 200 in direction (3,2).
3. Draw a circle using `sin()` and `cos()`

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