

Final Year Project

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Chapter 1

Literature Review

1.1 Background

The Prisoners Dilemma a large area of repeated games in Game Theory and has applications in the real world. This is due to the game being a good example of strategies that give a cooperative benefit to repeated games. It has been used to describe actions of people and governments in situations stretching from warfare [TC88, AH92], finance [CS97] and politics [Sni85] to sex and relationships [Low15]. Because of its applicability to real world scenarios there is a strong desire to understand how strategies are beaten to either exploit flaws or counter opponents in real world scenarios.

The Prisoners dilemma became a large research area in the combination of mathematics and computer science after Robert Axelrod published his work named "Effective Choice In The Prisoners Dilemma" [Axe80]. In it he makes an introduction to how tournaments are run and the properties of successful results. His method of experimenting became the standard for handling the IDP problem. After the tournament is complete he describes what successful strategies had in common; It turns out the majority of strategies have properties of niceness and forgiveness. This allowed them to thrive in the tournament and have overall scores that rose above strategies without niceness or forgiveness.

Since Axelrods' original tournament there have been many research papers on what makes a successful strategy for a specific objective. For example [PD12] looks at how to remove an opponents moves to reach a high score and [MD09] looks at a range of different objectives at once. These specific objectives are really the core part of applying Game Theory and The Prisoners Dilemma in the really word. Objectives are the wrapper for which we can work with real world scenarios, for example in a the cold war it was not the goal to get as many missiles as possible to pass an oppositions defence but to not allow any missiles through your own defence; i.e.minimising your opponents score. In a football tournament where winning is the number or goals your team scores it doesn't really matter how many goals you get in so long as you score the most overall. There are also some very useful applications such as rent splitting or work assignments [GP15], all of which were programmed and deployed to a web server for general use online¹

1.2 Strategy Structures

Strategies can be defined in multiple ways [HKJ⁺17]. Each method of representing a strategies has its benefits and drawbacks, though there is no method (or strategy in that case) that is best in all cases. There are methods of creating strategies that remove the way opponents play in order to create overall scores that desired. Subsection 1.3.7 discusses this in more detail. Ways of structuring a strategy are shown below:

- LookerUpper, find examples
- Gambler

¹<http://www.spliddit.org>

- Neural Networks
- Finite State Machines
- Hidden Markov Models
- Explicit Move List
- Mixtures

1.2.1 Equivalent strategies

When performing analysis of an opponent using a GA it can sometimes be useful to brute force the starting positions of our feature selection to create rare² starting points. Looking at certain opponents there are occasions some strategies that look indistinguishable from others; for example Alternator and Random (0.5) can very often create the same output. One of the ways we are able to identify strategies is the process of fingerprinting [AKK04, AK08].

Another way is to look up their structure definitions in the same model (for example...)

1.3 Strategies Of Interest

1.3.1 TitForTat

Tit For Tat was the

1.3.2 Alternator

See Subsection 1.3.4.

1.3.3 Grudger

See Subsection 1.3.4.

1.3.4 Random

For all the strategies () given above we are able to logically deduce that the best counter is a totality of Ds, no matter what their definition structure is. This leads us to consider what the common theme is between the strategies as to be beaten in the same way.

1.3.5 EvolvedFSM16

1.3.6 CollectiveStrategy

1.3.7 ZDExtortion

This is the paper [PD12]

²For example of a random sequence that is a totality of Cs is incredibly rare, with around 6.2^{-61} chance of occurring

1.3.8 Cycler

1.4 Genetic Algorithms

This work will focus on Genetic Algorithms (GAs) which are a specific form of Machine Learning (ML). Advanced techniques of machine learning can be combined and used together³ in many situations, lots of these techniques combine genetic algorithms or some sort of fitness testing within a larger scope. The field of mathematics research is one which has plenty of examples of GAs in action; for example the same techniques as we will using is used in [CB97] to solve the Generalised Assignment Problem. Another example, [BLM95], used neural networks when approaching the state regulation problem.

In Game Theory GAs are used in a couple of areas, most prominently in the study of the iterated prisoners dilemma.

³Techniques for teaching and versioning static algorithms such as building a ‘clever’ game AIs, where the core concept of the AI is fine tuned using GA in an development environment but isn’t implemented into the game [BSVdH09].

Chapter 2

Developing The Codebase

2.1 Research Environments

A mix of Jupyter Notebooks and integrated development environments¹ were used to write and execute code. The main analysis was run using a factory class pattern, called `AnalysisRun` in the `full_analysis` module, show in Figure ?? in Appendix Chapter A. This class was used to wrap a query to the Axelrod-Dojo functionality and subsequently to the Axelrod Library in such a way that was easy to control batch executions.

The analysis itself was done using native multi-threading on a Linux OS to improve individual opponent analysis run times and the overall scalability of the project. This will be discussed further in Subsection 2.1.

This section will go into detail on the set up and reasoning behind using specific development environments. This tutorial will also assume you're working with a local development station; analysis on remote cloud instances of Jupyter Notebooks is possible but the set up is different.

Installing Basic Libraries Your first step should be to download and install the Anaconda distribution for your OS here: <https://www.anaconda.com/download/>. This will allow you to use the integrated c++ libraries python has to offer without needing to mess about too much. Anaconda also has them majority of Functional Libraries above and Jupyter Notebooks pre installed to make setting up much easier. From here, follow the instructions the install wizard has to add any environment variables to allow CMD/Bash access to binaries.

Installing the Axelrod and Axelrod-Dojo libraries uses the pip tool that already comes with Anaconda and should be ready to execute after the last step. Running 'pip install axelrod' then 'pip install axelrod-dojo' will install these.

Once this is installed the `full_analysis.py` file has to be downloaded from github², it can be found in the code directory. This can just be copied and pasted if needed all were interested in is the class to generate a sequence for an opponent.

Running a Test Figure 2.1 is some sample code that will run an analysis with the following settings:

- Override the default mutation frequency of 0.1 to 0.3.
- Set the prefix for all the files to be 'example-'.
- Analysing 3 opponents. (Random will have multiple instances for different seeds.)

¹Pycharm Professional Edition and Microsoft VS Code.

²<https://github.com/GitToby/FinalYearProject>

```

from full_analysis import NewAnalysisRun
import axelrod as axl

run = NewAnalysisRun(mutation_frequency=0.33)
run.save_file_prefix = "example-"

run.add_opponent(axl.TitForTat())
run.add_opponent(axl.Random())
run.add_opponent(axl.Grudger())

run.start()

```

Figure 2.1: Code to create a sequence result to optimise best score for 3 opponents

After it has run the generated data output should be stored in the ‘./output’ directory. If the code fails to run there may be issues with this directory being created. There should be multiple output files, each with one opponents evolution stages through the generations.

Cloud Notebook Setup If you want to use a Cloud service, such as Azure Notebooks or AWS Sagemaker, the set up is similar to the above just executed differently. Installing Anaconda is not needed, the environment has the required installs. Using pip to install the Axelrod libraries and download the full analysis script can be done in an integrated web terminal or directly in a notebook. Figure 2.2 shows an example in azure, copy these in your top few cells of your jupyter notebook and it will work as required.

```

# ----- CELL 1 -----
# The ! means 'run this as a bash script'
! pip install axelrod
! pip install git+https://github.com/Axelrod-Python/axelrod-dojo.git
! wget https://raw.githubusercontent.com/GitToby/FinalYearProject/master/code/full_analysis.py

# ----- CELL 2 -----
import axelrod as axl
import full_analysis as fa

run = fa.NewAnalysisRun(population_size=40)

run.add_opponent(axl.TitForTat())
run.add_opponent(axl.Random())
run.add_opponent(axl.Grudger())

run.start()

```

Figure 2.2: Cells for creating the jupyter instance of a research environment

2.2 Codebase Contributions

Throughout the project I had split time between writing my own code and expanding the Axelrod Dojo codebase. In modern software development there is a commitment in the developer community to track and reuse as much code as possible, typically using a version control system such as Git or Subversion. The majority of open source community development is conducted on github[Tor] using Git, the Axelrod libraries, along with most other libraries I used, are hosted here.

When writing code in a professional environment there is a predetermined scope that all parties agree upon before the work commences. This, however, is not the case for research development which is more organic; final products that are created when conducting research for papers are highly personalised and typically cannot be used in other areas. This leads to more flexible products that operate more as platforms or tools for further research, this is why the Axelrod, Axelrod Dojo and other libraries mentioned in table 2.1 exist. My final code will only be used for researching the IPD in my projects specific direction, this meant extending the platforms to handle what I needed them to do, subsection 2.2.1 looks into how this was completed in my project.

2.2.1 Version Control

During the project I created a ‘fork’ of the axelrod dojo in order to add content to the open source community. This resulted in using Git to create a new ‘branch’ on which to write my code before asking the owners of the repository to pull my work back into the core product using a ‘pull request’(PR). The PR opened for my code³ resulted in changes to 457 lines of code and 14 files, adding classes and fixing bug that had been previously flagged. Figure 2.3 shows the initial scope of the PR and Figure 2.4 shows a section of the commits made before merging the branch.

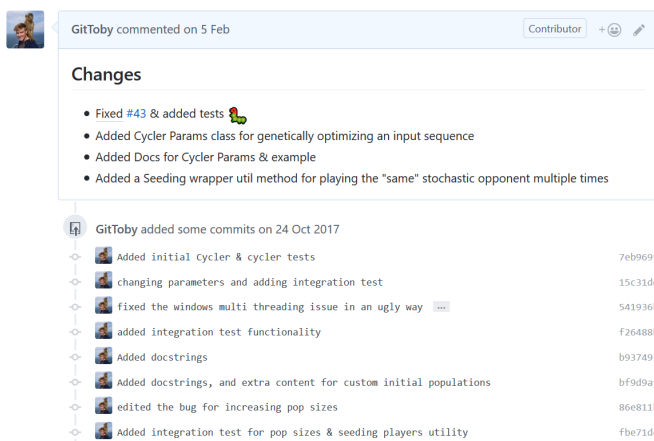


Figure 2.3: Description and commits for PR on Github

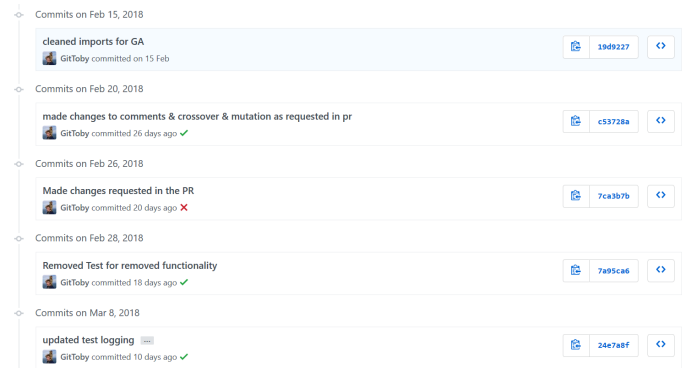


Figure 2.4: Tail of code commit log as shown on Github

After a request is opened there is a period of reviews by the owners, this is to ensure code quality and scope coverage. As my work progressed I continued to add to the PR which lead to more and more requests being added for features that fell outside the scope of the PR. Eventually we decided to create another fork of the PR that contained specialist code for my project that wouldn’t benefit the codebase as a whole. Figures 2.5 & 2.6 show examples of requests and discussions around features and code quality. Because of this review process the overall quality of the codebase can be kept high and the owners can decide on how their platforms are developed.

2.2.2 Testing

Code testing and version control go hand in hand. When new releases of code are made it is important to ensure that the new changes dont break previous functionality. This is kept in check by the presence of continues integrations (CI) and test environments. As of this project the CI for the Axelrod Dojo keeps the libraries tests running on a linux environment and the output fed to the Github page. During my development I had to implement tests for any functionality I wrote to ensure the library still worked as intended. Following the mixed practice of test driven design (TDD) and behaviour driven design (BDD) the code written to extend the Axelrod Dojo had tests to cover examples of what happens in a production environment. Snippet 2.7 shows an example of a test.

³<https://github.com/Axelrod-Python/axelrod-dojo/pull/45>

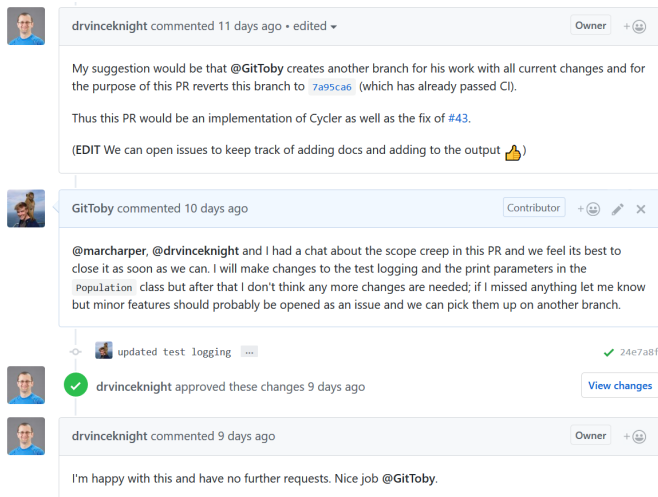


Figure 2.5: Feature discussions in PR on Github

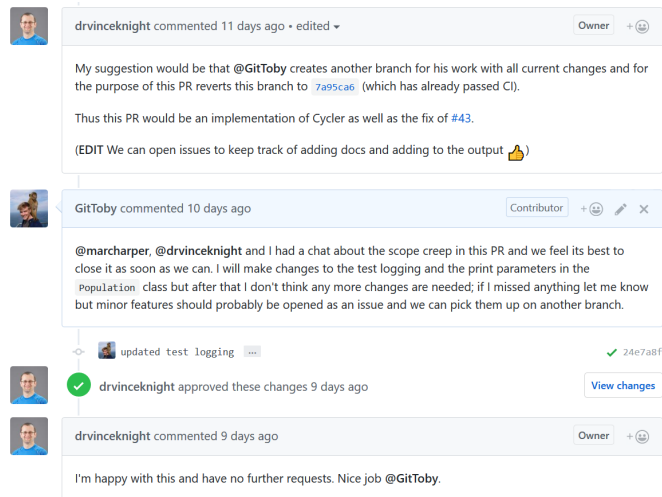


Figure 2.6: Scope Discussion in PR on Github

```
def test_creation_seqLen(self):
    axl.seed(0)
    test_length = 10
    self.instance = CyclerParams(sequence_length=test_length)
    self.assertEqual(self.instance.sequence, [D, C, C, D, C, C, C, C, C, C])
    self.assertEqual(self.instance.sequence_length, test_length)
    self.assertEqual(len(self.instance.sequence), test_length)
```

Figure 2.7: An Example of a test in the Axelrod Dojo

2.3 Libraries And External Modules

Main Research Libraries

- **Axelrod** — Used for the core of the prisoners dilemma and iterated prisoners dilemma functionality code.[pd17]
- **Axelrod-Dojo** — Applied machine learning techniques that revolve around generating solutions to questions relating to the Axelrod library.

Functional libraries Table 2.1 shows the external functional libraries used, while Table 2.2 shows the internal python built in modules that where leveraged during development. These are libraries which are not involved in the core functionality of the IPD.

Library	Reason
matplotlib	For plotting graphs and images with data
pyplot	For data manipulation.
pandas	For reducing complexity of numerical calculations.
numpy	

Table 2.1: Functional Python libraries for analysis

Library	Reason
os	For operating system functionality.
time	For time calculations.
itertools	For easier iterations over data structures.

Table 2.2: Internal built in python modules used

Chapter 3

Results and Discussion

3.1 Background

loading, cleaning data, description data

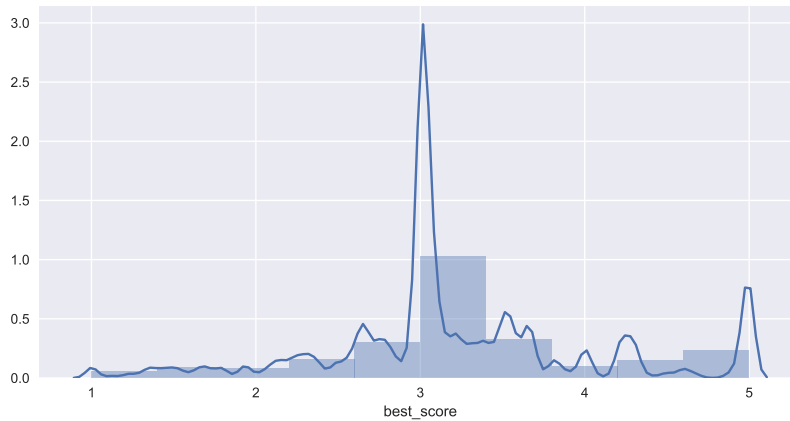


Figure 3.1: A histogram showing the distribution of best scores with overlaid kernel density estimate

3.2 Solution Distance Matrices

The first avenue of analysis, after constructing descriptive data, was to look at the relationship strategies have with each other. A distance matrix shows how much an opponent differs from every other, if 2 sequences are similar with respect to the distance function then they will score lower than 2 sequences that are more distinct. Certain distance functions have been selected because of their connotations.

In each matrix we order the sequences by score; $S_{O_0} \geq S_{O_1} \geq \dots \geq S_{O_n}$, but we will shorten S_{O_i} to S_i for simplicity. The solution sequence for the i th opponent, S_i vs the solution sequence for the j th, S_j is scored using our distance function, for example $d(S_0, S_n)$ is the distance between the best and worst scoring sequences respectively. The Matrix itself will be symmetric, so looking across rows vs columns tends to make more logical sense; the top rows are the highest scoring opponents, and the lower rows the worse scorers.

Hamming Distance

$$d(S_i, S_j) = S_i \cdot S_j^T \text{ or } 1 - \frac{\sum_{i,j=0}^n \delta_{ij}}{n} \text{ where } \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

The Hamming Distance represents the count of the elements that differ in any two sequences. A Hamming Distance will thus correspond to the number of places we have to play different moves against opponents to get out best score. Figure 3.2 shows the matrix generated by the code in Figure A.2.

By observing the graph row by row, we can build an idea of how similar each sequence is to the range of others. the top section of the plot shows the best scoring opponents vs themselves on the left and the worse scoring opponents on the right. At the very top left there is lots of darker rows, suggesting that there are lots of very similar or dissimilar solutions at the high end of the score level, as we move to the right we have many slightly similar opponents then on the far right lots of very different sequences. This is shown again as we move to the bottom of the diagram. The large blocks or red covering the bottom 100 or so rows show that the lowest scoring opponents have very different sequences to the higher scorers, but as a group on the bottom right they're quite similar.

The result of an average score means that we can estimate the distribution of move combinations because we know, if we are the first player: $(C, C) = 3$, $(C, D) = 0$, $(D, C) = 5$, $(D, D) = 1$. Thus if we assume scoring in the midrange (3 ish) means mostly cooperating or a combination of (C, D) and (D, C) whereas at the high and low ends its more likely to be defecting. Looking at the high vs low and low vs high scoring sections of the graph this hypothesis isn't supported, we would expect them to have $d(S_i, S_j) \approx 0$, whereas were seeing a mix of distances. Section 3.3 looks at patterns of move density vs best score.

Cosine Distance

$$d(S_i, S_j) = \cos(\theta) = \frac{S_i \cdot S_j}{\|S_i\| \|S_j\|}$$

The cosine of two vectors constructed by using the dot product formula as shown above. In our interpretation each dimension represents a sequence element so we are working in \mathbb{R}^{200} with every value taking a 1(C) or a 0(D). Figure 3.3 shows the distance matrix generated from the data files using code in Figure A.2

The Matrix for Cosine and Hamming are almost exactly similar, the only difference being the value of the distance between sequences. This is due to the two measures being similar in their relationship of space [CCT]. We can conclude that there is no extra information shown in this diagram.

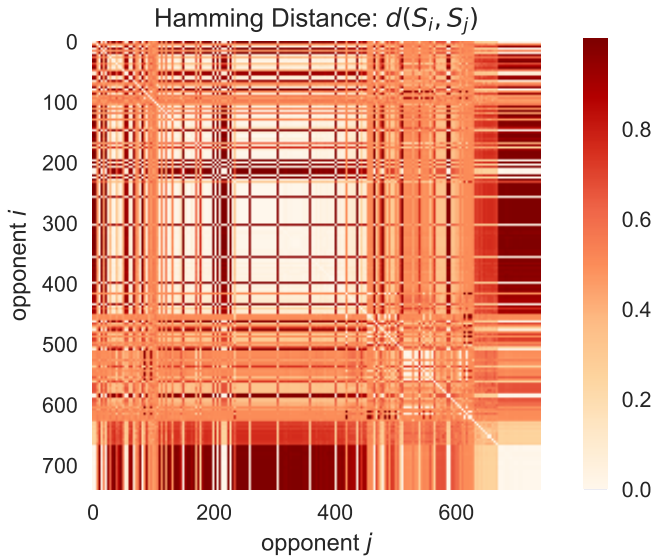


Figure 3.2: Distance Matrix for Hamming Distance

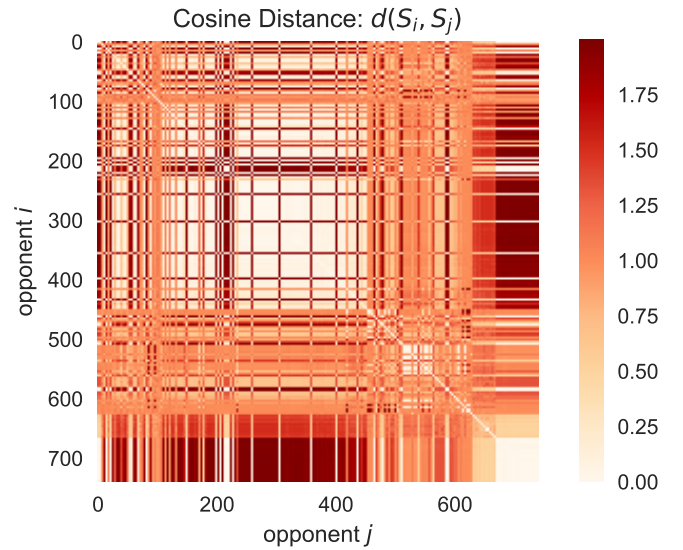


Figure 3.3: Distance Matrix for Cosine Distance

3.3 Solution Groups

If we want to group opponents together, the most obvious way is to look at which opponents have the same best score sequence. Appendix section C has full details, but figure 3.4 shows a plot of the trends. We can observe that there are lots of sequences with solutions that don't contain many blocks (≤ 25)

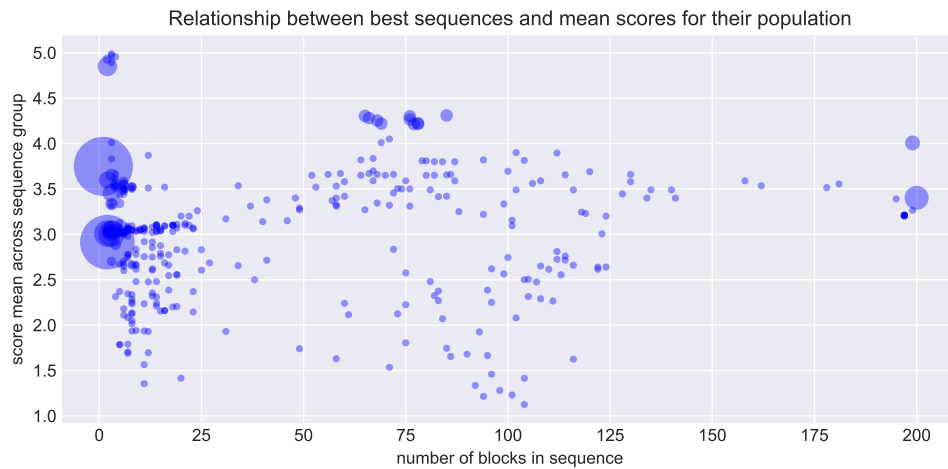


Figure 3.4: Trends for opponents grouped by their best sequence

3.4 Scoring Groups

3.5 Clustering Analysis

Here we will look at ways of grouping opponents and what that means for potential scoring outcomes.

Appendix A

Code Appendix

```

class NewAnalysisRun:
    # default options
    opponent_list = []
    output_files = {}
    save_directory = "output/"
    save_file_prefix = ""
    save_file_suffix = ""
    global_seed = 0
    overwrite_files = True
    stochastic_seeds = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]

    def __init__(self, sequence_length=20,
                  population_size=25,
                  generation_length=20,
                  mutation_frequency=0.1,
                  mutation_potency=1):
        self.sequence_length = sequence_length
        self.population_size = population_size
        self.generation_length = generation_length
        self.mutation_frequency = mutation_frequency
        self.mutation_potency = mutation_potency

    def get_pre_made_pop(self, pop_size: int):
        pop = []

        # Totalities & Handshakes
        handshake_leng = 5
        for start in itertools.product("CD", repeat=handshake_leng):
            pop.append(axl_dojo.CyclerParams(
                list(start) + [C] * (200 - handshake_leng)))
            pop.append(axl_dojo.CyclerParams(
                list(start) + [D] * (200 - handshake_leng)))

        # 50-50
        pop.append(axl_dojo.CyclerParams([C] * 100 + [D] * 100))
        pop.append(axl_dojo.CyclerParams([D] * 100 + [C] * 100))

        # Single Change
        for i in range(1, 11):
            pop.append(axl_dojo.CyclerParams([C] * i + [D] * (200 - i)))
            pop.append(axl_dojo.CyclerParams([D] * i + [C] * (200 - i)))

        for i in range(1, 11):
            pop.append(axl_dojo.CyclerParams([C] * (200 - i) + [D] * i))
            pop.append(axl_dojo.CyclerParams([D] * (200 - i) + [C] * i))

        # Matching Tails
        for i in range(1, 6):
            for j in range(1, 6):
                pop.append(axl_dojo.CyclerParams(
                    [C] * i + [D] * (200 - (i + j)) + [C] * j))
                pop.append(axl_dojo.CyclerParams(
                    [D] * i + [C] * (200 - (i + j)) + [D] * j))

        # Alternating
        pop.append(axl_dojo.CyclerParams([C, D] * 100))
        pop.append(axl_dojo.CyclerParams([D, C] * 100))
        pop.append(axl_dojo.CyclerParams([C, C, D, D] * 50))
        pop.append(axl_dojo.CyclerParams([D, D, C, C] * 50))
        pop.append(axl_dojo.CyclerParams([C, C, C, C, D, D, D, D] * 25))
        pop.append(axl_dojo.CyclerParams([D, D, D, D, C, C, C, C] * 25))
        pop.append(axl_dojo.CyclerParams([C, C, C, C, C, D, D, D, D, D] * 20))
        pop.append(axl_dojo.CyclerParams([D, D, D, D, D, C, C, C, C, C] * 20))

        # Random Filler

```

```

while len(pop) < pop_size:
    random_moves = list(map(axl.Action, np.random.randint(
        0, 1 + 1, (self.sequence_length, 1))))
    pop.append(axl_dojo.CyclerParams(random_moves))

```



```

from sklearn import metrics

def CD_map_to_int(x):
    # Returns D:0 C:1
    return 68 - ord(x)

# Itertuples is a list of (index,col1,col2,...) for each row. We sort them by score first
df_vectors = pd.DataFrame()
df_vectors_strings = pd.DataFrame()
df_generation_max = df_generation_max.sort_values('opponent_name')
df_generation_max = df_generation_max.sort_values('best_score')
for tup in df_generation_max.itertuples():
    # index vals mean the indexing starts at 1 so +1 to these:
    #(0, 'generation'), (1, 'score_mean'), (2, 'score_median'), (3, 'score_pop_var'), (4, 'score_range'), (5, 'best_score')
    seq_str = tup[7]
    opponent_name = str(tup[8])
    best_score = tup[6]
    bins = tup[15]
    # Mapping to integers 0 & 1
    df_vectors[opponent_name] = list(
        map(CD_map_to_int, seq_str)) + [best_score] + [seq_str] + [bins]

    # No Mapping Cs & Ds
    df_vectors_strings[opponent_name] = list(
        seq_str) + [best_score] + [seq_str] + [bins]

move_cols = ['move ' + str(x+1) for x in range(200)]
# Transposing and labeling moves as were forming it in a rotated way.
# (Its easier to just make 2 dfs than map one to the other)
df_vectors = df_vectors.transpose()
df_vectors.columns = move_cols + ["best_score", "best_sequence", "score_bins"]

df_vectors_strings = df_vectors_strings.transpose()
df_vectors_strings.columns = move_cols + \
    ["best_score", "best_sequence", "score_bins"]

# Manhattan Distance == Hamming Distance in this example
# Intresting examples: cosine(?), hamming, jaccard sim
dist_array_ham = metrics.pairwise.pairwise_distances(
    df_vectors[move_cols], metric='hamming')
# add labels
dist_array_ham = pd.DataFrame(
    dist_array_ham, index=df_vectors.index, columns=df_vectors.index)

dist_array_cos = metrics.pairwise.pairwise_distances(
    df_vectors[move_cols], metric='cosine')
dist_array_cos = pd.DataFrame(
    dist_array_cos, index=df_vectors.index, columns=df_vectors.index)

dist_array_jac = metrics.pairwise.pairwise_distances(
    df_vectors[move_cols], metric='jaccard')
dist_array_cos = pd.DataFrame(
    dist_array_cos, index=df_vectors.index, columns=df_vectors.index)

dist_array_other = metrics.pairwise.pairwise_distances(
    df_vectors[move_cols], metric='correlation')
dist_array_cos = pd.DataFrame(
    dist_array_cos, index=df_vectors.index, columns=df_vectors.index)

```

Figure A.2: Distance Matrix generation code using SciKitLearn

Appendix B

List of Axelrod Opponents

* means Stochastic Opponent, (LRT) means Long Run Time.

- ϕ
- π
- e
- ALLCorALLD*
- Adaptive
- Adaptive Pavlov 2006
- Adaptive Pavlov 2011
- Adaptive Tit For Tat
- Aggravater
- Alexei
- Alternator
- Alternator Hunter
- Anti Tit For Tat
- AntiCycler
- Appeaser
- Arrogant QLearner*
- Average Copier*
- BackStabber
- Better and Better*
- Black*
- Borufsen
- Bully
- Bush Mosteller*
- Calculator*
- Cautious QLearner*
- Cave*
- Champion*
- Colbert
- CollectiveStrategy
- Contrite Tit For Tat
- Cooperator
- Cooperator Hunter
- Cycle Hunter
- Cyclor CCCCCD
- Cyclor CCCD
- Cyclor CCCDCD
- Cyclor CCD
- Cyclor DC
- Cyclor DDC
- DBS (LRT)
- Davis
- Defector
- Defector Hunter
- Desperate*
- DoubleCrosser
- DoubleResurrection
- Doubler
- Dynamic Two Tits For Tat*
- EasyGo
- Eatherley*
- EugeneNier
- Eventual Cycle Hunter
- Evolved ANN
- Evolved ANN 5
- Evolved ANN 5 Noise 05
- Evolved FSM 16
- Evolved FSM 16 Noise 05
- Evolved FSM 4
- Evolved HMM 5*
- EvolvedLookerUp1_1_1
- EvolvedLookerUp2_2_2
- Feld*
- Firm But Fair*
- Fool Me Forever
- Fool Me Once
- Forgetful Fool Me Once*
- Forgetful Grudger
- Forgiver
- Forgiving Tit For Tat
- Fortress3
- Fortress4
- GTFT*
- General Soft Grudger
- Getzler*
- Gladstein
- Go By Majority
- Go By Majority 10
- Go By Majority 20
- Go By Majority 40
- Go By Majority 5
- GraaskampKatzen
- Gradual
- Gradual Killer
- Grofman*
- Grudger
- GrudgerAlternator
- Grumpy
- Handshake
- Hard Go By Majority
- Hard Go By Majority 10

- Hard Go By Majority 20
- Hard Go By Majority 40
- Hard Go By Majority 5
- Hard Prober
- Hard Tit For 2 Tats
- Hard Tit For Tat
- Harrington*
- Hesitant QLearner*
- Hopeless*
- Inverse*
- Inverse Punisher
- Joss*
- Kluepfel*
- Knowledgeable Worse and Worse*
- Level Punisher
- Leyvraz*
- Limited Retaliate
- Limited Retaliate 2
- Limited Retaliate 3
- MEM2
- Math Constant Hunter
- Meta Hunter
- Meta Hunter Aggressive
- Meta Majority* (LRT)
- Meta Majority Finite Memory* (LRT)
- Meta Majority Long Memory* (LRT)
- Meta Majority Memory One* (LRT)
- Meta Minority* (LRT)
- Meta Mixer* (LRT)
- Meta Winner* (LRT)
- Meta Winner Deterministic* (LRT)
- Meta Winner Ensemble* (LRT)
- Meta Winner Finite Memory* (LRT)
- Meta Winner Long Memory* (LRT)
- Meta Winner Memory One* (LRT)
- Meta Winner Stochastic* (LRT)
- Michaelos*
- More Tideman and Chieruzzi
- MoreGrofman
- N Tit(s) For M Tat(s)
- NMWE Deterministic* (LRT)
- NMWE Finite Memory* (LRT)
- NMWE Long Memory* (LRT)
- NMWE Memory One* (LRT)
- NMWE Stochastic* (LRT)
- Naive Prober*
- Negation*
- Nice Average Copier*
- Nice Meta Winner* (LRT)
- Nice Meta Winner Ensemble* (LRT)
- Nydegger
- Omega TFT
- Once Bitten
- Opposite Grudger
- PSO Gambler 1.1.1*
- PSO Gambler 2.2.2*
- PSO Gambler 2.2.2 Noise 05*
- PSO Gambler Mem1*
- Predator
- Prober
- Prober 2
- Prober 3
- Prober 4
- Pun1
- Punisher
- Raider
- Random*
- Random Hunter
- Random Tit for Tat*
- Remorseful Prober*
- Resurrection
- Retaliate
- Retaliate 2
- Retaliate 3
- Revised Downing
- RichardHufford
- Ripoff
- Risky QLearner*
- SelfSteem*
- ShortMem
- Shubik
- Slow Tit For Two Tats 2
- Sneaky Tit For Tat
- Soft Grudger
- Soft Joss*
- SolutionB1
- SolutionB5
- Spiteful Tit For Tat
- Stalker*
- Stein and Rapoport
- Stochastic Cooperator*
- Stochastic WSLs*
- Suspicious Tit For Tat
- TF1
- TF2
- TF3
- Tester
- ThueMorse
- ThueMorseInverse
- Thumper
- Tideman and Chieruzzi
- Tit For 2 Tats
- Tit For Tat
- Tranquilizer*
- Tricky Cooperator
- Tricky Defector
- Tullock*
- Two Tits For Tat
- VeryBad
- Weiner
- White
- Willing*
- Win-Shift Lose-Stay
- Win-Stay Lose-Shift
- Winner12
- Winner21
- WmAdams*
- Worse and Worse*
- Worse and Worse 2*
- Worse and Worse 3*
- Yamachi
- ZD-Extort-2*
- ZD-Extort-2 v2*
- ZD-Extort-4*
- ZD-Extort3*
- ZD-Extortion*
- ZD-GEN-2*
- ZD-GTFT-2*
- ZD-Mem2*
- ZD-Mischief*
- ZD-SET-2*

Appendix C

List of Best Solution Sequences

- C199,1 for 96 opponents:

- | | | |
|--|---|--------------------------------------|
| – Aggravater (score:2.965) | – Inverse@_3 (score:3.01) | – Naive_Prober@_0 (score:2.665) |
| – Borufsen (score:3.01) | – Inverse@_4 (score:3.01) | – Naive_Prober@_1 (score:2.8) |
| – Cave@_5 (score:3.01) | – Inverse@_5 (score:3.01) | – Naive_Prober@_2 (score:2.65) |
| – Contrite_Tit_For_Tat (score:3.01) | – Inverse@_6 (score:3.01) | – Naive_Prober@_3 (score:2.695) |
| – Davis@_10 (score:3.01) | – Inverse@_7 (score:3.01) | – Naive_Prober@_4 (score:2.635) |
| – EvolvedLookerUp1_1_1 (score:3.01) | – Inverse@_8 (score:3.01) | – Naive_Prober@_5 (score:2.59) |
| – Feld@_3 (score:2.29) | – Inverse@_9 (score:3.01) | – Naive_Prober@_7 (score:2.77) |
| – Firm_But_Fair@_0 (score:3.01) | – Inverse_Punisher (score:3.01) | – Naive_Prober@_8 (score:2.65) |
| – Firm_But_Fair@_1 (score:3.01) | – Joss@_0 (score:2.665) | – Naive_Prober@_9 (score:2.755) |
| – Firm_But_Fair@_2 (score:3.01) | – Joss@_1 (score:2.8) | – PSO_Gambler_1_1_1@_0 (score:3.01) |
| – Firm_But_Fair@_3 (score:3.01) | – Joss@_2 (score:2.65) | – PSO_Gambler_1_1_1@_2 (score:3.01) |
| – Firm_But_Fair@_4 (score:3.01) | – Joss@_3 (score:2.695) | – PSO_Gambler_1_1_1@_5 (score:3.01) |
| – Firm_But_Fair@_5 (score:3.01) | – Joss@_4 (score:2.635) | – PSO_Gambler_1_1_1@_6 (score:3.01) |
| – Firm_But_Fair@_6 (score:3.01) | – Joss@_5 (score:2.59) | – PSO_Gambler_Mem1@_0 (score:3.01) |
| – Firm_But_Fair@_7 (score:3.01) | – Joss@_7 (score:2.77) | – PSO_Gambler_Mem1@_2 (score:3.01) |
| – Firm_But_Fair@_8 (score:3.01) | – Joss@_8 (score:2.65) | – PSO_Gambler_Mem1@_5 (score:3.01) |
| – Firm_But_Fair@_9 (score:3.01) | – Joss@_9 (score:2.755) | – PSO_Gambler_Mem1@_6 (score:3.01) |
| – Forgetful_Grudger (score:3.01) | – Kluepfel@_1 (score:3.01) | – Punisher (score:3.01) |
| – General_Soft_Grudger@_n=1,d=4,c=2 (score:3.01) | – Kluepfel@_4 (score:3.01) | – Remorseful_Prober@_1 (score:2.8) |
| – Getzler@_2 (score:3.01) | – Leyvraz@_0 (score:3.01) | – Remorseful_Prober@_9 (score:2.755) |
| – GraaskampKatzen (score:3.01) | – Limited_Retaliante@_0.1,_20 (score:3.01) | – Resurrection (score:3.01) |
| – Grudger (score:3.01) | – Limited_Retaliante_2@_0.08,_15 (score:3.01) | |
| – Hard_Tit_For_Tat (score:3.01) | – Limited_Retaliante_3@_0.05,_20 (score:3.01) | |
| – Inverse@_0 (score:3.01) | – MEM2 (score:3.01) | |
| – Inverse@_1 (score:3.01) | – Meta_Hunter@_6_players (score:3.01) | |
| – Inverse@_2 (score:3.01) | | |

- Retaliate@_0.1 (score:3.01)
- Retaliate_2@_0.08 (score:3.01)
- Retaliate_3@_0.05 (score:3.01)
- Shubik (score:3.01)
- Soft_Grudger (score:3.01)
- Soft_Joss@_0.9 (score:3.01)
- SolutionB5 (score:2.995)
- Spiteful_Tit_For_Tat (score:3.01)
- Suspicious_Tit_For_Tat (score:2.995)
- Thumper (score:3.01)
- Tideman_and_Chieruzzi (score:3.01)
- Tit_For_Tat (score:3.01)
- Two_Tits_For_Tat (score:3.01)
- ZD-Extort-4@_2 (score:1.825)
- ZD-Extort-4@_9 (score:1.945)
- ZD-GEN-2@_2 (score:3.01)
- ZD-GEN-2@_5 (score:3.01)
- ZD-GTFT-2@_0 (score:3.01)
- ZD-GTFT-2@_1 (score:3.01)
- ZD-GTFT-2@_3 (score:3.01)
- ZD-GTFT-2@_5 (score:3.01)
- ZD-GTFT-2@_6 (score:3.01)
- ZD-GTFT-2@_8 (score:3.01)
- ZD-GTFT-2@_9 (score:3.01)

• C198,2 for 21 opponents:

- Alexei@_(D,) (score:3.0)
- EugeneNier@_(D,) (score:3.0)
- Fool_Me_Once (score:3.02)
- Kluepfel@_0 (score:3.02)
- Kluepfel@_5 (score:3.02)
- Michaelos@_0@_(D,) (score:3.0)
- Michaelos@_1@_(D,) (score:3.0)
- Michaelos@_2@_(D,) (score:3.0)
- Michaelos@_3@_(D,) (score:3.0)
- Michaelos@_4@_(D,) (score:3.0)
- Michaelos@_5@_(D,) (score:3.0)
- Michaelos@_6@_(D,) (score:3.0)
- Michaelos@_7@_(D,) (score:3.0)
- Michaelos@_8@_(D,) (score:3.0)
- Michaelos@_9@_(D,) (score:3.0)
- PSO_Gambler_2_2_2@_2 (score:3.02)
- PSO_Gambler_2_2_2@_6 (score:3.02)
- PSO_Gambler_Mem1@_1 (score:3.02)
- PSO_Gambler_Mem1@_3 (score:3.02)
- PSO_Gambler_Mem1@_8 (score:3.02)
- TF3 (score:3.02)

• C196,4 for 3 opponents:

- Gradual (score:3.02)
- More_Tideman_and_Chieruzzi (score:3.02)
- ZD-GEN-2@_3 (score:3.04)

• C194,6 for 1 opponents:

- BackStabber@_(D,D) (score:3.02)

• C193,7 for 2 opponents:

- PSO_Gambler_Mem1@_4 (score:3.07)
- PSO_Gambler_Mem1@_9 (score:3.05)

• C151,1,47,1 for 1 opponents:

- Feld@_2 (score:2.315)

• C100,26,1,9,1,63 for 3 opponents:

- Dynamic_Two_Tits_For_Tat@_9 (score:3.545)
- Nice_Average_Copier@_9 (score:3.545)
- Worse_and_Worse_3@_9 (score:3.545)

• C100,36,1,63 for 1 opponents:

- Average_Copier@_9 (score:3.535)
- C100,100 for 8 opponents:
 - Average_Copier@_5 (score:3.365)
 - Dynamic_Two_Tits_For_Tat@_3 (score:3.3)
 - Dynamic_Two_Tits_For_Tat@_5 (score:3.38)
 - Hard_Go_By_Majority (score:3.985)
 - Nice_Average_Copier@_5 (score:3.38)
 - Soft_Go_By_Majority (score:4.0)
 - VeryBad (score:4.0)
 - Worse_and_Worse_3@_5 (score:3.38)
- C99,1,3,1,1,1,2,2,3,1,2,1,1,2,10,2,1,1,1,1,2,1,3,1,1,1,3,1,1,1,3,1,3,5,1,2,1,1,2,1,3,1,1,1,1,3,1,1,1,1,4,1,2,1,1,2 for 1 opponents:
 - Level_Punisher (score:3.4)
- C98,2,1,4,1,9,2,1,3,2,2,2,1,1,1,1,1,2,4,1,1,1,2,1,1,2,4,1,5,1,1,2,4,4,5,2,1,6,4,2,1,1,1,1,1,3,1,1 for 1 opponents:
 - Dynamic_Two_Tits_For_Tat@_2 (score:3.29)
- C95,3,1,1,1,3,1,3,1,2,1,1,3,1,5,1,1,1,7,3,6,1,1,3,3,2,2,6,1,2,1,1,1,1,1,4,6,5,1,3,4,1,1,1,2,1,1,2,1 for 1 opponents:
 - Dynamic_Two_Tits_For_Tat@_0 (score:3.27)
- C92,3,3,1,2,1,2,1,2,1,11,1,1,4,7,1,5,1,1,3,4,1,1,2,4,1,1,1,1,1,1,1,2,2,1,2,2,3,2,4,1,4,1,5,6,1,1 for 1 opponents:
 - White (score:3.4)
- C87,1,1,3,1,1,2,2,1,1,1,3,1,3,2,1,2,2,4,1,1,1,3,3,2,1,4,1,1,3,2,2,2,2,4,5,1,2,1,2,4,1,3,3,1,1,2,1,2,1,2,3,1,5,1,1,1,1,1 for 1 opponents:
 - Feld@_8 (score:2.24)
- C87,3,2,2,6,12,1,87 for 2 opponents:
 - Nice_Average_Copier@_6 (score:3.52)
 - Worse_and_Worse_3@_6 (score:3.52)
- C87,3,2,2,6,100 for 1 opponents:
 - Average_Copier@_6 (score:3.495)
- C86,1,93,1,18,1 for 1 opponents:
 - Remorseful_Prober@_4 (score:2.645)
- C85,1,12,1,1,74,1,25 for 1 opponents:
 - Dynamic_Two_Tits_For_Tat@_8 (score:3.495)
- C84,1,5,1,1,1,1,3,1,3,1,1,1,3,3,2,1,1,1,2,2,1,1,1,1,7,1,6,1,2,1,4,1,1,4,2,2,4,5,6,1,2,3,1,3,1,1,4,6,5,1,2 for 1 opponents:

- [illegible]

- [illegible]

- [illegible]

- [illegible]

- [illegible]

- [illegible]

- C4,89,1,106 for 1 opponents:
 - Stochastic_WSLs@_7 (score:3.01)

- C3,1,44,1,4,1,15,1,12,2,10,1,16,1,12,1,71,4 for 1 opponents:
 - ZD-Extort-2@_5 (score:2.2)

- C3,1,10,1,37,1,32,1,33,1,11,1,16,1,49,2 for 1 opponents:
 - Forgetful_Fool_Me_Once@_3 (score:3.09)

- C3,1,6,1,38,1,2,2,1,1,1,1,4,1,2,3,1,3,1,2,2,1,1,1,1,1,3,1,1,1,1,3,2,1,2,1,3,1,2,4,1,1,2,1,1,1,4,2,5,1,7,2,1,1,7,2,2,1,4,1,1,1,3,1,2,1,1,1,1,
for 1 opponents:
- Stalker@_8@(D,) (score:3.655)

- [illegible]

- C3,1,2,2,1,2,3,3,2,5,3,2,169,2 for 1 opponents:
 - Cave@.7 (score:3.105)

- C3,1,2,2,1,4,4,1,1,2,29,1,148,1 for 1 opponents:
 - Cave@ 6 (score:3,035)

- [illegible]

- C3,2,45,1,29,1,24,1,6,1,1,1,14,1,63,7 for 1 opponents:
 - ZD-Extort-2 v2@ 6 (score:2 16)

- C3,2,26,1,6,1,30,1,13,1,10,1,102,3 for 1 opponents:
 - ZD-Extort-2 v2@ 5 (score:2 24)

- [illegible]

- C3,2,3,3,1,1,1,2,1,2,3,3,5,4,1,5,2,1,1,1,1,1,2,1,1,1,7,1,1,1,2,1,3,1,2,2,5,1,1,5,3,1,1,3,2,1,2,1,3,4,1,1,3,4,1,2,2,2,1,4,3,2,4,1,1,1,1,1,4,3
for 1 opponents:

- ZD-Extort3@_5 (score:1.925)
- C3,2,2,1,3,1,3,1,1,2,3,1,3,1,3,3,4,1,2,3,6,3,3,1,3,1,1,1,2,2,3,1,1,1,2,1,4,1,1,2,1,1,5,2,4,1,2,2,1,1,4,1,3,1,1,1,2,1,2,1,3,1,2,2,2,1,3,1,3,2
for 1 opponents:
 - Hard_Go_By_Majority@_10 (score:3.695)
- C3,2,1,4,2,2,4,3,4,3,3,4,1,2,2,1,5,1,3,1,3,5,1,2,1,3,2,2,1,2,3,5,1,1,1,1,2,3,2,1,1,1,2,1,1,3,1,4,2,1,1,1,1,5,1,2,2,5,2,2,3,1,1,2,2,1,3,2,1,
for 1 opponents:
 - ZD-Mischief@_5 (score:1.215)
- C3,6,24,1,25,1,8,1,11,1,9,1,14,1,23,1,4,1,21,1,28,1,4,10 for 1 opponents:
 - Evolved_ANN (score:3.26)
- C3,97,24,1,34,1,40 for 1 opponents:
 - Stochastic_Cooperator@_5 (score:2.345)
- C2,1,196,1 for 3 opponents:
 - Remorseful_Prober@_8 (score:2.655) - TF1 (score:3.0) - Tester (score:3.005)
- C2,1,97,26,1,73 for 1 opponents:
 - Average_Copier@_2 (score:3.495)
- C2,1,61,1,132,3 for 1 opponents:
 - Cave@_2 (score:3.02)
- C2,1,53,1,22,1,9,1,27,1,8,1,14,1,49,1,6,2 for 1 opponents:
 - Forgetful_Fool_Me_Once@_4 (score:3.1)
- C2,1,4,1,125,1,63,3 for 1 opponents:
 - Getzler@_1 (score:3.045)
- C2,1,4,1,18,1,73,1,97,2 for 1 opponents:
 - Forgetful_Fool_Me_Once@_7 (score:3.06)
- C2,1,4,1,4,2,2,184 for 1 opponents:
 - Worse_and_Worse_2@_9 (score:2.235)
- C2,1,2,1,2,1,1,3,2,1,1,2,2,1,2,2,1,1,1,1,1,2,3,1,1,1,1,1,3,2,3,3,3,2,2,1,1,2,2,2,3,2,2,1,2,1,4,2,1,1,1,3,3,2,1,1,3,1,1,4,1,1,3,1,4,1,1,1,1,1,
for 1 opponents:

- Hard_Go_By_Majority@_10 (score:3.695)

- ZD-Mischief@_5 (score:1.215)

- Evolved_ANN (score:3.26)

- Stochastic_Cooperator@_5 (score:2.345)

- | | | |
|---|-------------------|------------------------|
| - Remorseful_Prober@_8
(score:2.655) | - TF1 (score:3.0) | - Tester (score:3.005) |
|---|-------------------|------------------------|

- Average_Copier@_2 (score:3.495)

- Cave@_2 (score:3.02)

- Forgetful_Fool_Me_Once@_4 (score:3.1)

- Getzler@_1 (score:3.045)

- Forgetful_Fool_Me_Once@_7 (score:3.06)

- Worse_and_Worse_2@_9 (score:2.235)

- C2,1,2,1,2,1,1,3,2,1,1,2,2,1,2,2,1,1,1,1,1,2,3,1,1,1,1,1,3,2,3,3,3,2,2,1,1,2,2,2,3,2,2,1,2,1,4,2,1,1,1,3,3,2,1,1,3,1,1,4,1,1,3,1,4,1,1,1,1,1,
for 1 opponents:

- [illegible]

- ZD-Extortion@_9 (score:1.74)
- C2,3,54,1,36,1,29,1,23,1,31,1,15,2 for 1 opponents:
 - Forgetful_Fool_Me_Once@_9 (score:3.1)
- C2,3,22,1,27,2,1,1,1,2,2,2,1,2,3,3,2,1,2,1,1,2,1,1,1,2,5,3,2,1,1,1,1,3,5,1,6,1,4,1,1,2,2,1,1,1,1,1,1,1,2,2,2,1,1,1,1,1,1,3,1,1,2, for 1 opponents:
 - ZD-Extort-2@_6 (score:2.325)
- C2,3,22,1,22,1,4,2,1,2,3,1,2,6,1,1,3,2,1,1,3,1,2,1,1,1,3,1,3,3,1,1,1,3,3,4,1,1,1,1,2,1,1,2,2,1,1,1,1,4,2,1,1,3,1,1,2,2,1,1,1,3,1,4,4,3,1,1, for 1 opponents:
 - ZD-Extort3@_6 (score:2.07)
- C2,3,3,2,2,1,1,2,1,1,3,2,175,2 for 1 opponents:
 - ZD-GTFT-2@_7 (score:3.03)
- C2,3,1,2,2,1,5,2,1,2,3,1,2,1,3,1,3,1,1,1,2,1,1,1,2,6,1,1,2,1,3,1,1,4,2,1,1,2,1,1,1,1,1,1,3,2,2,1,2,4,2,1,2,1,1,2,2,1,1,1,3,1,3,1,2,2,1,3,3,4, for 1 opponents:
 - ZD-SET-2@_9 (score:2.745)
- C2,6,2,6,1,4,8,1,16,4,1,1,11,9,20,100,8 for 1 opponents:
 - Stochastic_Cooperator@_0 (score:2.66)
- C2,6,1,3,1,1,1,1,1,1,1,1,1,3,1,7,1,7,3,2,3,1,3,1,4,3,1,1,1,1,1,1,2,6,4,1,1,1,1,3,1,2,1,2,1,1,1,2,1,1,1,1,1,3,1,1,3,2,4,2,3,1,2,5,1,2,4,2,1,1, for 1 opponents:
 - ZD-SET-2@_5 (score:2.29)
- C2,13,1,21,1,47,1,1,4,3,1,1,4,3,3,1,4,3,2,4,2,6,1,1,1,3,2,1,1,1,2,2,1,1,3,1,2,1,1,2,3,2,6,3,1,6,3,4,1,1,1,1,1,1,3,1,1,1,2,1,3 for 1 opponents:
 - Bush_Mosteller@_3 (score:3.42)
- C2,198 for 2 opponents:
 - Adaptive (score:4.88)
 - Nydegger (score:4.98)
- C1,1,197,1 for 3 opponents:
 - CollectiveStrategy (score:3.0)
 - Gladstein (score:3.005)
 - Ripoff (score:3.005)
- C1,1,196,2 for 2 opponents:
 - PSO_Gambler_2_2_2@_5 (score:3.03)
 - PSO_Gambler_2_2_2@_9 (score:3.03)
- C1,1,5,1,3,2,5,1,2,1,1,4,1,5,2,3,2,1,1,3,3,2,2,1,1,3,1,1,1,1,1,2,2,2,2,2,2,2,3,1,1,1,4,2,1,1,2,6,1,2,2,1,1,2,3,1,1,4,1,1,3,1,1,1,1,2,1,1,1, for 1 opponents:

[illegible]

- ZD-Extortion@_7 (score:1.695)
- C1,7,1,1,2,1,1,1,1,2,1,1,2,3,2,1,1,1,2,1,2,1,3,3,1,1,1,2,2,3,2,2,1,4,3,1,1,1,1,1,2,2,1,1,2,1,1,1,5,1,2,1,1,1,1,1,102,5 for 1 opponents:
 - ZD-Extortion@_8 (score:1.63)
- C1,45,1,35,1,6,2,1,1,1,1,5,1,4,1,1,2,7,5,1,2,4,2,2,1,4,1,1,1,5,2,5,2,1,4,1,4,2,1,6,1,2,1,1,2,1,1,2,1,2,5,5,3 for 1 opponents:
 - Bush_Mosteller@_8 (score:3.52)
- C1,140,1,58 for 1 opponents:
 - Stochastic_WSLS@_1 (score:3.06)
- C1,199 for 11 opponents:

– Defector_Hunter (score:4.99)	– SolutionB1 (score:4.955)	– Willing@_5 (score:4.975)
– Evolved_FSM_4 (score:3.67)	– Tricky_Defector (score:4.915)	– Willing@_6 (score:4.975)
– Handshake (score:4.97)	– Willing@_0 (score:4.975)	– Win-Shift_Lose-Stay@_D (score:4.975)
– Opposite_Grudge (score:4.975)	– Willing@_2 (score:4.975)	
- D1,198,1 for 21 opponents:

– Calculator@_2 (score:2.945)	– Evolved_HMM_5@_2 (score:3.02)	– Evolved_HMM_5@_8 (score:3.02)
– Calculator@_3 (score:2.99)	– Evolved_HMM_5@_3 (score:3.02)	– Evolved_HMM_5@_9 (score:3.02)
– Calculator@_4 (score:2.975)	– Evolved_HMM_5@_4 (score:3.02)	– PSO_Gambler_1_1_1@_1 (score:3.02)
– Calculator@_5 (score:2.945)	– Evolved_HMM_5@_5 (score:3.02)	– PSO_Gambler_1_1_1@_8 (score:3.02)
– Calculator@_6 (score:3.005)	– Evolved_HMM_5@_6 (score:3.02)	– Winner12 (score:3.02)
– Calculator@_8 (score:2.975)	– Evolved_HMM_5@_7 (score:3.02)	– Winner21 (score:3.0)
– Calculator@_9 (score:3.005)		
– Evolved_HMM_5@_0 (score:3.02)		
– Evolved_HMM_5@_1 (score:3.02)		
- D1,196,3 for 1 opponents:
 - PSO_Gambler_1_1_1@_3 (score:3.04)
- D1,194,5 for 2 opponents:
 - Feld@_7 (score:2.38)
 - ZD-GTFT-2@_4 (score:3.025)
- D1,193,1,4,1 for 1 opponents:
 - ZD-Extort-4@_8 (score:1.785)
- D1,99,1,88,1,8,2 for 1 opponents:

- Remorseful_Prober@_7 (score:2.78)
- D1,99,100 for 1 opponents:
 - Revised_Downing@_True (score:4.01)
- D1,79,1,117,2 for 1 opponents:
 - Remorseful_Prober@_6 (score:2.67)
- D1,76,1,34,1,86,1 for 1 opponents:
 - ZD-Extort-4@_6 (score:1.69)
- D1,70,1,127,1 for 1 opponents:
 - Feld@_1 (score:2.37)
- D1,68,1,10,1,13,1,101,4 for 1 opponents:
 - Getzler@_5 (score:3.03)
- D1,60,1,95,1,39,3 for 1 opponents:
 - Forgetful_Fool_Me_Once@_0 (score:3.06)
- D1,45,1,35,1,19,1,34,1,60,2 for 1 opponents:
 - Forgetful_Fool_Me_Once@_8 (score:3.07)
- D1,42,1,7,1,5,2,3,1,1,1,2,1,2,1,1,1,1,1,4,1,1,1,1,2,3,2,1,1,5,2,2,2,3,1,1,2,2,2,1,1,2,1,7,2,2,1,1,3,2,1,3,3,1,8,1,1,2,1,1,1,1,2,3,3,1,1,3 for 1 opponents:
 - Champion@_2 (score:3.81)
- D1,21,1,20,1,75,1,6,1,24,1,46,2 for 1 opponents:
 - Forgetful_Fool_Me_Once@_2 (score:3.08)
- D1,14,1,183,1 for 1 opponents:
 - ZD-Extort-4@_1 (score:1.785)
- D1,10,1,25,1,14,1,78,1,16,1,49,2 for 1 opponents:
 - ZD-Mem2@_3 (score:2.605)
- D1,9,1,1,1,1,1,3,1,3,1,7,1,19,1,6,1,13,1,11,1,24,1,24,1,7,1,11,1,44,2 for 1 opponents:
 - Forgetful_Fool_Me_Once@_1 (score:3.17)
- D1,7,1,1,1,7,182 for 1 opponents:

- [illegible]

- [illegible]

- [illegible]

- [illegible]

- [illegible]

- [illegible]

- [illegible]

[illegible]

- [illegible]

- D3,192,5 for 8 opponents:

- D3,177,1,2,1,13,3 for 1 opponents:

- D3,97,100 for 1 opponents:

- D3,56,1,36,1,32,1,6,1,45,1,9,2,1,5 for 1 opponents:

- D3,24,1,2,1,4,2,1,1,2,2,1,1,1,1,1,5,2,1,2,3,2,3,3,1,1,1,3,1,1,1,1,1,3,1,3,2,1,1,1,4,1,2,2,5,1,1,5,1,2,2,2,3,1,2,4,3,1,1,1,2,1,3,2,1,2 for 1 opponents:

- [illegible]

- D3,12,1,151,1,22,10 for 1 opponents:

- D3,8,1,1,2,7,21,1,156 for 1 opponents:

- D3,4,4,1,2,2,5,1,1,3,2,2,3,2,2,1,5,3,4,2,2,1,1,1,2,1,1,6,2,2,1,2,2,1,1,5,1,1,1,4,3,1,2,1,1,1,4,1,1,1,1,2,1,1,5,1,1,1,1,1,2,5,5,1,4,2,3,1,2 for 1 opponents:

- D3,2,9,2,3,2,1,3,2,1,1,7,4,4,1,2,1,2,3,1,1,1,2,1,2,1,2,1,2,2,1,1,4,2,1,1,1,1,2,2,1,1,1,1,2,1,2,1,5,1,4,2,1,1,1,1,3,2,2,3,2,1,1,1,2,4,1,1,3 for 1 opponents:

- ZD-Extort-2@_9 (score:2.555)
- D3,1,1,54,1,36,1,3,50,1,43,1,5 for 1 opponents:
 - Stochastic_Cooperator@_9 (score:2.75)
- D3,1,1,8,4,1,1,25,1,147,8 for 1 opponents:
 - Tullock@_7 (score:2.92)
- D3,1,1,4,2,2,2,2,1,1,1,2,2,3,2,1,1,1,2,2,7,28,1,2,2,4,1,1,1,4,1,1,2,3,1,1,1,1,3,1,1,2,1,2,4,6,2,4,2,2,1,4,1,3,1,2,5,3,1,2,1,2,3,1,3,3,2 for 1 opponents:
 - ZD-Mischief@_1 (score:1.655)
- D3,1,1,3,1,3,1,2,1,2,2,1,2,1,5,1,1,1,1,17,1,32,1,11,5,2,1,2,1,2,8,2,1,1,1,1,1,1,1,5,2,4,1,5,1,1,2,1,3,4,2,2,3,5,3,2,1,2,1,5,2,2,2,2,1,1,3,2 for 1 opponents:
 - ZD-Extortion@_1 (score:1.535)
- D3,1,1,1,1,2,1,1,1,2,1,1,2,1,1,1,2,1,1,1,2,1,1,2,1,1,1,2,1,1,1,2,1,1,1,4,1,1,2,4,1,1,1,2,1,1,1,2,1,1,1,2,1,1,3,4,1,2,1,4,1,1,2 for 1 opponents:
 - SelfSteem@_4 (score:2.64)
- D3,1,2,1,6,1,16,1,1,1,2,1,9,1,8,1,5,1,6,1,7,1,6,2,10,1,2,3,3,2,2,2,2,1,3,1,2,1,5,1,6,1,1,1,2,1,5,4,5,1,5,2,5,1,2,1,2,1,1,1,4,1,2,2,3,1,1,3, for 4 opponents:

<ul style="list-style-type: none"> – Arrogant_QLearner@_6 (score:4.98) – Hopeless@_3 (score:5.0) – Hopeless@_4 (score:5.0) – Hopeless@_5 (score:4.98) – Hopeless@_6 (score:4.98) – Hopeless@_7 (score:5.0) – Hopeless@_8 (score:5.0) – Hopeless@_9 (score:5.0) – Knowledgeable_Worse_and_Worse@_0 (score:2.88) – Knowledgeable_Worse_and_Worse@_1 (score:3.04) – Knowledgeable_Worse_and_Worse@_2 (score:3.04) – Knowledgeable_Worse_and_Worse@_3 (score:2.82) – Knowledgeable_Worse_and_Worse@_4 (score:3.12) – Knowledgeable_Worse_and_Worse@_5 (score:2.74) – Knowledgeable_Worse_and_Worse@_6 (score:3.12) 	<ul style="list-style-type: none"> – Knowledgeable_Worse_and_Worse@_7 (score:3.14) – Knowledgeable_Worse_and_Worse@_8 (score:2.94) – Knowledgeable_Worse_and_Worse@_9 (score:3.18) – Math_Constant_Hunter (score:5.0) – Negation@_0 (score:4.98) – Negation@_1 (score:5.0) – Negation@_2 (score:4.98) – Negation@_3 (score:5.0) – Negation@_4 (score:5.0) – Negation@_5 (score:4.98) – Negation@_6 (score:4.98) – Negation@_7 (score:5.0) – Negation@_8 (score:5.0) – Negation@_9 (score:5.0) – Random@_0 (score:2.86) – Random@_1 (score:3.02) – Random@_2 (score:2.96) 	<ul style="list-style-type: none"> – Random@_3 (score:2.92) – Random@_4 (score:3.04) – Random@_5 (score:2.82) – Random@_6 (score:3.0) – Random@_7 (score:3.2) – Random@_8 (score:3.14) – Random@_9 (score:3.1) – Random_Hunter (score:5.0) – ThueMorse (score:3.0) – ThueMorseInverse (score:3.0) – Tricky_Cooperator (score:5.0) – Willing@_1 (score:5.0) – Willing@_3 (score:5.0) – Willing@_4 (score:5.0) – Willing@_7 (score:5.0) – Willing@_8 (score:5.0) – Willing@_9 (score:5.0) – Worse_and_Worse@_0 (score:4.6) – Worse_and_Worse@_1 (score:4.68) – Worse_and_Worse@_2 (score:4.66)
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- Worse_and_Worse@_3 (score:4.48)
- Worse_and_Worse@_4 (score:4.52)
- Worse_and_Worse@_5 (score:4.62)
- Worse_and_Worse@_6 (score:4.48)
- Worse_and_Worse@_7 (score:4.5)
- Worse_and_Worse@_8 (score:4.6)
- Worse_and_Worse@_9 (score:1.02)
- ZD-Mischief@_0 (score:1.02)

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