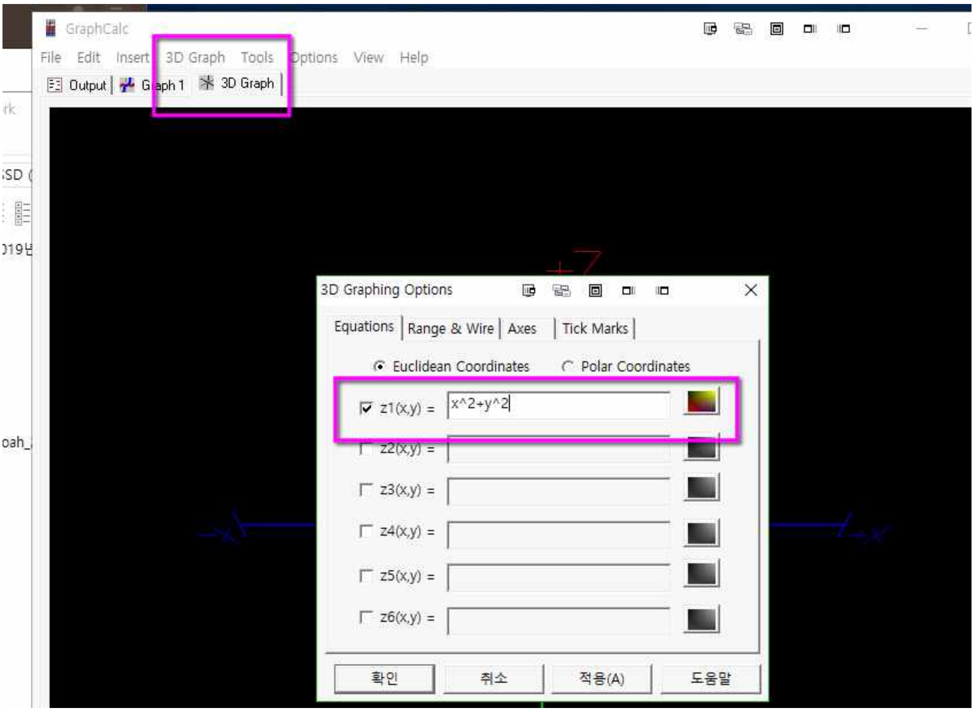
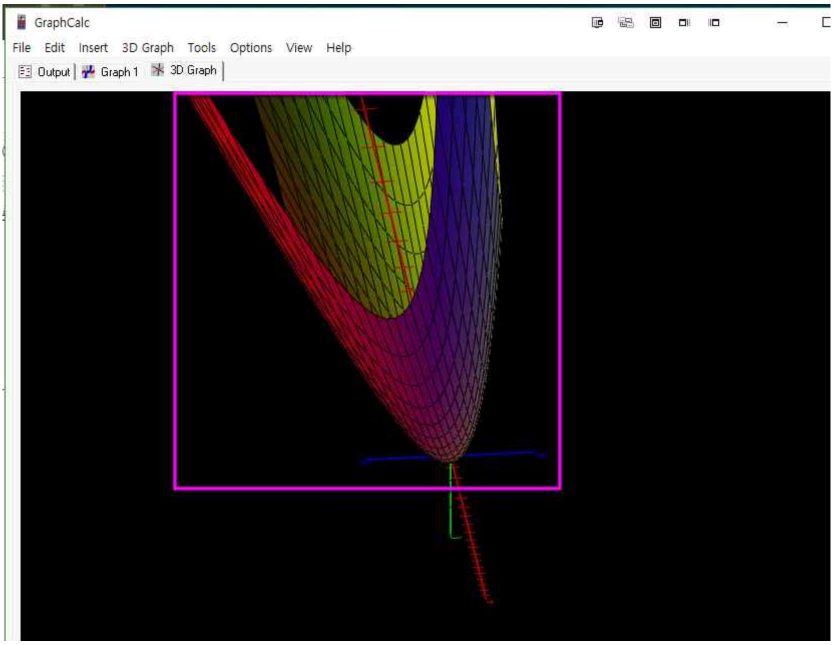
인공 신경망: Back-propagation

> 2019년1월31일, 서진택

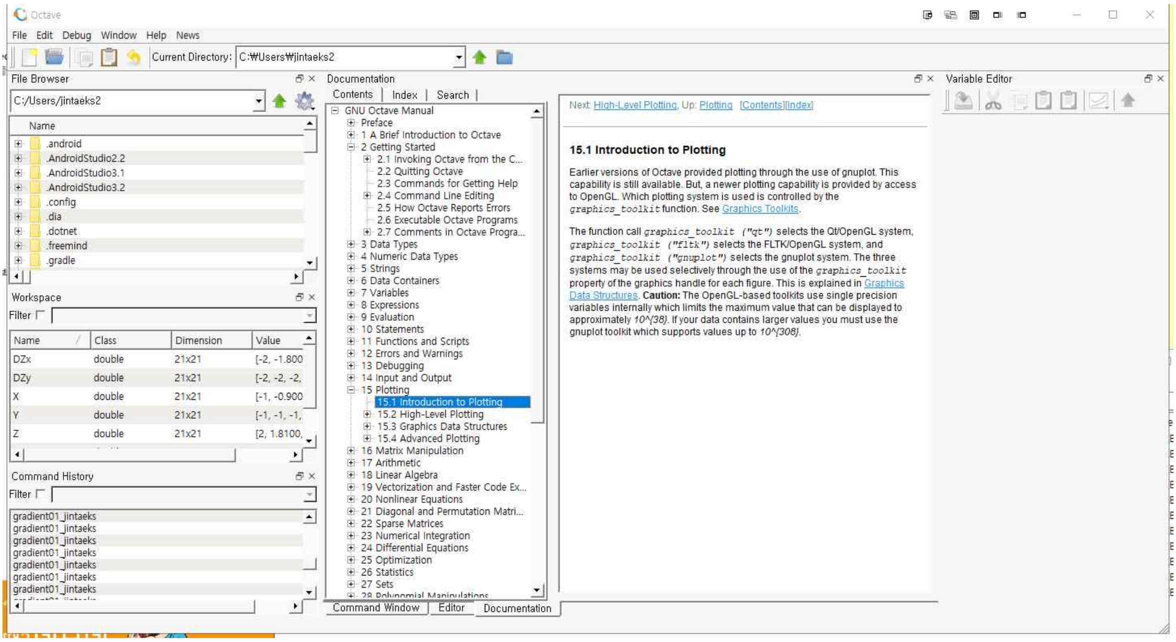
Plotting 3D Graph

GraphCalc

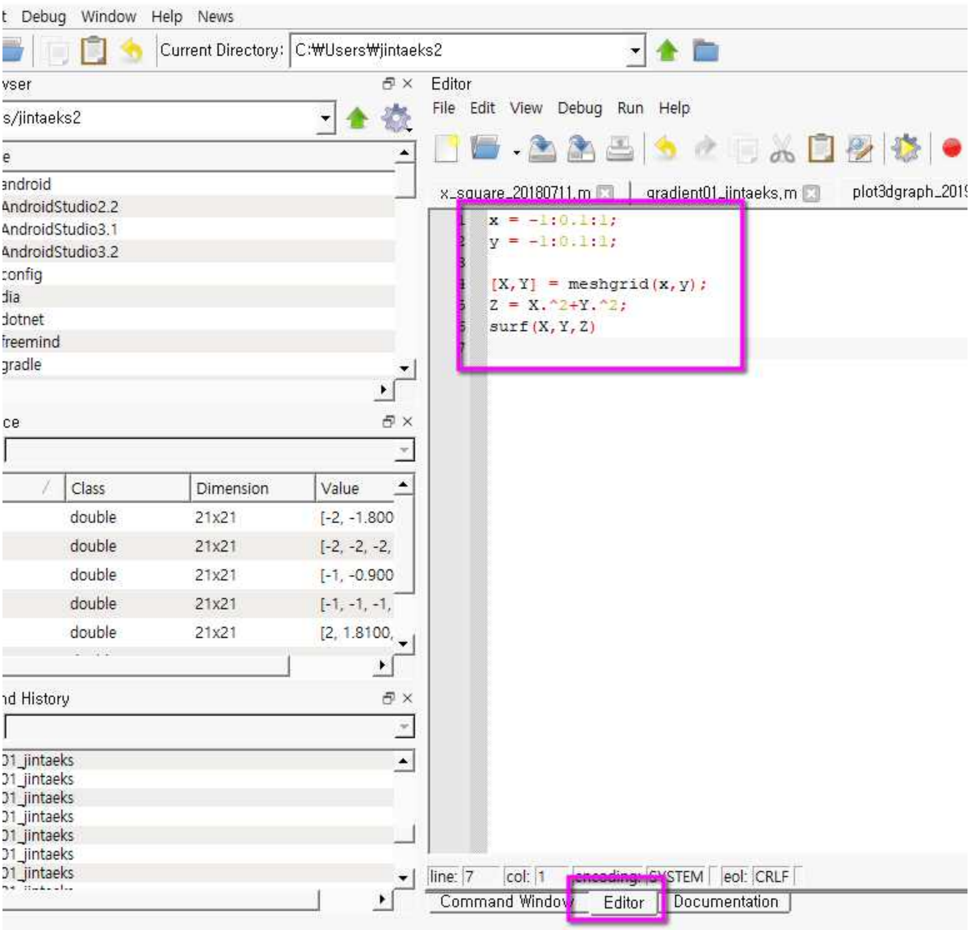




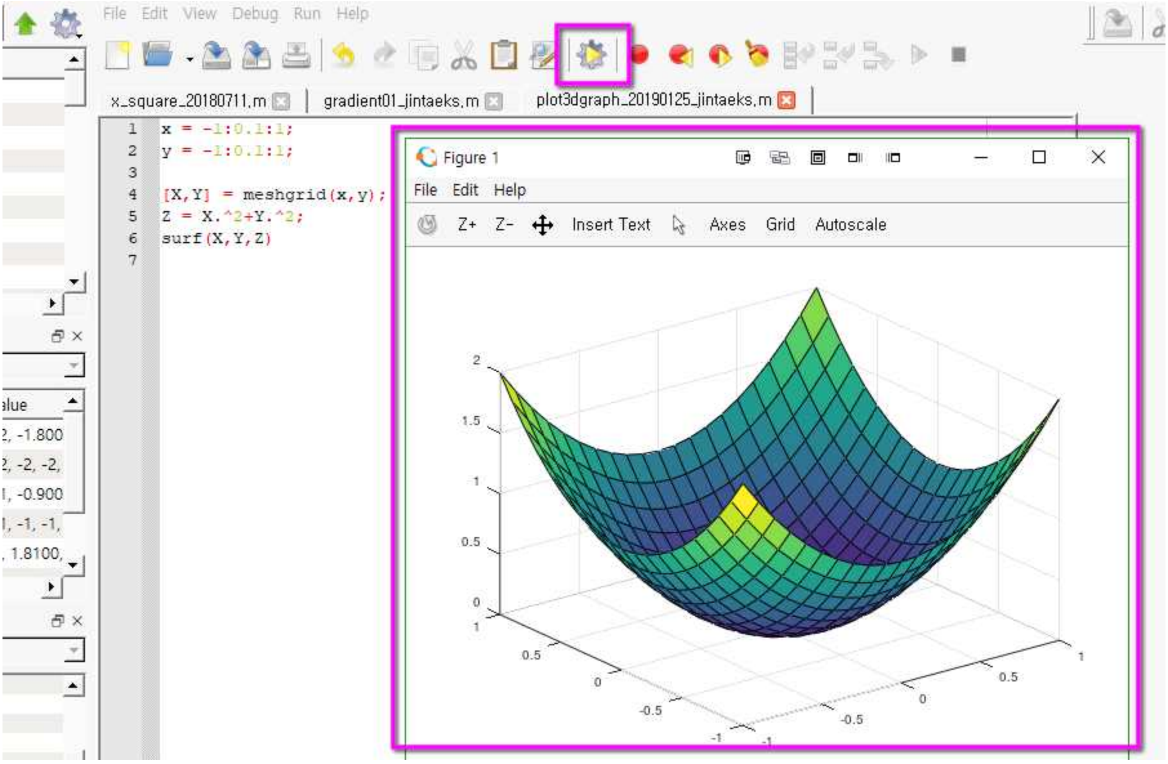
Octave



15.1 Introduction to Plotting
Earlier versions of Octave provided plotting through the use of gnuplot. This capability is still available. But, a newer plotting capability is provided by access to OpenGL. Which plotting system is used is controlled by the graphics_toolkit function. See Graphics Toolkits.



x = -1:0.1:1;
y = -1:0.1:1;
[X,Y] = meshgrid(x,y);
Z = X.^2+Y.^2;
surf(X,Y,Z)



Differentiation

Product Rule

$$[f \times g]' = f'g + f \times g'$$

$$\left[(2x+3)^4(x+1)^2\right]'$$
$$f(x) = (2x+3)^4$$
$$g(x) = (x+1)^2$$

$$f'(x) = 4(2x+3)^3 \times 2 = 8(2x+3)^3$$
$$g'(x) = 2(x+1)^1 \times 1 = 2x+2$$

$$[f \times g]' = f'g + f \times g'$$
$$= 8(2x+3)^3(x+1)^2 + (2x+3)^4(2x+2)$$

Quotient Rule

$$\left[\frac{f}{g}\right]' = \frac{f'g - fg'}{g^2}$$

$$\frac{d}{dx}\left(\frac{3x-5}{x^2+4}\right)$$
$$= \frac{3(x^2+4)-(3x-5)(2x)}{(x^2+4)^2}$$

Chain Rule

$$\frac{d}{dx}(x^2+3)^4$$
$$u = (x^2+3)$$
$$y = u^4$$
$$\frac{dy}{du} = 4u^3$$
$$\frac{du}{dx} = 2x$$

$$\begin{aligned} y &= (x^2+3)^4 \\ \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= 4u^3 \times 2x \\ &= 4(x^2+3)^3 \times 2x \\ &= 8x(x^2+3)^3 \end{aligned}$$

Differentiation of Trigonometric Functions

$$\frac{d}{dx} sin(x) = \cos(x)$$

$$\frac{d}{dx} cos(x) = -\sin(x)$$

$$\frac{d}{dx}tan(x) = \left(\frac{\sin(x)}{\cos(x)}\right)' = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$$

Differentiation of Exponential Functions

$$\left(e^x\right)' = e^x$$

$$\begin{aligned}
&\left(e^{-x}\right)' \\
&= e^{-x} \times (-1) \\
&= -e^{-x}
\end{aligned}$$

Differentiation of Sigmoid Function

$$sigmoid(x) = \frac{1}{1 + e^{-x}}$$

$$\begin{aligned}
\frac{d}{dx}sigmoid(x) &= \left((1 + e^{-x})^{-1}\right)' \\
&= -1 \times (1 + e^{-x})^{-2} \times (-e^{-x}) \\
&= \frac{e^{-x}}{(1 + e^{-x})^2} \\
&= \frac{1}{(1 + e^{-x})} \times \frac{(1 + e^{-x}) - 1}{(1 + e^{-x})} \\
&= sigmoid(x) \times (1 - sigmoid(x))
\end{aligned}$$

Gradient

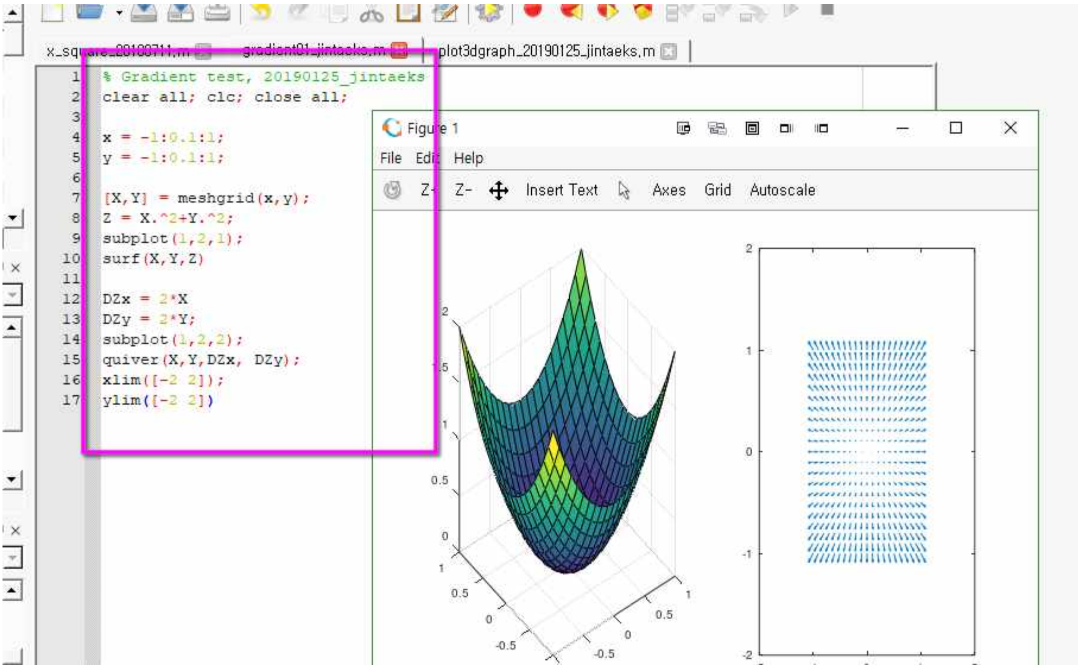
$$f(x,y) = x^2 \sin(y)$$

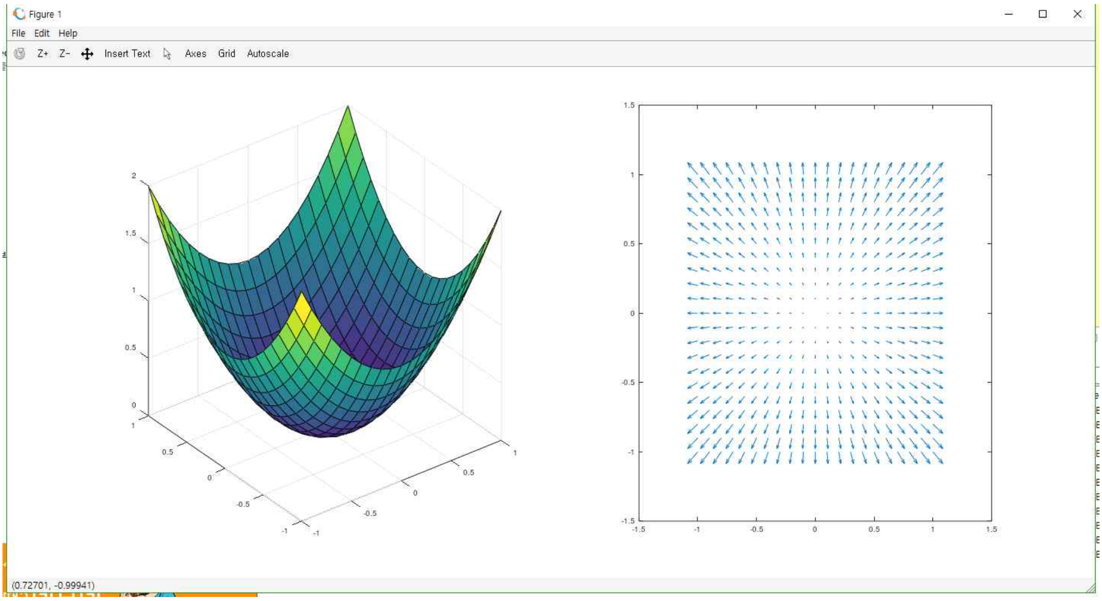
$$\frac{df}{dx} = 2x\sin(y), \frac{df}{dy} = x^2\cos(y)$$

$$\nabla f(x,y) = \begin{bmatrix} 2x sin(y) \\ x^2 \cos(y) \end{bmatrix}$$

example: $f(x,y) = x^2 + y^2$

$$\nabla f(x,y) = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$



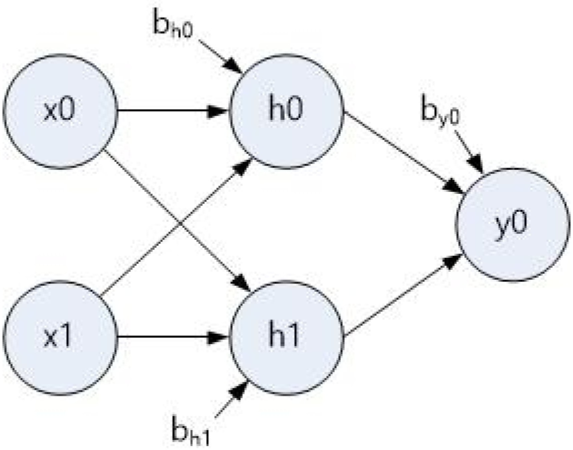


Neural Network

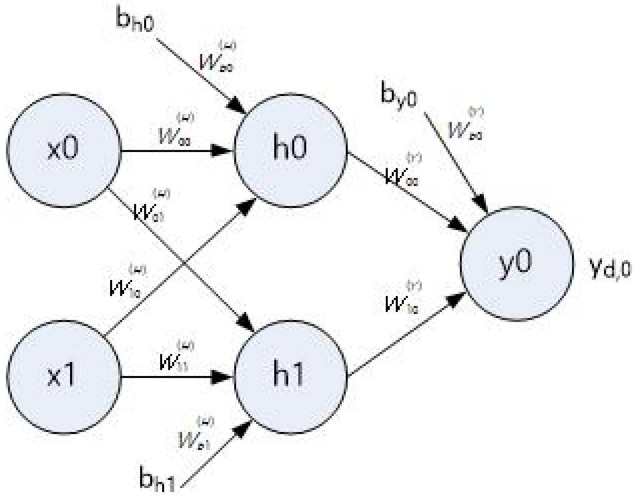
Differentiation of Sigmoid Function

$$sigmoid(x) = \frac{1}{1+e^{-x}}$$

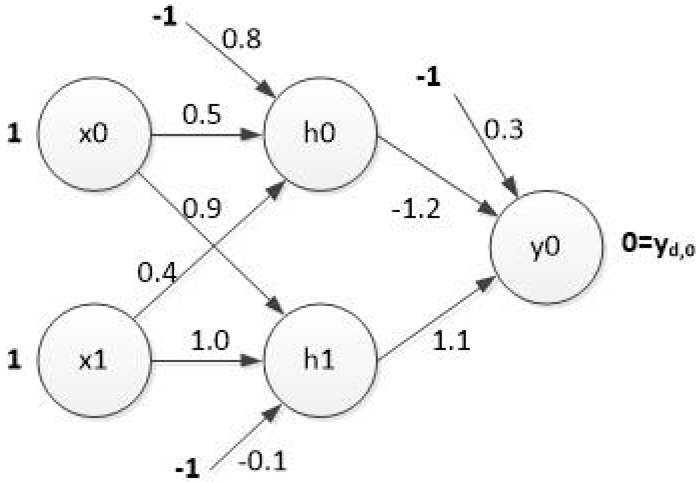
$$\begin{aligned}
\frac{d}{dx} sigmoid(x) &= \left((1+e^{-x})^{-1}\right)' \\
&= -1 \times (1+e^{-x})^{-2} \times (-e^{-x}) \\
&= \frac{e^{-x}}{(1+e^{-x})^2} \\
&= \frac{1}{(1+e^{-x})} \times \frac{(1+e^{-x})-1}{(1+e^{-x})} \\
&= sigmoid(x) \times (1 - sigmoid(x))
\end{aligned}$$



Feed Forward

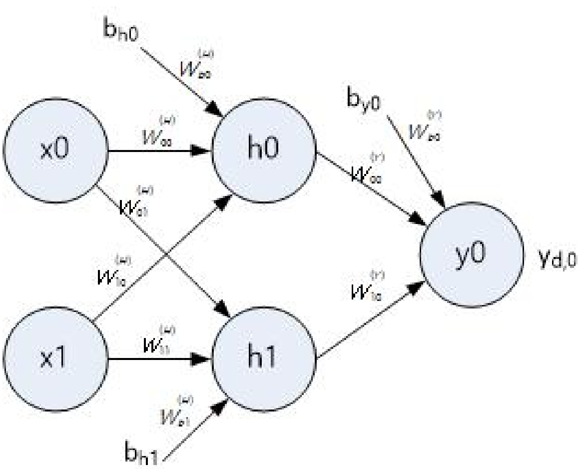


x0
x1
h0
h1
y0
b_{h0}
b_{h1}
b_{y0}
$y_{d,0}$

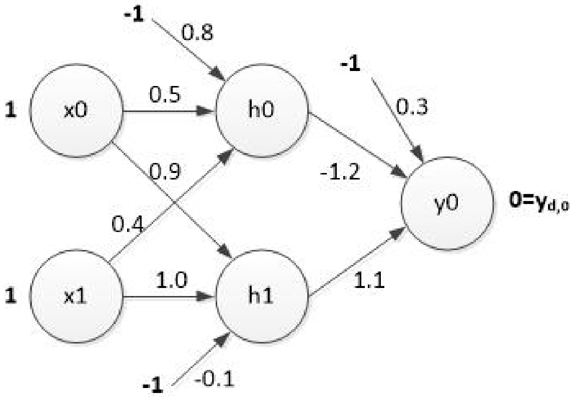


-1
0.8
-1
0.3
1
x0
0.5
h0
0.9
-1.2
y0
0=$y_{d,0}$
0.4
1
x1
1.0
h1
1.1
-1
-0.1

Steps

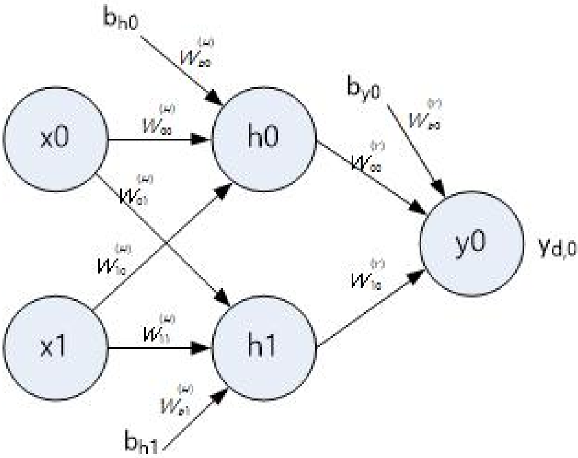


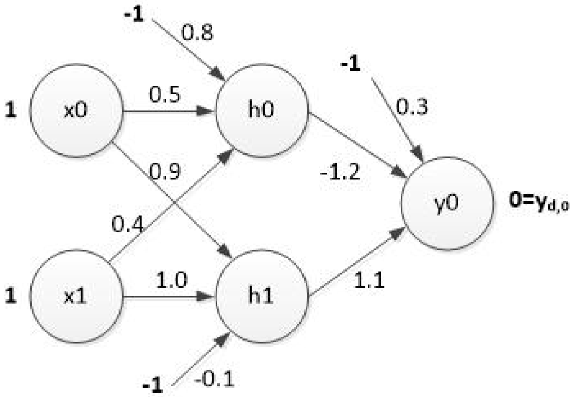
x0
x1
h0
h1
y0
b_{h0}
b_{h1}
b_{y0}
$y_{d,0}$



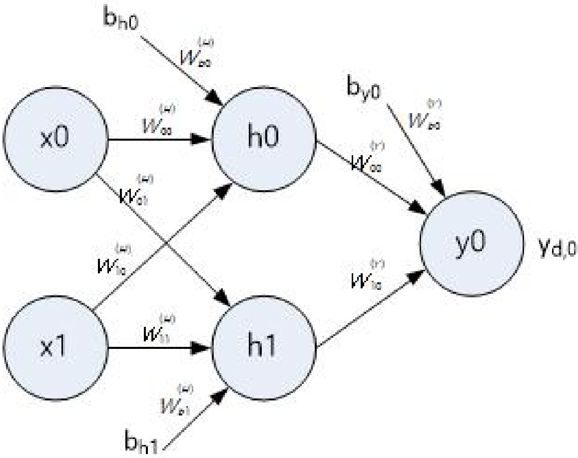
-1
0.8
-1
0.3
1
x0
0.5
h0
0.9
-1.2
y0
0=$y_{d,0}$
0.4
1
x1
1.0
h1
1.1
-1
-0.1

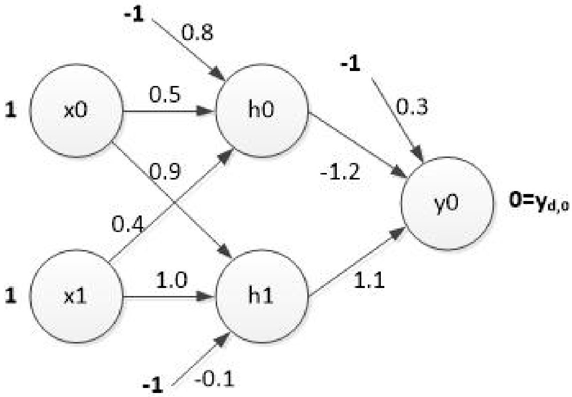
$$
\begin{aligned}
h0 &= sigmoid(x_0 w_{00}^{(H)} + x_1 w_{10}^{(H)} + b_{h0} w_{b0}^{(H)}) \\
&= 1/(1+e^{-(x_0 w_{00}^{(H)} + x_1 w_{10}^{(H)} + b_{h0} w_{b0}^{(H)})}) \\
&= 1/(1+e^{-(1\times 0.5 + 1\times 0.4 + (-1)\times 0.8)}) \\
&= 0.5250
\end{aligned}
$$





$$
\begin{aligned}
h1 &= sigmoid(x_0 w_{01}^{(H)} + x_1 w_{11}^{(H)} + b_{h1} w_{b1}^{(H)}) \\
&= 1/(1+e^{-(x_0 w_{01}^{(H)} + x_1 w_{11}^{(H)} + b_{h1} w_{b1}^{(H)})}) \\
&= 1/(1+e^{-(1\times 0.9 + 1\times 1.0 + (-1)\times(-0.1))}) \\
&= 0.8808
\end{aligned}
$$

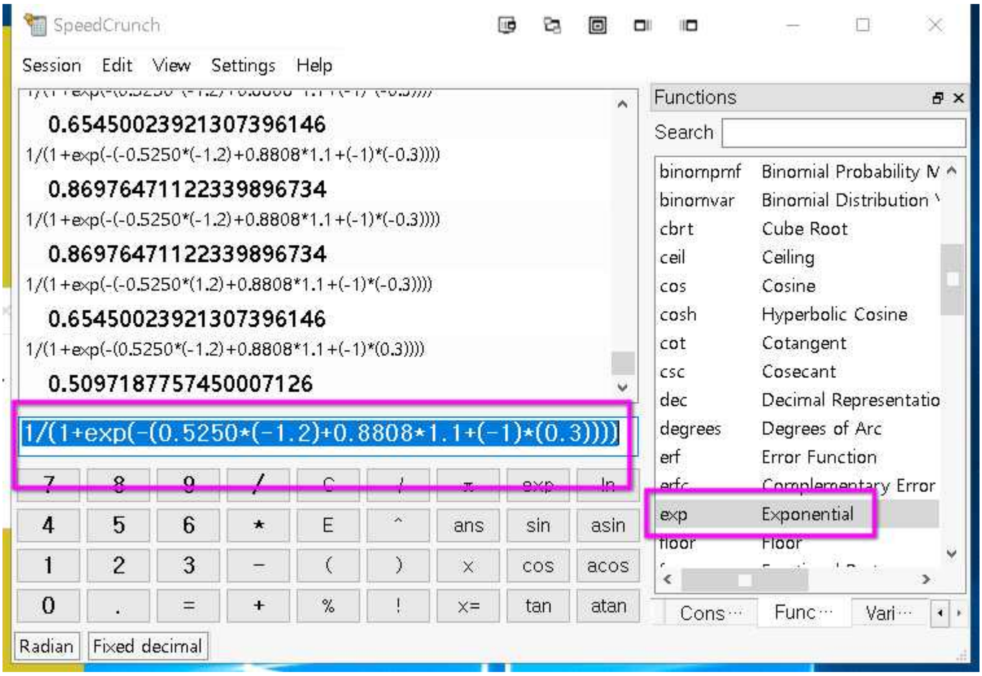




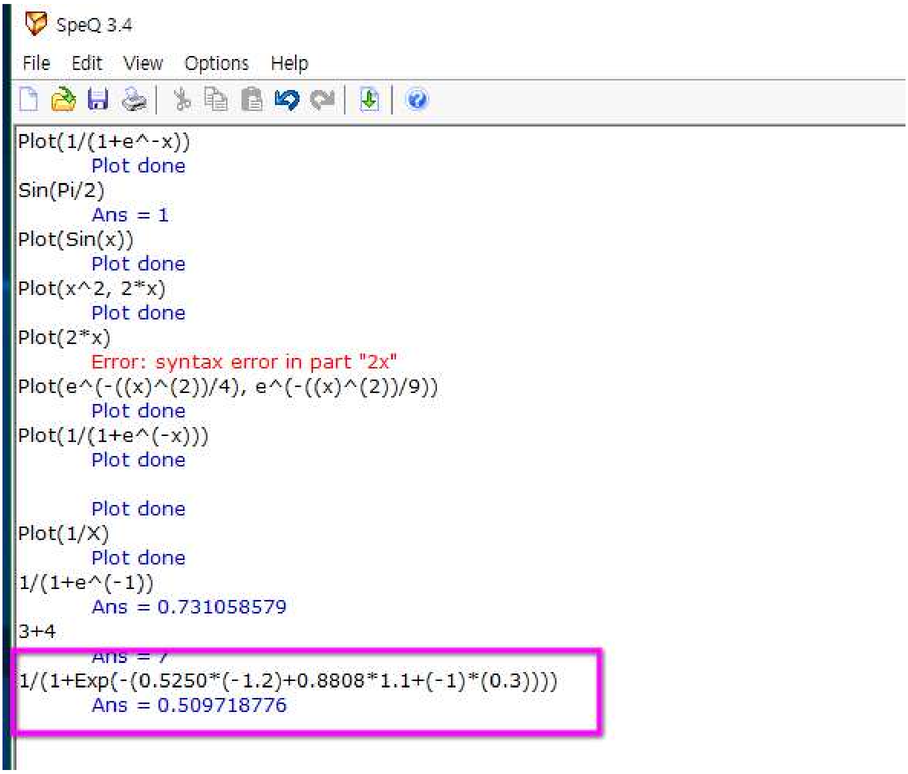
$$
\begin{aligned}
y0 &= sigmoid(h_0 w_{00}^{(Y)} + h_1 w_{10}^{(Y)} + b_{y0} w_{b0}^{(Y)}) \\
&= 1/(1+e^{-(0.5250\times(-1.2) + 0.8808\times 1.1 + (-1)\times 0.3)}) \\
&= 0.5097
\end{aligned}
$$

$$e = y_{d,0} - y_0 = 0 - 0.5097 = -0.5097$$

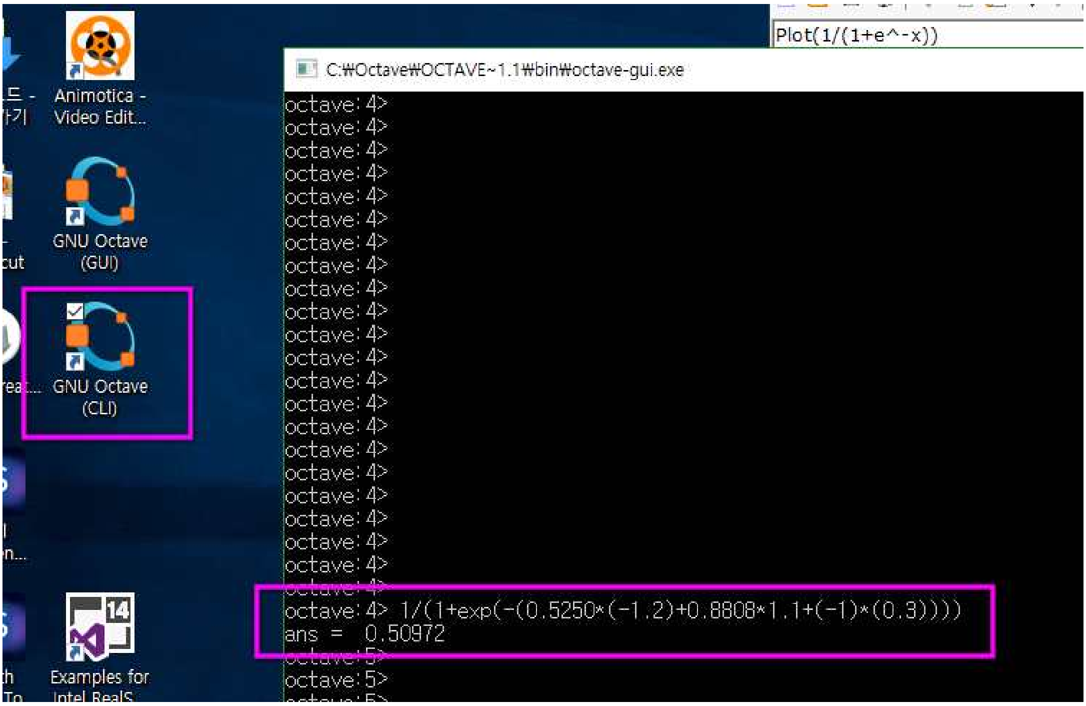
SpeedCrunch



SpeQ



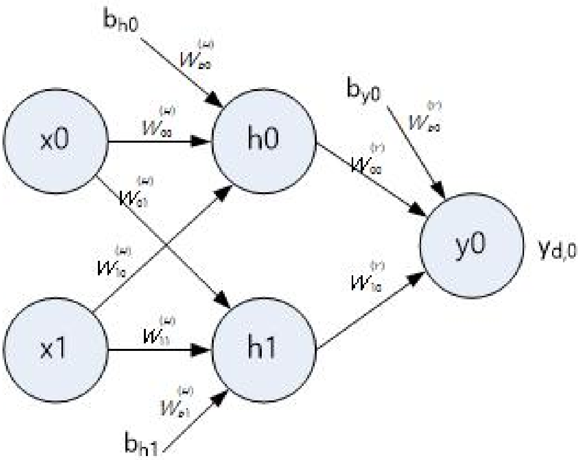
Octave CLI

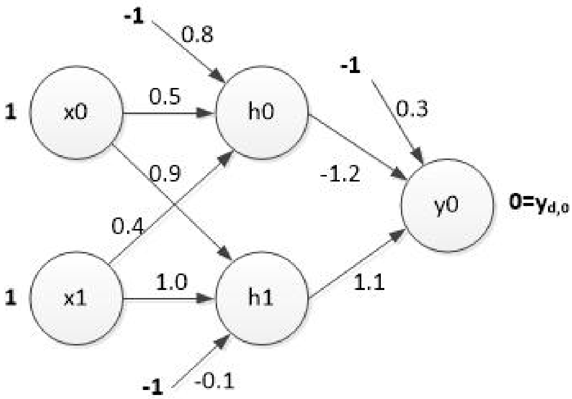


Backward Propagation

Concept

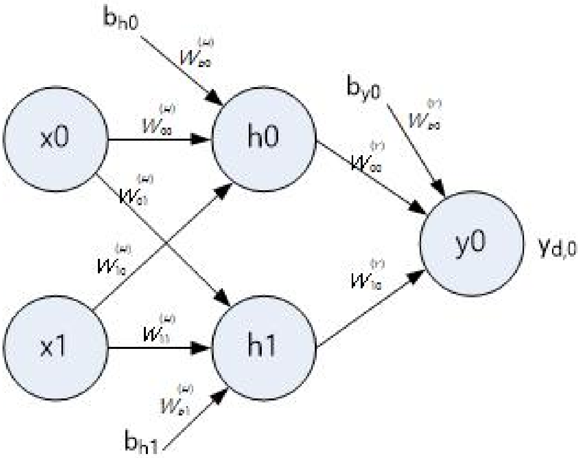
Output Layer

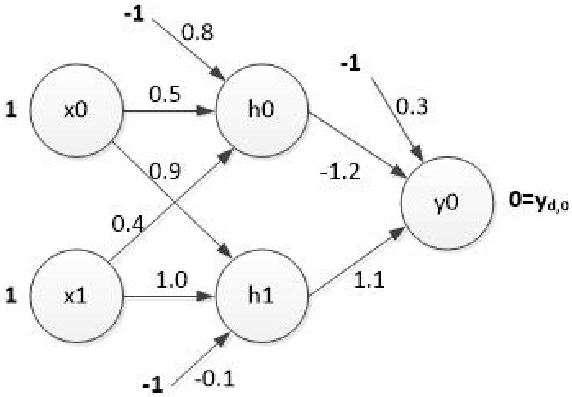




error gradient

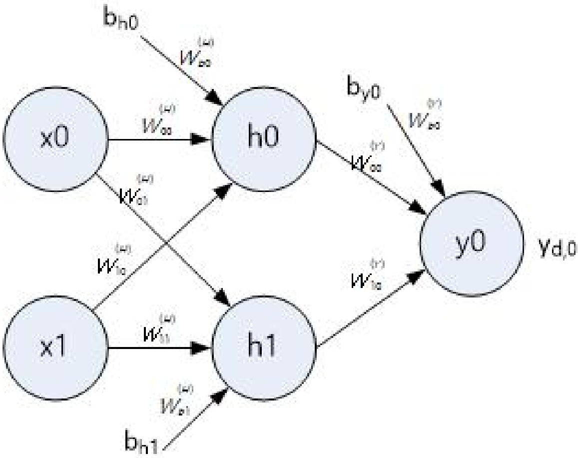
$$\delta_0^Y = y_0(1-y_0)e$$
$$= 0.5097 \times (1-0.5097) \times (-0.5097)$$
$$= -0.1274$$

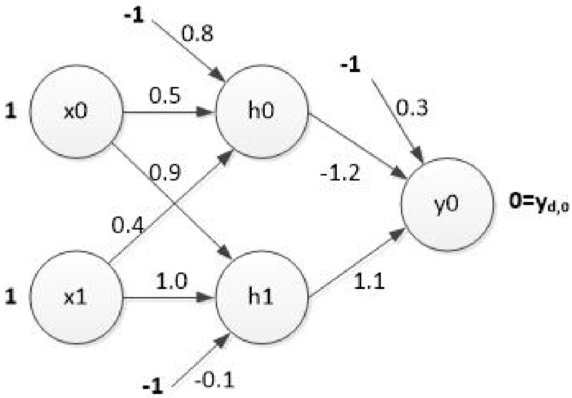




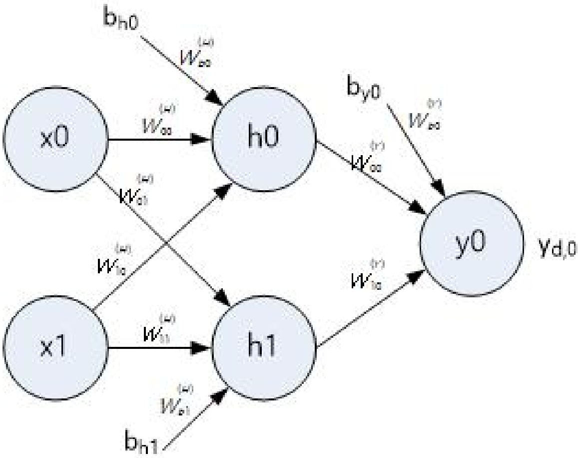
learning ratio $\alpha = 0.1$

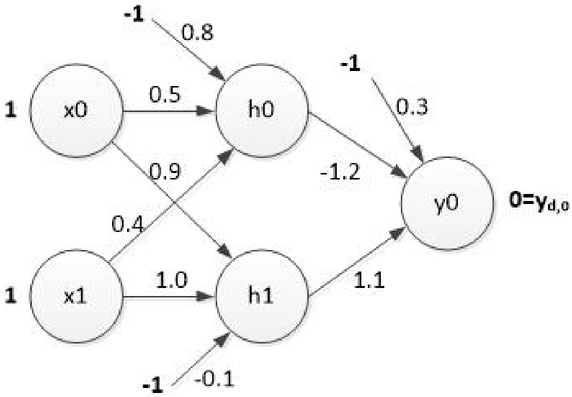
$$\Delta w_{00}^Y = \alpha \times h_0 \times \delta_0$$
$$= 0.1 \times 0.5250 \times (-0.1274)$$
$$= -0.0067$$



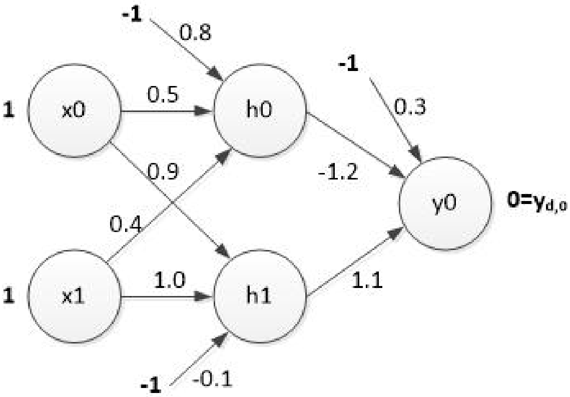


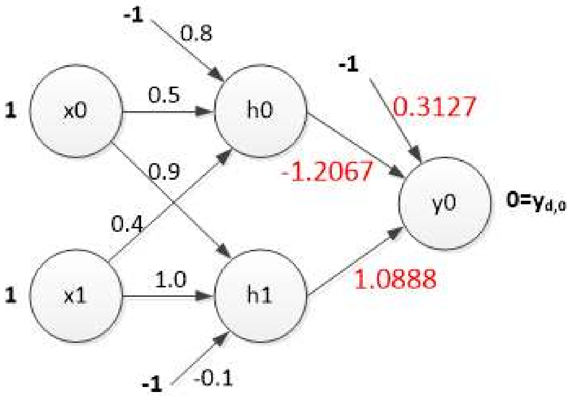
$$\Delta w_{10}^Y = \alpha \times h_1 \times \delta_0$$
$$= 0.1 \times 0.8808 \times (-0.1274)$$
$$= -0.0112$$





$$\begin{aligned}\Delta w_{b0}^{Y} &= \alpha \times b_{y0} \times \delta_0 \\ &= 0.1 \times (-1) \times (-0.1274) \\ &= -0.0127\end{aligned}$$





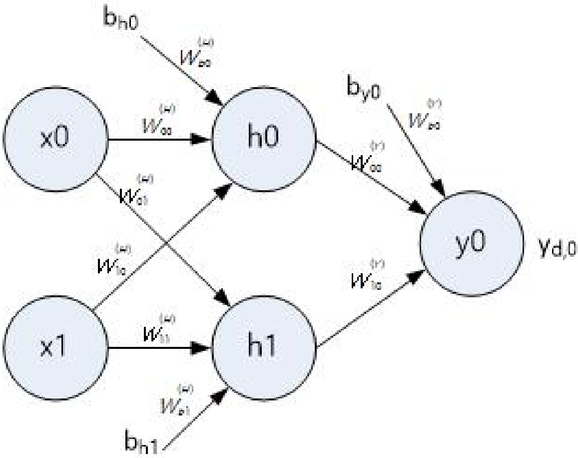
Update Output Weights

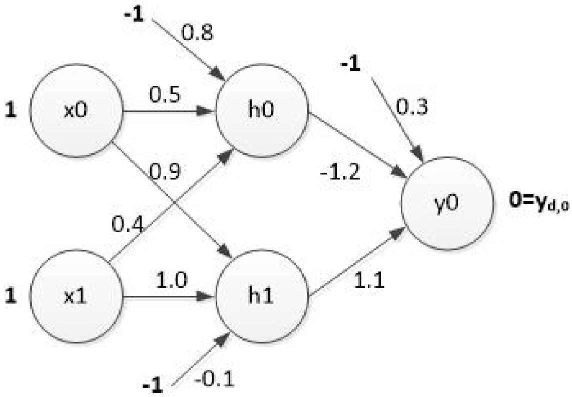
$$w_{00}^{Y} = w_{00}^{Y} + \Delta w_{00}^{Y} = -1.2 + (-0.0067) = -1.2067$$

$$w_{10}^{Y} = w_{10}^{Y} + \Delta w_{10}^{Y} = -1.1 + (-0.0112) = 1.0888$$

$$w_{b0}^{Y} = w_{b0}^{Y} + \Delta w_{b0}^{Y} = 0.3 + (0.0127) = 0.3127$$

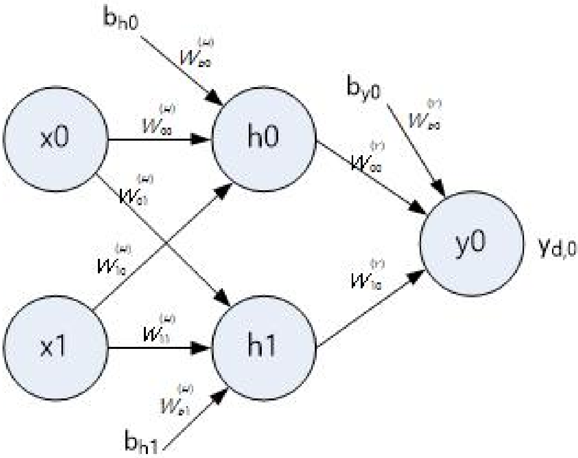
Hidden Layer

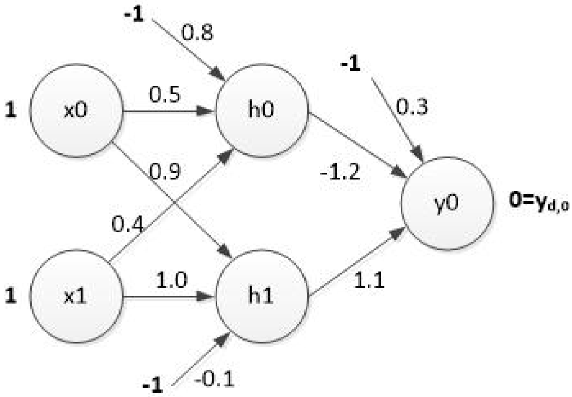




$$
\begin{aligned}
\delta_0^H &= h_0(1-h_0)\times\delta_0^Y\times w_{00}^Y \\
&= 0.5250\times(1-0.5250)\times(-0.1274)\times(-1.2) \\
&= 0.0381
\end{aligned}
$$

$$
\begin{aligned}
\delta_1^H &= h_1(1-h_1)\times\delta_0^Y\times w_{10}^Y \\
&= 0.8808\times(1-0.8808)\times(-0.1274)\times(1.1) \\
&= -0.0147
\end{aligned}
$$





$$\varDelta w_{00}^H = \alpha\times x_0\times\delta_0^H = 0.1\times 1\times 0.0381 = 0.0038$$

$$\varDelta w_{10}^H = \alpha\times x_1\times\delta_0^H = 0.1\times 1\times 0.0381 = 0.0038$$

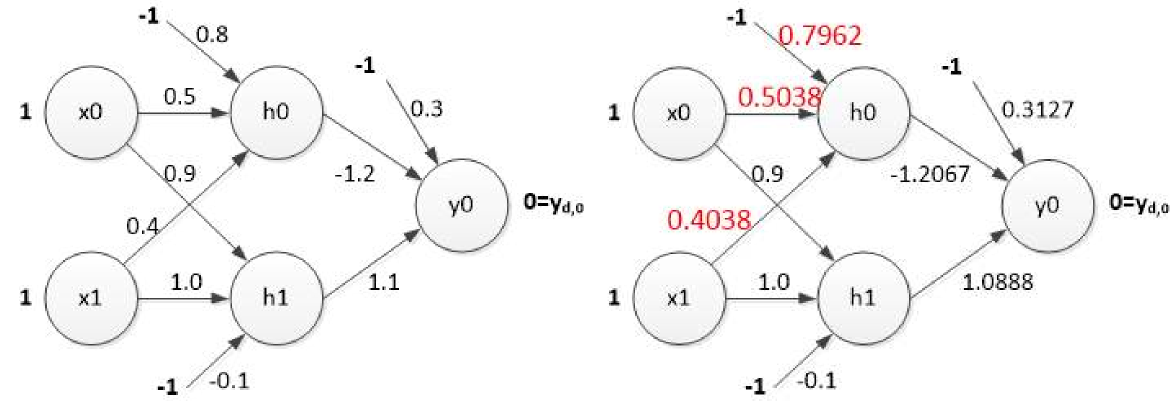
$$\varDelta w_{b0}^H = \alpha\times b_{h0}\times\delta_0^H = 0.1\times(-1)\times 0.0381 = -0.0038$$

$$\varDelta w_{01}^H = \alpha\times x_0\times\delta_1^H = 0.1\times 1\times(-0.0147) = -0.0015$$

$$\Delta w_{11}^{H} = \alpha \times x_1 \times \delta_1^H = 0.1 \times 1 \times (-0.0147) = -0.0015$$

$$\Delta w_{b1}^{H} = \alpha \times b_{h1} \times \delta_1^H = 0.1 \times (-1) \times (-0.0147) = 0.0015$$

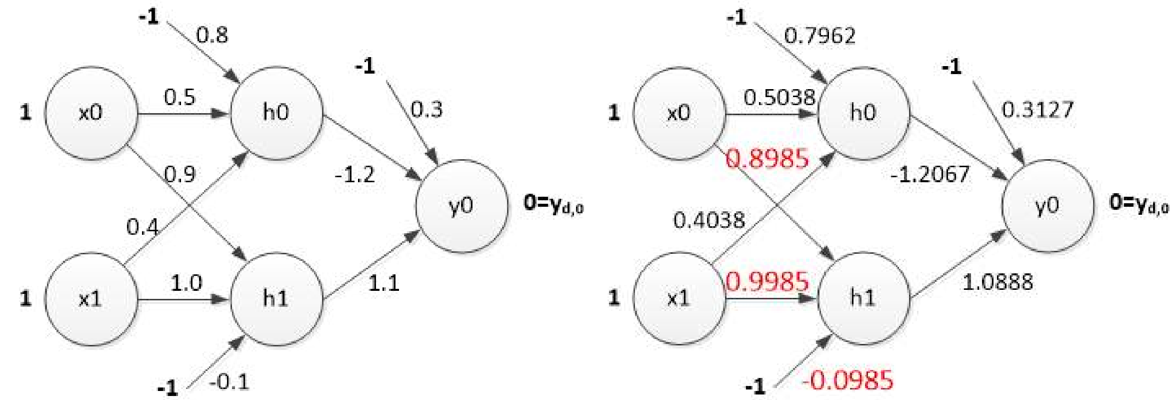
Update Hidden Weights



$$w_{00}^{H} = w_{00}^{H} + \Delta w_{00}^{H} = 0.5 + (0.0038) = 0.5038$$

$$w_{10}^{H} = w_{10}^{H} + \Delta w_{10}^{H} = 0.4 + (0.0038) = 0.4038$$

$$w_{b0}^{H} = w_{b0}^{H} + \Delta w_{b0}^{H} = 0.8 + (-0.0038) = 0.7962$$



$$w_{01}^{H} = w_{01}^{H} + \Delta w_{01}^{H} = 0.9 + (-0.0015) = 0.8985$$

$$w_{11}^{H} = w_{11}^{H} + \Delta w_{11}^{H} = 1.0 + (-0.0015) = 0.9985$$

$$w_{b1}^{H} = w_{b1}^{H} + \Delta w_{b1}^{H} = (-0.1) + 0.0015 = -0.0985$$

Sum of Square Errors

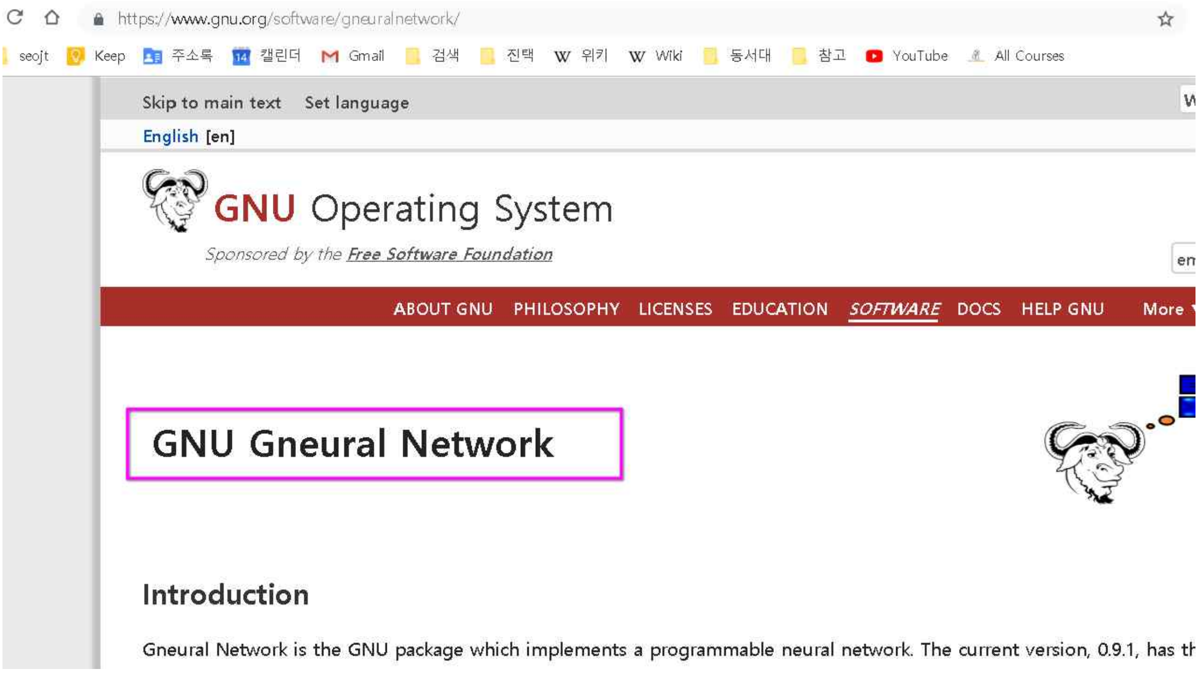
$$SSE = \sum_{i=1}^{n} (x_i - \overline{x})^2$$

$$D_i = \sum_{i=0}^{n} (y_i - y_{d,i})^2$$

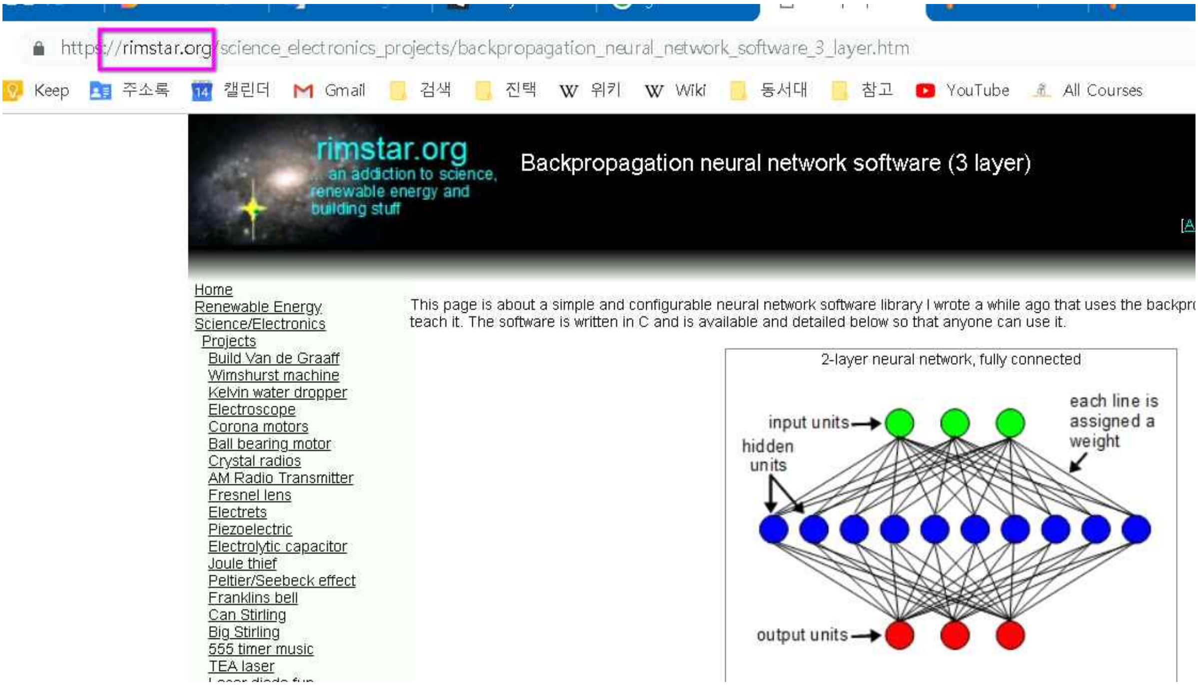
$$\begin{aligned} SSE &= \sum_{i=0}^{n} (y_i - \overline{y})^2 + \sum_{i=0}^{n} (y_{d,i} - \overline{y})^2 \\ &= D_i/2 = \left\{ \sum_{i=0}^{n} (y_i - y_{d,i})^2 \right\} /2 \end{aligned}$$

Naive Implementation

https://www.gnu.org/software/gneuralnetwork/



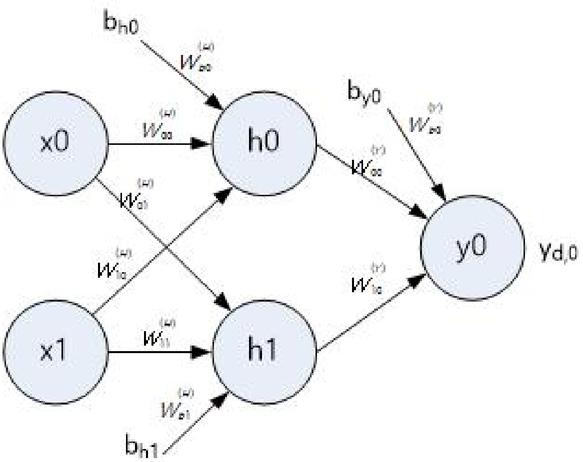
https://rimstar.org/science_electronics_projects/backpropagation_neural_network_software_3_layer.htm



C++ conversion

(github링크를 명시할 것)

Implementation Issues



$$h0 = sigmoid(x_0 w_{00}^{(H)} + x_1 w_{10}^{(H)} + b_{h0} w_{b0}^{(H)})$$

$$h1 = sigmoid(x_0 w_{01}^{(H)} + x_1 w_{11}^{(H)} + b_{h1} w_{b1}^{(H)})$$

$$\begin{bmatrix} x_0 & x_1 \end{bmatrix} \begin{bmatrix} W_{00} & W_{01} \\ W_{10} & W_{11} \end{bmatrix} + \begin{bmatrix} bias_0 \\ bias_1 \end{bmatrix} = \begin{bmatrix} WeightSum_0 \\ WeightSum_1 \end{bmatrix}$$

$$\begin{bmatrix} W_{00} & W_{10} \\ W_{01} & W_{11} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} + \begin{bmatrix} bias_0 \\ bias_1 \end{bmatrix} = \begin{bmatrix} WeightSum_0 \\ WeightSum_1 \end{bmatrix}$$

Covector

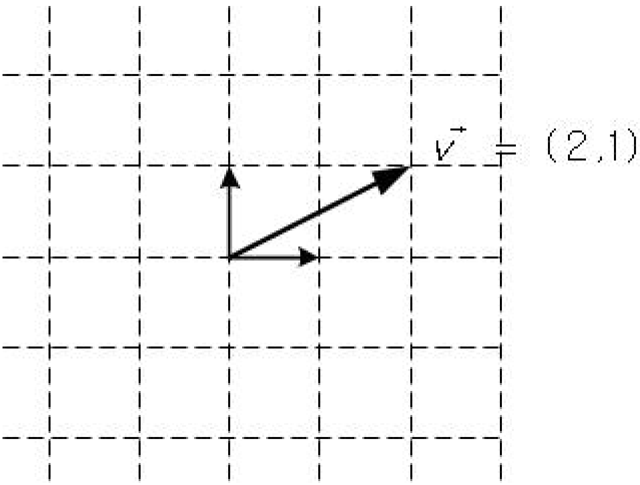
a function for a column vector.

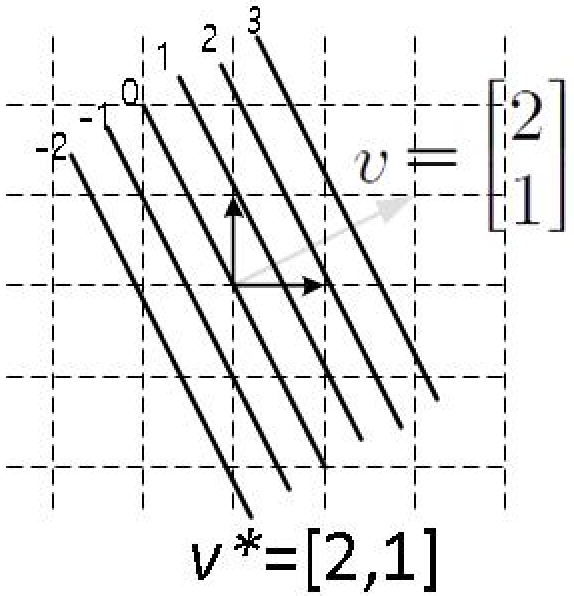
$$\begin{bmatrix} 2 & 1 \end{bmatrix} \left(\begin{bmatrix} 4 \\ 5 \end{bmatrix} \right) = 2 \times 4 + 1 \times 5 = 13$$

$$\begin{bmatrix} 2 & 1 \end{bmatrix} \left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = 2x + 1y$$

$2x + 1y = -2$
$2x + 1y = -1$
$2x + 1y = 0$
$2x + 1y = 1$
$2x + 1y = 2$

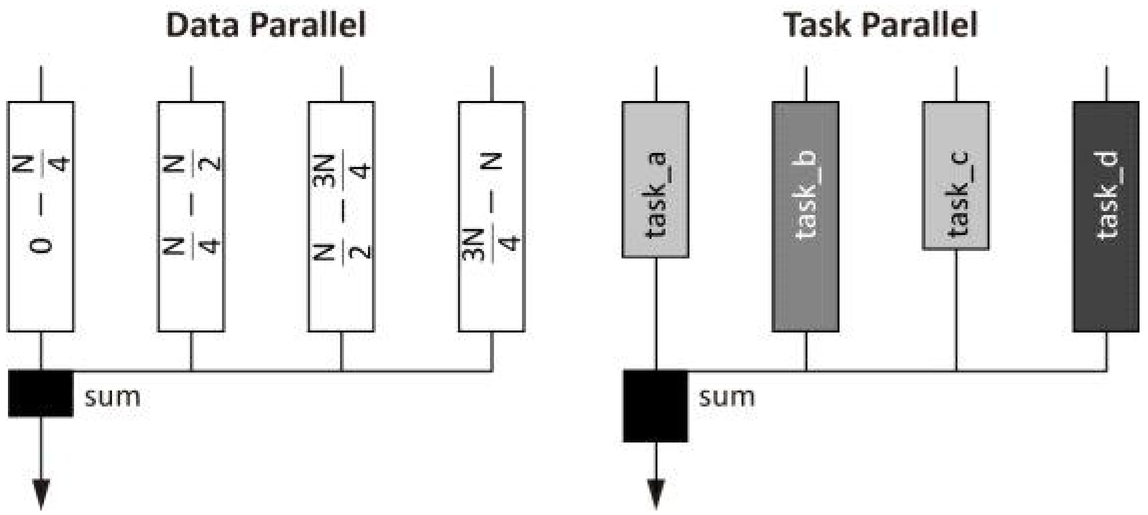
$$v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$





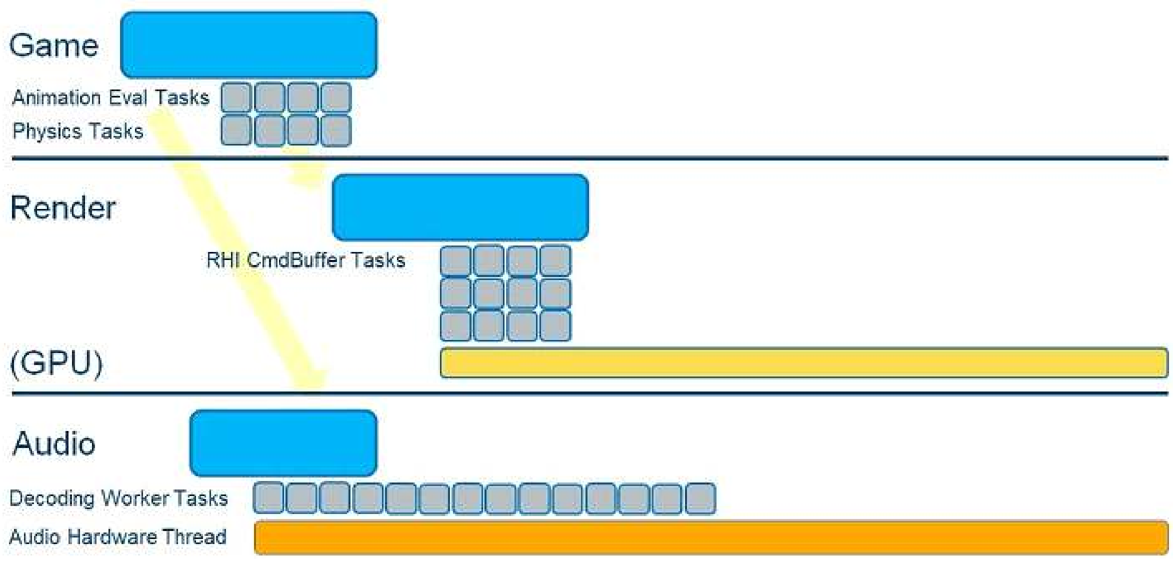
Machine Learning Library

How to implement with multi-thread?

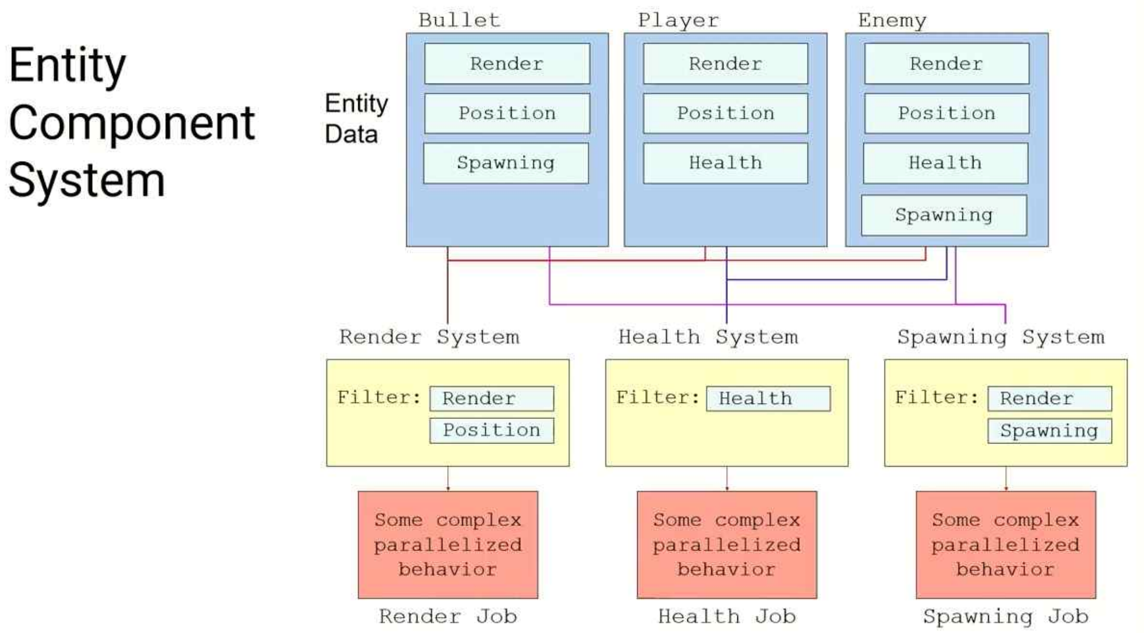


Unreal Engine 4 threading model

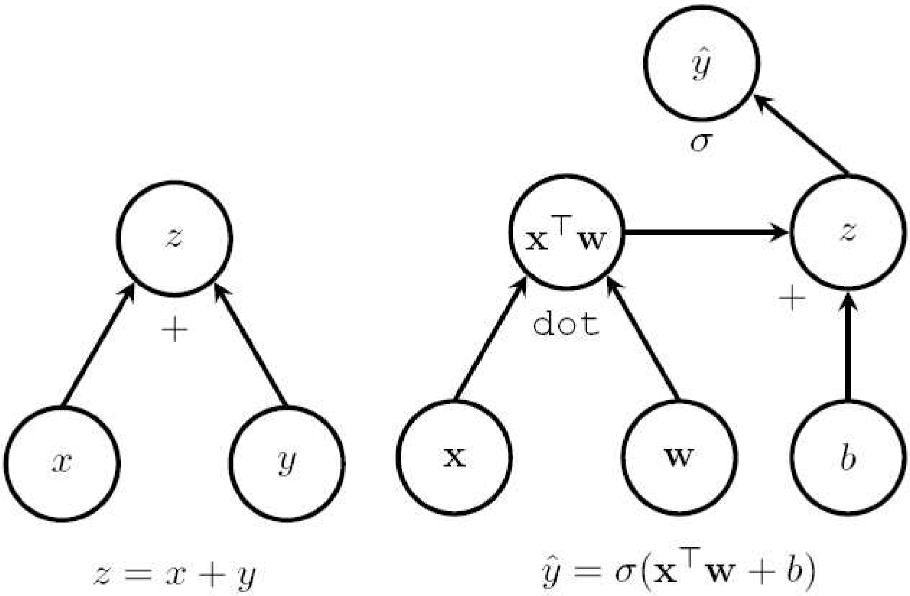
https://software.intel.com/en-us/articles/intel-software-engineers-assist-with-unreal-engine-419-optimizations



Unity ECS(Entity Component System)



Computational Graph



Ex)

$$\begin{bmatrix} W_{00}\, W_{10} \\ W_{01}\, W_{11} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} + \begin{bmatrix} bias_0 \\ bias_1 \end{bmatrix} = \begin{bmatrix} WeightSum_0 \\ WeightSum_1 \end{bmatrix}$$

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