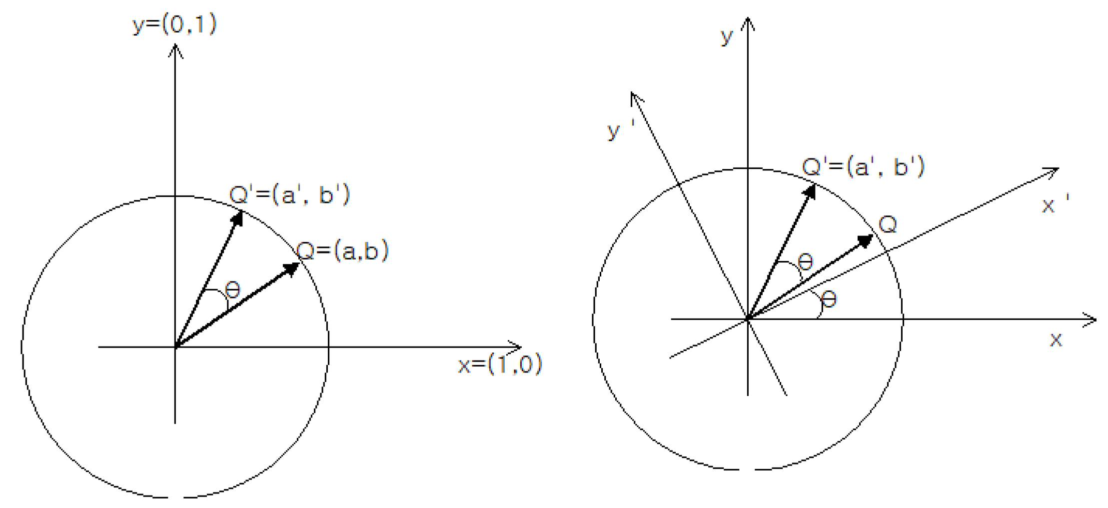
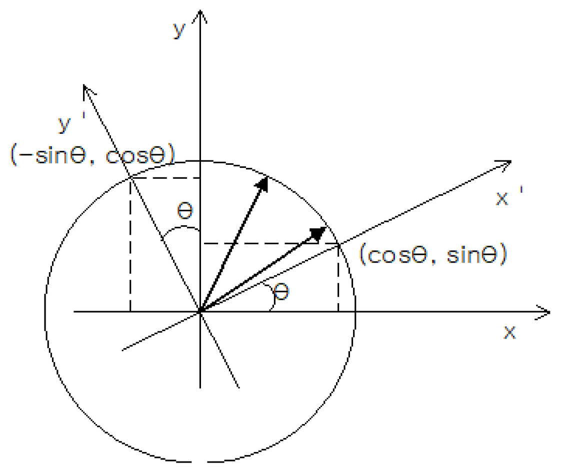
Rotation matrix https://en.wikipedia.org/wiki/Rotation_matrix#Direction

From Wikipedia, the free encyclopedia

In linear algebra, a **rotation matrix** is a matrix that is used to perform a rotation in Euclidean space. For example, using the convention below, the matrix

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$





$$Q' = \begin{bmatrix} a' \\ b' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

Properties [edit]

For any n-dimensional rotation matrix R acting on $\mathbb{R}^n$,

- $R^{\mathrm{T}} = R^{-1}$ (The rotation is an orthogonal matrix)

Quaternion [edit]

Main article: Quaternions and spatial rotation

Given the unit quaternion $\mathbf{q} = w + x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, the equivalent left-handed (Post-Multiplied) 3 × 3 rotation matrix is

$$Q = \begin{bmatrix} 1-2y^2-2z^2 & 2xy-2zw & 2xz+2yw \\ 2xy+2zw & 1-2x^2-2z^2 & 2yz-2xw \\ 2xz-2yw & 2yz+2xw & 1-2x^2-2y^2 \end{bmatrix}.$$

Orthogonal matrix https://en.wikipedia.org/wiki/Orthogonal_matrix

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For matrices with orthogonality over the complex number field, see unitary matrix.

In linear algebra, an **orthogonal matrix** is a real square matrix whose columns and rows are orthogonal unit vectors (orthonormal vectors).

One way to express this is

$$Q^{\mathrm{T}}Q = QQ^{\mathrm{T}} = I,$$

where Q^{T} is the transpose of Q and I is the identity matrix.

This leads to the equivalent characterization: a matrix Q is orthogonal if its transpose is equal to its inverse:

$$Q^{\mathrm{T}} = Q^{-1},$$

where Q^{-1} is the inverse of Q.

Matrix properties [edit]

The determinant of any orthogonal matrix is +1 or −1. This follows from basic facts about determinants, as follows:

$$1 = \det(I) = \det\left(Q^{\mathrm{T}}Q\right) = \det\left(Q^{\mathrm{T}}\right)\det(Q) = \left(\det(Q)\right)^2.$$

Stronger than the determinant restriction is the fact that an orthogonal matrix can always be diagonalized over the complex numbers to exhibit a full set of eigenvalues, all of which must have (complex) modulus 1.

Diagonalizable matrix https://en.wikipedia.org/wiki/Diagonalizable_matrix

From Wikipedia, the free encyclopedia

This article is about matrix diagonalization in linear algebra. For other uses, see Diagonalization.

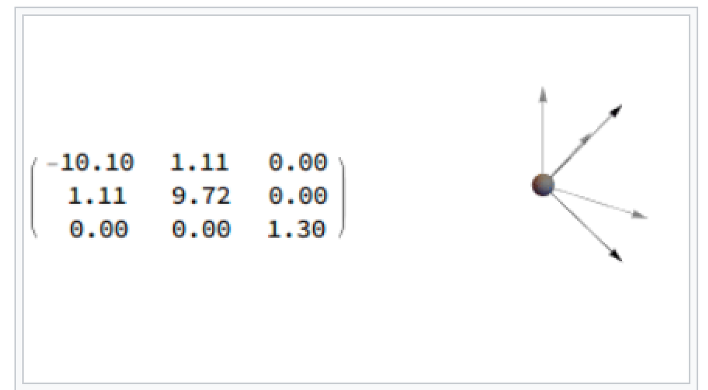
In linear algebra, a square matrix A is called **diagonalizable** or **nondefective** if it is similar to a diagonal matrix, i.e., if there exists an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$, or equivalently $A = PDP^{-1}$. (Such P, D are not unique.) For a finite-

Diagonalization [edit]

See also: Eigendecomposition of a matrix

If a matrix A can be diagonalized, that is,

$$P^{-1}AP = \begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_n \end{pmatrix},$$



The diagonalization of a matrix can be interpreted as a rotation of the axes to align them with the eigenvectors.

then:

$$AP = P\begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_n \end{pmatrix}.$$

Writing P as a block matrix of its column vectors $\vec{\alpha}_i$

$$P = \begin{pmatrix} \vec{\alpha}_1 & \vec{\alpha}_2 & \cdots & \vec{\alpha}_n \end{pmatrix},$$

the above equation can be rewritten as

$$A\vec{\alpha}_i = \lambda_i\vec{\alpha}_i \qquad (i = 1, 2, \cdots, n).$$

So the column vectors of P are right eigenvectors of A, and the corresponding diagonal entry is the corresponding eigenvalue. The invertibility of P also suggests that the eigenvectors are linearly independent

Examples [edit]

Diagonalizable matrices [edit]

- Involutions are diagonalizable over the reals (and indeed any field of characteristic not 2), with ±1 on the diagonal.
- Finite order endomorphisms are diagonalisable over $\mathbb{C}$ (or any algebraically closed field where the characteristic of the field does not divide the order of the endomorphism) with roots of unity on the diagonal. This follows since the minimal polynomial is separable, because the roots of unity are distinct.
- Projections are diagonalizable, with 0s and 1s on the diagonal.
- Real symmetric matrices are diagonalizable by orthogonal matrices; i.e., given a real symmetric matrix A, $Q^{\mathrm{T}}AQ$ is diagonal for some orthogonal matrix Q. More generally, matrices are diagonalizable by unitary matrices if and only if they are normal. In the case of the real symmetric matrix, we see that $A = A^{\mathrm{T}}$, so clearly $AA^{\mathrm{T}} = A^{\mathrm{T}}A$ holds. Examples of normal matrices are real symmetric (or skew-symmetric) matrices (e.g. covariance matrices) and Hermitian matrices (or skew-Hermitian matrices). See spectral theorems for generalizations to infinite-dimensional vector spaces.

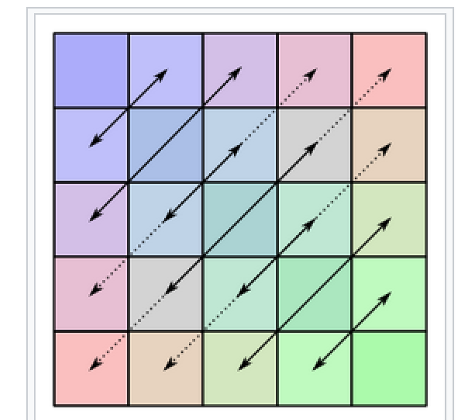
Symmetric matrix https://en.wikipedia.org/wiki/Symmetric_matrix#Properties

From Wikipedia, the free encyclopedia

For matrices with symmetry over the complex number field, see Hermitian matrix.

In linear algebra, a **symmetric matrix** is a square matrix that is equal to its transpose. Formally,

$$\boxed{A \text{ is symmetric} \iff A = A^{\mathrm{T}}.}$$



Symmetry of a 5×5 matrix

Because equal matrices have equal dimensions, only square matrices can be symmetric.

The entries of a symmetric matrix are symmetric with respect to the main diagonal. So if a_{ij} denotes the entry in the i-th row and j-th column then

$$\boxed{A \text{ is symmetric} \iff \text{for every } i, j, \quad a_{ji} = a_{ij}}$$

for all indices i and j.

Example [edit]

The following 3×3 matrix is symmetric:

$$A = \begin{bmatrix} 1 & 7 & 3 \\ 7 & 4 & -5 \\ 3 & -5 & 6 \end{bmatrix}$$

Real symmetric matrices [edit]

Denote by $\langle \cdot, \cdot \rangle$ the standard inner product on $\mathbb{R}^n$. The real $n \times n$ matrix A is symmetric if and only if

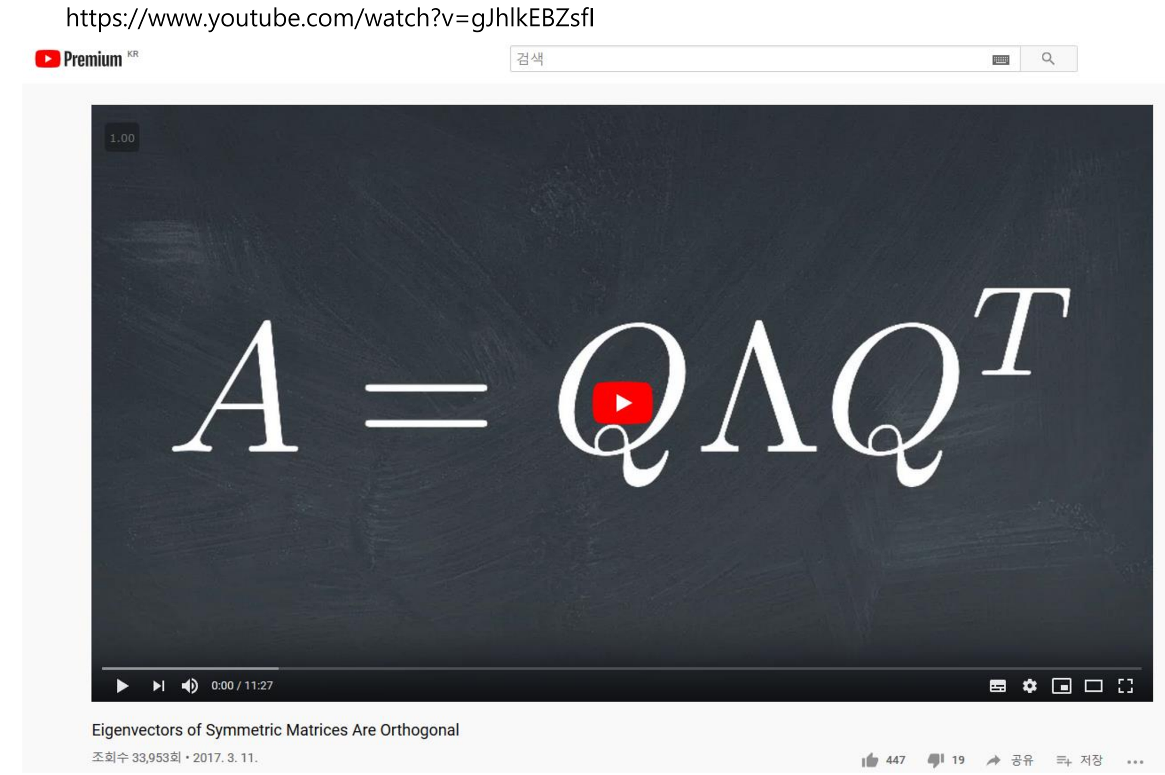
$$\langle Ax, y \rangle = \langle x, Ay \rangle \quad \forall x, y \in \mathbb{R}^n.$$

Since this definition is independent of the choice of basis, symmetry is a property that depends only on the linear operator A and a choice of inner product. This characterization of symmetry is useful, for example, in differential geometry, for each tangent space to a manifold may be endowed with an inner product, giving rise to what is called a Riemannian manifold. Another area where this formulation is used is in Hilbert spaces.

The finite-dimensional spectral theorem says that any symmetric matrix whose entries are real can be diagonalized by an orthogonal matrix. More explicitly: For every symmetric real matrix A there exists a real orthogonal matrix Q such that $D = Q^{\mathrm{T}}AQ$ is a diagonal matrix. Every symmetric matrix is thus, up to choice of an orthonormal basis, a diagonal matrix.

Eigenvectors of Symmetric Matrices are orthogonal.

https://www.youtube.com/watch?v=gJhlkEBZsfI



When a vector is multiplied with a symmetry matrix, what does it means?