Fast and Simple Physics using Sequential Impulses

Erin Catto
Crystal Dynamics

Game Developers Conference

Physics Engine Checklist

- Collision and contact
- Friction: static and dynamic
- Stacking
- Joints
- Fast, simple, and robust

Box2D Demo

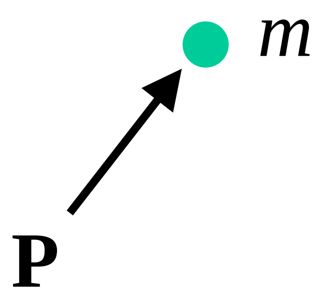
- It’s got collision
- It’s got friction
- It’s got stacking
- It’s got joints
- Check the code, it’s simple!

Fast and Simple Physics

- Penalty method?
  - Nope
- Linear complementarity (LCP)?
  - Nope
- Joint coordinates (Featherstone)?
  - Nope
- Particles (Jakobsen)?
  - Nope
- Impulses?
  - Bingo!

Why Impulses?

- Most people don't hate impulses
- The math is almost understandable
- Intuition often works
- Impulses can be robust



$$\Delta \mathbf{v} = \frac{\mathbf{P}}{m}$$

Making Impulses not Suck

- Impulses are good at making things bounce.
- Many attempts to use impulses leads to bouncy simulations (aka jitter).
- Forget static friction.
- Forget stacking.

Impulses without the Bounce

- Forget bounces for a moment.
- Let's concentrate on keeping things still.
- It's always easy to add back in the bounce.

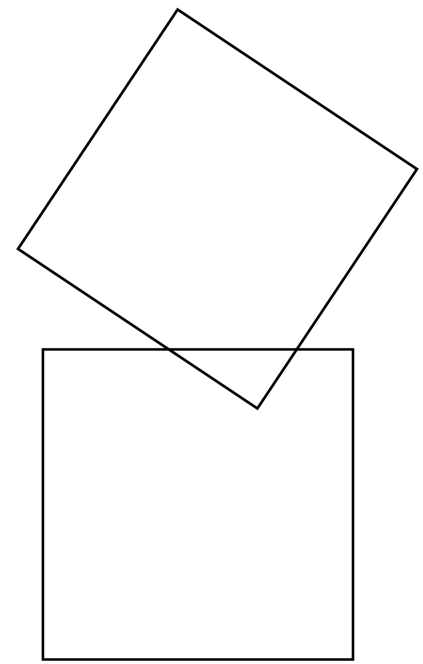
The 5 Step Program

(for taking the jitter out of impulses)

- Accept penetration
- Remember the past
- Apply impulses early and often
- Pursue the true impulse
- Update position last

Penetration

- Performance
- Simplicity
- Coherence
- Game logic
- Fewer cracks



Algorithm Overview

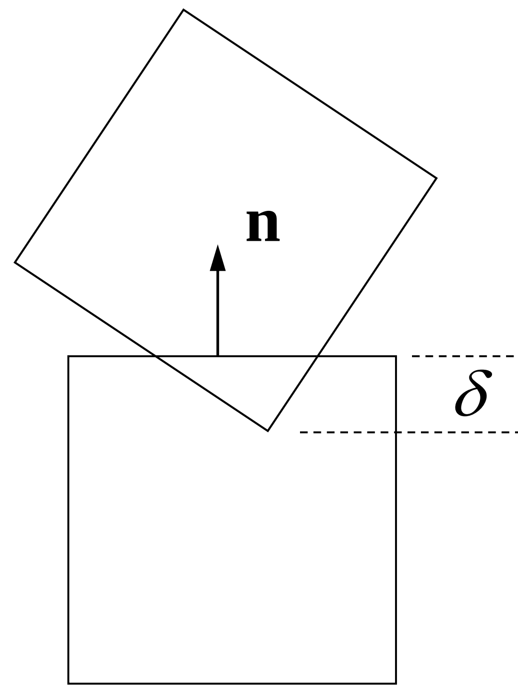
- Compute contact points
- Apply forces (gravity)
- Apply impulses
- Update position
- Loop

Contact Points

- Position, normal, and penetration
- Box-box using the SAT
- Find the axis of minimum penetration
- Find the incident face on the other box
- Clip

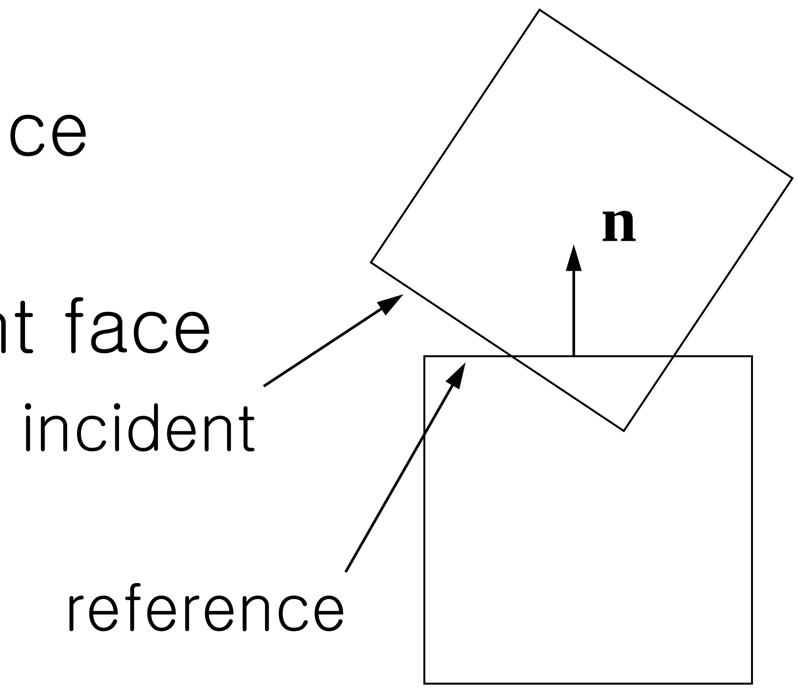
Box-Box SAT

- First find the separating axis with the minimum penetration.
- In 2D the separating axis is a face normal.



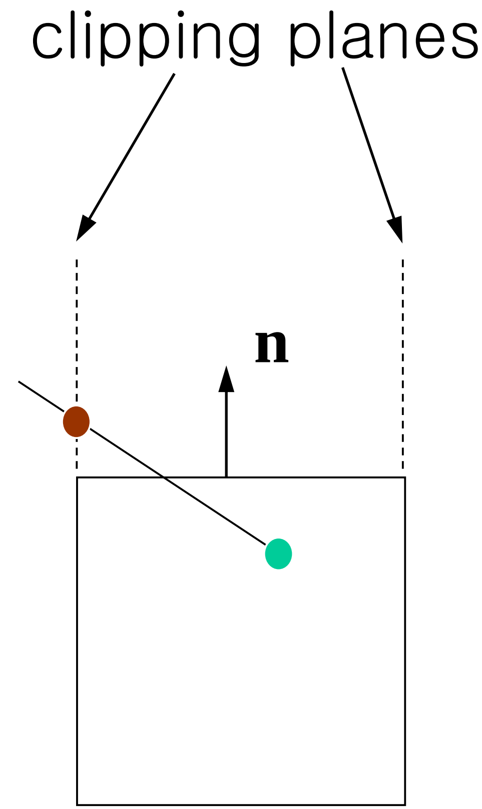
Box-Box Clipping Setup

- Identify reference face
- Identify incident face



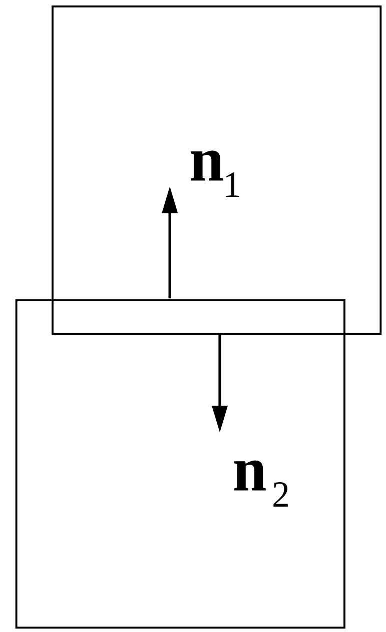
Box-Box Clipping

- Clip incident face against reference face side planes (but not the reference face).
- Consider clip points with positive penetration.



Feature Flip-Flop

- Which normal is the separating axis?
- Apply weightings to prefer one axis over another.
- Improved coherence.



Apply Forces

Newton's Law

$$m\dot{\mathbf{v}} = \mathbf{F}$$

$$I\dot{\boldsymbol{\omega}} + \cancel{\boldsymbol{\omega} \times I\boldsymbol{\omega}} = \mathbf{T}$$

Ignore gyroscopic term for improved stability

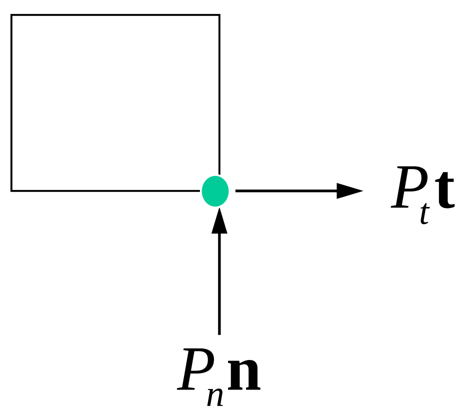
Use Euler's rule

$$\mathbf{v}_2 = \mathbf{v}_1 + \Delta t\, m^{-1}\mathbf{F}$$

$$\boldsymbol{\omega}_2 = \boldsymbol{\omega}_1 + \Delta t\, I^{-1}\mathbf{T}$$

Impulses

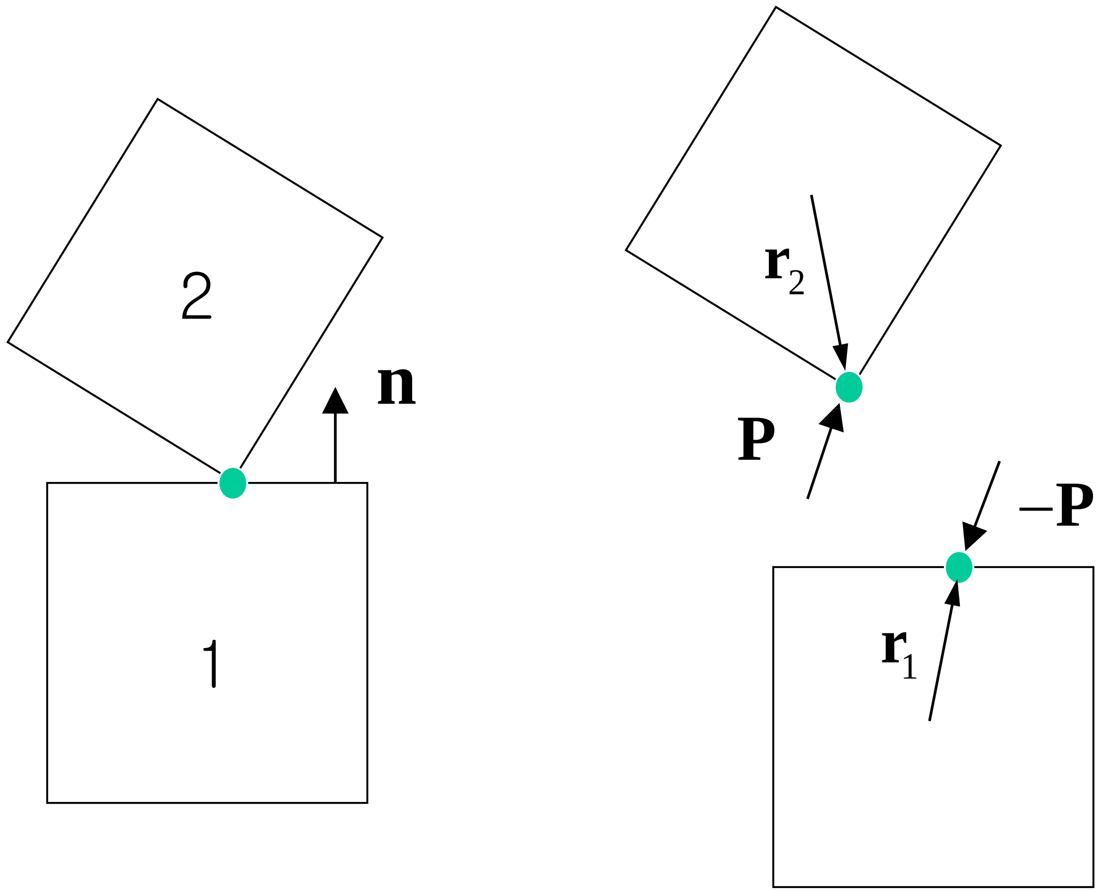
- Impulses are applied at each contact point.
- Normal impulses to prevent penetration.
- Tangent impulses to impose friction.



$$P_n \geq 0$$

$$\left|P_t\right| \leq \mu P_n$$

Computing the Impulse



Linear Momentum

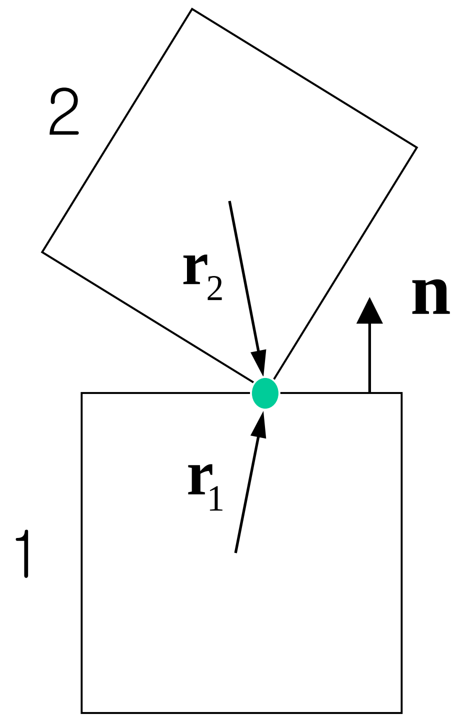
The normal impulse causes an instant change in velocity.

$$\mathbf{v}_1 = \bar{\mathbf{v}}_1 - \mathbf{P}/m_1$$
$$\boldsymbol{\omega}_1 = \bar{\boldsymbol{\omega}}_1 - I_1^{-1}\mathbf{r}_1 \times \mathbf{P}$$
$$\mathbf{v}_2 = \bar{\mathbf{v}}_2 + \mathbf{P}/m_2$$
$$\boldsymbol{\omega}_2 = \bar{\boldsymbol{\omega}}_2 + I_2^{-1}\mathbf{r}_2 \times \mathbf{P}$$

We know the direction of the normal impulse. We only need it's magnitude.

$$\mathbf{P} = P_n\mathbf{n}$$

Relative Velocity



$$\Delta\mathbf{v} = \mathbf{v}_2 + \boldsymbol{\omega}_2 \times \mathbf{r}_2 - \mathbf{v}_1 - \boldsymbol{\omega}_1 \times \mathbf{r}_1$$

Along Normal:

$$v_n = \Delta\mathbf{v} \cdot \mathbf{n}$$

The Normal Impulse

Want: $v_n = 0$ $P_n \geq 0$

Get: $$P_n = \max\left(\frac{-\Delta\bar{\mathbf{v}}\cdot\mathbf{n}}{k_n}, 0\right)$$

Fine Print:

$$\Delta\bar{\mathbf{v}} = \bar{\mathbf{v}}_2 + \bar{\boldsymbol{\omega}}_2 \times \mathbf{r}_2 - \bar{\mathbf{v}}_1 - \bar{\boldsymbol{\omega}}_1 \times \mathbf{r}_1$$

$$k_n = \frac{1}{m_1} + \frac{1}{m_2} + \left[I_1^{-1}\left(\mathbf{r}_1 \times \mathbf{n}\right) \times \mathbf{r}_1 + I_2^{-1}\left(\mathbf{r}_2 \times \mathbf{n}\right) \times \mathbf{r}_2 \right] \cdot \mathbf{n}$$

Bias Impulse

- Give the normal impulse some extra oomph.
- Proportional to the penetration.
- Allow some slop.
- Be gentle.

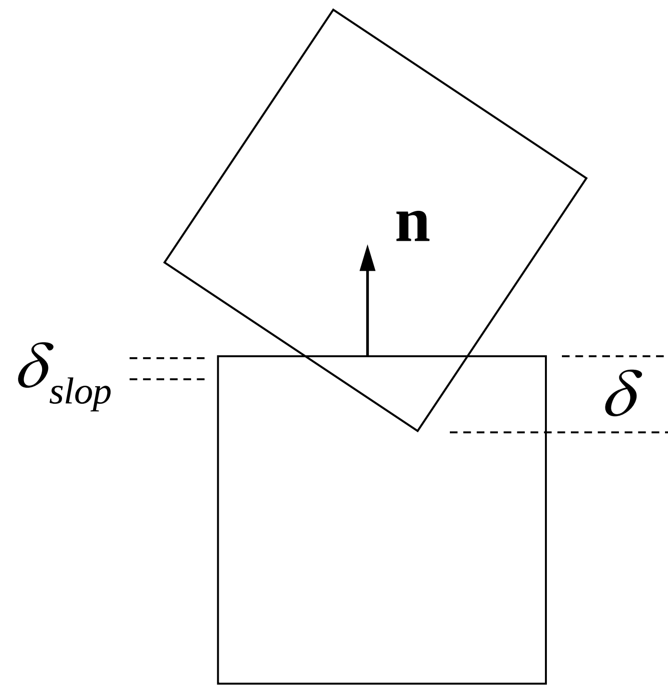
Bias Velocity

Slop: δ_{slop}

Bias Factor: $\beta \approx [0.1, 0.3]$

Bias velocity:

$$v_{bias} = \frac{\beta}{\Delta t} \max\left(0, \delta - \delta_{slop}\right)$$



Bias Impulse

With bias velocity, this:

$$P_n = \max\left(\frac{-\Delta\bar{\mathbf{v}}\cdot\mathbf{n}}{k_n}, 0\right)$$

Becomes:

$$P_n = \max\left(\frac{-\Delta\bar{\mathbf{v}}\cdot\mathbf{n} + v_{bias}}{k_n}, 0\right)$$

Friction Impulse

Tangent Velocity: $v_t = \Delta\mathbf{v} \cdot \mathbf{t}$

Want: $v_t = 0$ $\quad -\mu P_n \le P_t \le \mu P_n$

Get: $P_t = \text{clamp}\left(\frac{-\Delta\bar{\mathbf{v}} \cdot \mathbf{t}}{k_t}, -\mu P_n, \mu P_n\right)$

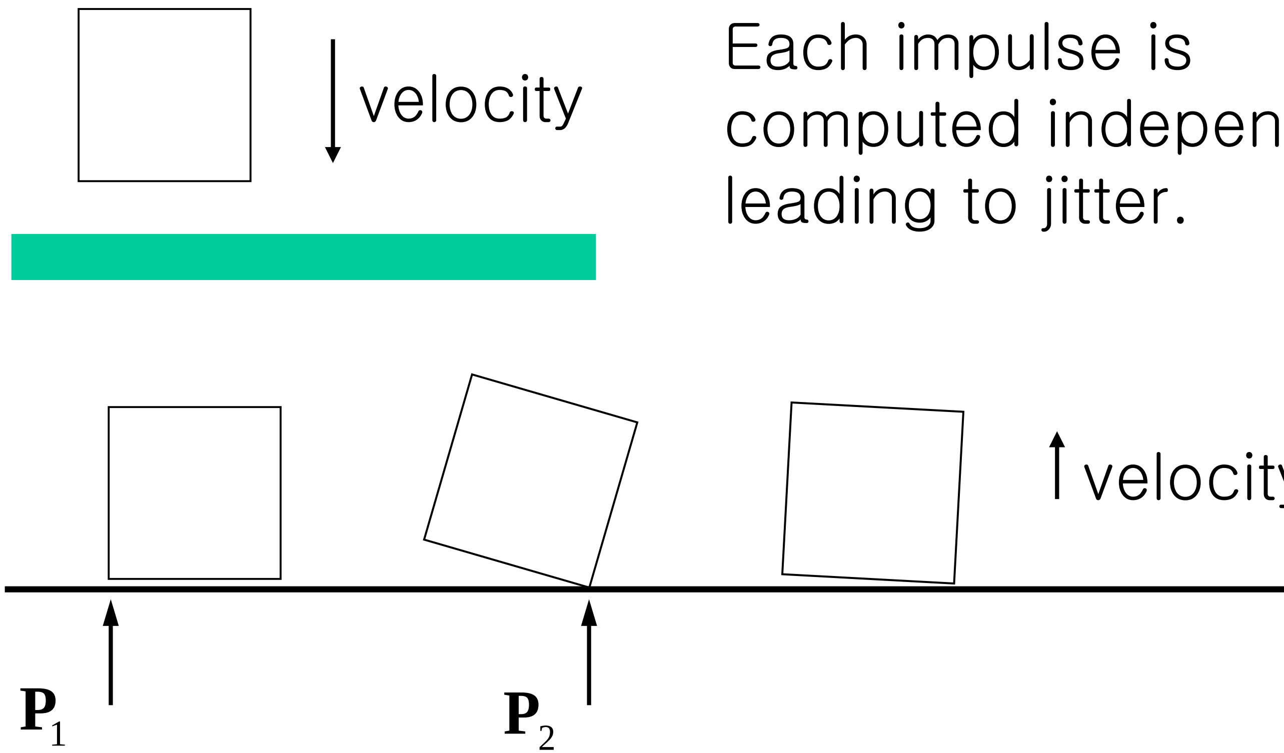
Fine Print:

$$k_t = \frac{1}{m_1} + \frac{1}{m_2} + \left[I_1^{-1}\left(\mathbf{r}_1 \times \mathbf{t}\right) \times \mathbf{r}_1 + I_2^{-1}\left(\mathbf{r}_2 \times \mathbf{t}\right) \times \mathbf{r}_2 \right] \cdot \mathbf{t}$$

Sequential Impulses

- Apply an impulse at each contact point.
- Continue applying impulses for several iterations.
- Terminate after:
  - fixed number of iterations
  - impulses become small

Naïve Impulses

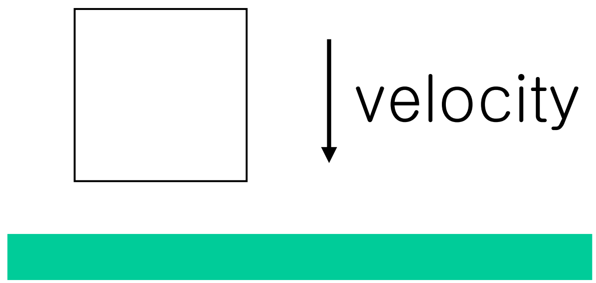


Each impulse is computed independently, leading to jitter.

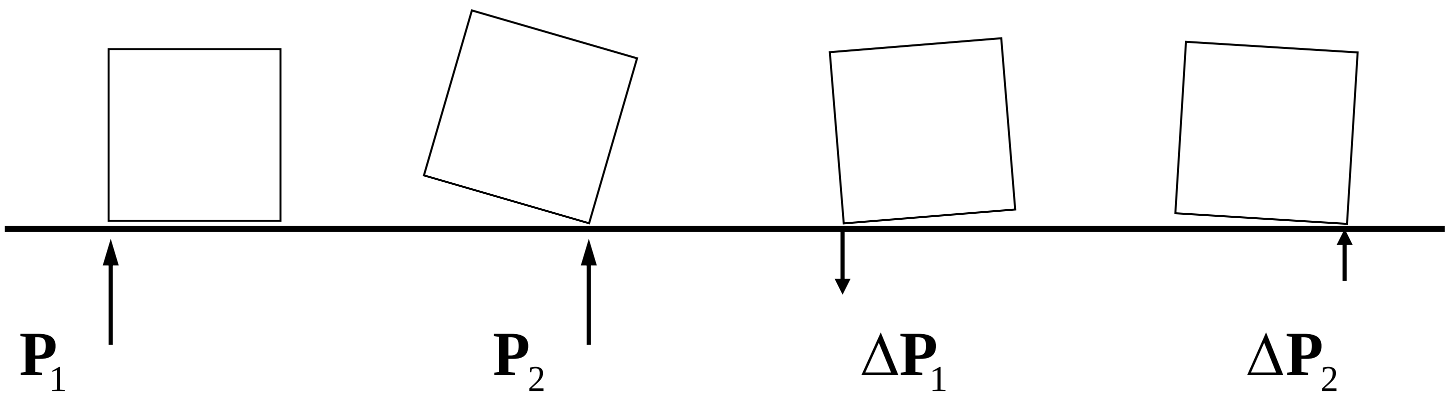
Where Did We Go Wrong?

- Each contact point forgets its impulse history.
- Each contact point requires that every impulse be positive.
- There is no way to recover from a bad impulse.

Accumulated Impulses



Each impulse adds to the total. Increments can be negative.



The True Impulse

- Each impulse adds to an accumulated impulse for each contact point.
- The accumulated impulse approaches the true impulse (hopefully).
- True impulse: an exact global solution.

Accumulated Impulse

- Clamp the accumulated impulse, not the incremental impulses.

Accumulated impulses:

$$P_{\Sigma n} \qquad\qquad P_{\Sigma t}$$

Correct Clamping

Normal Clamping:

$$temp = P_{\Sigma n}$$
$$P_{\Sigma n} = \max\left(P_{\Sigma n} + P_n, 0\right)$$
$$P_n = P_{\Sigma n} - temp$$

Friction Clamping:

$$temp = P_{\Sigma t}$$
$$P_{\Sigma t} = \text{clamp}\left(P_{\Sigma t} + P_t, -\mu P_{\Sigma n}, \mu P_{\Sigma n}\right)$$
$$P_t = P_{\Sigma t} - temp$$

Position Update

- Use the new velocities to integrate the positions.
- The time step is complete.

Extras

- Coherence
- Feature-based contact points
- Joints
- Engine layout
- Loose ends
- 3D Issues

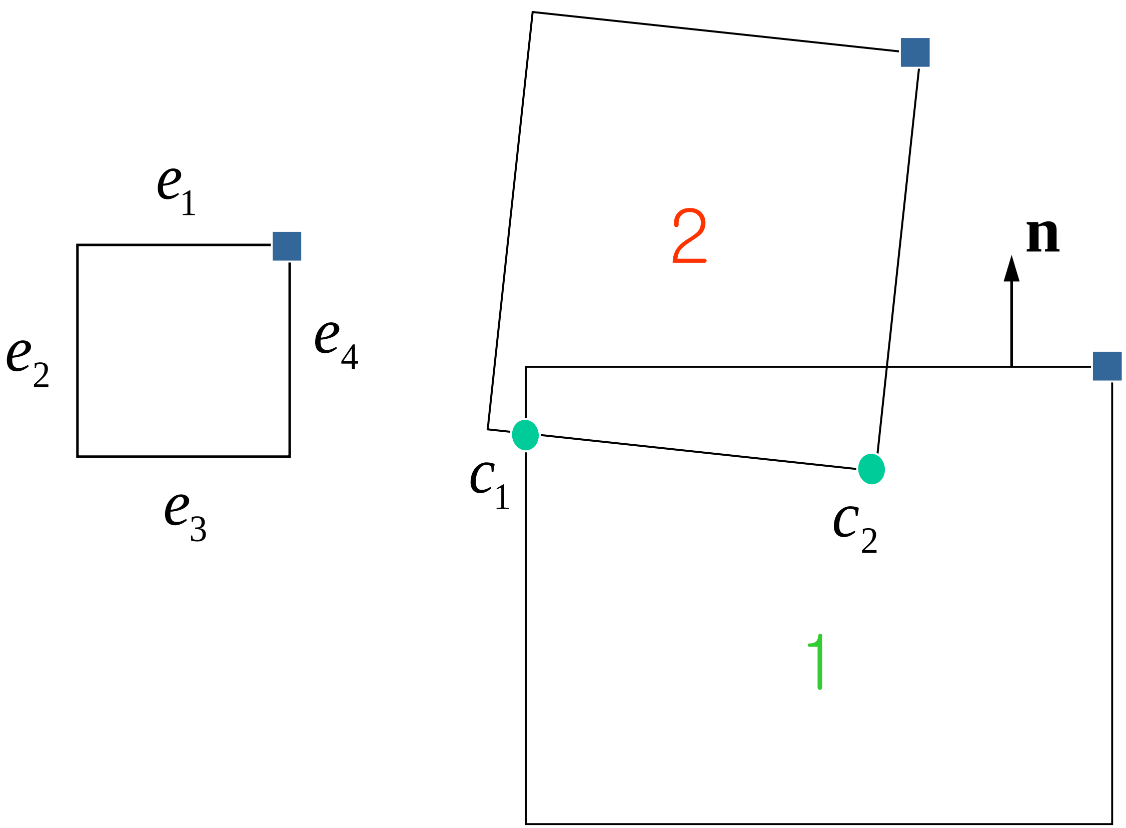
Coherence

- Apply old accumulated impulses at the beginning of the step.
- Less iterations and greater stability.
- We need a way to match old and new contacts.

Feature-Based Contact Points

- Each contact point is the result of clipping.
- It is the junction of two different edges.
- An edge may come from either box.
- Store the two edge numbers with each contact point – this is the Contact ID.

Contact Point IDs



c_1

box 1 edge 2

box 2 edge 3

c_2

box 2 edge 3

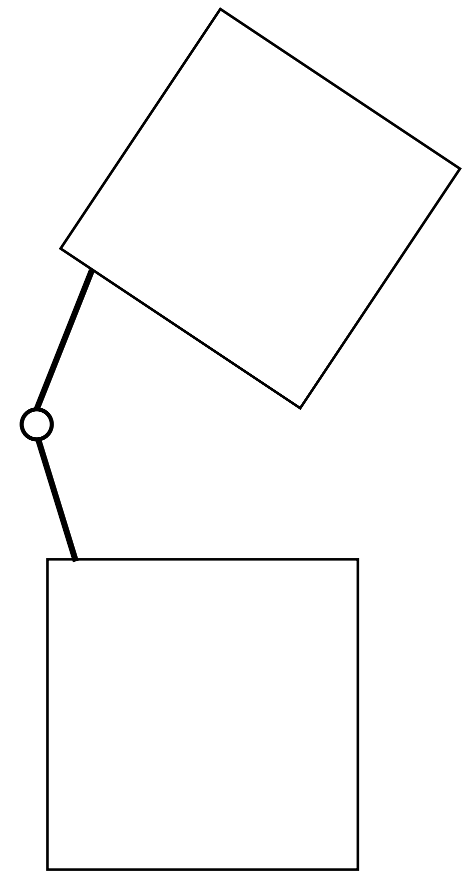
box 2 edge 4

Joints

- Specify (constrain) part of the motion.
- Compute the impulse necessary to achieve the constraint.
- Use an accumulator to pursue the true impulse.
- Bias impulse to prevent separation.

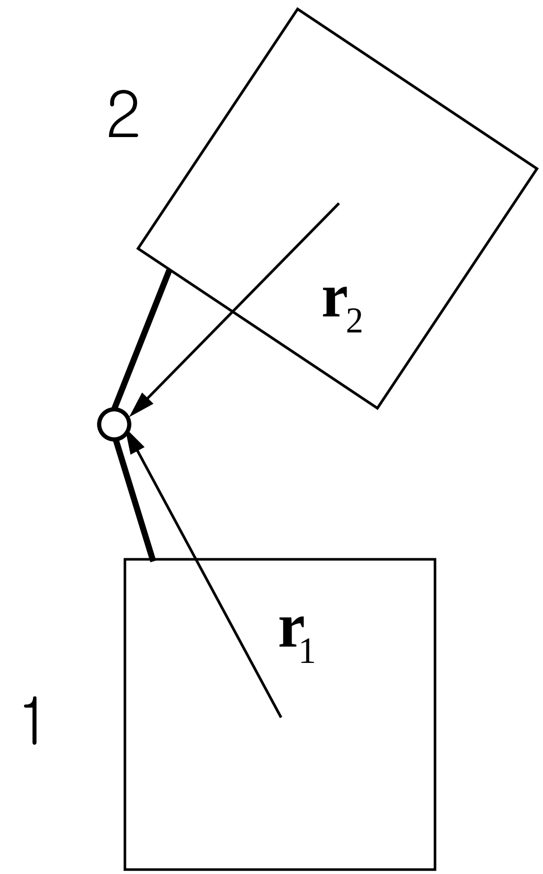
Revolute Joint

- Two bodies share a common point.
- They rotate freely about the point.



Revolute Joint

- The joint knows the local anchor point for both bodies.



Relative Velocity

- The relative velocity of the anchor points is zero.

$$\Delta\mathbf{v} = \mathbf{v}_2 + \boldsymbol{\omega}_2 \times \mathbf{r}_2 - \mathbf{v}_1 - \boldsymbol{\omega}_1 \times \mathbf{r}_1 = 0$$

- An impulse is applied to the two bodies.

$$\mathbf{P}$$

Linear Momentum

- Apply linear momentum to the relative velocity to get:

$$K\mathbf{P} = -\Delta\bar{\mathbf{v}}$$

- Fine Print:

$$K = \left(\frac{1}{m_1} + \frac{1}{m_2}\right)\mathbf{1} - \tilde{\mathbf{r}}_1 I_1^{-1}\tilde{\mathbf{r}}_1 - \tilde{\mathbf{r}}_2 I_2^{-1}\tilde{\mathbf{r}}_2$$

- Tilde (~) for the cross-product matrix.

K Matrix

- 2-by-2 matrix in 2D, 3-by-3 in 3D.
- Symmetric positive definite.
- Think of K as the inverse mass matrix of the constraint.

$$M_c = K^{-1}$$

Bias Impulse

- The error is the separation between the anchor points

$$\Delta\mathbf{p} = \mathbf{x}_2 + \mathbf{r}_2 - \mathbf{x}_1 - \mathbf{r}_1$$

- Center of mass: x
- Bias velocity and impulse:

$$\mathbf{v}_{bias} = -\frac{\beta}{\Delta t}\Delta\mathbf{p}$$

$$K\mathbf{P} = -\Delta\bar{\mathbf{v}} + \mathbf{v}_{bias}$$

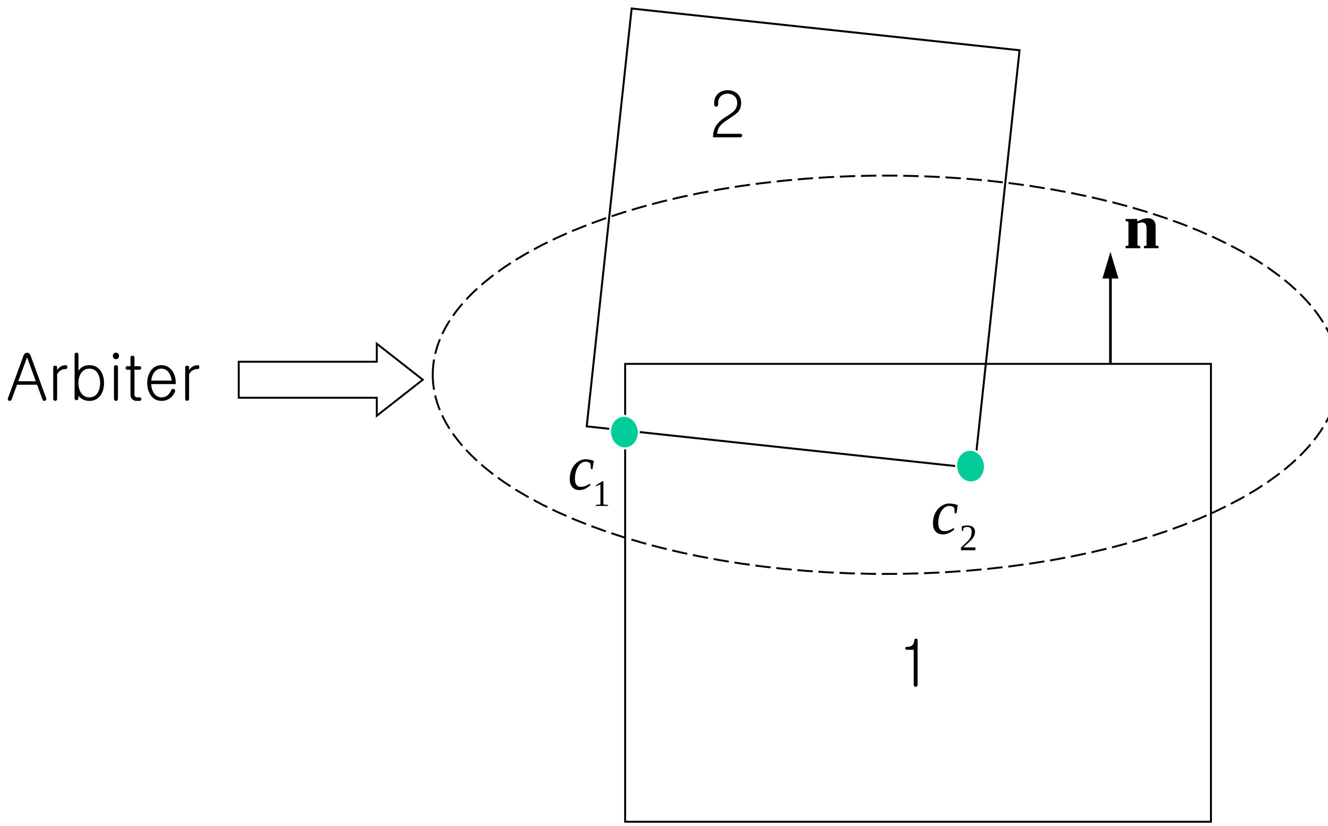
Engine Layout

- The *World* class contains all bodies, contacts, and joints.
- Contacts are maintained by the *Arbiter* class.

Arbiter

- An arbiter exists for every touching pair of boxes.
- Provides coherence.
- Matches new and old contact points using the Contact ID.
- Persistence of accumulated impulses.

Arbiters



Collision Coherence

- Use the arbiter to store the separating axis.
- Improve performance at the cost of memory.
- Use with broad-phase.

More on Arbiters

- Arbiters are stored in a set according to the ordered body pointers.
- Use time-stamping to remove stale arbiters.
- Joints are permanent arbiters.
- Arbiters can be used for game logic.

Loose Ends

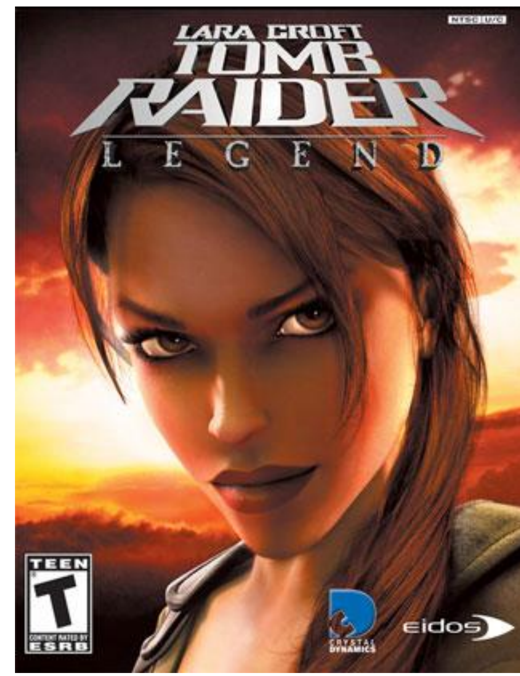
- Ground is represented with bodies whose inverse mass is zero.
- Contact mass can be computed as a pre-step.
- Bias impulses shouldn't affect the velocity state (TODO).

3D Issues

- Friction requires two axes.
- Align the axes with velocity if it is non-zero.
- Identify a *contact patch* (manifold) and apply friction at the center.
- This requires a *twist friction*.
- Big CPU savings.

Questions?

- http://www.gphysics.com
- erincatto at that domain
- Download the code there.
- Buy Tomb Raider Legend!



References

- Physics-Based Animation by Kenny Erleben et al.
- Real-Time Collision Detection by Christer Ericson.
- Collision Detection in Interactive 3D Environments by Gino van den Bergen.
- Fast Contact Reduction for Dynamics Simulation by Adam Moravanszky and Pierre Terdiman in Game Programming Gems 4.