

4 THE SCREENING EXERCISE

4.1 Introduction

Mathematical models are often very complex, computationally expensive to evaluate, and involve a large number of input factors. In these cases, one of the aims in modelling is to come up with a *short list* of important factors (this is sometimes called the principle of parsimony or Occam's razor). The question to address is: 'Which factors – among the many potentially important ones – are really important?'

Answering this question is important for a number of reasons. When a few important factors are identified, the modeller may choose to simplify the model structure by eliminating parts that appear to be irrelevant or he may decide to proceed with model lumping and extract a simpler model from the complex one. The identification of the input factors driving most of the variation in the output is also a mean of quality assurance. If the model shows strong dependencies on factors that are supposed not to be influential, or the other way around, one may rethink the model and eventually decide to revise its structure. Furthermore, additional studies may be devoted to improving the estimates of the most influential factors, so as to increase the accuracy of model predictions.

To identify the most important factors from among a large number, the choice of a well-designed experiment is essential. The experiment must be designed to be *computationally cheap*, i.e.

requiring a relatively small number of model evaluations. Screening designs fulfil this requirement. These designs are conceived to deal with models containing tens or hundreds of input factors efficiently. As a drawback, these methods tend to provide qualitative sensitivity measures, i.e. they rank the input factors in order of importance, but do not quantify how much a given factor is more important than another.

Screening designs are a convenient choice when the problem setting is that defined in Chapter 2 as Factors' Fixing (FF). In the FF setting the objective is to identify the subset of input factors that can be fixed at any given value over their range of uncertainty without significantly reducing the output variance. The screening methods provide a list of factors ranked in order of decreasing importance, allowing the modeller to identify the subset of less influential ones.

Screening techniques have been applied to several practical simulation studies in different domains, providing good results. In general, screening designs perform better when the number of important factors in the model is small compared with the total number of factors. In other words, they perform better under the assumption that the influence of factors in the model is distributed as the wealth in nations, i.e. it follows Pareto's law, with a few, very influential factors and a majority of non-influential ones. In practice this is often verified and the results of screening exercises are generally rather satisfactory.

Several screening designs have been proposed in the literature (for a review see Saltelli *et al.* (2000b, p. 65)). In this chapter we shall focus on the design proposed by Morris (1991), and on some extensions of it (Campolongo *et al.*, 2003), as we believe this design to be the most appealing in several problem settings.

The method of Morris varies one-factor-at-a-time and is therefore referred to as an OAT method. Each input factor may assume a discrete number of values, called *levels*, which are chosen within the factor range of variation. Two sensitivity measures are proposed by Morris for each factor: a measure μ that estimates the overall effect of the factor on the output, and a measure σ that, according to Morris, estimates the ensemble of the second- and

higher-order effects in which the factor is involved (including curvatures and interaction effects). The Morris measure, μ , is obtained by computing a number, r , of incremental ratios at different points $\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(r)}$ of the input space, and than taking their average. The number, r , of selected points is called the *sample size* of the experiment. Here we also describe a third measure, μ^* , proposed by Campolongo *et al.* (2003), which is a revised version of the Morris μ . μ^* is very successful in ranking factors in order of importance and performs capably when the setting is that of Factor's Fixing.

The method illustrated in this chapter is simple, easy to implement, and the results are easily interpreted. It is economic in the sense that it requires a number of model evaluations that are linear in the number of model factors. As a drawback, the method relies on a sensitivity measure, called the *elementary effect*, which uses incremental ratios and is apparently a local measure. However, the final measure, μ and μ^* , are obtained respectively by averaging several elementary effects and their absolute values computed at different points of the input space, so as to lose the dependence on the specific points at which the elementary effects are computed. In this sense, as it attempts to explore several regions of the input space, the method can be regarded as global.

Other screening methods that it is worth mentioning are: the design of Cotter (1979), the Iterated Fractional Factorial Designs, IFFDs (Andres and Hajas, 1993), and the sequential bifurcation proposed by Bettonvil (Bettonvil, 1990; Bettonvil and Kleijnen, 1997). However, with respect to each of these methods, the Morris's design has the benefit of a greater applicability. While the design of Cotter performs well when factors do not have effects that cancel each other out, the IFFD is recommended when only a restricted number of factors is important, and sequential bifurcation is ideal when factor effects have *known signs* (which means that the analyst knows whether a specific individual factor has a positive or negative effect on the simulation response); the Morris's design does not rely on restricted assumptions and is therefore model independent.

A description of these methods and a discussion on their properties can be found in Saltelli *et al.* (2000b, p. 65).

4.2 The method of Morris

The guiding philosophy of the Morris method (Morris, 1991) is to determine which factors may be considered to have effects which are (a) negligible, (b) linear and additive, or (c) non-linear or involved in interactions with other factors. The experimental plan proposed by Morris is composed of individually randomised ‘one-factor-at-a-time’ experiments; the impact of changing one factor at a time is evaluated in turn.

In order to illustrate this experimental plan, assume that the k -dimensional vector \mathbf{X} of the model input has components X_i each of which can assume integer values in the set $\{0, 1/(p - 1), 2/(p - 1), \dots, 1\}$. The region of experimentation, Ω , will then be a k -dimensional p -level grid.¹

The method suggested by Morris is based on what is called an *elementary effect*. The elementary effect for the i th input is defined as follows. Let Δ be a predetermined multiple of $1/(p - 1)$. For a given value \mathbf{x} of \mathbf{X} , the elementary effect of the i th input factor is defined as

$$d_i(\mathbf{x}) = \frac{[y(x_1, \dots, x_{i-1}, x_i + \Delta, x_{i+1}, \dots, x_k) - y(\mathbf{x})]}{\Delta} \quad (4.1)$$

where $\mathbf{x} = (x_1, x_2, \dots, x_k)$ is any selected value in Ω such that the transformed point $(\mathbf{x} + \mathbf{e}_i \Delta)$, where \mathbf{e}_i is a vector of zeros but with a unit as its i th component, is still in Ω for each index $i = 1, \dots, k$.

The finite distribution of elementary effects associated with the i th input factor, is obtained by randomly sampling different \mathbf{x} from Ω , and is denoted by F_i . The number of elements of each F_i is $p^{k-1} [p - \Delta(p - 1)]$. Assume for instance that $k = 2$, $p = 5$, and $\Delta = 1/4$, for a total number of 20 elements for each F_i . The five-level greed in the input space is represented in Figure 4.1. The total number of elementary effects can be counted from the grid by simply keeping in mind that each elementary effect relative to a factor i is computed by using two points whose relative distance in the coordinate X_i is Δ .

¹ In practical applications, the values sampled in Ω are subsequently rescaled to generate the actual values assumed by the input factors.

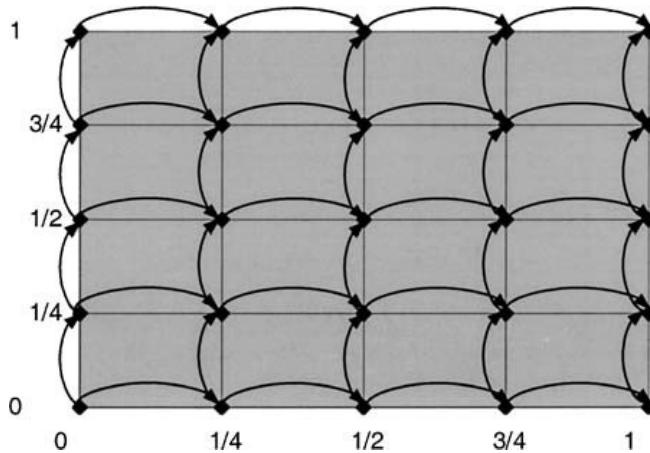


Figure 4.1 Representation of the five-level grid ($p = 5$) in the two-dimensional input space ($k = 2$). The value of Δ is $1/4$. Each arrow identifies the couple of points needed to compute one elementary effect. The horizontal arrows identify the 20 elementary effects relative to X_1 , while the vertical ones identify the 20 elementary effects relative to X_2 .

Campolongo *et al.* (2003) proposed that the distribution of the absolute values of the elementary effects, namely G_i , should also be considered. The examination of the distributions F_i and G_i provides useful information about the influence of the i th input factor on the output.

Here we take as the most informative sensitivity measures μ^* , the mean of the distribution G_i , and σ , the standard deviation of F_i . μ^* is used to detect input factors with an important overall influence on the output. σ is used to detect factors involved in interaction with other factors or whose effect is non-linear.

Note that in the original work of Morris (Morris, 1991) the two sensitivity measures proposed were respectively the mean, μ , and the standard deviation, σ , of F_i . However, choosing Morris has the drawback that, if the distribution, F_i , contains negative elements, which occurs when the model is non-monotonic, when computing the mean some effects may cancel each other out. Thus, the measure μ on its own is not reliable for ranking factors in order of importance. It is necessary to consider at the same time the values of μ and σ , as a factor with elementary effects of different signs (that cancel each other out) would have a low value of μ but a considerable value of σ that avoids underestimating the factors'

importance. For interpreting results by simultaneously taking into account the two sensitivity measures, Morris suggested a graphical representation. The estimated mean and standard deviation of each sample of elementary effects are displayed in the (σ, μ) plane (see examples in Figures 4.4 and 4.5). The plotted values may thus be examined relative to each other to see which input factor appears to be the most important.

When the goal is to rank factors in order of importance by making use of a single sensitivity measure, our advice is to use μ^* , which by making use of the absolute value, avoids the occurrence of effects of opposite signs.

The mean of the distribution F_i , which comes out at no extra computational cost, can still be used to detect additional information on the signs of the effects that the factor has on the output. If the mean of F_i is high, it implies not only that the factor has a large effect on the output but also that the sign of this effect is always the same. If, in contrast, the mean of F_i is low, while the mean of G_i is high, it means that the factor examined has effects of different signs depending on the point in space at which the effect is computed.

To examine the effects due to interactions we use the original measure proposed by Morris and consider the standard deviation of the distribution F_i . An intuitive interpretation of its meaning is the following. Assume that, for factor X_i , we get a high value of σ . This means that the elementary effects relative to this factor are significantly different from each other, i.e. the value of an elementary effect is strongly affected by the choice of the point in the input space at which it is computed, i.e. by the choice of the other factor's values. In contrast, a low σ indicates very similar values of the elementary effects, implying that the effect of X_i is almost independent of the values taken by the other factors.

The sensitivity measures preferred here are therefore μ^* , the mean of the distribution G_i , and σ , the standard deviation of F_i . If we attempt to make a comparison with the variance-based measures proposed in Chapter 5, we see that μ^* is the best parallel of the total sensitivity index S_{T_i} . In fact, if we were to express μ^* in terms of variance operators, we would write $\mu^* = E[\psi(Y|X_{-i})]$ where $\psi(\bullet)$ is the operator taking the absolute local variation.

Therefore, μ^* is the best parallel to S_{T_i} as far as the operator ψ can be assimilated into the variance operator.

The Morris design focuses on the problem of sampling a number, r , of elementary effects from each distribution F_i (and hence from each G_i) in order to estimate the distribution's statistics. In the simplest form, since each elementary effect requires the evaluation of y twice, the total computational effort required for a random sample of r values from each F_i , is $n = 2rk$ runs, where k is the number of input factors. The *economy* of the design, defined by Morris as the number of elementary effects produced by the design divided by the number of experimental runs necessary to produce them, is then $rk/2rk$, i.e. $1/2$.

Morris suggests a more efficient design, with a larger value of the economy. Note that the larger the value of the economy for a particular design or method, the better it is in terms of providing information for sensitivity. The design proposed by Morris is based on the construction of a matrix, B^* , of dimension k -by- $(k + 1)$, whose rows represent input vectors $\mathbf{x}s$, for which the corresponding experiment provides k elementary effects, one for each input factor, from $(k + 1)$ runs. The economy of the design is therefore increased to $k/(k + 1)$.

A convenient choice for the parameters p and Δ of the design is p even and Δ equal to $p/[2(p - 1)]$. This choice has the advantage that, although the design sampling strategy does not guarantee equal-probability sampling from each F_i , at least a certain symmetric treatment of inputs that may be desirable is ensured (for details see Morris (1991)).

The Morris designs starts by randomly selecting a ‘base’ value \mathbf{x}^* for the vector \mathbf{X} . Each component x_i of \mathbf{x}^* is sampled from the set $\{0, 1/(p - 1), 2/(p - 1), \dots, 1\}$. Note that the vector \mathbf{x}^* is used to generate the other sampling points but it is not one of them. The model is never evaluated at \mathbf{x}^* . The first sampling point, $\mathbf{x}^{(1)}$, is obtained by increasing one or more components of \mathbf{x}^* by Δ . The choice of the components of \mathbf{x}^* to be increased is conditioned by $\mathbf{x}^{(1)}$ still being in Ω . The second sampling point is generated from \mathbf{x}^* with the property that it differs from $\mathbf{x}^{(1)}$ in its i th component that has been either increased or decreased by Δ . The index i is randomly selected in the set $\{1, 2, \dots, k\}$. In

mathematical notation $\mathbf{x}^{(2)} = (x_1^{(1)}, \dots, x_{i-1}^{(1)}, x_i^{(1)} \pm \Delta, x_{i+1}^{(1)}, \dots, x_k^{(1)}) = (\mathbf{x}^{(1)} \pm \mathbf{e}_i \Delta)$. The third sampling point, $\mathbf{x}^{(3)}$, is again generated from the ‘base’ value \mathbf{x}^* . One or more of the k components of \mathbf{x}^* are increased by Δ , with the property that $\mathbf{x}^{(3)}$ differs from $\mathbf{x}^{(2)}$ for only one component j , for any $j \neq i$. It can be either $x_j^{(3)} = x_j^{(2)} + \Delta$ or $x_j^{(3)} = x_j^{(2)} - \Delta$. The design proceeds producing a succession of $(k+1)$ sampling points $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \dots, \mathbf{x}^{(k+1)}$, with the key property that two consecutive points differ in only one component. Furthermore any component i of the ‘base vector’ \mathbf{x}^* has been selected at least once to be increased by Δ in order to calculate one elementary effect for each factor.

Note that while each component of the ‘base’ vector \mathbf{x}^* can only be increased (and not decreased) by Δ , a sampling point $\mathbf{x}^{(l+1)}$, with l in $\{1, \dots, k\}$, may be different from $\mathbf{x}^{(l)}$ also because one of its components has been decreased (see example below).

The succession of sampling points $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \dots, \mathbf{x}^{(k+1)}$ defines what is called a *trajectory* in the input space. It also defines a matrix \mathbf{B}^* , with dimension $(k+1) \times k$, whose rows are the vectors $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \dots, \mathbf{x}^{(k+1)}$. \mathbf{B}^* represents the design matrix and is called the Orientation matrix. An example of a trajectory is given in Figure 4.2 for $k = 3$.

Once a trajectory has been constructed and the model evaluated at its points, an elementary effect for each factor i , $i = 1, \dots, k$, can be computed. If $\mathbf{x}^{(l)}$ and $\mathbf{x}^{(l+1)}$, with l in the set $\{1, \dots, k\}$, are two

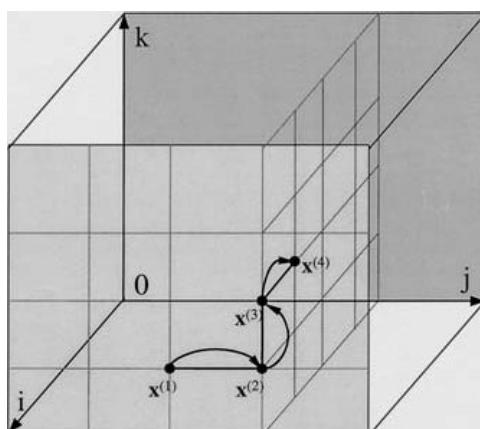


Figure 4.2 An example of trajectory in the input factor space when $k = 3$

sampling points differing in their i th component, the elementary effect associated with the factor i is either

$$d_i(\mathbf{x}^{(l)}) = \frac{[y(\mathbf{x}^{(l+1)}) - y(\mathbf{x}^{(l)})]}{\Delta}, \quad (4.2)$$

if the i th component of $\mathbf{x}^{(l)}$ has been increased by Δ or

$$d_i(\mathbf{x}^{(l)}) = \frac{[y(\mathbf{x}^{(l)}) - y(\mathbf{x}^{(l+1)})]}{\Delta}, \quad (4.3)$$

if the i th component of $\mathbf{x}^{(l)}$ has been decreased by Δ .

In other words, the orientation matrix \mathbf{B}^* provides a single elementary effect per input factor and corresponds to a trajectory of k steps, in the input space, with starting point $\mathbf{x}^{(1)}$. Technicalities on how to build a design orientation matrix are given in Section 4.3.

The goal of the experiment is to estimate the mean and the variance of the distributions F_i and G_i , $i = 1, \dots, k$. To this end a random sample of r elements from each F_i has to be selected, thus automatically providing a corresponding sample of r elements belonging to G_i . The extraction of such a sample requires the construction of r orientation matrices, independently generated, corresponding to r different trajectories in the input space. Each trajectory has a different starting point that is randomly generated. Since each orientation matrix provides an elementary effect per factor, the r matrices all together provide k r -dimensional samples, one for each F_i .

Although a characteristic of this sampling method is that points belonging to the same trajectory are not independent, the r points sampled from each F_i belong to different trajectories and are therefore independent. The same obviously applies to G_i . Therefore, the mean and standard deviation of each distribution F_i and G_i can be estimated by using the same estimators that would be used with independent random samples, i.e. as

$$\mu = \sum_{i=1}^r d_i / r \quad (4.4)$$

$$\sigma = \sqrt{\sum_{i=1}^r (d_i - \mu)^2 / r} \quad (4.5)$$

where $d_i, i = 1, \dots, r$, are the r elementary effects (or their absolute values) sampled from F_i (or from G_i).

Results of the Morris experiment can be easily interpreted. A large (absolute) measure of central tendency for G_i , i.e. a value of mean that is substantially different from zero, indicates an input with an important ‘overall’ influence on the output.

A large measure of spread, i.e. a high value of the standard deviation of F_i , indicates an input with a non-linear effect on the output, or an input involved in interaction with other factors. To rank factors in order of importance it is advisable to use μ^* , as this measure provides an estimate of the overall factor importance.

4.3 Implementing the method

To implement the Morris design, a number, r , of orientation matrices B^* have to be constructed. To build a matrix B^* , the first step is the selection of a matrix B , whose dimensions are $(k+1) \times k$, with elements that are 0s and 1s and the key property that for every column index j , $j = 1, \dots, k$, there are two rows of B that differ only in the j th entry. A convenient choice for B is a strictly lower triangular matrix of 1s.

The matrix B' , given by,

$$B' = J_{k+1,k}x^* + \Delta B, \quad (4.6)$$

where $J_{k+1,k}$ is a $(k+1) \times k$ matrix of 1s, and x^* is a randomly chosen ‘base value’ of X , could be used as a design matrix, since the corresponding experiment would provide k elementary effects, one for each input factor, with a computational cost of $(k+1)$ runs. However, the problem with B' is that the k elementary effects that it produces would not be randomly selected.

Assume that D^* is a k -dimensional diagonal matrix in which each element is either $+1$ or -1 with equal probability, and P^* is a k -by- k random permutation matrix in which each column contains one element equal to 1 and all others equal to 0 and no two columns have 1s in the same position.

A randomized version of the sampling matrix is given by

$$\mathbf{B}^* = (\mathbf{J}_{k+1,1} \mathbf{x}^* + (\Delta/2)[(2\mathbf{B} - \mathbf{J}_{k+1,k})\mathbf{D}^* + \mathbf{J}_{k+1,k}])\mathbf{P}^*. \quad (4.7)$$

\mathbf{B}^* provides one elementary effect per input, which is randomly selected.

Example

Consider a model with two input factors taking values in the set $\{0, 1/3, 2/3, 1\}$. In this case $k = 2$, $p = 4$, and $\Delta = 2/3$.

The matrix \mathbf{B} is given by

$$\mathbf{B} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix},$$

and the randomly generated \mathbf{x}^* , \mathbf{D}^* and \mathbf{P}^* are

$$\mathbf{x}^* = (0, 1/3); \quad \mathbf{D}^* = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}; \quad \mathbf{P}^* = \mathbf{I}.$$

For these values, then

$$(\Delta/2)[(2\mathbf{B} - \mathbf{J}_{k+1,k})\mathbf{D}^* + \mathbf{J}_{k+1,k}] = \begin{bmatrix} 0 & \Delta \\ \Delta & \Delta \\ \Delta & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2/3 \\ 2/3 & 2/3 \\ 2/3 & 0 \end{bmatrix}$$

and

$$\mathbf{B}^* = \begin{bmatrix} 0 & 1 \\ 2/3 & 1 \\ 2/3 & 1/3 \end{bmatrix},$$

so that

$$\mathbf{x}^{(1)} = (0, 1); \quad \mathbf{x}^{(2)} = (2/3, 1); \quad \mathbf{x}^{(3)} = (2/3, 1/3).$$

Figure 4.3 shows the resulting trajectory in the input space.

When implementing the Morris exercise on a model, the first problem to be addressed concerns the choice of the p levels among which each input factor is varied. For a factor following a uniform distribution, the levels are simply obtained by dividing the interval in which each factor varies into equal parts. For a factor following

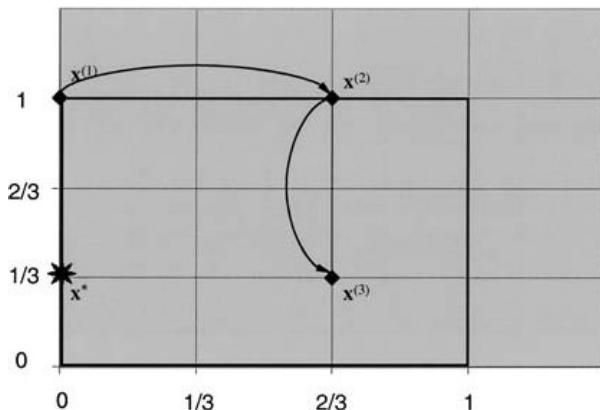


Figure 4.3 An example of trajectory in the two-dimensional space.

distributions other than uniform, it is opportune to select the levels in the space of the quantiles of the distribution.

Input values are then not sampled directly. Instead, the sampling is carried out in the space of the quantiles of the distributions, which is a k -dimensional hyper-cube (each quantile varies in $[0, 1]$). Then, given a quantile value for a given input factor, the actual value taken by the factor is derived from its known statistical distribution (Campolongo *et al.*, 1999).

The choice of the number of levels, p , or, in other words, the choice of the sampling step Δ , which is linked to p by the relation $\Delta = p/2(p - 1)$, is an open problem. The choice of p is strictly linked to the choice of r . When the sampling size r is small, it is likely that not all the possible factor levels are explored within the experiment. For instance, in the above example, if $r = 1$, factor 1 never gets the values $1/3$ and 1 while factor 2 never gets the values 0 and $2/3$. Increasing the sampling size, thus reproducing the matrix B^* r times, would increase the probability that all the levels are explored at least once. Considering a high value of p , thus producing a high number of possible levels to be explored, only appears to augment the accuracy of the sampling. If this is not coupled with the choice of a high value of r , the effort will be wasted as many possible levels will remain unexplored. Previous experiments (Campolongo and Saltelli, 1997; Campolongo *et al.*, 1999; Saltelli *et al.*, 2000b, p. 367) have demonstrated that the choice of $p = 4$ and $r = 10$ has produced valuable results. Morris (1991) used a sample size of $r = 4$, this is

probably the minimum value to place confidence in the experiment results.

The Morris method is implemented in SIMLAB.

4.4 Putting the method to work: an analytical example

The twenty factor analytical example used to test the performance of the Morris method, which is taken from Morris (1991), was described in Section 3.7. Parameters of the Morris experiment were set respectively to $p = 4$, $\Delta = 2/3$ and $r = 4$. Using the same representation as in Morris (1991), the values obtained for the sensitivity measures μ and σ are displayed in Figure 4.4.

The pattern described in Figure 4.4 almost reproduces the one shown in Figure 1 of Morris (1991). Input variables 1–10, which are supposed to have a significant effect on the output, are well separated from the others. In particular, as shown in Morris (1991), variables 8, 9 and 10 are separated from the others because of their high mean (abscissa) values. Hence, considering both means and standard deviations together, one can conclude that the first ten factors are important; of these, the first seven have significant effects that involve either interactions or curvatures; the other three are important mainly because of their first-order effect.

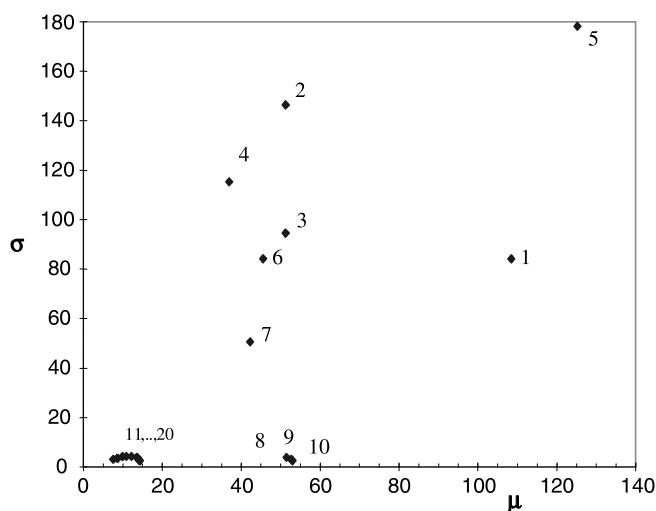


Figure 4.4 Results of the Morris experiment on the analytical model described in Chapter 3.

4.5 Putting the method to work: sensitivity analysis of a fish population model

The method of Morris in its extended version is applied to the model of fish population dynamics described in Chapter 3. The goal of the experiment is twofold: (i) to establish the relative importance of the various physical and ecological process involved in the dynamics of the fish population, so improving our understanding of the system; and/or (ii) to eliminate those factors or group of factors that seem to be irrelevant in order to reduce the complexity of the model and increase its efficiency.

To start the analysis, the quantities of interest have to be determined. First, it is essential to specify which is the model response (or model output) regarded as the most informative for the goal of the analysis. In this study, we focus on λ_{\max} , which is the dominant eigenvalue of the population matrix. The eigenvalue λ_{\max} represents the population growth rate. If $\lambda_{\max} = 1$ the population is stationary. In particular, the quantity of interest is λ^{365} , which is λ_{\max} after one year simulation time, representing the annual population growth. The total number of model input factors is 103. Of these, 72 are factors that represent the daily natural mortality (Z), duration (D), and daily fecundity (F) of each of the three species under study (sardines (I), anchovies (J) and mackerel (K)). Their best values and ranges of variation are specified in the Tables 3.2–3.4 in Chapter 3. Best values are chosen to produce a dominant eigenvalue in the population matrix equal to 1. Daily fecundity factors of early development stages with $\min = \max = 0$ are not considered.

To simplify the notation, these 72 factors are denoted by two capital letters, the first indicating the type of factor (Z, D or F), and the second indicating the species to which it is referring (I, J or K). The numbers between brackets denote the life-stage: $i = 1, 13$ for sardines, i.e. from egg to late adult following the life stages given in Table 3.2, and $i = 1, 9$ for anchovies and mackerel, from egg to late adult following the life stages given respectively in Tables 3.3 and 3.4. For example, ZJ(3) denotes the mortality (Z) of anchovies (J) in the early larvae stage. The remaining 31 inputs are factors

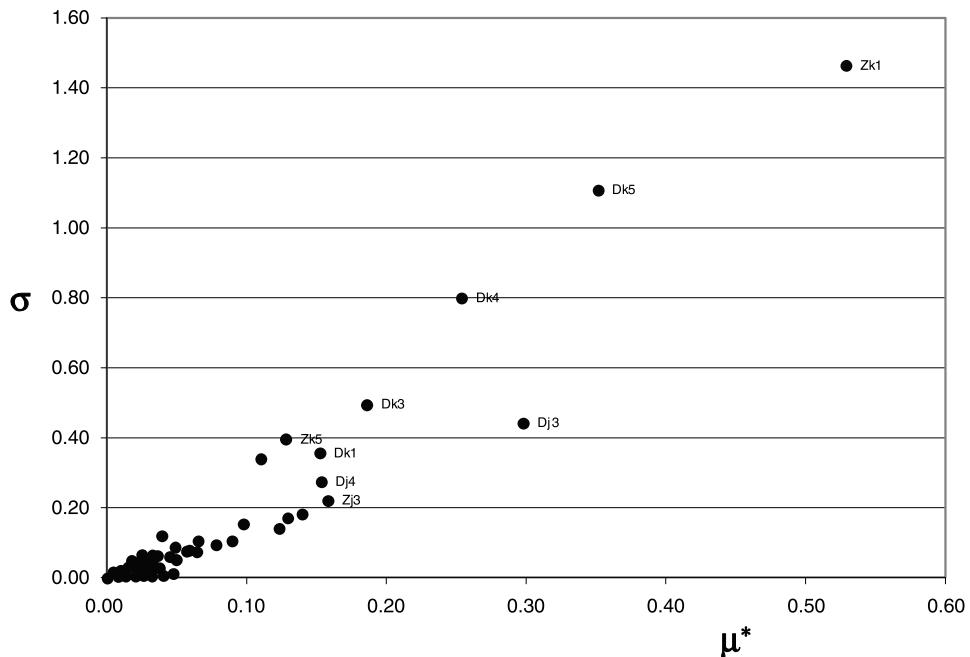


Figure 4.5 Graph displaying the Morris sensitivity measures μ^* and σ for the 103 model input factors. Only the most important factors are labelled.

involved in the migration and interspecies competition between larvae and juveniles. Lower case letters denote these factors.

The extended Morris method is applied to the fish population model with a sample size $r = 10$. Each of the 103 input factors is assumed to follow a uniform distribution between its extreme values, reported in Tables 3.2–3.4. In the design, each factor is varied across four levels ($p = 4$). A total number $N = 1040$ of model evaluations is performed ($N = r \times (k + 1)$, where k is the number of input factors).

The results of the experiment exercise are shown in Figure 4.5, where the sensitivity measures μ^* and σ are plotted for the 103 input factors. Labels indicating the names of the factors are given only for the ten most important factors.

Factors can also be ranked in (decreasing) order of importance according to μ^* , which is a measure of the overall factor importance. However, it is worth noting that in this case the order of importance that would have been obtained by using σ instead of

μ^* is very similar. Results of the experiment show that each input factor with a high value for the estimated mean, μ^* , also has a high value for the estimated standard deviation, σ , or in other words, none of the factors has a purely linear effect. This is also evident from Figure 4.5, where all the points lie around the diagonal.

A number of conclusions can be drawn by examining Figure 4.5. The first group of factors is clearly separated from the others, indicating a large influence on the population growth rate. These are (in decreasing order of importance): ZK(1), the daily natural mortality for mackerel at the egg stage; DK(5) and DK(4), the duration for mackerel at the juvenile and early juvenile stage respectively; DJ(3), the duration for anchovies at the early larvae stage, which is also the factor that is less involved in the interaction and/or curvature effects, as it does not lie exactly on the diagonal of the (μ^*, σ) plane but in the $\mu^* > \sigma$ zone. A second group of factors that are quite influential on the output include (not in order of importance): DJ(4), DJ(6) and DJ(7), i.e. the duration for anchovies at the late larvae, late juvenile and prerecruit stages; DK(1), DK(2) and DK(3), i.e. the duration for mackerel at the egg and early and late larvae stages; ZJ(1), ZJ(3), ZJ(4), ZJ(5) and ZJ(6), i.e. the mortality for anchovies at the egg, early and late larvae, and early and late juvenile stages; and ZK(5), i.e. the mortality for mackerel at the juvenile stage. Then other factors follow immediately: DJ(2), i.e. the duration for anchovies at the yolk-sac larvae stage, ZJ(7), i.e. the mortality for anchovies at the prerecruit stage, and so on. Values of μ^* for these remaining factors decrease smoothly, without any discontinuity, indicating that it is very difficult to distinguish a group of important factors from a group of non-important ones.

Input factors involved in interspecies competition between larvae and juveniles are not very important with respect to the others. None of them appear among the first thirty that were identified by Morris, even if they were responsible for the fluctuations observed in the simulations. This means they do not influence the magnitude of the fluctuations. The conclusion is that, if our modelling approach is correct, it would be difficult to identify interspecies competition in real time series data, as it will be masked by environmental fluctuations.

Other relevant conclusions include:

1. the daily fecundity (F) factors are not very significant for any of the three species at any life stage;
2. none of the most important twenty factors are related to adult life stages;
3. the daily natural mortality and duration factors play a more substantial role on the dynamics of the three populations;
4. the sardine population appears to be less influential on the overall population dynamics than do other populations.

The results of the screening experiment have, on the one hand, contributed to improving our understanding of the fish population dynamics; on the other hand, they may be used to update the model in order to make it more efficient and/or more consistent with observed data. For instance, point (4) above may call for a revision of the role of sardines in the model or one may think of focusing on a simplified version of the model obtained by eliminating the parameters relative to interspecies competition between larvae and juveniles that do not seem to play a substantial role. Furthermore, results can be used to prioritise further research and experiments by addressing the estimates of those parameters that have the greatest effect on the output of interest.

4.6 Conclusions

We suggest that one uses the method discussed in this chapter when the problem is that of screening a few important input factors among a large number contained in the model, or in other words for setting Factor's Fixing. Operatively, when factors with small values of μ^* are identified, these can be fixed at any value in their uncertainty distribution without any significant loss of information. At times, this may lead to segment of the model being dropped.

The main advantage of the method is its low computational cost: the plan then requires a total number of runs that is a linear function of the number of examined factors.

The method has a number of advantages with respect to other screening methods that are widely accepted in the literature. For example, with respect to methods based on Fractional Factorials (Saltelli *et al.*, 2000b, p. 53), the Morris design is computationally more efficient. A fractional factorial design with high resolution (high resolution is desirable to avoid confounding effects of different orders) may be too expensive to perform.

A more economical design such as that proposed by Cotter (1979) relies on strict assumptions and fails when these assumptions are not fulfilled.

The Iterated Fractional Factorial Design proposed by Andres and Hajas (1993) is based on the idea of grouping factors and performs appropriately when the number of factors that is important is restricted. The sequential bifurcation proposed by Bettonvil (1990) is applicable only when factor effects have *known signs*, which means the analyst knows whether a specific individual factor has a positive or negative effect on the simulation response, an assumption that is rarely fulfilled.

The Morris method, as all screening methods, provide sensitivity measures that tend to be qualitative, i.e. capable of ranking the input factors in order of importance, but do not attempt to quantify by how much one given factor is more important than another. A quantitative method would provide an estimate, for example, of the exact percentage of total output variance that each factor (or group of factors) accounts for. However, quantitative methods are more computational expensive (see Chapter 5) and not affordable when a large number of input factors are involved in the analysis or the model is time consuming.