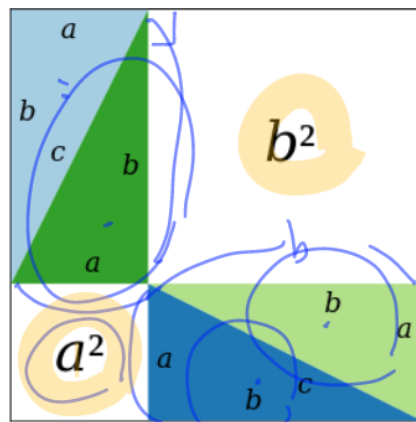
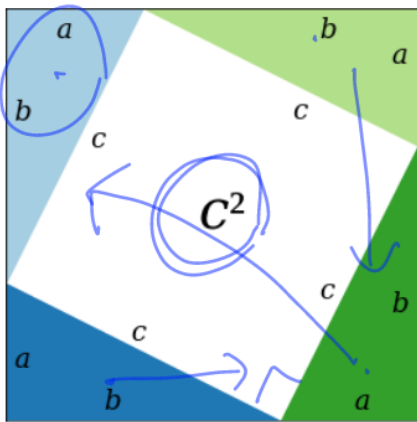


1. Pythagorean theorem
2. Trigonometric functions
3. Number space.
4. 2-dimension rotation.

$$h * h = a * a + b * b$$

$$\underline{h^2 = a^2 + b^2}$$



$$\textcircled{c^2} = \textcircled{a^2} + \textcircled{b^2}$$

$$\sqrt{4} = 2$$

$$\sqrt{9} = 3$$

$$c = \sqrt{c^2 = a^2 + b^2}$$

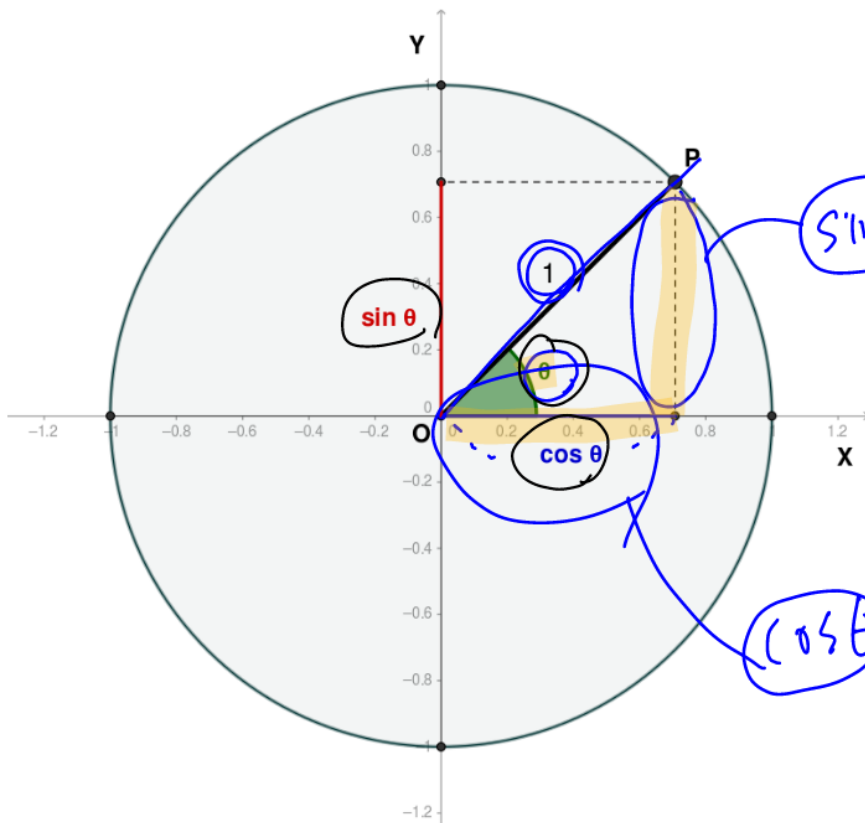
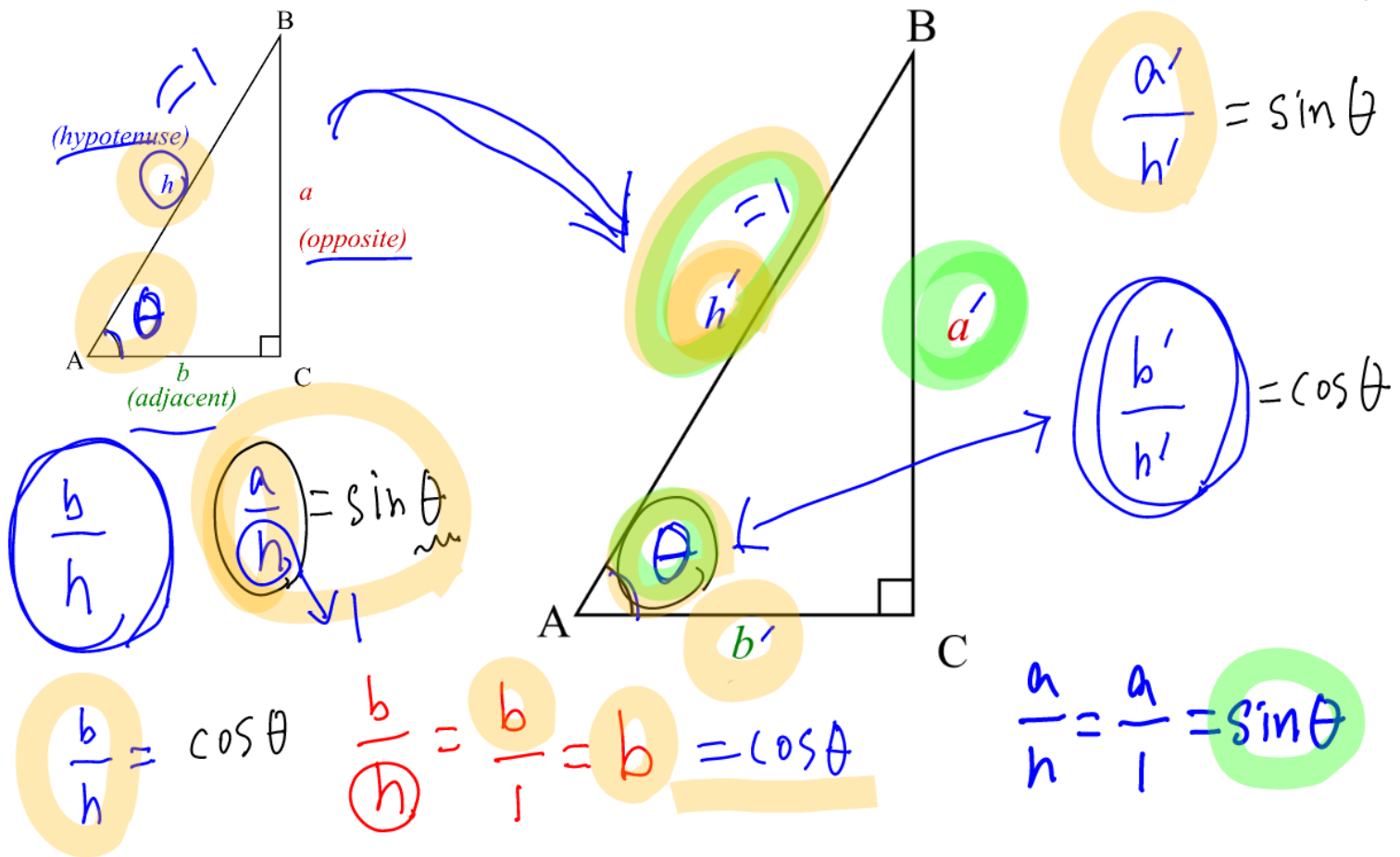
$$\textcircled{2^2} = 4$$

$$\sqrt{4} \rightarrow 2^2$$

$$\sqrt{9} \rightarrow 3^2$$

$$\boxed{c = \sqrt{a^2 + b^2}}$$

(2)



$$1^2 = (\sin \theta)^2 + (\cos \theta)^2$$

$$\downarrow$$

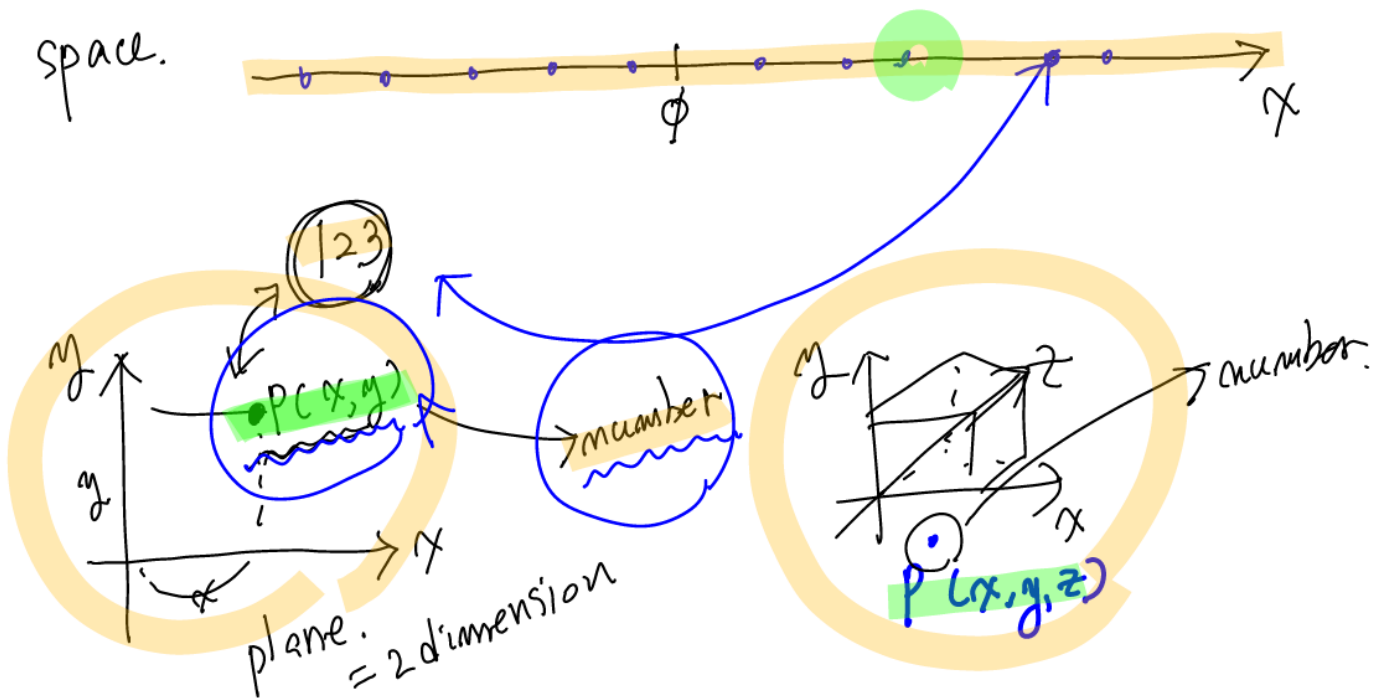
$$1 = \sin^2 \theta + \cos^2 \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

123 number
 ↓
 digit

number space.

space.



$$\sqrt{4} = 2$$

$$\sqrt{9} = 3$$

$$\sqrt{1} = 1$$

$$\sqrt{0} \quad n \geq 0$$

~~$$\sqrt{-4} = ?$$~~

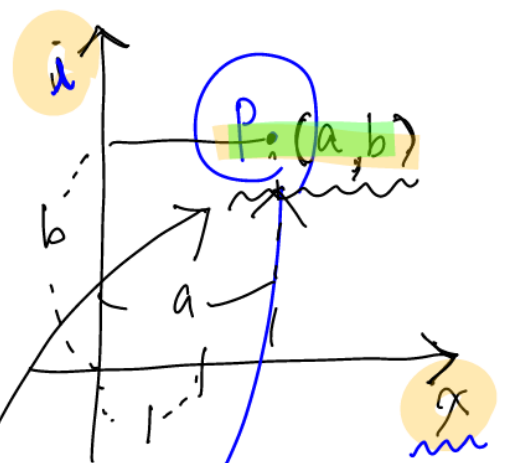
$$\sqrt{-1} = \text{imaginary number}$$

$$i = \sqrt{-1}$$

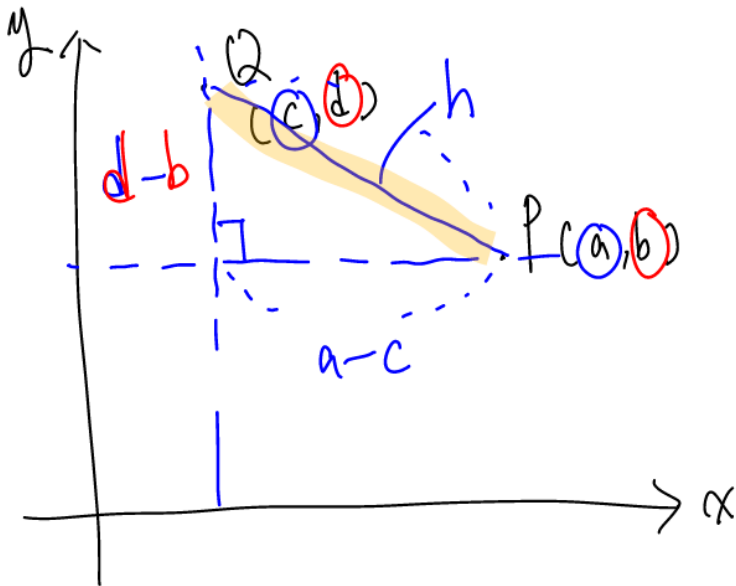
$$i = \sqrt{-1}$$

complex number

↓
 $\frac{1}{2} \pm \frac{1}{2}i$

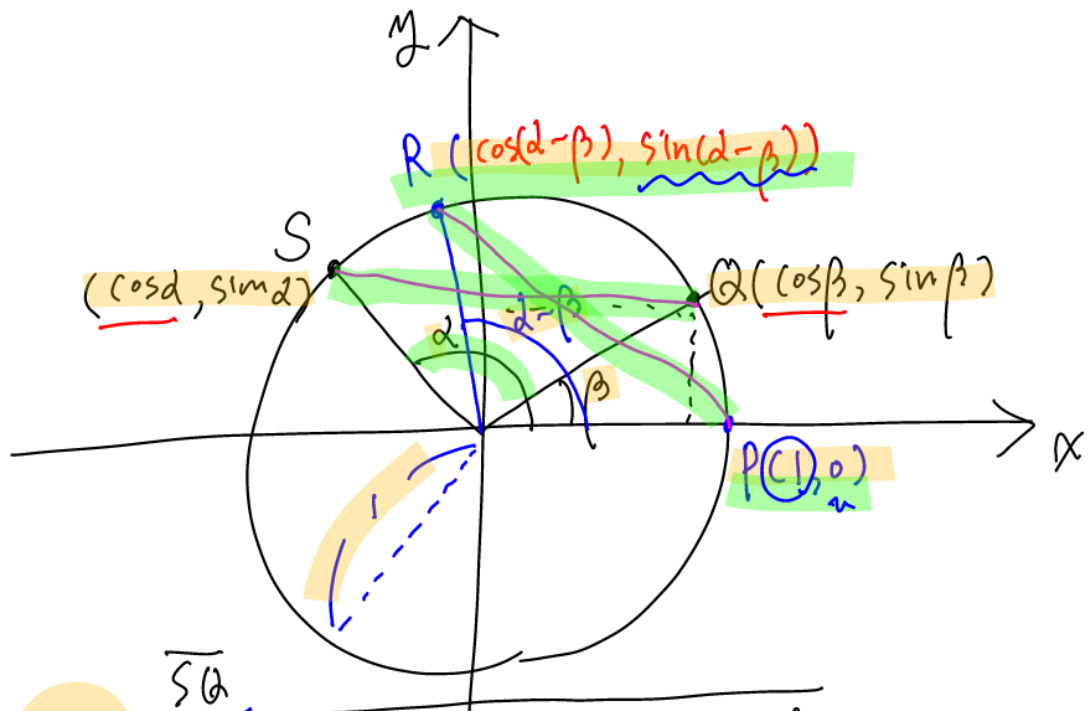


$$ax + b i \text{ complex number} \\ = a + b i$$



$$h^2 = \underbrace{(a-c)^2} + \underbrace{(d-b)^2}$$

$$h = \sqrt{(a-c)^2 + (d-b)^2}$$



$$\begin{aligned} \overline{SQ} &\equiv \overline{RP} \\ &= \sqrt{(\cos \alpha - \cos \beta)^2 + (\sin \alpha - \sin \beta)^2} \\ &= \sqrt{(\cos(\alpha - \beta) - 1)^2 + (\sin(\alpha - \beta) - 0)^2} \end{aligned}$$

$$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2$$

$$= a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(\cancel{\cos^2 \alpha} - 2\cos \alpha \cos \beta + \cancel{\cos^2 \beta}) + (\cancel{\sin^2 \alpha} - 2\sin \alpha \sin \beta + \cancel{\sin^2 \beta})$$

$$= (\cos^2(\alpha - \beta) - 2\cos(\alpha - \beta) + 1) + \sin^2(\alpha - \beta)$$

$$\cancel{\cos^2 \alpha + \sin^2 \alpha + \cos^2 \beta + \sin^2 \beta} - 2\cos \alpha \cos \beta - 2\sin \alpha \sin \beta$$

$$= \cancel{\cos^2(\alpha - \beta) + \sin^2(\alpha - \beta)} - 2\cos(\alpha - \beta) + \cancel{1}$$

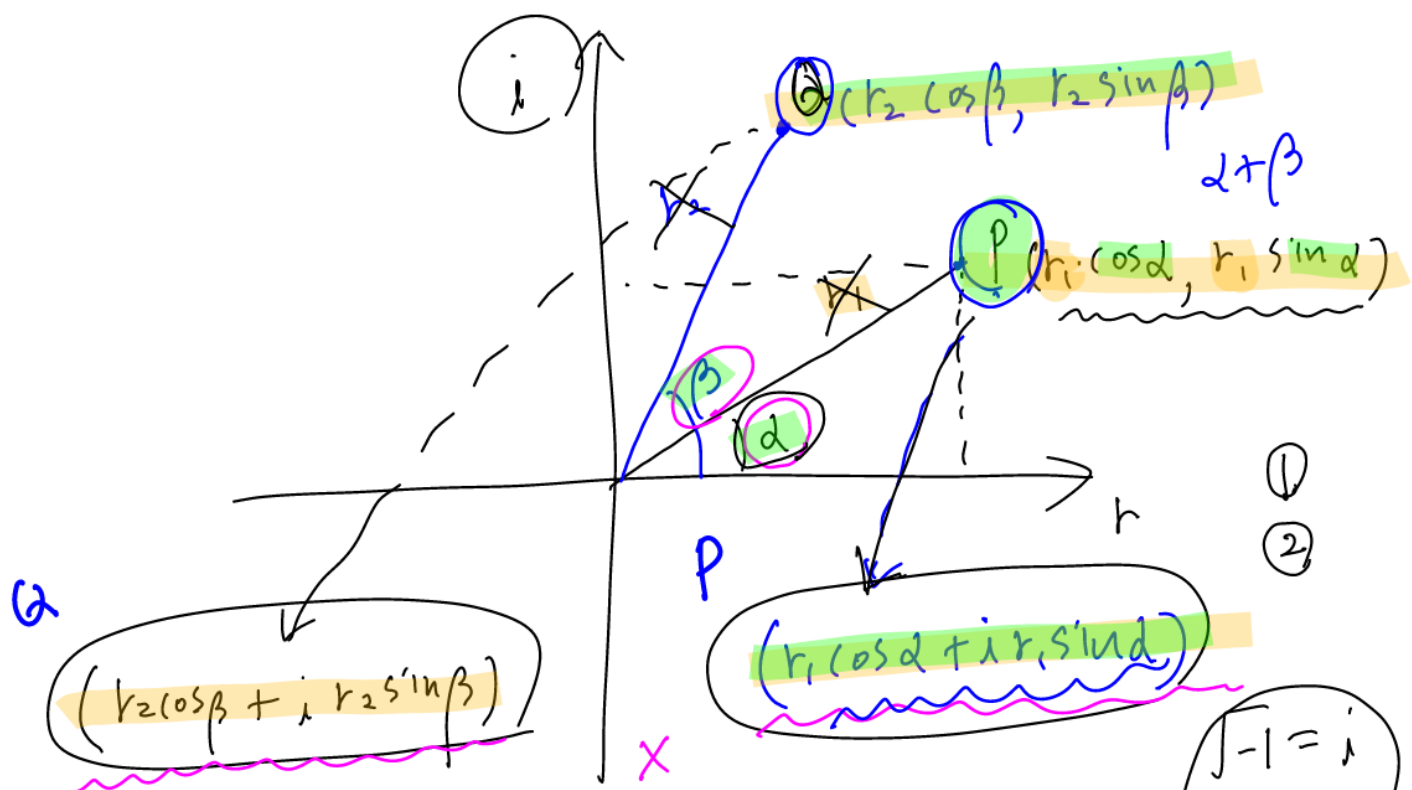
$$\cancel{2\cos \alpha \cos \beta} - \cancel{2\sin \alpha \sin \beta} = \cancel{2\cos(\alpha - \beta)}$$

$$\cos \alpha \cos \beta + \sin \alpha \sin \beta = \cos(\alpha - \beta)$$

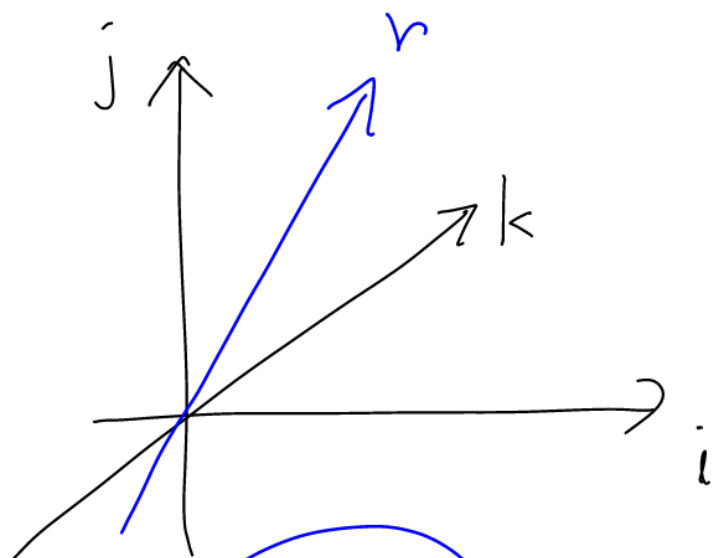
$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$



$$\begin{aligned}
 Q * P &= (r_2(\cos \beta + i \sin \beta)) * (r_1(\cos \alpha + i \sin \alpha)) \\
 &= r_1 r_2 (\cos \alpha \cos \beta + i r_1 r_2 \sin \alpha \cos \beta + i r_1 r_2 \cos \alpha \sin \beta \\
 &\quad + i^2 r_1 r_2 \sin \alpha \sin \beta) \\
 &= r_1 r_2 (\cos \alpha \cos \beta - r_1 r_2 \sin \alpha \sin \beta \\
 &\quad + i r_1 r_2 (\sin \alpha \cos \beta + \cos \alpha \sin \beta)) \\
 &= r_1 r_2 (\cos \alpha \cos \beta - \sin \alpha \sin \beta + i (\sin \alpha \cos \beta + \cos \alpha \sin \beta)) \\
 &= (\cos(\alpha + \beta) + i \sin(\alpha + \beta)) \\
 &\Rightarrow (\cos \beta + i \sin \beta) * (\cos \alpha + i \sin \alpha) \\
 &= \cos(\alpha + \beta) + i \sin(\alpha + \beta)
 \end{aligned}$$



$$i = \sqrt{-1}$$

$$j = \sqrt{-1}$$

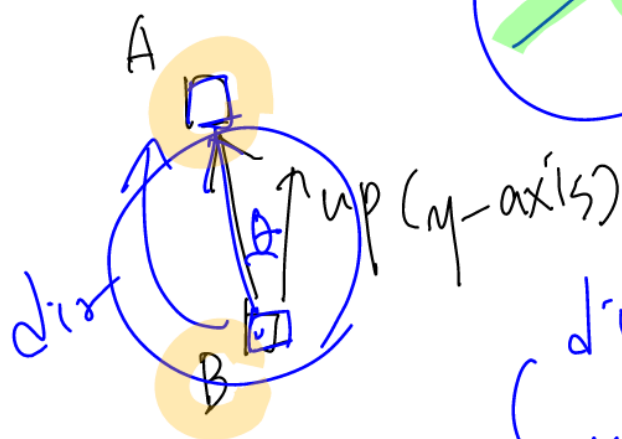
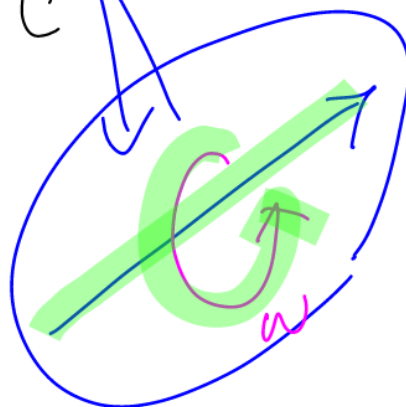
$r = \text{real number}$

$$k = \sqrt{-1}$$

$A = (i, j, k, w)$ quaternion.

$B \quad A * B = C$

real number



$dir = A.\text{position} - B.\text{position}$
 $(up \rightarrow \text{Quaternion})$

$B.\text{rotation} =$