두 벡터의 각을 구하는 몇가지 방법

> 2018년11월13일, 서진택

내적(Dot Product)

$$u\cdot v = \begin{cases} |u||v|\cos\theta, & \text{if } u \neq 0 \text{ and } v \neq 0 \\ 0, & \text{if } u = 0 \text{ or } v = 0 \end{cases}$$

코사인 법칙(law of cosines)

$$|\overrightarrow{PQ}|^2 = |u|^2 + |v|^2 - 2|u||v|\cos\theta$$

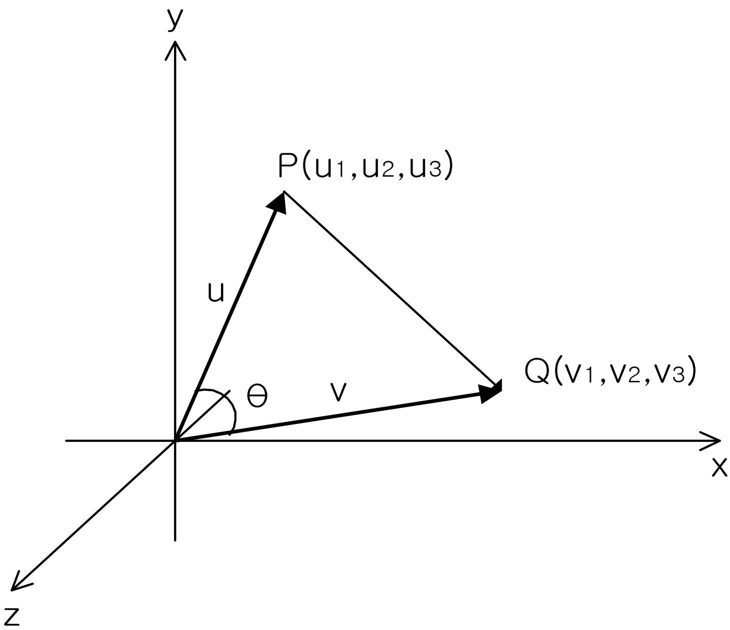


그림. 코사인 법칙: $|\overrightarrow{PQ}|^2 = |u|^2 + |v|^2 - 2|u||v|\cos\theta$가 성립합니다.

$$v_1^2 - 2v_1u_1 + u_1^2 + v_2^2 - 2v_2u_2 + u_2^2 + v_3^2 - 2v_3u_3 + u_3^2$$
$$= u_1^2 + u_2^2 + u_3^2 + v_1^2 + v_2^2 + v_3^2 - 2|u||v|\cos\theta$$

$$-2v_1u_1 - 2v_2u_2 - 2v_3u_3$$
$$= -2|u||v|\cos\theta$$

$$|u||v|\cos\theta = u_1v_1 + u_2v_2 + u_3v_3$$

$|u||v|\cos\theta$의 의미

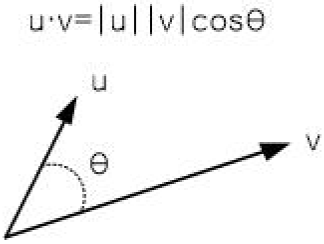


그림. 벡터의 내적

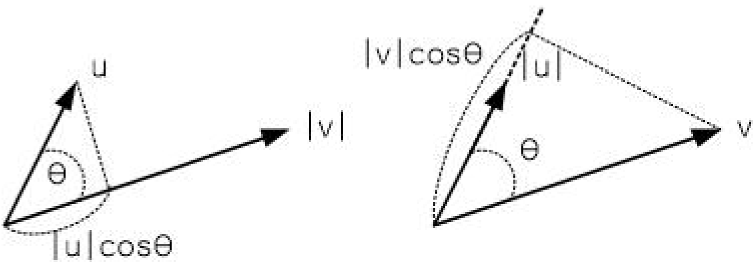


그림. 두 벡터의 내적: 벡터의 내적은 한 벡터를 대상 벡터에 직교 투영했을 때의 길이와 대상 벡터의 길이를 곱한 값입니다.

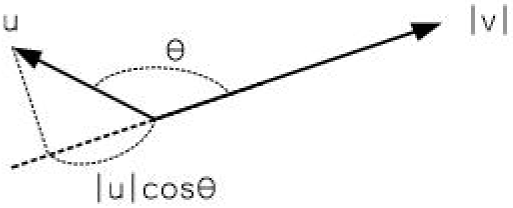


그림. 내적이 음수(negative number)인 경우: 내적이 음수라면 두 벡터는 $\pi/2$(90도)보다 큰 각으로 벌어져 있음을 나타냅니다.

$$\mathrm{u}\cdot\mathrm{v} = u_1v_1 + u_2v_2 + u_3v_3 = |u||v|\cos\theta$$

$$\cos\theta = \frac{u \cdot v}{|u||v|}$$

std::acos

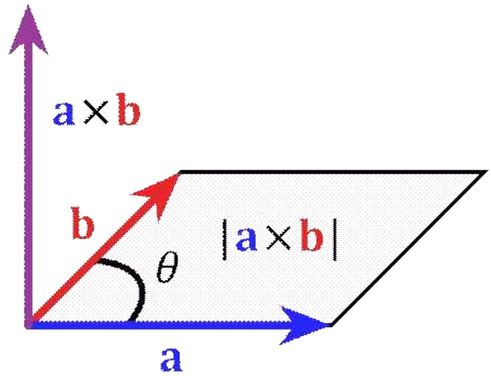
Defined in header <cmath>

```
float       acos( float arg );
double      acos( double arg );
long double acos( long double arg );
```

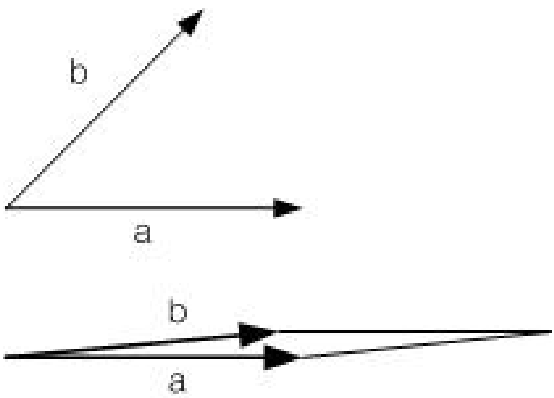
비율을 알 때 각 구하기

$$u\cdot v = |u||v|\cos(\theta)$$
$$|u\times v| = |u||v|\sin(\theta)$$

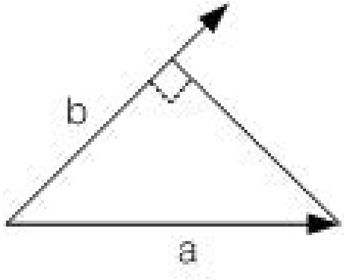
$$\tan(\theta) = \frac{|u\times v|}{u\cdot v}$$

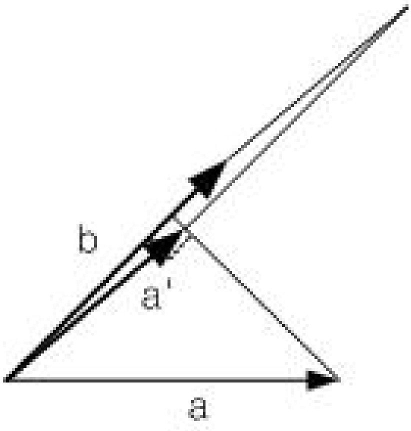


Cross Product as Length



Dot Product as Length





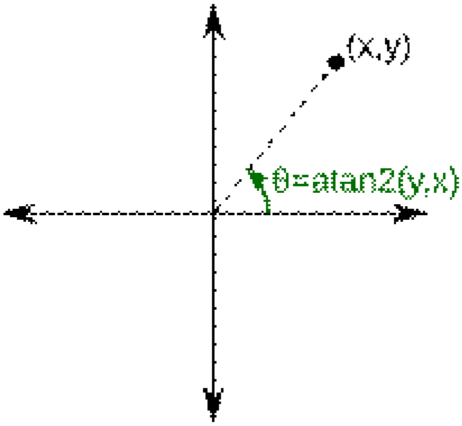
$$\tan(\theta) = \frac{|u \times v|}{u \cdot v}$$

std::atan2

$$-\pi < atan2(y,x) < \pi$$

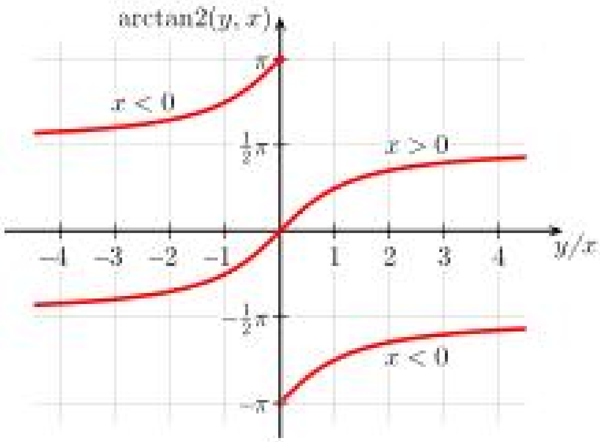
Defined in header <cmath>

```
float       atan2( float y, float x );
double      atan2( double y, double x );
long double atan2( long double y, long double x );
```



atan2(y,x) returns the angle θ between the ray to the point (x,y) and the positive x-axis, confined to (−π, π].

The single-argument arctangent function cannot distinguish between diametrically opposite directions. For example, the anticlockwise angle from the x-axis to the vector (1, 1), calculated in the usual way as arctan(1/1), is π/4 (radians), or 45°. However, the angle between the x-axis and the vector (−1, −1) appears, by the same method, to be arctan(−1/−1), again π/4, even though one might expect the answers −3π/4, or 5π/4, −135° or 225°. In addition, an attempt to find the angle between the x-axis and the vectors (0, y), y ≠ 0 requires evaluation of arctan(y/0), which fails on division by zero.



Graph of $atan2(y,x)$ over y/x

The atan2 function calculates one unique arc tangent value from two variables y and x, where the signs of both arguments are used to determine the quadrant of the result, thereby selecting the desired branch of the arc tangent of y/x.

e.g.,

atan2 (1, 1) = $\pi/4$

atan2 (−1, −1) = $-3\pi/4$.

atan2 (1, 0)= $\pi/2$.

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